surface diffusion ratio (value above which $(\theta/c)_2$ becomes excessive), blades should be designed for low friction coefficient (high Reynolds number and low relative surface roughness) and minimum laminar flow (high Reynolds number and high free-stream turbulence).

The experimental correlation between wake momentum thickness and suction-surface diffusion ratio was made applicable for conventional blade-design use by empirically establishing an equivalent diffusion ratio expressible in terms of only the design velocity triangle and the blade solidity. The possibility of blade stall at minimum loss and greater angles of attack, as evidenced by a sharp rise in wake momentum thickness, occurred when the equivalent diffusion ratio attained a value of about 2. With the use of the equivalent diffusion ratio, calculations were made of the total-pressure loss and unstalled range of operation of conventional cascade sections as functions of solidity, air inlet angle, and air turning angle.

APPENDIX

For partly laminar and partly turbulent flow, the flat-plate friction coefficient is given by [from 6, p. 434],

$$C_f' = C_{f,\,\mathrm{tb}}' - \left(\frac{x}{c}\right)_{\mathrm{tr}} (C_{f,\,\mathrm{tb}}^{**} - C_{f,\,\mathrm{lm}}^{**}) \qquad (18)$$

where $(x/c)_{\mathrm{tr}}$ is the extent of the laminar region, $C_{f,\,\mathrm{tb}}'$ is the turbulent total friction coefficient based on the chord-length Reynolds number, and $C_{f,\,\mathrm{lm}}^{**}$ and $C_{f,\,\mathrm{tb}}^{**}$ are the laminar and turbulent total friction coefficients, respectively, based on the Reynolds number at $(x/c)_{\mathrm{tr}}$.

In order to obtain an equation in which all friction coefficients are based on blade-chord Reynolds number, use is made of the general relations [6, pp. 108 and 433]

$$C_{f,\,\mathrm{lm},\,x}' = \frac{1.328}{\sqrt{\mathrm{Re}_{c,\,x}}} \quad \text{and} \quad C_{f,\,\mathrm{tb},\,x}' = \frac{0.074}{\sqrt[5]{\mathrm{Re}_{c,\,x}}}$$

to give

$$C_{f,\,\mathrm{lm}}^{**} = \frac{C_{f,\,\mathrm{lm}}'}{\sqrt{\left(\frac{x}{c}\right)_{\mathrm{tr}}}} \quad \text{and} \quad C_{f,\,\mathrm{tb}}^{**} = \frac{C_{f,\,\mathrm{tb}}'}{\sqrt[5]{\left(\frac{x}{c}\right)_{\mathrm{tr}}}}$$

Substitution in Equation (18) then yields, for partly laminar and partly turbulent flow

$$C_f' = C_{f,\,\mathrm{tb}}' \left[1 - \left(\frac{x}{c}\right)_{\mathrm{tr}}^{4/5}\right] + C_{f,\,\mathrm{lm}}' \left(\frac{x}{c}\right)_{\mathrm{tr}}^{1/2} \qquad (19)$$

where $C_{f,\,\mathrm{tb}}'$ and $C_{f,\,\mathrm{lm}}'$ are obtained for the blade-chord Reynolds number in question.

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DISCUSSION

H. C. Eatock[7]

The author is to be congratulated on his ingenious and useful work. He has had to make a number of simplifying assumptions while carrying his calculations through quite a few steps. His work is very well repaid by the excellent experimental agreement he has achieved!

For designers working with a body of experimental cascade results this paper will be useful in confirming observed trends and in extrapolating beyond the test limits. The curves of unstalled-incidence range against blade loading, Fig. 16, are immediately useful. They show at once that, even for low-speed compressors, the operating range narrows as the design is changed to higher work per stage.

The writer's company has been collecting cascade data since 1948.[8] Most of our results have been on basic C7, C4, or two-arc compressor cascades, and on T6 turbine cascades. Our aim has been to cover the entire operating field including inlet Mach number up to about 1. We have tested literally hundreds of configurations on a fairly high-speed basis. Inlet conditions are controlled by upstream air injection and a large number of blades are tested in each cascade. No suction is applied on the walls of the blade passage so that boundary-layer build-up on these walls causes the axial velocity to increase through the cascade in a similar manner to the early stages of a compressor. Thus we are quite interested in comparing our results with NASA data deduced using fully two-dimensional tests.

The loss curves of Fig. 14 check well with the analysis of our data. We have covered solidities from 2.0 to 0.5 so we cannot check the stalls occurring at very low solidities. All the other trends were checked and with absolute values of loss coefficient usually within ±0.005 of the author's predicted values.

The limit $D_{\mathrm{eq}}^* = 2.0$ is of particular interest to the writer. This can be taken as a load function which, if exceeded, predicts a bad cascade. During our early testing we found some of these bad cascades. The writer developed several loading functions which distinguished the bad cascades from their brothers. One of these was

$$C_{l_{v_2 m}} \sqrt{\sigma} \leq 2.0$$

where

[7] Preliminary Aerodynamics Analyst, Orenda Engines Limited, Malton, Ontario, Canada.

[8] F. H. Keast, "High-Speed Cascade Testing Techniques," Trans. ASME, vol. 74, 1952, p. 685.

$$C_{l_{v2m}} = \frac{2}{\sigma} (\tan \beta_{1m} - \tan \beta_2) \frac{\cos^2 \beta_2}{\cos \beta_{vm}}$$

and the vector mean angle, β_{vm} is from

$$\tan \beta_{vm} = {}^1/_2(\tan \beta_1 + \tan \beta_2)$$

$C_{l_{v2}}$ is the lift coefficient based on vector mean direction and outlet velocity and is borrowed from Howell.[9] $C_{l_{v2m}}$ is taken at that incidence which gives the maximum limiting (stalling) Mach number. This incidence is reasonably close to the author's minimum-loss incidence except at very low staggers. It was found, entirely empirically, that, if this loading function was greater than 2, the cascade would be marginal and, if it was greater than 2.2, it would be bad. The intriguing thing is that $C_{l_{v2m}} \sqrt{\sigma} = 2.0$ gives limits very close to $D_{eq}{}^* = 2.0$. The author's loading function thus agrees very well with our test results. This function was developed in early 1953 and covered blades up to 50 deg camber over solidities from 2.0 to 0.5. Our testing since then has not shown up any cascade which violates this rule. The agreement between the author's rule, solidly based on theory, and this purely empirical one is remarkable in that it extends to solidities well beyond those tested.

A brief check of the author's theory with our test results has shown excellent agreement. This is a further check on his results and also shows that the NACA 65 series and the C7 profile are comparable as far as minimum loss and ultimate loading are concerned.

J. F. Klapproth[10]

The author is to be commended on this extension and improvement of his original diffusion parameter. The paper is clearly written and well organized, characteristic of the high standards the author has established in all of his reports.

The use of a diffusion parameter to evaluate proposed compressor designs and assist in the analysis of performance is, of course, common practice. The earlier diffusion parameter or *D*-factor of the author was quickly accepted and very widely used among all designers as a primary design and analysis tool. We can anticipate that this latest contribution will be as quickly and as widely used.

Most useful applications of the diffusion-parameter concept are concerned with the compressor rather than the cascade. Reference [5] of the paper, also by the author, contains a brief consideration of the deviations from the cascade that will exist in the compressor. He lists these as compressibility, changes in axial velocity across the blade row, change in radius across the blade, secondary flows, spanwise boundary-layer flows, end effects, and so on. These, of course, should be considered whenever possible. The writer's comments are concerned primarily with accounting in an approximate manner for the effects of changes in axial velocity and radius through the blade row, in an effort to make the proposed diffusion parameter more directly applicable to compressors.

As reference [5] points out, the change in outlet velocity for a change in axial velocity can be corrected by

$$\frac{V_1}{V_2} = \frac{\cos \beta_2 V_{z,1}}{\cos \beta_1 V_{z,2}}$$

However, for a given inflow and outflow angle, a change in axial velocity also results in a change in circulation. Similarly, a change in radius through the blade row will change the circulation. By following the suggestion of reference [5], the circulation parameter is written as:

$$\frac{\cos^2 \beta_1}{\sigma} (\tan \beta_1 - \tan \beta_2) \equiv \frac{\cos^2 \beta_1}{r_1 \sigma V_{z1}} (r_1 V_{u1} - r_2 V_{u2})$$

where V_u is the absolute tangential component. When a change in axial velocity or radius is assumed, the equation takes the following form:

For stators,

$$\frac{\cos^2 \beta_1}{\sigma} (\tan \beta_1 - \tan \beta_2) \equiv \frac{\cos^2 \beta_1}{\sigma} \left(\tan \beta_1 - \frac{r_2}{r_1} \frac{V_{z2}}{V_{z1}} \tan \beta_2 \right)$$

For rotors,

$$\frac{\cos^2 \beta_1}{\sigma} (\tan \beta_1 - \tan \beta_2) \equiv \frac{\cos^2 \beta_1}{\sigma} \left[\tan \beta_1 - \frac{r_2}{r_1} \frac{V_{z2}}{V_{z1}} \tan \beta_2 - \frac{\omega r_1}{V_{z1}} \left(1 - \frac{r_2^2}{r_1^2} \right) \right] \quad (20)$$

The effect of the change in circulation caused by changes in axial velocity and radius will result, of course, in a change in the velocity around the airfoil. If it is assumed that the resultant velocity distribution is similar to that for an airfoil with an increased camber to produce the same absolute circulation, then Equation (20) can be used to express the circulation parameter. The expression for equivalent diffusion ratio corresponding to Equation (13) of the paper, but accounting for changes in axial velocity and radius in an approximate manner, would then be

$$D_{eq}{}^* = \frac{\cos \beta_2}{\cos \beta_1} \frac{V_{z1}}{V_{z2}} \left[1.12 + 0.61 \frac{\cos^2 \beta_1}{\sigma} K \right]$$

where

$$K = \tan \beta_1 - \frac{r_2}{r_1} \frac{V_{z2}}{V_{z1}} \tan \beta_2 - \frac{\omega r_1}{V_{z1}} \left(1 - \frac{r_2^2}{r_1^2} \right)$$

For stators, ωr is, of course, zero.

In view of the undoubtedly wide application that this latest work of the author will find, it is hoped that it will soon be extended to account for compressibility effects.

L. H. Smith, Jr.[11]

The author is to be commended for an excellent application of basic scientific principles to a down-to-earth engineering problem.

In this discussion the writer will use the results found by the author to calculate the efficiency of certain ideal compressor stages. All of the stages considered here are so-called repeating stages, made up of one rotor and one stator with stator-discharge conditions the same as rotor-inlet conditions. All blade rows operate at the reference minimum-loss condition. The blade rows are spaced far enough apart axially so that there is no unsteady interference between them, but the loss assigned to each row is taken as only that which has occurred up to the cascade-outlet measuring station so that Equation (11) can be used. Equation (13) and Fig. 13 are then used with Equation (11) to relate the blade-row losses to the stage-velocity-diagram properties and the solidities. The stage efficiency is defined here as

$$\eta = \frac{(\text{work input}) - (\text{rotor loss}) - (\text{stator loss})}{\text{work input}}$$

The work input is taken as proportional to the product of rotor-blade speed and change across the rotor of tangential velocity, according to the Euler equation of turbomachinery.

Since the author has shown that the equivalent diffusion ratio

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[9] A. R. Howell and A. D. S. Carter, "Fluid Flow Through Cascades of Aerofoils," NGTE, R6, 1946.

[10] Supervisor, Compressor Development Unit, General Electric Company, Cincinnati, Ohio.

[11] Compressor Aerodynamicist, General Electric Company, Cincinnati, Ohio. Assoc. Mem. ASME.

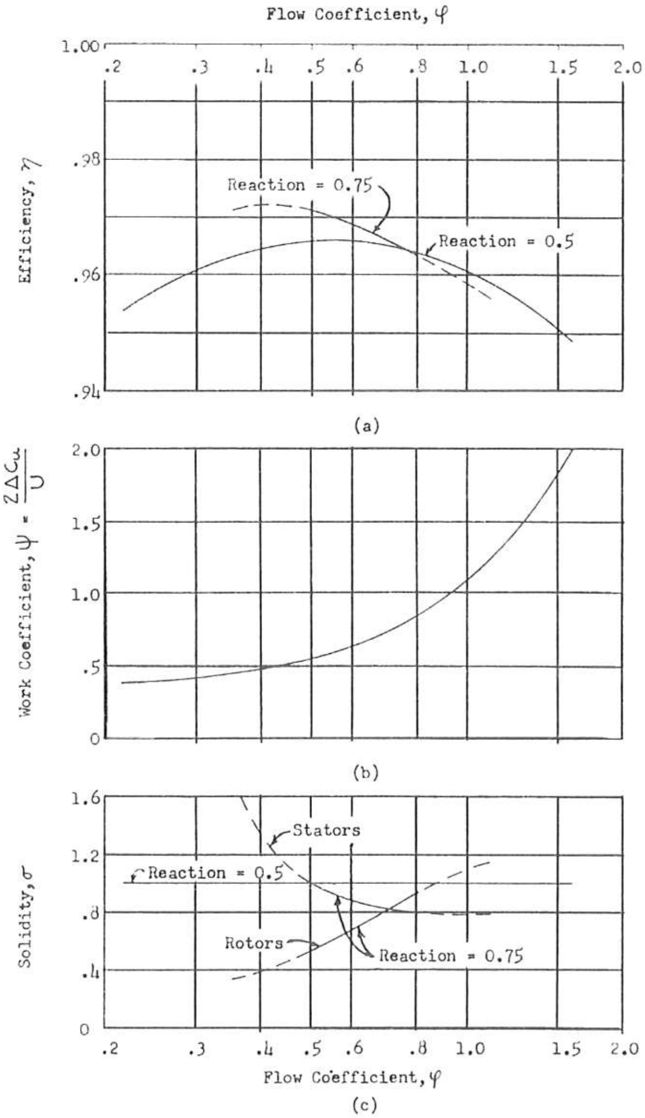

Fig. 17 Properties of stages having $D_{eq}^* = 1.64$. Dashed lines are extrapolations into regions not completely covered by cascade tests.

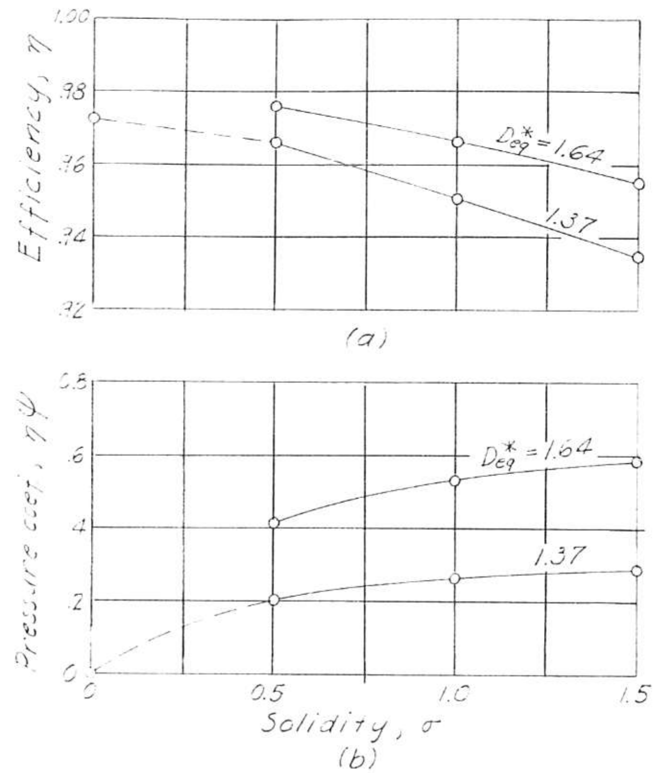

Fig. 18 Effect of solidity on efficiency and pressure coefficient. Reaction ratio = 0.5 and flow coefficient = 0.5.

is a good measure of the aerodynamic loading and range capabilities of a cascade, it is of interest to compare different stages while holding this parameter fixed. We then have three further independent quantities that can be varied; e.g., flow coefficient (ratio of axial velocity to blade speed), reaction ratio (ratio of static pressure rise in the rotor to that in the whole stage), and either rotor or stator solidity.

In the first study that was made the equivalent diffusion was fixed at a moderate value, $D_{eq}* = 1.64$. A reaction ratio of 0.5 (symmetrical stages) was chosen and a solidity of unity was used for both rotors and stators. The resulting variation of efficiency with flow coefficient is shown in Fig. 17(*a*), herewith. It is seen that the efficiency is maximum at a flow coefficient of about 0.55. This is in surprisingly good agreement with the classical method of C. Keller which predicts, based on the assumption of constant drag/lift ratio, that the optimum flow coefficient is just under 0.5. In Fig. 17(*b*), the work coefficient (the ratio of twice the work input to the squared blade speed) that results from these assumptions is shown.

The effect of reaction ratio was studied by designing stages with 75 per cent reaction to have the same work coefficients as the 50 per cent reaction stages for the same flow coefficients. Since the same D_{eq}^* was maintained also, the rotor and stator solidities were no longer unity. The values of these solidities are shown in Fig. 17(*c*) and the derived efficiency variation is given in Fig. 17(*a*). It is interesting to note that for the lower flow coefficients the 75 per cent reaction blading is more efficient than the symmetrical blading. This occurs because the solidity has been significantly reduced in the rotor, and the consequent reduction in drag/lift ratio overshadows the effect of the less-favorable skewed velocity diagram.

In order to investigate separately the effect of solidity on efficiency, 50 per cent reaction stages having a flow coefficient of 0.5, $D_{eq}^* = 1.64$, and solidities of 0.5 and 1.5, were examined for comparison with the previously discussed stages with unity solidity. The result is a plot of efficiency versus solidity, Fig. 18(*a*). The drop-off in efficiency with increasing solidity is seen to be quite significant (the 100 per cent efficiency line is a pertinent reference). Also shown in Fig. 18(*a*) is a line for stages designed to have $D_{eq}^* = 1.37$. These stages have about 50 per cent more angle-of-attack range than the others.

The point at zero solidity requires further explanation. It was determined that the author's Equation (12), giving the ratio of maximum surface velocity to inlet velocity, was not applicable at solidities near zero. This was determined by expressing the author's circulation parameter in terms of lift and drag coefficients in order to remove the solidity, and then noting that the flow angle β continued to exert a significant influence on the expression. Since at zero solidity the result obviously must be independent of flow angle, it was concluded that it would be dangerous to use Equation (12), and hence Equation (13), when the solidity was lower than 0.5, the lowest value used in the correlation. Instead, the Bogdonoff measurements[12] were used to determine a relationship between lift coefficient and diffusion

[12] S. M. Bogdonoff and H. E. Bogdonoff, "Blade Design Data for Axial-Flow Fans and Compressors," NACA ACRL5F07a, L-635, July, 1945.

ratio. (In that memorandum[12] tests of isolated airfoils of high camber are reported.) For a diffusion ratio of 1.37, a lift co-efficient of 0.81 was found, and use of Fig. 13 led to a drag co-efficient of 0.0116. The resulting efficiency of 97.22 per cent is plotted in Fig. 18(a) and is connected to the rest of the points with a broken line. This efficiency is seen to be lower than an extrapolation of the solid curve would indicate. The zero-solidity point was not calculated for $D_{eq}^* = 1.64$ because the highest camber tested by the Bogdonoffs[12] ($C_{l,0} = 1.8$) was not sufficiently high to reach that diffusion value at the (estimated) reference minimum loss condition.

Fig. 18(b) displays the pressure coefficients for the stages whose efficiencies are shown in Fig. 18(a). The conclusion to be drawn from these figures is that the use of solidities much above unity is difficult to justify because only slightly higher pressure coefficients are obtained, but significantly lower efficiencies are suffered. It also appears that range comes at a high price; the 50 per cent increase in range obtained by decreasing the equivalent diffusion ratio from 1.64 to 1.37 caused the loss, i.e., $1 - \eta$, to increase by 45 per cent and the pressure coefficient to decrease by 50 per cent.

Walter C. Swan[13]

I wish to congratulate the author for the outstanding contribution this paper constitutes in the area of axial flow compressor design and performance analysis. This new technique of correlating blade element loading with viscous and wake mixing losses offers the compressor designer an extremely practical tool for achieving approximate complete radial equilibrium solutions for real flows in both the direct and inverse problems.

This writer has applied Mr. Lieblein's wake momentum thickness-equivalent diffusion concept to statistical correlations of both rotor and stator row element data and achieved similar characteristic curves to those discussed by Mr. Lieblein in the two-dimensional cascade case. It was found possible to obtain modifications to these curves for such items as compressibility and position along the blade span. Applying these results to a single stage design, using iterative techniques in a digital computer, yielded results which were very accurately reproduced on actual test. Off design performance was also found to be closely predictable, when applying the author's techniques to a stream filament performance method. Rotating stall was observed by hot wire on test when the calculated equivalent diffusion reached a magnitude of approximately 2.20.

I am sure that those who are involved with the task of designing axial flow compressors which must accurately achieve both a specified design condition and a required range will find this paper to be a real milestone toward the practical design of turbomachines.

[13] Supervisor, Staff Engineering, Transport Division, Boeing Aircraft Corpo ration, Renton, Wash.

Author's Closure

It is indeed gratifying for a researcher to learn of the corroboration of his theory or of the application of his results to practical design studies. In this respect, the comments of Messrs. Eatock, Klapproth, Smith, and Swan are received with sincere appreciation.

The agreement found with the extensive cascade investigations mentioned by Mr. Eatock is considered quite significant in that it reveals an essential similarity in the minimum loss and loading limit characteristics of conventional cascade sections. It is intriguing indeed that the magnitude of the loading parameter C_{lv2m} used by Mr. Eatock is very nearly the same as the derived stalling equivalent diffusion ratio, D_{eq}^*. Although the theoretical equivalence of the two parameters is not immediately obvious, their numerical near equivalence can readily be illustrated by sample calculations of limiting cascade velocity triangles as indicated in the following table:

β_1	σ	$\Delta\beta$	D_{eq}^*	C_{lv2m}
45	0.5	24.77	2.00	1.91
45	1.0	43.18	2.00	2.17
60	0.5	17.14	2.00	2.03
60	1.0	22.14	2.00	1.91

The extension of the equivalent diffusion ratio to include changes in radius and axial velocity across the blade row derived by Mr. Klapproth is most welcome, and the author shares his hope for the ultimate inclusion of compressibility and other compressor effects. In this respect, it is interesting to find, from Mr. Swan's comment, that such an application of the diffusion ratio in the prediction of test results from an actual compressor single stage has proved useful.

The results achieved by Mr. Smith in his evaluations of compressor stage performance based on the diffusion ratio are indeed enlightening, and the author is in agreement with the derived conclusions. In particular, attention is called to his result showing high efficiencies for the higher design D_{eq}^*, since it demonstrates that higher blade loading in itself does not necessarily mean lower efficiency. The crux of the matter is the comparative rates of increase of work input and losses as diffusion ratio is increased.

The author is grateful to Mr. Smith for the caution concerning the extension of the equivalent diffusion ratio to solidities lower than 0.5. The reason for the difficulty here is that the circulation parameter in the expression for equivalent diffusion ratio is in reality the circulation multiplied by cos β_1. Thus the β dependence will remain when the circulation is converted to lift and drag coefficients. The need for the additional β_1 term, as established empirically, is not clear from the physics of the situation. However, the extension to zero solidity devised by Mr. Smith should be entirely adequate.