mensionless parameter describing the fluid bulk temperature and can be obtained from the velocity field and temperature field resulting from the calculation just described. An equation corresponding to Equation (25) describes the Nusselt number $\overline{Nu}$ when the parameter $\overline{\theta_0}^{++}$ describing the average wall temperature replaces the parameter θ_0^{++}.

The actual calculation following this procedure turns out to be quite tedious because the location O as well as the shape of the isotherms has to be varied in the course of the iteration procedure. We were therefore satisfied to replace the curved boundaries of ΔA by straight lines. A variation of O along the duct center line leads to a position for which Equation (22) was fulfilled around the duct periphery within ± 20 per cent. A study of the convergence obtained to that point indicated that the result is not accurate enough for a description of the local Nusselt numbers, but that it should approximate, on the other hand, the average Nusselt number fairly well. The value of the Nusselt number obtained in this way is inserted as a full point in Fig. 10.

DISCUSSION

L. C. Hoagland[3]

The authors should be commended for an excellent piece of experimental work and a very significant contribution to the rather sparse literature on the subject of heat transfer in noncircular ducts. Research on heat transfer to fluids in noncircular ducts is fraught with many difficulties not encountered in work with circular ducts. Perhaps the most significant difficulty is the sensitivity of the average heat-transfer coefficient or Nusselt number to changes in the thermal boundary condition applied around the duct periphery.

As stated by the authors, previous analytical work has shown that "the Nusselt number averaged over the periphery may differ by an order of magnitude depending on whether a constant (uniform) wall temperature or a constant (uniform) heat flux from the duct wall into the fluid is prescribed peripherally." These two extremes, of course, represent the limits of actual boundary conditions that may be achieved in practice. It appears that the authors have obtained experimental results for a boundary condition approaching the uniform peripheral heat-flux case. (It would be interesting to know how the heat flux into the fluid varied peripherally in these tests.)

Previous analytical work reported in the literature has indicated that for extremely noncircular shapes such as the 5 to 1 triangle, the average Nusselt numbers for uniform peripheral heat flux are considerably lower than those for uniform peripheral wall temperature. Therefore it is not too surprising that the average Nusselt numbers reported in Fig. 10 are considerably lower than one would predict from the Dittus-Boelter equation. It is hoped that the authors will continue their program to investigate other values of the parameter $k_w b/kD_h$ and, in particular, the uniform peripheral wall-temperature case to determine whether the Nusselt number for this case approaches the Dittus-Boelter correlation. This, of course, is easier said than done.

The experiments reported here indicated that very long duct lengths (measured in terms of number of hydraulic diameters) are required to develop the temperature profile along the axis of symmetry (across the long duct dimension). The author has recently been doing some work at M.I.T. on the thermal entrance region for laminar slug flow in rectangular ducts. Although it is impossible to make a direct comparison between laminar slug flow and turbulent flow, some of the results obtained to date may help to explain this seemingly strange discovery. Two thermal boundary conditions have been investigated, namely, (a) uniform

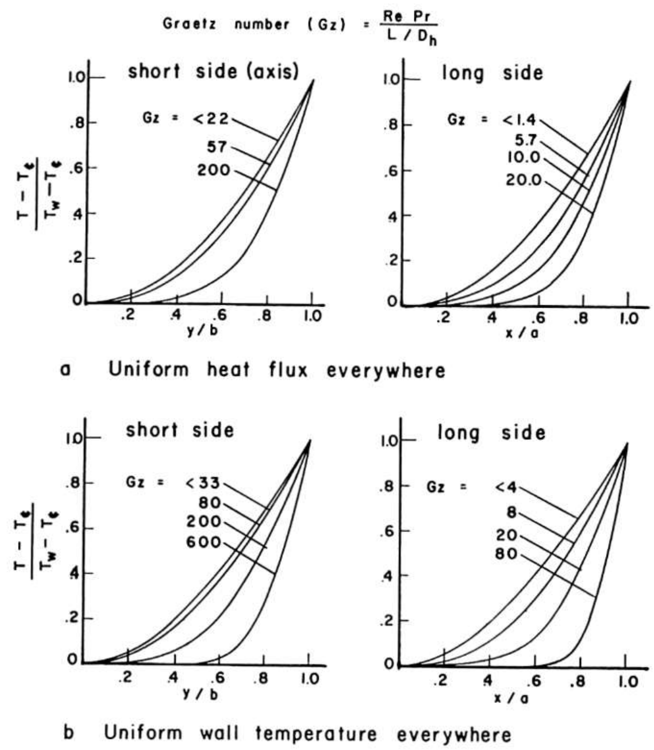

Fig. 14 Laminar slug flow in 4 to 1 rectangular duct—development of temperature profiles across long and short axes

heat flux to the fluid both peripherally and axially, and (b) uniform duct wall temperature both peripherally and axially. Fig. 14 shows for a 4:1 aspect ratio duct typical temperature distributions along the long and short axes of symmetry at various positions from the duct entrance [measured by the conventional Graetz number $Gz = Re \cdot Pr/(L/D_h)$]. These curves indicate that the temperature profiles along the long and short axes become fully developed at widely different values of the Graetz number (based on the conventional hydraulic diameter) or, in other words, at considerably different distances from the entrance to the heating section. This effect appears about the same qualitatively for both boundary conditions.

Of special interest is the manner in which the Graetz number required to develop the temperature distribution across the long and short axes varies with the duct aspect ratio. Fig. 15 shows this relationship using the criterion that the temperature parameter (see Fig. 14) must everywhere be within 5 per cent of its value when fully developed. The Graetz number required for developing the short side profile increases by a factor of 3 as the aspect ratio goes from one to ten whereas the Graetz number required to develop the long side profile simultaneously "decreases" by a factor of 30. Note that a lower Graetz number means a larger L/D_h for fixed values of Re and Pr. For comparison, the Graetz number required to develop the temperature profile in a circular duct is about 15 or 20.

At this point it is interesting to philosophize a bit. Should the thermal entrance length for noncircular ducts be comparable to that for circular ducts? Should the hydraulic diameter be a meaningful parameter determining the thermal entrance length for noncircular ducts? Physically, in order to obtain a fully developed temperature profile (say, across the long axis of symmetry), the effect of the sudden change in the thermal boundary condition at the entrance of the heating section must propagate across the long axis from the wall to the center, a distance a. The propagation phenomenon is accomplished physically by heat conduction and is inhibited by convection and thermal storage

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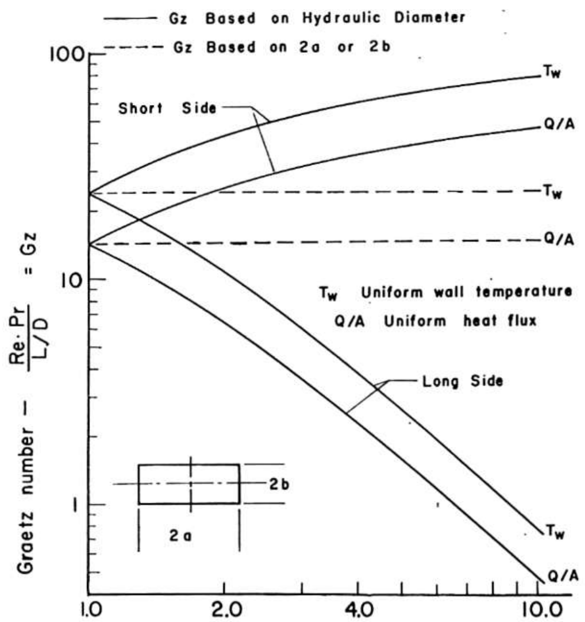

Fig. 15 Laminar slug flow in rectangular ducts—thermal entrance length required to develop temperature profiles across long and short axes of symmetry (within 5 per cent)

processes. Hence the thermal entrance length L required for the boundary effect to propagate across the distance a should be directly proportional to the fluid velocity u and the thermal capacity of the fluid through which the effect must propagate ($\rho c_p A_n \cdot a$) and inversely proportional to the thermal conductance of the fluid (kA_n/a) where A_n is the area normal to long axis or a. Thus

$$L \propto \frac{u\rho A_n \cdot ac_p}{kA_n/a} = \frac{u\rho a}{\mu} \cdot \frac{c_p\mu}{k} \cdot a$$

This physical notion suggests that a Graetz number built with the duct dimension representative of the temperature profile in question should provide a better thermal entrance length parameter than the Graetz number built with the hydraulic diameter. When the results shown in Fig. 15 are converted to Graetz numbers based on the width ($2a$) and height ($2b$) of the duct, respectively, the Graetz number required to obtain fully developed temperature profiles is indeed a constant (for a given boundary condition) and is the same for both long and short sides and independent of aspect ratio.

Although the solutions illustrated in Figs. 14 and 15 indicate large entrance lengths required to develop temperature profiles across the long duct dimension, another boundary condition of practical importance should be expected to have a much smaller thermal entrance length. Consider the boundary condition of uniform heat input per unit duct length with uniform peripheral wall temperature. For this case the fully developed temperature distribution across the long side is completely different in character from the "parabolic type" temperature profiles obtained with uniform wall temperature and uniform heat flux everywhere. Fig. 16 shows *fully developed* temperature profiles across the long axis of rectangular ducts with uniform heat input per unit length (uniform peripheral wall temperature). The temperature distribution is nearly parabolic for a square duct, but is markedly flattened in the center for the larger aspect ratios.[4] Because here

[4] This fact can also be visualized physically by noting that the membrane or soap film analogy (generally used for torsion of noncircular bars) is applicable to this problem.

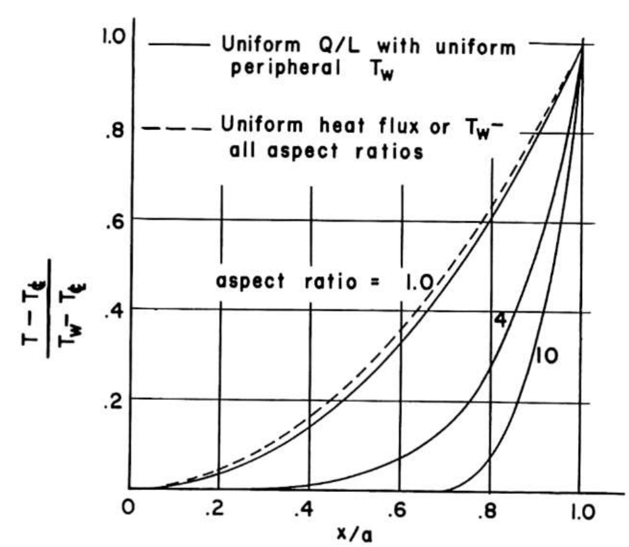

Fig. 16 Laminar slug flow in rectangular ducts—fully developed temperature profiles across long axis of symmetry

the ultimate effect of the boundary condition is localized, the thermal entrance length required to develop the long side temperature profile is probably considerably smaller for this boundary condition than for those mentioned previously. As yet, no analytical solution is available for the thermal entrance length under the influence of this boundary condition. It will be interesting to see if this notion can be verified by a solution. If so, it appears that a knowledge of the wall conductivity parameter ($k_w b/kD_h$) is essential for determining the thermal entrance length when a uniform heat flux is generated in the duct wall.

Finally, it is interesting to compare the duct length required to develop temperature profiles along the long and short sides based on the results shown in Fig. 15. Consider a rectangular duct with an aspect ratio of 5:1, a Reynolds number of 1000, and a Prandtl number of 1.0. For the short side, a Graetz number of 40 is required. The corresponding length to hydraulic diameter ratio is 25. For the long side, the required Graetz number is 1.5 and the necessary length-diameter ratio becomes 667. The writer plans to finish this work soon and submit a paper with a complete description to the ASME.

Authors' Closure

The preceding discussion of our paper by L. C. Hoagland focuses attention on the question: Which length should be used as basis for a dimensionless parameter characterizing the duct length required for the development of the temperature field? Mr. Hoagland was successful in finding such a parameter for laminar bulk flow through a rectangular duct the value of which is independent of the side ratio of the duct cross section. In a similar attempt, it was investigated in our paper whether the ratio of duct length L necessary for development of the temperature field to duct height would have a value which agrees better with the ratio of L to diameter D for a circular tube than the ratio L/D_h. The experiments showed, however, that the temperature field in turbulent flow through a triangular duct was not developed for a ratio of duct length to duct height equal 21.5, whereas it has been found to be developed for a ratio L/D from 10 to 15 in a circular pipe. This led us to the conclusion that the turbulent flow process itself must be different in the triangular duct and in the circular tube.

In answer to a suggestion by Mr. Hoagland, we are presenting in Fig. 17 a parameter which is proportional to the local heat flux q_w plotted over the distance x from the sharp corner of the

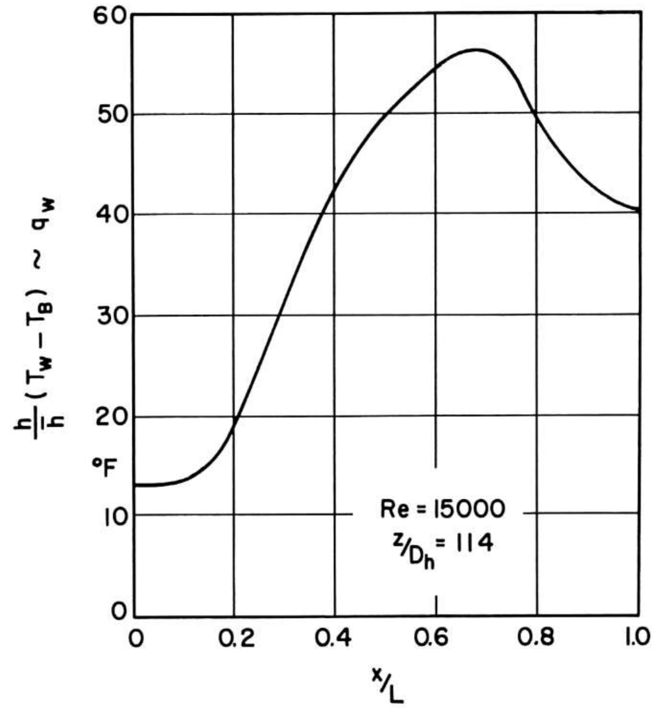

Fig. 17

duct for a specific Reynolds number. It may be observed that the heat flux varies considerably over the duct periphery indicating that the specific conditions applied in our experiments led to a situation approximately halfway between a constant heat flux and a constant wall temperature boundary condition.

Friction and heat transfer in turbulent flow through a rectangular duct are presently studied in our Heat Transfer Laboratory. A report on the friction characteristics is presently being prepared.