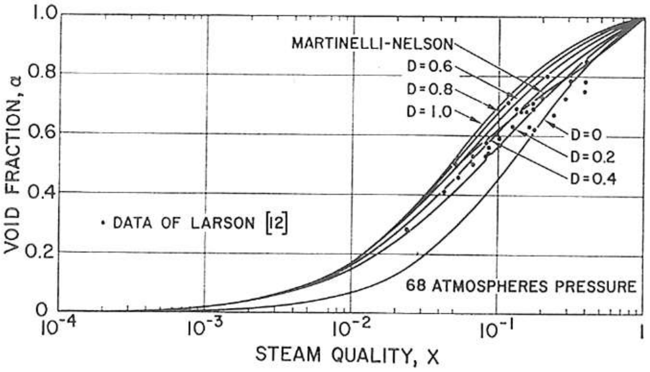

Fig. 7 Calculated steam void-fraction for various liquid entrainment quantities (D), 68 atmospheres pressure. Data points are measurements [12].

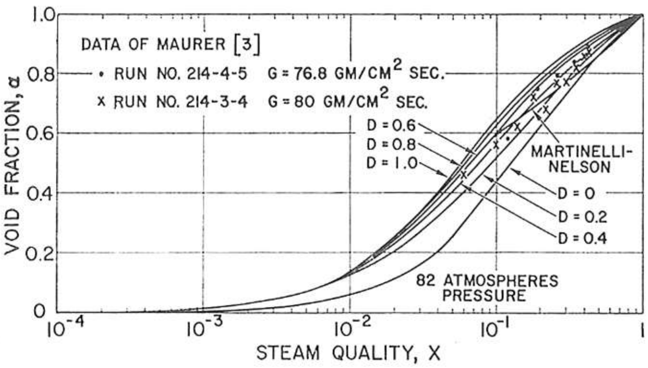

Fig. 8 Calculated steam void-fraction for various liquid entrainment quantities (D), 82 atmospheres pressure. Data points are measurements [3].

should bracket a steady-state two-phase flow with negligible wall friction, since these cases represent the two extreme conditions of mixing of the two phases. Therefore, it appears that liquid entrainment has a more pronounced effect on the void-fraction than did wall friction in the experiments considered. Wall friction would tend to cause the experimental data to lie below the curve for $D = 0$. It would then appear that a physical model for describing this two-phase flow data should include the effects of liquid entrainment. The importance of entrainment in predicting two-phase pressure drop has been emphasized by Dukler and co-workers [15, 16] whose experimental measurements of entrainment in water-air systems represent a major source of data on this subject.

Entrainment is also an important factor in the dynamics of two-phase flow, for the propagation of disturbances in flow, void-fraction and pressure will be strongly affected by the amount of entrainment and the velocity distribution of the entrained liquid particles. Further experimental data which may enable the entrainment quantity and velocity distribution to be related to the flow conditions are needed before two-phase flow models can become more fundamental.

Conclusion

The present analysis is based on a variational principle rather than on specific microscopic mechanisms. It has been found that the idealized steady-state annular flow can be described by the simple relation $u_g/u_f = (\rho_f/\rho_g)^{1/3}$ and that the corresponding void-fraction-quality curve forms the lower bound for much of the available experimental data, with the upper bound being given by the homogeneous flow assumption. Another important observation from the analysis is that these two bounding curves approach each other as pressure is increased. The quantity of liquid entrained in the vapor stream is seen to be the determining factor in interpolating between these limits.

Acknowledgment

This work was sponsored by the U.S.A.E.C. Nuclear Safety Research and Development Branch, as part of the project "Kinetic Studies of Heterogeneous Water Reactors." The author is grateful for discussions with Mr. R. W. Wright and Dr. F. N. Mastrup.

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DISCUSSION

S. Levy[2]

The theoretical treatment presented by S. M. Zivi is a welcome addition to a field which has been dominated by experimental correlations. The author offers predictions of void fraction for two-phase flow with and without entrainment. His solutions, given by equations (8) and (22), assume that the frictional and hydrostatic head losses can be neglected and that the liquid and gas can be represented by a single velocity u_f and u_g. It should be noted that, for these very special conditions, it is possible to write a momentum equation for the liquid and a momentum equation for the gas. The equations are:[3]

[2] General Electric Co., San Jose, Calif.

[3] S. Levy, "Steam Slip—Theoretical Prediction From Momentum Model," Journal of Heat Transfer, Trans. ASME, Series C, vol. 82, pp. 113–124.

$$dP + \rho_f u_f (du_f/g) = 0$$

$$dP + (1/g\alpha)d(\alpha\rho_g u_g^2) + (u_f/g\alpha)d[(1-\alpha)\rho_f u_f] = 0$$

Elimination of the pressure P from the above equations makes it possible to obtain the slip ratio or void fraction. The results were derived by the discusser in his "momentum exchange model." In other words, for the specific conditions treated by Zivi, namely, flow without friction and head losses, it is not necessary to employ the minimum enthropy principle to calculate the void fraction.

The author has made an attempt to evaluate the effects of friction in equation (14). It is not clear, however, why the frictional losses are proportional to the product of wall shear stress and liquid velocity. Should not the losses also depend upon the gas velocity and the interface shear?

Finally, it would be interesting to include in equation (14) the potential energy. Did the author try it?

G. B. Wallis[4]

It would be very convenient if the analysis which is presented in this paper could be shown to be valid since it would lead to a great simplification in the explanation of two-phase flow phenomena. However, it should be remembered that the principle of minimum entropy production has only been successfully applied up to now for the solution of problems of orderly streamline laminar flow. Turbulent single-phase flow has proved intractable because one simply does not know how to describe "entropy" completely in an apparently disorderly motion. Entropy is not identical with energy dissipation. In the case of two-phase flow one might be more skeptical because of the numerous possible flow regimes, turbulence and interfacial waves, wakes of drops and bubbles, and so on.

Of course the true test of any theory is its ability to predict experimental results. If Mr. Zivi's theories are more successful than previous work in this respect, they will have been adequately justified.

From the logical point of view, I find it difficult to see just why minimum entropy production should be equated with minimum kinetic energy flux. Surely the flow dynamics in a duct are determined by all the processes which create entropy *in the duct* and not by some hypothetical energy dissipation at the outlet. Furthermore, if the wall shear stress is negligible (as assumed in the first part of the paper) one would expect any motion of the tube wall to be irrelevant, in which case the *relative velocity* between the phases would be the characteristic of the motion and not the slip ratio. In the absence of body forces, which tend to produce slip, any interaction between the phases will promote uniform velocities, i.e. homogeneous flow. In fact, the only reason there is any slip in annular flow is because the wall slows down the liquid which is in contact with it. This phenomenon cannot be accounted for solely in terms of kinetic energy flux as is assumed in equations (3) and (4).

Author's Closure

I thank Dr. Wallis and Dr. Levy for their discussion. In this paper, the effects of only several energy dissipating processes have been considered in an idealized system. If other energy dissipating processes become relatively important in the real systems, the system analyzed would become less realistic. I share Dr. Wallis' skepticism regarding the applicability of the principle of minimum entropy production to a turbulent two-phase flow where expressions for the rate of entropy production are not available. My analysis is applied to hypothetical two-phase flows where this difficulty is defined to be absent, and the intent of the paper is to show that the void-fractions calculated for these hypothetical two-phase flows compare remarkably well with the available experimental data. It was stated in the paper that time variations in the local flow variables were excluded from the cases considered. The specification of a given steam quality implies that the thermal processes are invariant, so that the only variable process for producing entropy in the hypothetical flow is mechanical energy dissipation.

Dr. Wallis questions the importance of kinetic energy in a two-phase flow. This question brings out an assumption which I did not state in the paper, but which is implicit in the analysis. It is tacitly assumed that most of the kinetic energy of the two-phase flow is acquired in the duct, and that the work of accelerating the two phases to their ultimate velocities is a significant part of the total flow work. Under this assumption, the application of the principle of minimum entropy production leads to the conclusion that the processes (in the duct) which accelerate the hypothesized two-phase flow will do so with the least work. As the work of acceleration becomes negligible with respect to the energy dissipation associated with the relative velocity of the phases, I would agree with Dr. Wallis that the kinetic energy of the flow becomes unimportant. For most boiling water nuclear reactor applications, the acceleration pressure drop (and therefore, the work done in accelerating the flow) is important.

Dr. Levy states that the equations derived in his earlier paper completely define the slip ratio or void fraction, making further conditions (such as an energy principle) redundant. These equations consider the momentum change of the liquid and of the vapor as being simultaneously determined by a common pressure drop along the duct. It is assumed in these equations that there is no momentum exchange between the two phases, except for the increment of liquid which becomes vaporized and which then acquires the velocity of the vapor. As shown by Dr. Levy in his paper, these momentum equations can be solved by eliminating dP, with the assumption that X^2/α goes to zero as X and α go to zero. Following Dr. Levy's analysis, the slip ratio (for X much smaller than one) is found to be $\sqrt{\frac{\alpha}{2}\ \rho_f/\rho_g}$. One puzzling feature of this model is that the vapor velocity becomes less than the liquid velocity as the quality and void fraction become very small. Dr. Levy's analysis is based on a specific mechanism for accelerating the two-phase flow, whereas my paper is an attempt to treat the flow without assumptions as to the detailed mechanisms.

In treating the effect of wall friction, I assumed that the two-phase frictional pressure drop could be described by an expression of the same form as in single-phase flow (i.e., proportional to the square of the liquid velocity). To the extent that the frictional pressure drop is affected by the relative velocity of the two phases (the interfacial shear referred to by Levy), this assumption will be in error. The effect could be included in equation (11) by the addition of a dissipation term including $K(u_g - u_f)^2$, where K would be a coefficient to be explored along with N. The qualitative effect of this added dissipation would be to increase the void fraction. It appears difficult to evaluate interfacial frictional dissipation in experimental data because of the simultaneous interchange of entrained liquid between the vapor and liquid streams (which would involve momentum exchanges). I assumed that momentum interchange would dominate the interfacial phenomena, and did not attempt to include interfacial frictional dissipation. In reply to Dr. Levy's last question, I do not believe the potential energy of a two-phase flow plays a significant part in the determination of the void fraction, so I did not attempt to include it.

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