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Coalescence of Spinning Binary Neutron Stars of Equal Mass

—— 3D Numerical Simulations ——

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We have performed numerical simulations of coalescence of binary neutron stars using a Newtonian hydrodynamics code including radiation reaction by gravitational waves. In order to examine the effect of spin, we start the simulations from two distinct types of the initial conditions. In the first case, to see the dependence of results on the initial separation of binary, a Roche solution of separation 27 km, each mass $\sim 1.5\,M_\odot$ and each radius ~ 9 km, respectively, is given as the initial condition. We found that the evolution sequence and the wave form of gravitational waves are essentially the same as those in the previous simulations in which computations are started when two neutron stars just contact. In the second case, we include spin of each star with $-\Omega$, where Ω is Keplerian angular velocity of orbital motion, which is required from the conservation of the circulation. We found that the wave pattern has high amplitude oscillation after coalescence contrary to the former case. This means that it will be possible to determine the spin of coalescing neutron stars from the observed wave form of gravitational waves. The maximum amplitude of gravitational waves is 3.4×10^{-21} for a hypothetical event at the distance of 10 Mpc.

§ 1. Introduction

There are at least three neutron star-neutron star binaries, PSR 1913+16, PSR 2127+11C and PSR 1534+12 in our galaxy. This suggests that about one percent of neutron stars are in the binary system. These binaries will coalesce within $10^8\sim3\times10^9$ yr due to the emission of gravitational waves. The statistical analysis shows that the binary coalescence may occur ~1 event par year within the distance of ~100 Mpc.[1),2)] Therefore coalescing binary neutron stars is one of the most promising sources of gravitational waves. Nakamura and Oohara performed the post-Newtonian three dimensional simulations including the radiation reaction of gravitational waves to know the amplitude and the wave form of gravitational waves at coalescing events.[3)~6)] They found that the maximum amplitude of gravitational waves is $\sim10^{-21}$ for a hypothetical event at a distance of 50 Mpc. This suggests that the sensitivity of LIGO (Laser Interferometric Gravity Wave Observatory) project will enable us to know the final phase of the coalescence.

Calculations by Nakamura and Oohara were performed for the various initial conditions, but we must argue the following two points. One is that while the calculations were started when the neutron stars in Roche equilibrium just contact each other for saving CPU time, it is not clear whether they keep the equilibrium state until coalescence. This is because the equilibria are kept only when the radiation loss time scale of gravitational waves is much longer than the period of the orbital rotation. However when the neutron stars contact each other, these time scales are nearly equal. In the calculations of Nakamura and Oohara, the initial neutron stars are rigidly rotating around the center of mass. However as recently suggested by Kochanek,[7] if there is no or small viscosity in the system, the neutron stars must have spin from the conservation of the circulation while their separation decreases due to the emission of gravitational waves. Accordingly the initial conditions of Nakamura and Oohara is correct only when the viscosity works sufficiently in the system until the coalescence. If the sufficient viscosity does not exist in the system, we must consider the initial condition in which the neutron stars are spinning.

There is another kind of interest in coalescence of binary neutron stars. Recently it is proposed that γ-ray bursts are produced during or soon after the coalescence of the binary neutron stars.[8)~10)] The various mechanism to produce the γ-ray bursts associated with the evolution of the binary are suggested. Therefore it is required to know the feature of the evolution as well as the final coalescence event of the binary neutron stars from various initial conditions. As a first step we consider the following initial conditions in this paper. We use the initial model in which two neutron stars are separated each other far enough so that the radiation time scale becomes longer than the dynamical time scale. We also use the initial condition in which each neutron star has spin angular momentum assuming that the viscosity does not exist in the system.

Contents of the paper are as follows. In § 2 we show the basic equations and the initial model we use. In § 3 we show the numerical results. In § 4 we discuss the astrophysical implications of our results.

§ 2. Basic equations and initial models

2.1. *Basic equations*

The basic equations are the three dimensional hydrodynamics equations with a back reaction potential proposed by Blanchet et al.[11] Although we should evaluate post-Newtonian terms up to 2.5 PN order,[12] in this paper we only include the radiation reaction terms.

The basic equations are as follows:[5),6)]

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho v^i}{\partial x^i}=0\,, \tag{2·1}$$

$$\frac{\partial\rho w^i}{\partial t}+\frac{\partial\rho w^i v^j}{\partial x^j}=-\frac{\partial P}{\partial x^i}-\rho\frac{\partial(\psi+\psi_{\rm react})}{\partial x^i}\,, \tag{2·2}$$

$$\frac{\partial\rho\varepsilon}{\partial t}+\frac{\partial\rho\varepsilon v^j}{\partial x^j}=-P\frac{\partial v^i}{\partial x^i}\,, \tag{2·3}$$

$$P=(\Gamma-1)\rho\epsilon\,, \tag{2·4}$$

$$\Delta\psi=4\pi G\rho\,, \tag{2·5}$$

$$\Delta R=4\pi G\left(\frac{d^3}{dt^3}D_{ij}\right)x^j\frac{\partial\rho}{\partial x^i}\,, \tag{2·6}$$

$$v^i = w^i + \frac{4G}{5c^5}\left(\frac{d^3}{dt^3}D_{ij}\right)w^j\,, \tag{2·7}$$

$$\psi_{\text{react}} = \frac{2G}{5}\left(R - \left(\frac{d^3}{dt^3}D_{ij}\right)x^j\frac{\partial\psi}{\partial x^i}\right). \tag{2·8}$$

The symmetric trace-free part of the quadrupole moment D_{ij} and its third time derivative can be obtained by

$$D_{ij} = STF\left(\int \rho x^i x^j dV\right)$$

and

$$\frac{d^3}{dt^3}D_{ij} = \text{STF}\left[2\int\left(2P\frac{\partial v_i}{\partial x^j} - 2\rho v_i\frac{\partial\psi}{\partial x^j} + x_i\frac{\partial\psi}{\partial x^j}\frac{\partial\rho v^k}{\partial x^k} - \rho x_i\frac{\partial\dot{\psi}}{\partial x^j}\right)dV\right], \tag{2·9}$$

where the notation STF means

$$\text{STF}(Q_{ij}) = \frac{1}{2}(Q_{ij} + Q_{ji}) - \frac{1}{3}\delta_{ij}Q_{kk}\,.$$

The time derivative of the gravitational potential $\dot{\psi}$ is determined by

$$\Delta\dot{\psi} = -4\pi G\frac{\partial\rho v^k}{\partial x^k}\,. \tag{2·10}$$

We take the units of

$$M = M_\odot\,, \quad L = \frac{GM_\odot}{c^2} = 1.5\,\text{km}\,, \quad T = \frac{GM_\odot}{c^3} = 5\times10^{-6}\,\text{sec}\,. \tag{2·11}$$

We fix the polytropic index $\Gamma = 2$.

2.2. *Initial conditions*

In the previous work by Oohara and Nakamura,[5),6)] to save CPU time they started the simulations when two neutron stars just contact each other. In their initial conditions the binary system of two neutron stars is in a Roche equilibrium state with rigid rotation around the center of mass. However we should consider two points. 1) It is not clear whether each neutron star keeps the nearly equilibrium configuration until the contact. This is because when the neutron stars touch each other the radiation reaction time scale $a/\dot{a} = 5c^5a^4/(64G^3M^2\mu)$, where a is the separation of two neutron stars, M is the total mass and μ is the reduced mass of the system, becomes nearly equal to the orbital period $2\pi/\Omega = 2\pi\sqrt{a^3/GM}$, so that the Roche equilibrium state is impossible practically. 2) In the absence of the viscosity, the circulation of the system should be conserved. If the neutron stars are assumed to rotate rigidly around the center of mass at first, the circulation in the z=constant plane is $2\Omega_0 S$, where Ω_0 and S are the initial angular velocity and the area of the cross section of the neutron stars, respectively. While the two neutron stars approach each other by radiation reaction, the area of the cross section of the neutron stars does not change so much, but the angular velocity Ω becomes much larger than Ω_0 because $\Omega \propto a^{-3/2}$. Therefore to conserve the circulation, the neutron stars must have spin with the angular velocity $-\Omega$, i.e., retrograde to the orbital motion.[7)]

To take into account above two points, in this paper we adopt the different initial conditions from those in previous papers: 1) We begin all the simulations in this paper when the separation of two neutron stars is 18 in our unit (27 km). If the mass of each neutron star is assumed to be $\sim 1.4\ M_{\odot}$, the radiation reaction time scale is fifth times as large as the dynamical time scale. 2) We calculate not only the models in which the neutron stars initially rotate around the center of mass i.e., a Roche solution, but also the one in which the neutron stars have spin angular velocity retrograde to the orbital motion. In particular we consider the extreme model that there is no viscosity in the system, so that the spin angular velocity is $-\Omega$, where Ω is the Keplerian angular velocity of the orbital motion.

To see the effect of the tidal force in the initial condition, we use both the spherical polytropic star model and the Roche equilibrium model as the initial condition in the non-spinning model. As for the spherical polytropic star, the equation of state is given by

$$P = K\rho^2 , \tag{2·12}$$

where $K = 2r_0^2 G/\pi$ and r_0 is the radius of spherical stars. Then the density distribution of each star is given by

$$\rho = \frac{M}{4r_0^2 r}\sin\left(\frac{\pi r}{r_0}\right), \tag{2·13}$$

where r is the distance from the center of each star.

A Roche equilibrium model is determined by the method given by Oohara and Nakamura.[5)] It is obtained by solving the following equations:

$$\psi + h - \frac{1}{2}R^2\Omega^2 = C , \tag{2·14}$$

and

$$\Delta\psi = 4\pi G\rho . \tag{2·15}$$

Equation (2·14) is the integrated representation of equation of motion and $h = 2K\rho$. To determine the solution, we must fix three parameters. We fix the innermost points of the stars, the centers of the neutron stars and the density there. Solving the above equations by the iteration, we obtain an equilibrium model. In this case, the mass of two neutron stars and the orbital angular velocity are determined only from the equilibrium model.

In the case that two neutron stars have spin, we do not know how to determine an equilibrium model because this is a similar problem to determine the Dedekind configuration.[13)] However as for an axisymmetric rotating star, we can obtain the equilibrium configurations. We here consider axisymmetric equilibrium rotating stars as the initial condition for each spinning neutron star. This initial condition is consistent if the tidal force is much smaller than the self-gravity. An equilibrium axisymmetric rotating star is determined by

$$\rho=\frac{1}{2K}\left[\frac{1}{2}R^2\Omega^2+C-\psi\right], \tag{2·16}$$

and

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial\psi}{\partial x}\right]=4\pi G\rho, \tag{2·17}$$

where $x=\cos\theta$. To solve these equations, we must fix three quantities. In the absence of viscosity, the circulation should be conserved, so that each neutron star should have spin angular velocity the same as the Keplerian angular velocity with opposite sign. Hence the angular velocity is fixed contrary to the usual case. We also fix the density at the center and the major axis of the star.

A numerical method to solve the above equations is as follows. 1) We expect the trial density configuration and using this we solve the axisymmetric Poisson equation (2·17). 2) The constant C and the polytropic constant K are determined by

$$\psi_c+2K\rho_c=C,$$

$$\psi_e-\frac{1}{2}R_e^2\Omega^2=C, \tag{2·18}$$

where ψ_c, ψ_e, ρ_c and R_e are the Newtonian potential at the center, the Newtonian potential at the surface of the star in equatorial plane, the density at the center and the major axis of the star, respectively. 3) Using Eq. (2·16), we determine the new trial density configuration. The procedure of 1), 2) and 3) is repeated until convergence. We use an axisymmetric rotating star determined by the above method as each neutron star of the binary. As a result, it is found that the virial equilibrium is almost satisfied. This means that the solution seems to be almost in a true equilibrium.

§ 3. Numerical results

3.1. *Evolution sequence for non-spinning models*

We first show models in the case that the neutron stars rigidly rotate around the center of mass. We perform two simulations; one is that the two neutron stars are spherical polytropes, the other is that the two neutron stars are in Roche equilibrium. In the former case, we take $121\times121\times121$ grids. The grid covers $[-21, 21]$ in x, y and z directions. The mass, the radius of each neutron star and the initial separation are set as $m_1=m_2=1.4M_\odot$, $r_0=6$ and $l_0=18$, respectively. We assume that the neutron stars rigidly rotate around the center of mass with $\Omega_K=\sqrt{G(m_1+m_2)/l_0^3}$. We call this Model I (see the Table).

In the latter case, we take $141\times141\times131$ grids. The grid covers $[-31.5, 31.5]$ in x and y directions and $[0, 32.5]$ in z direction. The radius of each neutron star at $z=0$ is $r_0=5.85$. The mass and the angular velocity of the orbital rotation are determined by solving Eqs. (2·14) and (2·15). They are $m_1=m_2=1.5\,M_\odot$ and $\Omega=2.06\times10^{-2}$ in our unit, respectively. The initial separation is the same as in Model I. We call this Model II (see the Table). Note that if the system is assumed to be composed

Table. Models in numerical simulations. Spin, mass, major axis and orbital angular velocity of each neutron star and the grid number of each model in numerical simulation are denoted.

Model	spin	Mass $M_\odot$	Major axis	Orbital frequency	Grid number (x, y, z)	Figs.
I	no	1.4×1.4	9 km	Keplerian	(121, 121, 121)	1, 4
II	no	1.5×1.5	8.78 km	Roche model (2.06×10^{-2})	(141, 141, 131)	2, 5
III	yes	1.4×1.4	9 km	Keplerian	(121, 121, 121)	3, 6

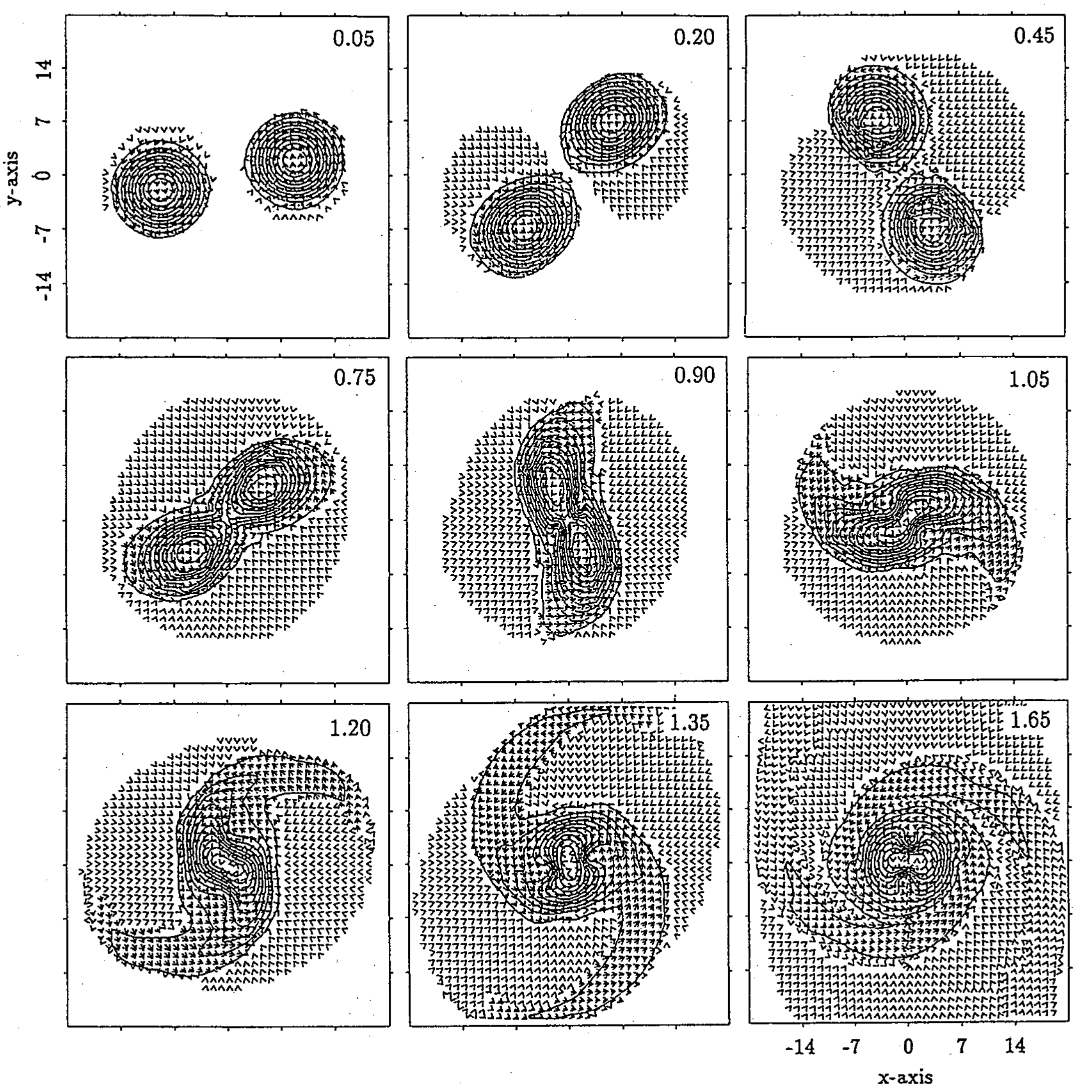


Fig. 1. Density contours on x-y plane for Model I. The time in unit of millisecond is indicated. Solid lines show the density contours. Outermost line is 1/100 of the maximum density and the inner lines are drawn by step of a tenth of the maximum density. Arrows indicate the velocity vectors of the matter.

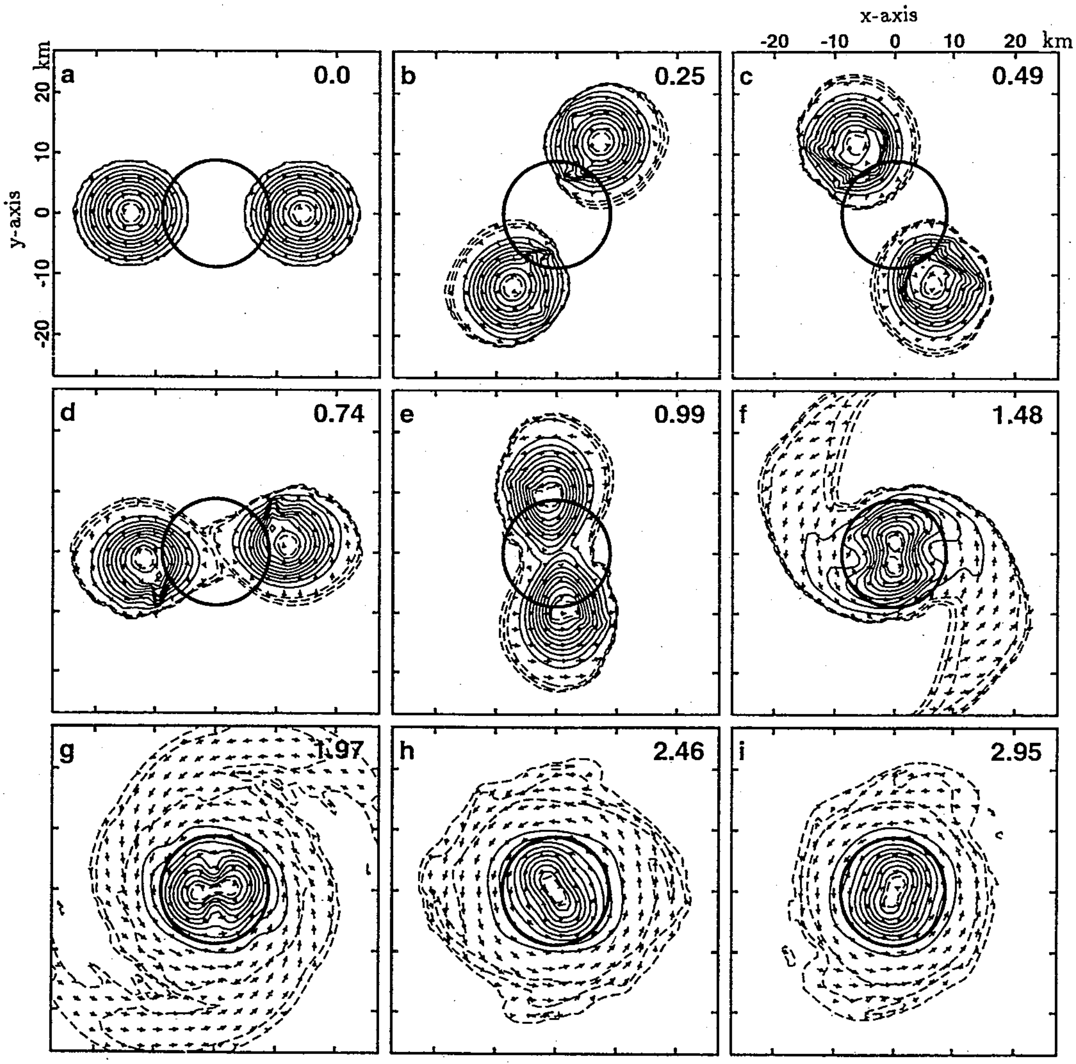


Fig. 2. Density contours on x-y plane for Model II. The time in unit of millisecond is indicated.

of the point masses with $m_1=m_2=1.5\ M_\odot$, the Keplerian angular velocity is $\Omega_K=2.26\times10^{-2}$. In model EQ0 of Ref. 5), Ω and Ω_K are respectively 3.3×10^{-2} and 5.5×10^{-2}. Therefore the tidal effect in the initial condition of Model II seems to be negligible compared with EQ0.

We show the contours of the density and the velocity vectors on the x-y plane for Models I and II in Figs. 1 and 2. When two point masses approach by radiation reaction, the time required to coalesce becomes

$$t_c=\frac{5c^5}{256G^3M^2\mu}(a_i^4-a_f^4)\,, \tag{3·1}$$

where a_i and a_f denote, respectively, the initial separation and the final separation of the neutron stars. In Models I and II, t_c becomes 1.5 and 1.25 msec, respectively, if we put $a_i=18$ and $a_f=2r_0$. In Model II it takes almost the above time for the neutron stars to coalesce. However in Model I the neutron stars coalesce at $t_c \lesssim 1 < 1.5$ msec. This seems to be mainly due to the initial condition for Model I, in which we do not give the equilibrium state and the tidal force is neglected. Although there is slight difference between Models I and II such as the coalescing time, as a whole both models have essentially the same evolution sequence; initially the orbit of the neutron stars shrinks radiating almost periodic gravitational waves ($t \lesssim 1$ msec). After the coalescence, in the outer region the spiral arms are formed and in the inner region the ellipsoidal core is formed ($t > 1$ msec). Then the spiral arms gradually wind round the core by differential rotation and become axially symmetric disk. Since the Models I and II have almost the same tendency, the spherical polytrope for each neutron star seems to be nearly in equilibrium and the tidal effect does not seem to be so important in our initial conditions. Therefore in spinning model discussed in next section, each axially symmetric neutron star seems to be nearly in equilibrium.

Now let us compare the results in the present paper with the previous calculations. Models I and II should be compared with EQ0 in Ref. 5). It is found that both calculations show essentially the same results as for the evolution. This means that if the mass of two neutron stars is equal and the neutron stars do not have spin angular momentum, they seculary shrink radiating gravitational waves until the coalescence in the Newtonian hydrodynamics.

3.2. *Evolution sequence for spinning model*

Next we argue the model in which each neutron star is spinning. As mentioned in § 2, we do not know how to determine an equilibrium model. Hence we set the model in which each neutron star is axially symmetric. The mass and the major axis of each neutron star are set as $m_1 = m_2 = 1.4\,M_\odot$ and $r_0 = 6$. We put the angular velocity of each star the same as that of the orbital one, $\Omega_K = \sqrt{G(m_1+m_2)/l_0^3}$, where the initial separation l_0 is the same as in Models I and II. In the numerical calculation, we take the same grid as Model I. We call this Model III (see the Table). Comparison of Models I and II strongly suggests Model III is almost in equilibrium because tidal force is not so important.

In Fig. 3 we show the contours of the density and the velocity vectors on the $x-y$, $y-z$ and $z-x$ plane for Model III. Initially the orbital velocity v_K is given as

$$v_K{}^x = -y\Omega\,,$$

$$v_K{}^y = x\Omega\,,$$

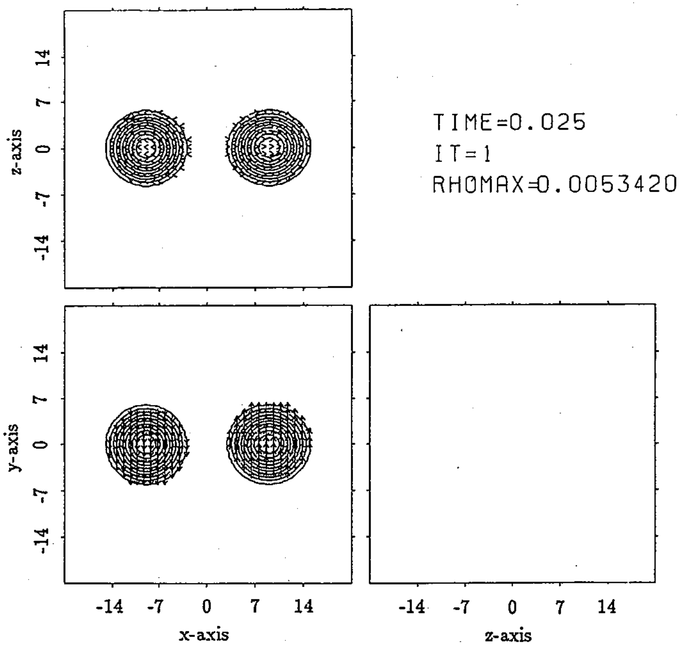
and the spin velocity v_s is given as

$$v_s{}^x = y\Omega\,,$$

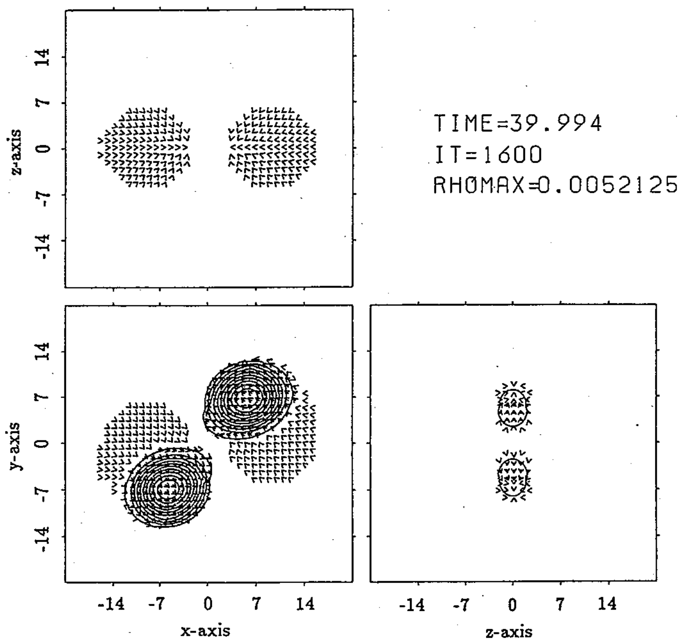
$$v_s{}^y = -\left(x - \frac{l_0}{2}\right)\Omega\,, \qquad (x>0)$$

$$= -\left(x + \frac{l_0}{2}\right)\Omega\,. \qquad (x<0)$$

Since $v^x = v_K{}^x + v_s{}^x = 0$ in the inertial frame, the velocity vectors point to only



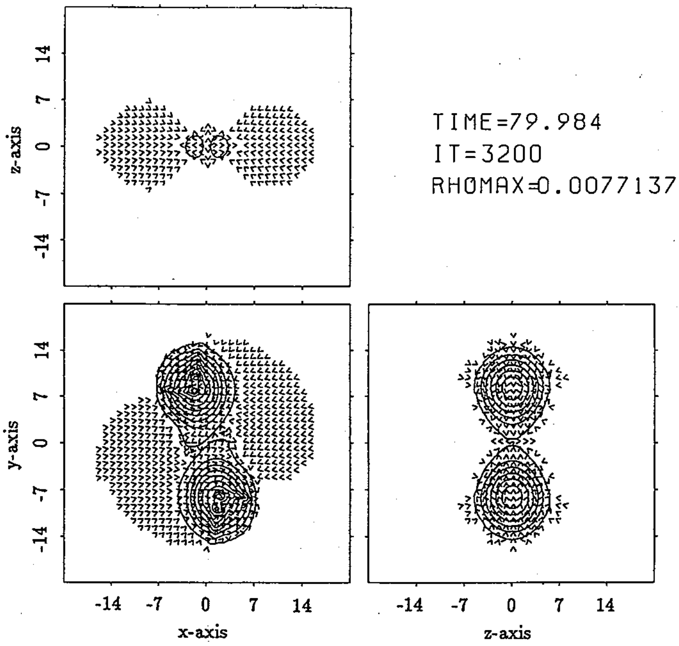
(a)



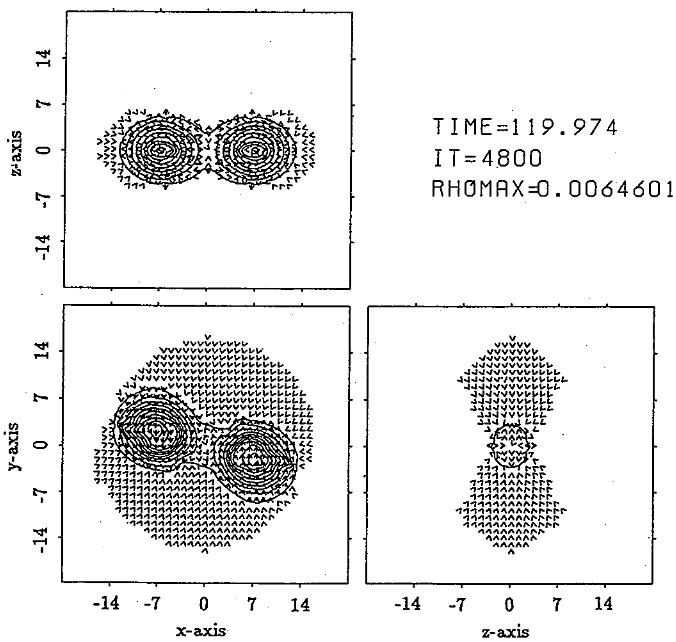
(b)

Fig. 3. (continued)

y-direction. In Fig. 3(a) we show the initial state in the inertial frame in which the neutron stars move only toward the y-direction. Until the coalescence, the evolution sequence is almost the same as in Models I and II. However after the coalescence, the evolution of the system in Model III is quite different from those in Models I and



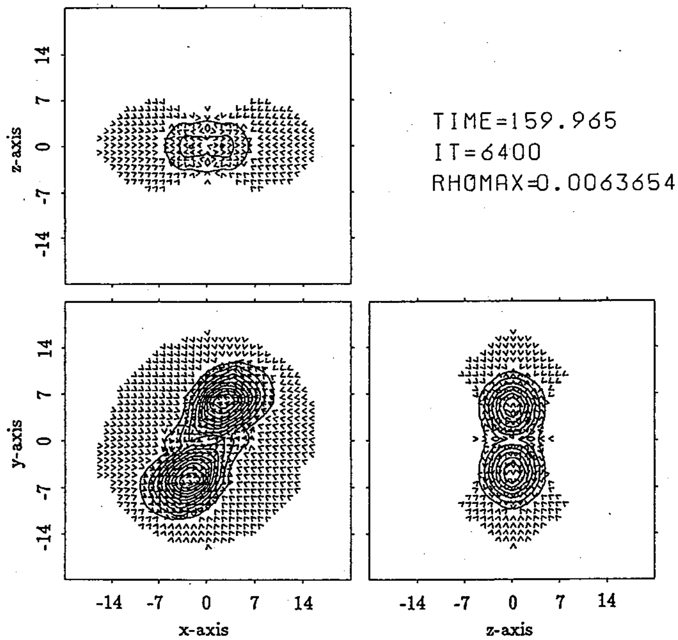
(c)



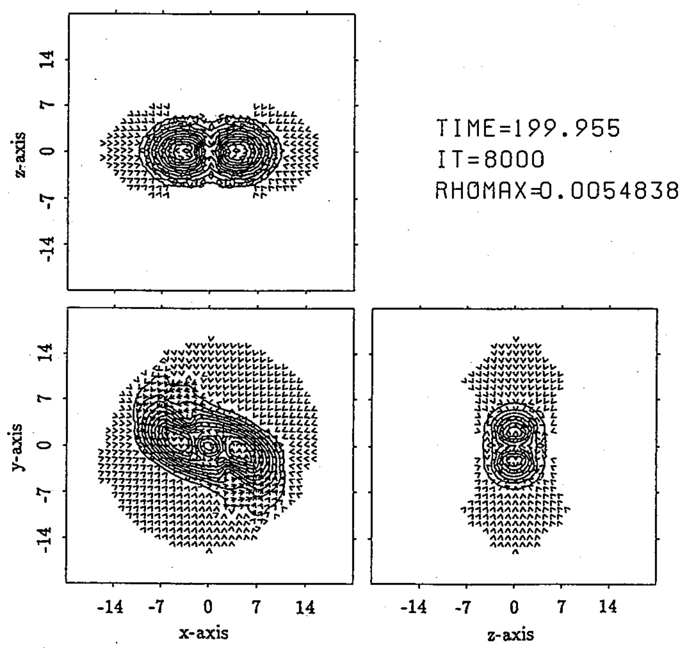
(d)

Fig. 3. (continued)

II. In the outer region, there is no spiral arm, so the configuration becomes a nearly axially symmetric disk soon. This result seems to be reasonable. Since the velocity of the outer region is smaller than in Models I and II due to the spin retrograde to the orbital motion for each neutron star, the centrifugal force is weaker than in Models



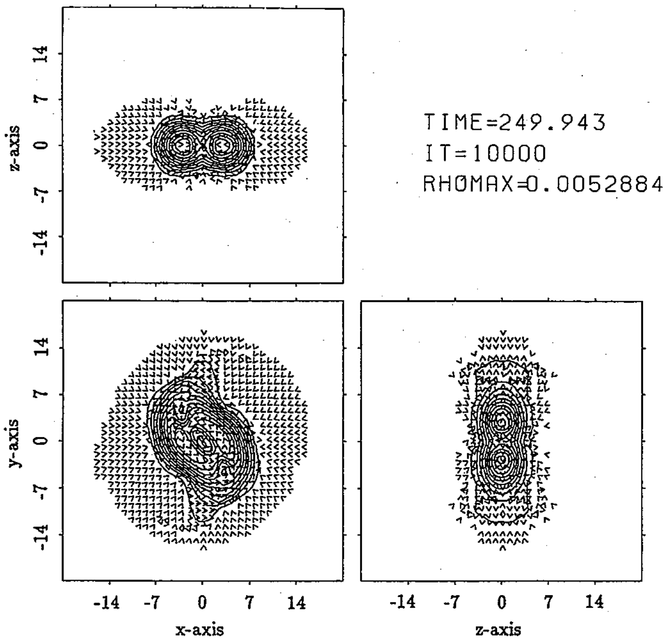
(e)



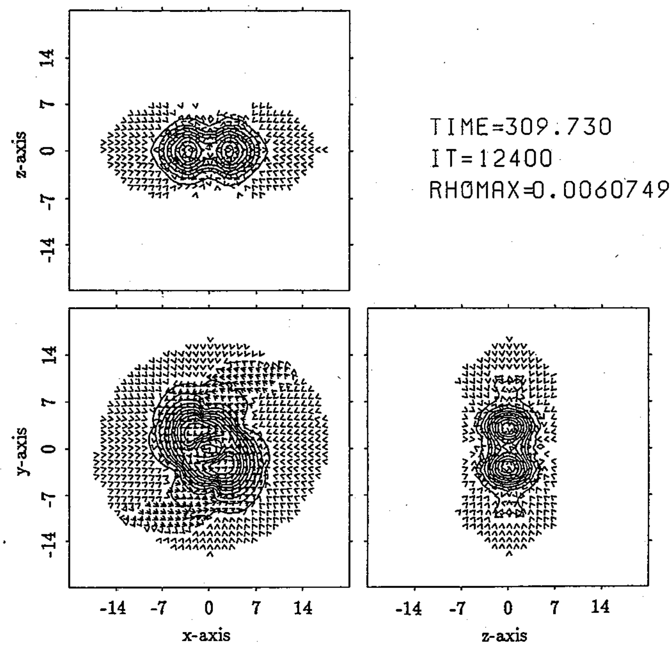
(f)

Fig. 3. (continued)

I and II. In the inner region, the neutron stars are gradually coalescing because of the enhanced centrifugal force, so that the double core structure is kept for a long time. By the radiation reaction, the double cores merge at last. It does not become the rotating ellipsoid, but the ring. At $t \sim 3$ msec, $T/|W|$ of the system is 0.143. This



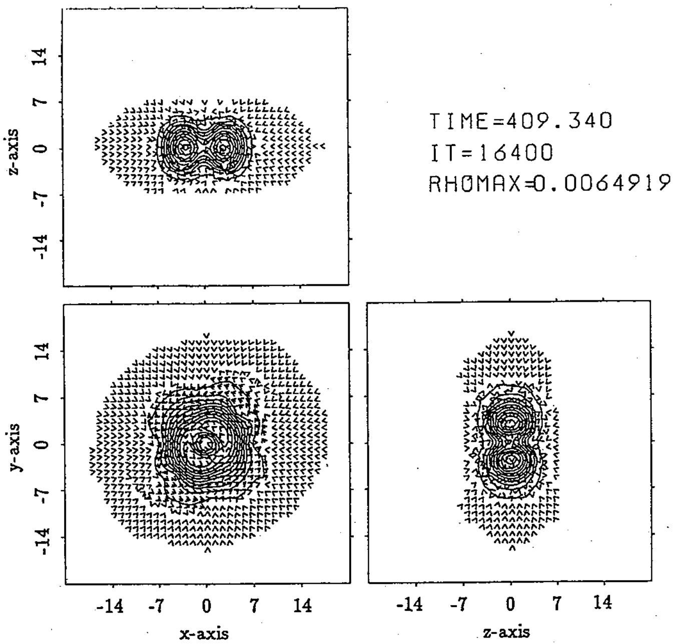
(g)



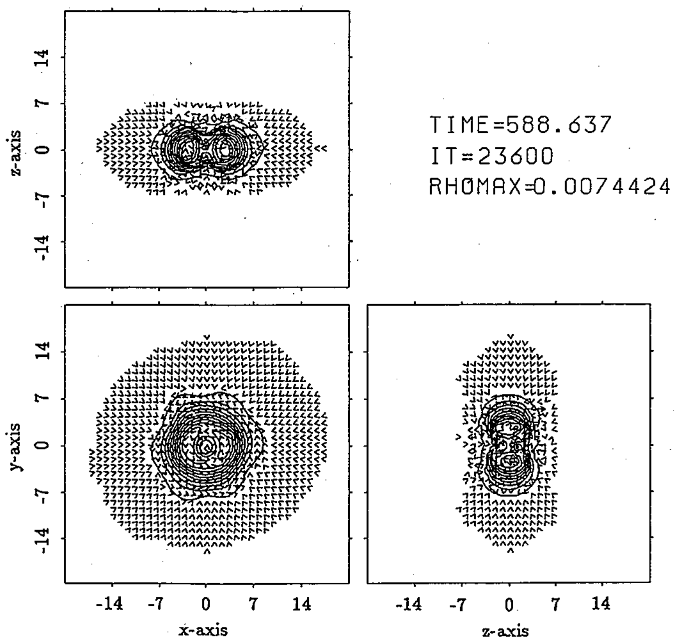
(h)

Fig. 3. (continued)

value is slightly larger than that at the secular instability limit[14] ($T/|W|\lesssim 0.14$). In the model EQ8 of Ref. 5) the ring slowly evolved to a disk, so that it is expected that the disk is formed in the subsequent evolution.



(i)



(j)

Fig. 3. Density contours on x-y, y-z and z-x planes for Model III. The non-dimensional time, time steps and the non-dimensional maximum density are indicated. Note that 200 in the non-dimensional time is equal to 1msec.

3.3. *Gravitational waves*

In Figs. 4, 5(a) and 6(a) we show the wave forms h_+ and $h_\times$ of gravitational waves observed on the z-axis at 10 Mpc in Models I, II and III. They are defined by

$$h_+=\frac{1}{r}(\ddot{D}_{xx}-\ddot{D}_{yy}),$$

$$h_\times=\frac{2}{r}\ddot{D}_{xy}. \tag{3·2}$$

In Figs. 5(b) and 6(b) we also show the luminosity of gravitational waves calculated by

$$\frac{dE}{dt}=\frac{1}{5}\left(\frac{d^3D_{ij}}{dt^3}\right)^2. \tag{3·3}$$

Since the evolution sequences of Models I and II are essentially the same, the wave forms of them are also the same. The wave forms are made of the three part. One is the periodic wave part ($t\lesssim 1$ msec). The second is the burst part induced by the coalescence of the neutron stars ($t\lesssim 1$ msec). The third is the damping part ($t\gtrsim 1$ msec). Compared with EQ0 in Ref. 5), the wave forms are essentially the same. This is reasonable because the density evolution sequences of three models Models I, II and EQ0 are essentially the same. Contrary to Models I and II, the wave form of Model III is composed of the four part. The first, the second and the last part have the same tendency as Models I and II. However after the coalescence ($t\lesssim 1$ msec), it shows different tendency: The high amplitude oscillation continues for several times. This difference of the wave form also affects the time variation of the luminosity. In Fig. 5(b) the peak of luminosity appears at the moment of the contact of the neutron stars and then the luminosity decrease exponentially. On the other hand, in Fig.6(b) after the first peak appears, the second large peak appears again. This is because in Model III the double core structure is formed after the coalescence and it oscillates before merging. In

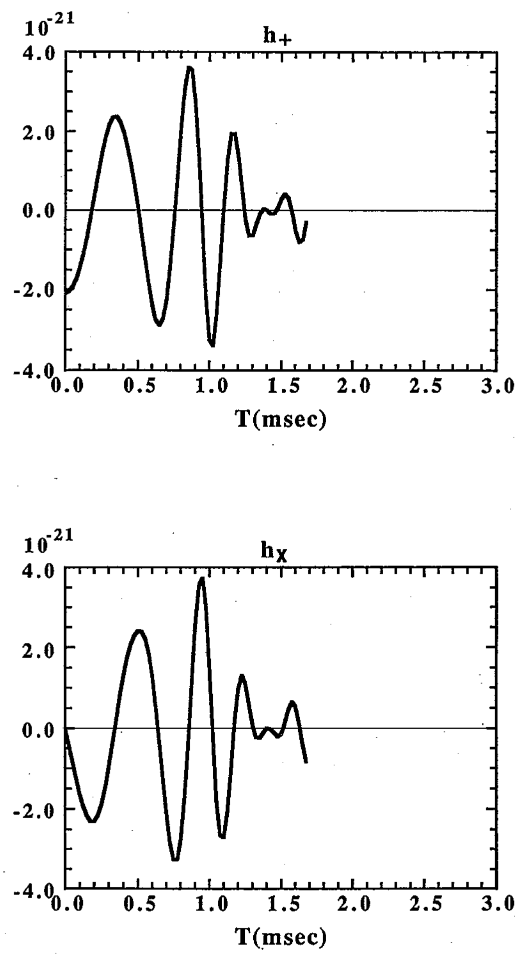


Fig. 4. The wave forms of gravitational waves observed on the z-axis at 10 Mpc for Model I.

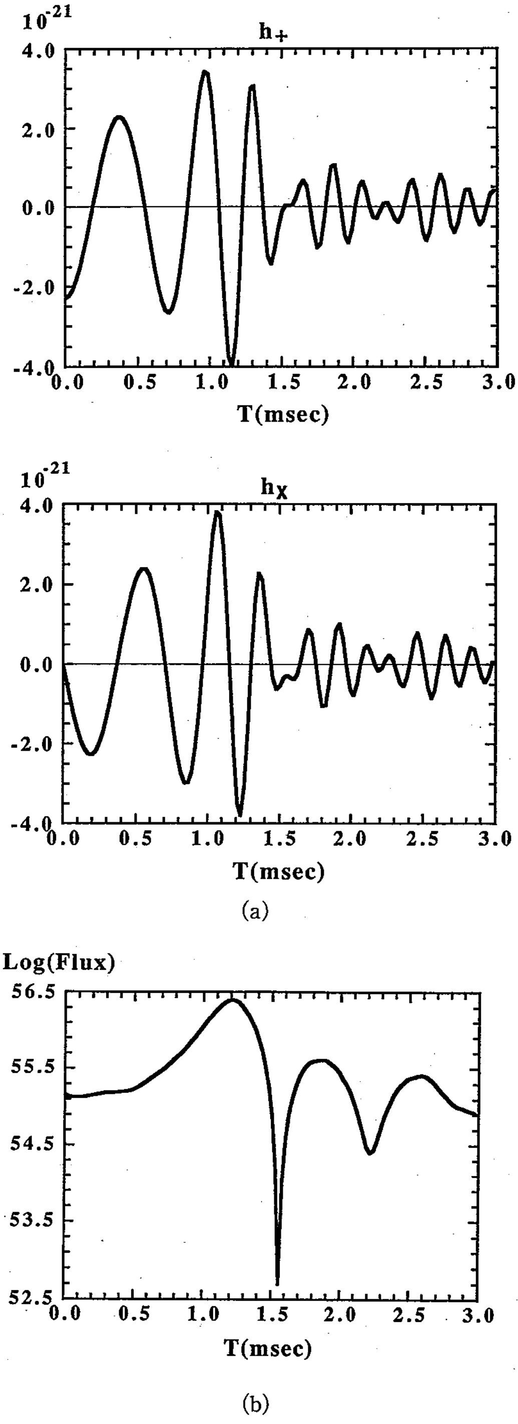


Fig. 5. The wave forms and the luminosity of gravitational waves for Model II. (a) +mode and × mode gravitational waves observed on the z-axis at 10 Mpc. (b) The luminosity of gravitational waves is indicated in cgs unit.

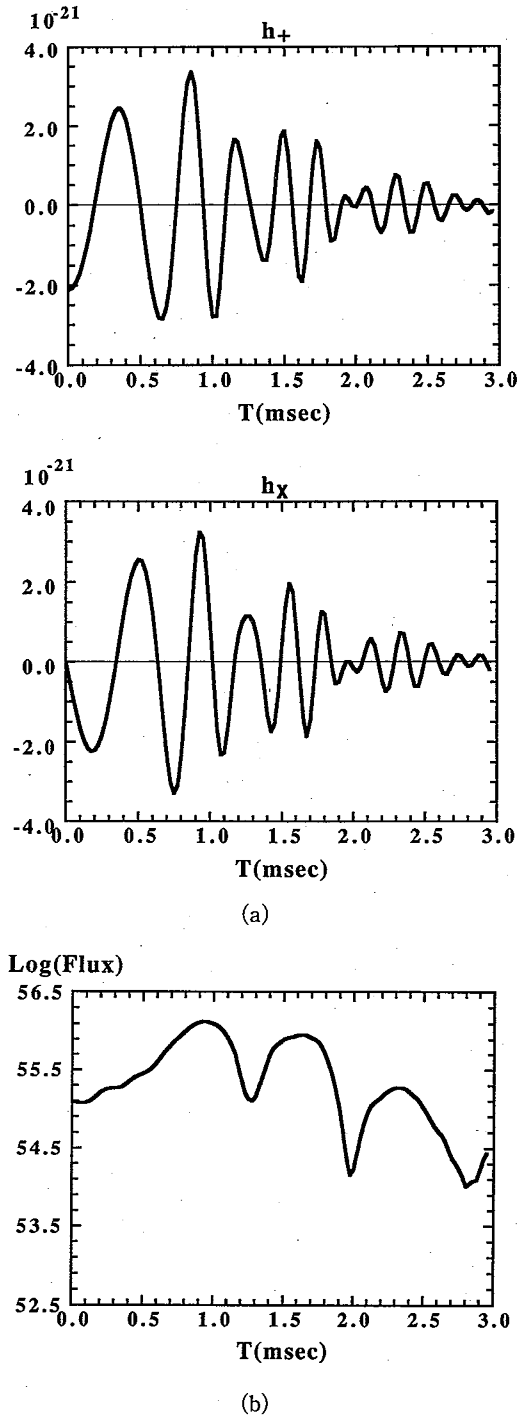


Fig. 6. The wave form and the luminosity of gravitational waves for Model III. (a) + mode and × mode gravitational waves observed on the z-axis at 10 Mpc. (b) The luminosity of gravitational waves is indicated in cgs unit.

conclusion, the wave forms are considerably different in the case that the effect of the conservation of the circulation is included. Inversely if gravitational waves from coalescing binary neutron stars are detected, we may know whether each neutron star had spin before the coalescence or not. If the spin is not so large, this means that the viscosity was very effective before the coalescence.

§ 4. Discussion

The maximum amplitude of gravitational waves observed on the z-axis at 10 Mpc is 4.0×10^{-21} and 3.4×10^{-21} for Models II and III. This value is somewhat smaller than the results of EQ0 in Ref. 5) (4.2×10^{-21}). In EQ0, the maximum amplitude is recorded at the first bounce and oscillation of the coalescence. However in the calculation of this paper the maximum amplitudes are recorded when the neutron stars just contact each other. This means that if the initial separation of the neutron stars is increased, the bounce and the oscillation which occur after the coalescence are suppressed. The total energy emitted until the end of the calculations are 0.077 and 0.063 for Models II and III and the efficiency of the gravitational wave emission amounts to 2.6 % and 2.3 % for Models II and III. This is smaller than that of EQ0. This also means that the bounce and the oscillation are suppressed compared with EQ0. It seems that the evolution of the binary neutron stars becomes more secular when the initial separation of the neutron stars increases.

In Models II and III the initial angular momentum are 6.3 and 5.0 and the final angular momentum are 3.6 and 3.1 in our unit. A fraction of the angular momentum loss is 43 % and 38 % in Models II and III, respectively. More angular momentum is lost, more gravitational waves are emitted, so that our results are reasonable. $J/M^2(=a/M)$ is 0.4 in both Models II and III and total mass of the coalescing objects exceed the neutron star mass limit, so the final products are expected to be slowly rotating black holes.

Let us shortly discuss the implication of our results to the γ-ray bursts. Some authors are suggesting[8),10)] that the γ-ray bursts are produced in the merging disk formed during the binary coalescence. As far as the calculations of this paper are concerned, the disk is not formed sufficiently. That is, the radius of the disk is smaller than the radius of the last stable circular orbit($5M\sim14$). Therefore if the sufficient viscosity does not exist in the system, the disk needed to produce the γ-ray bursts in their scenarios does not seem to be formed. However we need to consider the following two possibilities. 1) If there is large enough viscosity in the system, the thermal pressure produced by the viscosity may blow off the crust of the neutron stars[9)] and the disk may be efficiently formed. 2) Lincoln and Will[15)] and Kidder et al.[16)] showed that there exists the last stable circular orbit for binary neutron stars similar to that for a test particle in Schwarzschild space-time. Therefore if we take into account the post-Newtonian correction or full effect of general relativity, in the final phase of the coalescence the approaching velocity of binaries is almost the same as the orbital velocity, that is, they have almost the plunging orbit. Furthermore in the post-Newtonian calculation both radial and angular velocity is much larger than those in the Newtonian case.[15)] These mean that the coalescence event may be more violent than in the Newtonian case. Thus we need the further probe for this problem.

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