DISCUSSION

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The approach used by the author is very nearly that used in reference [6]. Since the authors present both velocity and temperature profiles for flow with mass injection, it would be worthwhile to compare these results with those of [6] for similar conditions, i.e.: comparison of velocity and temperature profiles, displacement thicknesses, and thermal boundary layer growth.

The discussion presented for the boundary-layer temperature distribution as shown in Fig. 5 would lead one to believe that one finds the same phenomenon in other data. Cited were references [2, 3, and 6]. However, it is not apparent at all that such is the case. There appears to be no systematic trend as shown in Fig. 5. Further comparison of temperature profiles can be made with results presented in [1, 12, and 13][5] and once again this effect is not present.

Finally, in a recent Russian publication, Kutateladze and Leont'ev [14] derive a parameter K which is related to the film effectiveness η by

$$\eta = \frac{1}{K + 1} \qquad (10)$$

where

$$K = 0.33 \frac{\mu_\infty}{\rho_2 u_2 h} \mathrm{Re}_x^{8/10} - 1 \qquad (11)$$

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Upon substitution of equation (11) into (10)

$$\eta = 3.03 \left[\frac{\rho_2 u_2 h}{\mu_\infty}\right]^{2/10} [X/Mh]^{-8/10}$$

Using the range of values for $\rho_2 u_2/\mu_\infty$ in the present paper, the following equation is obtained

$$\eta = [14.2 \text{ to } 16.0][X/Mh]^{-8/10} \qquad (12)$$

over the large of X/Mh from 40 to 1000, equation (12) agrees with equation (4) to within approximate ±15 percent.

Additional References

12 J. P. Hartnett, R. C. Birkebak, and E. R. G. Eckert, "Velocity Distributions, Temperature Distributions, Effectiveness and Heat Transfer in Cooling of a Surface With a Pressure Gradient," 1961 International Heat Transfer Conference, part IV, no. 81, The American Society of Mechanical Engineers.

13 R. A. Seban and L. H. Back, "Velocity and Temperature Profiles in Turbulent Boundary Layer With Tangential Injection," Journal of Heat Transfer, Trans. ASME, Series C, vol. 84, 1962, pp. 45–54.

14 S. S. Kutateladze and A. I. Leont'ev, "The Heat Curtain in the Turbulent Boundary Layer of a Gas," Translated from *Teplofiziko Vysokikh Temperature*, vol. 1, no. 2, September-October, 1963; *High Temperature*, vol. 1, Sept.-Oct., 1963.

Authors' Closure

The authors would like to thank Dr. Birkebak for his comments and interest in the paper.

Because of the different experimental conditions encountered in the present studies, a direct comparison under similar conditions with the work reported in reference [6] is rather difficult. However, qualitative agreement is observed in the experimental results of velocity profiles, temperature profiles, and the boundary layer growth as well as in the film cooling effectiveness which is illustrated in Fig. 10.

The remark on the similarity of temperature profiles was apparently misunderstood by the discusser. The statement in the paper refers to the sharp turn of the temperature profile near the wall. Since the wall is adiabatic, the temperature gradient at the wall-gas interface must be zero. This gradient is not zero a

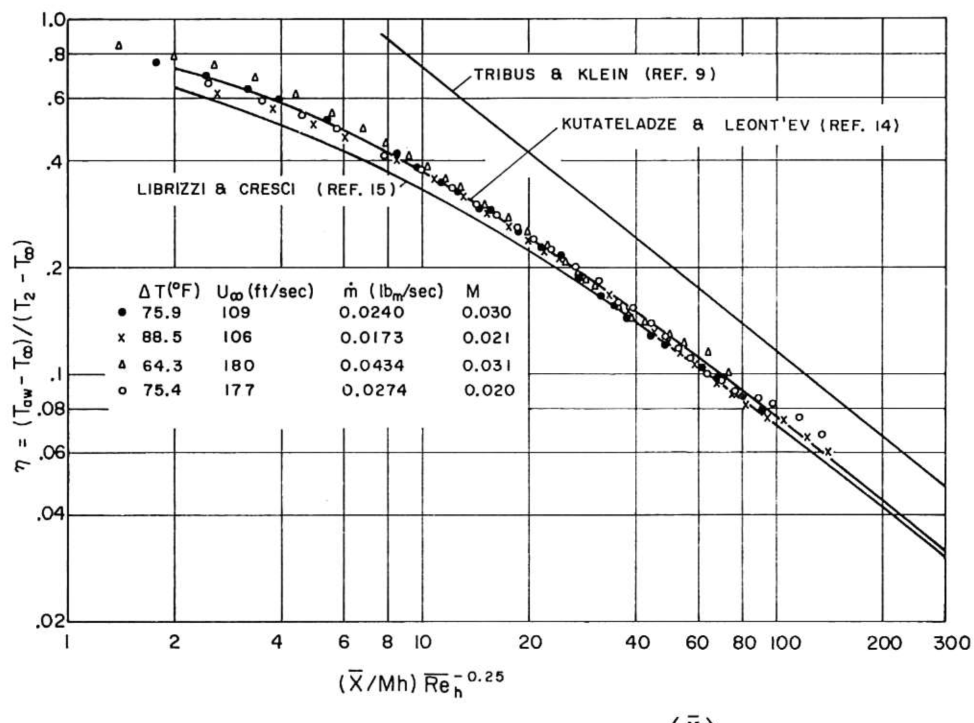

Fig. 11 Film cooling effectiveness as a function of $\left(\frac{\bar{X}}{Mh}\right) \overline{\mathrm{Re}}_h^{-0.25}$

small distance from the wall, as might be expected, and this has also been found in the cited references.

As long as the subject of the intersection of the temperature profiles has been raised, it may be well to reiterate that one would expect to find such an intersection when the dimensionless temperature difference is plotted against the actual distance from the wall. In particular, this would be true with tangential injection of a gas (of uniform temperature) through a slot of finite width if one includes temperature profiles near the point of injection. Unfortunately, these graphs are not available in all of the references cited, but they are shown in Fig. 20 of reference [2] and Fig. 7 (compare curves B and C) of reference [12]. The uniform temperature region near the wall should have the same thickness as the slot at the point of injection and decrease to a very thin region as one goes downstream from the slot. With injection through a porous wall as in the present study, a similar effect is encountered. With still larger blowing rates than that shown in Fig. 5 similar curves result but with an intersection point a still greater distance away from the wall. Thus we expect that some crossing of the curves would in general occur.

Since this paper was submitted, several works have been published which include simplified analyses on the film cooling effectiveness with subsonic flow which are basically similar to reference [10]. Librizzi and Cresci [15] assume that the injected fluid and the fluid in the boundary are completely mixed (yielding a mean temperature) and that the only difference from a turbulent boundary layer with no injection is the addition of the mass of the injected fluid. This last assumption has the advantage that it predicts an effectiveness of unity at the point of injection. For constant property air to air injection their calculation would yield

$$\eta = \left[1 + .329\left(\frac{\bar{X}}{Mh}\right)^{0.8} \overline{\mathrm{Re}}_h{}^{-0.2}\right]^{-1} \tag{13}$$

where $\overline{\mathrm{Re}}_h = U_2h/\nu_2$

and $\bar{X} = X - h$

Kutateladze and Leont'ev [14] using a similar approach find,

$$\eta = \left[1 + 0.24\left(\frac{\bar{X}}{Mh}\right) \overline{\mathrm{Re}}_h{}^{-0.25}\right]^{-0.8} \tag{14}$$

The data for a number of tests had been recalculated to obtain the parameter $(\bar{X}/Mh)\overline{\mathrm{Re}}_h{}^{-0.25}$ which appears in equations (13) and (14) and also equation (7) after rearrangement of the terms. The results are shown in Fig. 11. The good fit is perhaps surprising as the two basic assumptions in the Librizzi and Cresci[6] model are not met. Thus, one can see from Fig. 5 or Fig. 6 that the temperature is not uniform in the boundary layer, and from Fig. 3 we see that the growth of the boundary layer is quite different with blowing than without blowing. These deviations from the assumptions appear fortuitously to cancel each other out in the final expressions.

[6] Joseph Librizzi and Robert J. Cresci, "Transpiration Cooling of a Turbulent Boundary Layer in an Axisymmetric Nozzle," Reprinted from *AIAA Journal*, vol. 2, no. 4, 1964, pp. 617–624.