Coalition Lattice: A Data Structure considering Robustness for Robust Coalition Structure Generation Problem

Katsuya Nakano[a,*], Shun Shiramatsu[a], Tadachika Ozono[a], Toramatsu Shintani[a]

[a]Nagoya Institute of Technology, Gokiso-cho, Showa-ku, Nagoya 466-8555, Japan

*Corresponding Author: nkatsu@toralab.org

Abstract

Forming effective coalitions is a major problem in multiagent systems. Coalition structure generation (CSG) involves partitioning a set of agents into teams, i.e. coalitions. A coalition structure is a set of coalitions. In the context of the CSG research, our goal is forming the coalition structure that maximizes the social surplus. The social surplus is the sum of utilities obtained by forming coalitions. Calculating the optimal coalition structure is time consuming, therefore recalculating the optimal coalition structure should be avoided when an agent leaves a coalition structure because of sudden reasons such as an accident or an illness. Robust coalition structure generation (RCSG) is a variant of CSG focused on the robustness of a coalition structure. The robustness of coalition structure is the property that the social surplus is kept at the maximum when any agents leave the coalition structure. We focused on the robustness of each coalition in a coalition structure to solve an RCSG problem. Our method finds the optimal coalition structure considering the robustness of each coalition in the optimal coalition structure. We proposed a coalition lattice, which is a novel data structure to represent the robustness of coalitions. The paper presents an algorithm to construct the coalition lattice from a CSG problem and the result of our evaluation.

Keywords: robust coalition structure generation, robustness of coalitions, partial linear model, coalition lattice.

1. Introduction

Forming effective coalitions is a major problem in multiagent systems. Coalition structure generation (CSG) involves partitioning a set of agents into teams, i.e. coalitions[1,2]. A coalition structure is a set of coalitions. In the context of the CSG research, our goal is forming the coalition structure that maximizes the social surplus, the sum of utilities obtained by forming coalitions.

Algorithms to find the optimal coalition structure for CSG have been proposed. Yeh et al.[3] proposed an algorithm based on dynamic programming. Rahwan et al.[4] proposed Integer Partition (IP) algorithm that is one example of anytime algorithms. Rahwan and Jennings[5] proposed an algorithm that consists of IP algorithm and dynamic programming. Michalak et al.[6] proposed a decentralized algorithm for optimal coalition structure generation problem.

The robustness of coalition structures is a new important problem. The robustness of a coalition structure is the property that the social surplus is kept at the maximum when any agents leave the coalition structure. Robust coalition structure generation (RCSG) is CSG focused on the robustness of coalition structures. Okimoto et al.[7] defined the framework for robust team formation problem (RTFP). RTFP is equal to forming a robust coalition in RCSG. Okimoto et al. presented the computational complexity of RTFP that the order of computational complexity is not increased even if we consider the robustness of teams.

We focus on the robustness of coalitions to solve an RCSG problem. We developed a conversion algorithm from a CSG problem to an RCSG problem under the assumption that the characteristic function based on the partial linear model, i.e., the characteristic function does not satisfy superadditivity and monotonicity partially. We propose a coalition lattice, which is a novel data structure to represent the robustness of coalitions and present the method to solve an RCSG problem.

DOI: 10.12792/iciae2015.009

2. Robust Coalition Structure Generation (RCSG) Problem

Robust coalition structure generation (RCSG) problem is one of coalition structure generation (CSG) problems considering the robustness of the solution [8].

Let n be the number of agents, and let $A=\{a_1,a_2,\ldots,a_n\}$ be the set of agents. A subset of agents, i.e., a coalition, is denoted by $S \subseteq A$. Let CS be a partition of A where CS is satisfying (1).

$$\forall i,j(i \neq j),\quad S_i \cap S_j = \varnothing,\quad \bigcup_{S_i \in CS} S_i = A \qquad (1)$$

Each agent belongs to only one coalition at the same time. A characteristic function $v: 2^A \rightarrow \mathrm{N}$ is given. The N is the utility of the cooperation with agents of coalition S, denoted by $v(S)$. We assume that v can be calculated in polynomial time. The utility of CS, i.e. the social surplus, denoted by $V(CS)$. The value of $V(CS)$ is calculated by (2).

$$V(CS) = \sum_{S_i \in CS} v(S_i) \qquad (2)$$

The optimal coalition structure CS^* satisfies (3).

$$\forall CS : V(CS) \leq V(CS^*) \qquad (3)$$

Let k be a non-negative integer and A' be a subset of A. CS is the k-robust coalition structure if any coalition structure CS' of $A \setminus A'$ does not satisfy (4) where $k \leq |A'|$ $(0 \leq k \leq |A|-2)$. In CSG, any coalition structure should be the 0-robust coalition structure. And the range of k is $0 \leq k \leq |A|-2$ because all coalition structures are $(|A|-1)$-robust coalition structures.

$$V(CS \setminus A') < V(CS') \qquad (4)$$

We should consider ${}_{|A|}C_k$ patterns to distinguish whether the coalition structure is the k-robust coalition structure. Fig. 1 shows the example of the judgment of the 1-robust coalition structure in the case that the number of agents is 4, i.e., $|A|=4$. In Fig. 1, coalition structure (a) is one of coalition structures of "Agent 1 to 4." For (a) to be the 1-robust coalition structure, the social surplus of (a) should be maximum if any agent leaves (a). Therefore we should consider the cases of each agent leave (a) to distinguish whether (a) is the 1-robust coalition structure or not. Fig. 1 shows the case that "Agent 1" leaves (a). Then, we should calculate the social surpluses of all coalition structures formed by "Agent 2 to 4." In Fig. 1, coalition structure (b), (c), (d), and (e) are coalition structures of "Agent 2 to 4" (except (a)). The social surplus of (a) without "Agent 1" should be more than the social surpluses of (b), (c), (d), and (e). The social surplus of (a) without another agent should be the greatest.

Now, let CS_R be the k-robust coalition structure and CS_R' be the k'-robust coalition structure. In RCSG, CS_R dominates CS_R' if and only if $k \geq k'$ and $V(CS_R) > V(CS_R')$, or $k > k'$ and $V(CS_R) \geq V(CS_R')$. And CS_R is the Pareto optimal and robust coalition structure if CS_R' that dominates CS does not exist. In RCSG, our goal is to find Pareto optimal and k-robust coalition structures where $k \leq |A'|$ $(0 \leq k \leq |A|-2)$. Finding Pareto optimal and k-robust coalition structures takes enormous times clearly. Therefore the effective method to find the k-robust coalition structure is necessary.

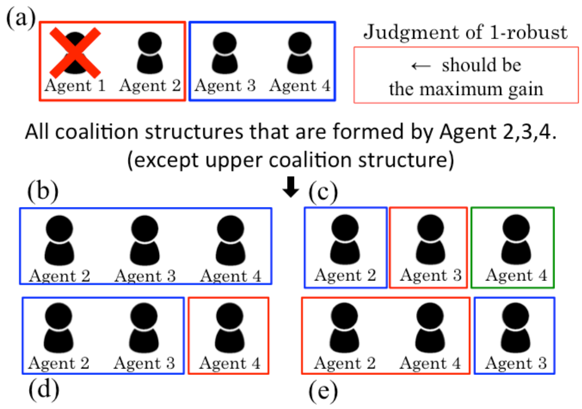

Fig. 1. The example of the judgment of 1-robust.

TABLE I: All coalitions and utilities of them. (n = 4)

v	$v(S)$	P/N	v	$v(S)$	P/N
$v(\{a_1\})$	1	P	$v(\{a_2, a_4\})$	4	N
$v(\{a_2\})$	5	P	$v(\{a_3, a_4\})$	9	P
$v(\{a_3\})$	3	P	$v(\{a_1, a_2, a_3\})$	10	P
$v(\{a_4\})$	4	P	$v(\{a_1, a_2, a_4\})$	7	N
$v(\{a_1, a_2\})$	8	P	$v(\{a_1, a_3, a_4\})$	10	P
$v(\{a_1, a_3\})$	5	P	$v(\{a_2, a_3, a_4\})$	8	N
$v(\{a_1, a_4\})$	3	N	$v(\{a_1, a_2, a_3, a_4\})$	19	P
$v(\{a_2, a_3\})$	5	N			

3. Characteristic Function Based on Partial Linear Model

In coalition structure generation (CSG) problem, there are researches focused on the notation of the characteristic function[9]. There are properties such as superadditivity and monotonicity in the characteristic function. When the characteristic function satisfies the superadditivity, any coalitions S_i and S_j where $S_i \cap S_j = \varnothing$ should satisfy $v(S_i)+v(S_j) \leq v(S_i \cap S_j)$. When the characteristic function satisfies the monotonicity, any coalitions S and S' where

$S' \subseteq S$ should satisfy $v(S) \geq v(S')$.

We assume the characteristic function does not satisfy superadditivity and monotonicity partially. We apply a sparse characteristic function for simplification, which is a characteristic function based on the partial linear model with a few non-linear parts. In the linear model, the utility of *S* is the sum of utilities of all coalitions formed by one agent in *S*. In this paper, the percentage of non-linear utilities of coalitions is called non-linear degree *(%)*. And we premise that the non-linear degree is small enough.

4. Robustness of Coalitions

Let a be the agent. The utility of coalition $\{a\}$ is denoted by $v(\{a\})$. In this paper, coalition S_p satisfying (5) is called a positive coalition, and coalition S_n satisfying (6) is called a negative coalition. All coalitions that consist of a single agent are positive coalitions.

$$v(S_p) > \sum_{a_i \in S_p} v(\{a_i\}) \qquad (5)$$

$$v(S_n) < \sum_{a_i \in S_n} v(\{a_i\}) \qquad (6)$$

Fig. 2 shows the example of the classification of coalitions. In Fig. 2, there are two agents, "Agent 1" and "Agent 2." The utility of the coalition formed by only "Agent 1" is one, and the utility of the coalition formed by only "Agent 2" is two. Therefore the sum of them is three. If the utility of the coalition formed by "Agent 1 and 2" is five, the coalition is a positive coalition because the utility is greater than three. If the utility of the coalition formed by "Agent 1 and 2" is two, the coalition is a negative coalition because the utility is smaller than three.

CSG and RCSG problem are the same in respect to forming the coalition structure that makes the social surplus greater. For example, let CS_1 be the coalition structure including negative coalition $S_n = \{a_1, a_4\}$, and CS_2 be the coalition structure including $\{\{a_1\},\{a_4\}\}$ instead of S_n. Then $V(CS_1)$ is less than $V(CS_2)$. Coalition structures should have positive coalitions as possible. And coalition structure should not have negative coalitions. Because the social surplus of the coalition structure that includes negative coalitions becomes less than the one that does not. In CSG, we can find the optimal coalition structure efficiently by considering only positive coalitions. In addition, we should consider the robustness of the coalition structure in RCSG. *CS* is the *k*-robust coalition structure in RCSG if no coalition structure *CS'* of $A \setminus A'$ satisfying (4) exists. Computational complexity becomes clearly more enormous in RCSG than in CSG. Because we need to calculate all optimal coalition structures formed by elements of $A \setminus A'$ to find the *k*-robust coalition structure.

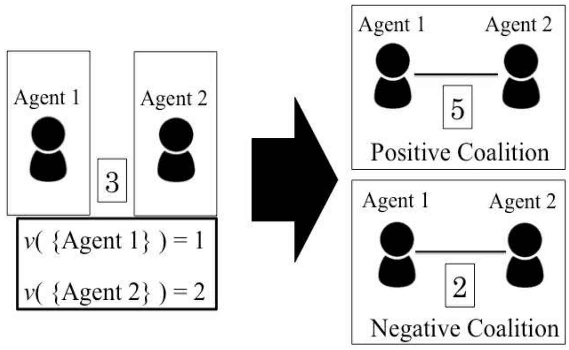

Fig. 2. Positive and negative coalition.

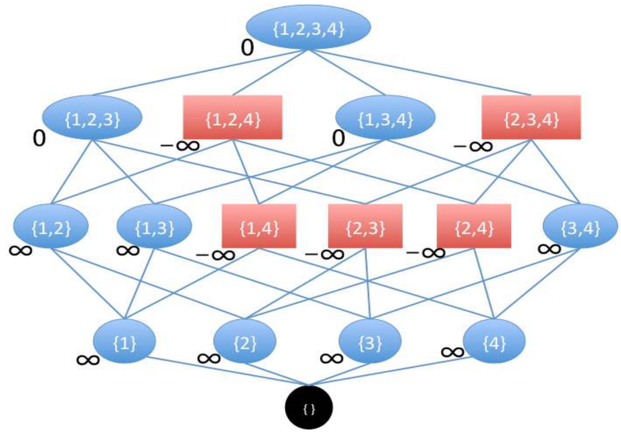

Fig. 3. Coalition lattice in case of Table I.

We need to consider all cases that any *k* agents leave *CS* to distinguish whether *CS* is the *k*-robust coalition structure or not. *CS* consists of one or more coalitions. Therefore at most *k* agents potentially leave one of the coalitions in RCSG. Then let *S'* be the subset of *S* where *S'* is formed by remaining agents except *k* agents in *S*. We should consider the utility of *S'*. In our research, we focus on the robustness of coalitions to find coalition structures with robustness efficiently. In this paper, the robustness of *S* is the property that *S'* is not a negative coalition if any agents leave *S*.

Let S_p be a positive coalition. To be a *k*-robust coalition structure, *CS* including S_p should keep the utility maximum if *k* agents leave *CS*. Therefore at most *k* agents may leave S_p. Let S_p' be a subset of S_p. And let CS' be the coalition structure including S_p'. S_p' should not be a negative coalition. Because $V(CS')$ is less than that includes coalitions which S_p' is dissolved instead of S_p'. Let $S_n (\subset S_p)$ be the negative coalition. S_n^* is calculated by (7). And the value of *k* is calculated by (8). Then we call S_p *k-L-robust* coalition. When *S* is a *k*-L-robust coalition, $S' (\subset S)$ is not a negative coalition if any *k* agents leave *S*.

Algorithm 1 Construct a Coalition Lattice

Require: $k \geq 0$
Ensure: Necessary parts of *CL*
$CL \leftarrow \{\}$
for $i = 0$ to $|A|$ **do**
  $CL \leftarrow CL \cup \{\langle i, GetS(i), +\infty \rangle\}$
end for
for $i = |A| + 1$ to $2^{|A|} - 1$ **do**
  $S \leftarrow GetS(i)$
  if S is positive **then**
    $CL \leftarrow CL \cup \{\langle i, S, Calculate(S) \rangle\}$
  else if S is negative **then**
    $CL \leftarrow CL \cup \{\langle i, S, -\infty \rangle\}$
  end if
end for
$CL \leftarrow \{\langle i, S, l \rangle | k \leq l, \langle i, S, l \rangle \in CL\}$

Algorithm 2 $l = Calculate(S)$ in Algorithm 1

Require: $|S| > 1$
Ensure: $l_{\min} = Calculate(S)$
$l_{\min} \leftarrow +\infty$
for all $\langle S_{child}, l_{child} \rangle \in Child(S)$ **do**
  if S_{child} is negative **then**
    $l' \leftarrow |S| - |S_{child}| - 1$
  else
    $l' \leftarrow l_{child} + |S| - |S_{child}|$
  end if
  if $l_{\min} > l'$ **then**
    $l_{\min} \leftarrow l'$
  end if
end for

We can find the optimal coalition structure in RCSG by calculating the robustness of all coalitions.

$$S_n^* = \arg\max_{S_n} |S_n| \qquad (7)$$

$$k = |S_p| - |S_n^*| - 1 \qquad (8)$$

5. Coalition Lattice

We need to clarify the inclusive relation of each coalition to calculate the robustness of coalitions. In our research, we propose *coalition lattice CL* that is a new data structure for the robustness of coalitions. On *CL*, a node is a coalition described in the characteristic function. And *CL* is the data structure that connected two nodes with inclusive relation by an arc.

5.1 Data Structure

We summarize the example of the characteristic function in case that the number of agents is four ($A = \{a_1, a_2, a_3, a_4\}$) in Table I. And Fig. 3 is an image of *CL* in case of Table I. *CL* in Fig. 3 shows the inclusive relations of all coalitions in case that the number of agents is four in Table I. And *CL* is assumed that the upper limit is the whole coalition and that the lower limit is the empty set. Therefore, *CL* is divided a floor with the number of the agents in the coalitions. In Fig. 3, a circular node shows a positive coalition and a rectangle node shows a negative coalition. The left lower value of each node represents the number digitized the robustness of the node. For example, if the value of a node is zero, then the coalition is 0-L-robust. However, the robustness of an empty coalition is not given because the empty set has no value in the characteristic function. We can calculate the robustness of each coalition easily by constructing *CL* such as Fig. 3 because we can obtain the inclusive relation of each coalition.

We describe a method to calculate the robustness of each coalition, i.e. value *k* of *k*-L-robust, on *CL*. Let *a* be an agent. And let *CS* be the optimal coalition structure where *CS* includes the coalition $\{a\}$. When *a* leaves *CS*, $V(CS \setminus \{a\})$ is kept maximum because other coalitions in *CS* are not affected by the secession of *a* at all. Therefore the robustness of the coalition formed by a single agent is infinity, i.e. $k = \infty$. In addition, the robustness of the negative coalition is $k = -\infty$ as a matter of convenience. Any negative coalition should not exist in *CS*. When negative coalition S_n exists in the child node that is the subset of positive coalition S_p on *CL*, the robustness of S_p is calculated by (8) where $S_n = S_n^*$. We assume the case that there is not negative coalition in the child node of S_p. Let $k_{\min}$ be the minimum value of *k* in the child node of S_p. And let $S_{\min}$ be a k_{min}-L-robust coalition. The difference of the number of agents between S_p and $S_{\min}$ is *M*, i.e., $M=|S_p|-|S_{\min}|$. Even if *M* agents leave S_p, S_p without *M* agents is not a negative coalition. Therefore, we can calculate the robustness of S_p by $k = k_{\min} + M$ when there is not negative coalition in the child node of S_p.

5.2 Algorithm

We describe an algorithm to construct a coalition lattice in Algorithm 1. Let *CL* be a coalition lattice, *A* be a set of agents, *S* be a coalition, and *l* be the numerical value of the robustness of *S*. Algorithm 1 returns a coalition lattice with robustness *k*, a set of agent *A*, and characteristic function *v*. *CL* is a set of $\langle i, S, l \rangle$. $\forall S \subset A$ are sorted ascending order

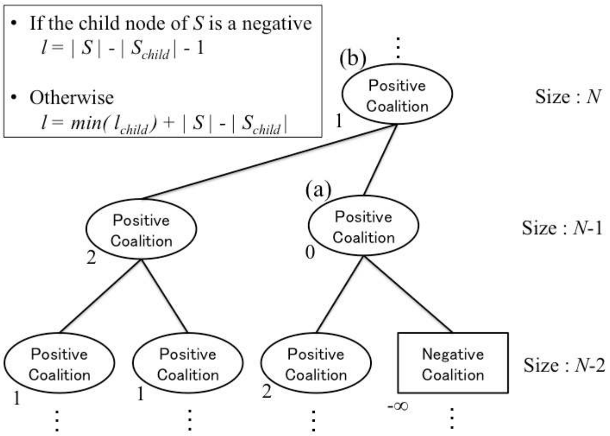

Fig. 4. Calculation of the robustness of coalitions on *CL*

by $|S|$. And index *i* corresponds to all coalitions.

CL is initialized in line 1 to 4 in Algorithm 1. The function $GetS(i)$ returns *S* corresponding to *i*. The value of *i* from 1 to $|A|$ represents the case where *S* is formed by a single agent. At line 3, Algorithm 1 adds the node with the robustness of *S* to *CL* as $+\infty$.

For the coalition with more than two agents, *CL* is constructed in bottom up (as described in line 5 to 12 in Algorithm 1). For all coalitions, Algorithm 1 distinguishes *S* whether it is a positive or negative coalition, and adds each node to *CL* in line 7 to 11. If *S* is a positive coalition, Algorithm 1 adds the node to *CL* after calculating the robustness of *S* by the function $Calculate(S)$ shown in Algorithm 2. $Calculate(S)$ returns the robustness of *S* by considering the child nodes of *S*. If *S* is a negative coalition, Algorithm 1 adds the node with the robustness of *S* to *CL* as $-\infty$ at line 10.

The required coalitions to find the *k*-robust coalition structure are not all coalitions on *CL*. To form the *k*-robust coalition structure, we should use only the coalitions that the robustness is greater than *k*. At line 13, Algorithm 1 prunes the extra part of *CL* by using *k*.

Fig. 4 shows the calculation of the robustness on a coalition lattice. The function $Calculate(S)$ in Algorithm 2 takes (a) (the positive coalition in Fig. 4) as input. The child nodes of (a) are the positive coalition and the negative coalition. Let l_a be the robustness of (a). And let S_{child} be the child node of (a). In this case, l_a is calculated by $l_a = |S| - |S_{child}| - 1$. Therefore l_a is $(N-1)-(N-2)-1 = 0$. Next, $Calculate(S)$ in Algorithm 2 takes (b) (the positive coalition in Fig. 4) as input. The child nodes of (b) are two positive coalitions. Let l_b be the robustness of (b). And let S_{child} be the child node of (b). In this case, l_b is calculated by $l_b = \min(l_{child}) + |S| - |S_{child}|$. The function $\min(l_{child})$ returns the smallest value of the robustness among the child nodes. In Fig. 4, the robustness of the child nodes are 2 and 0. Therefore l_b is $0 + N - (N-1) = 1$.

TABLE II: All coalition structures. (n = 4)

CS	$V(CS)$	CS	$V(CS)$
$\{\{a_1\}\{a_2\}\{a_3\}\{a_4\}\}$	13	$\{\{a_1, a_4\}\{a_2, a_3\}\}$	8
$\{\{a_1, a_2\}\{a_3\}\{a_4\}\}$	15	$\{\{a_1\}\{a_4\}\{a_2, a_3\}\}$	10
$\{\{a_1, a_2\}\{a_3, a_4\}\}$	17	$\{\{a_1, a_2, a_3\}\{a_4\}\}$	14
$\{\{a_1\}\{a_2\}\{a_3, a_4\}\}$	15	$\{\{a_1, a_2, a_4\}\{a_3\}\}$	10
$\{\{a_1, a_3\}\{a_2\}\{a_4\}\}$	14	$\{\{a_1, a_3, a_4\}\{a_2\}\}$	15
$\{\{a_1, a_3\}\{a_2, a_4\}\}$	9	$\{\{a_2, a_3, a_4\}\{a_1\}\}$	9
$\{\{a_1\}\{a_3\}\{a_2, a_4\}\}$	8	$\{\{a_1, a_2, a_3, a_4\}\}$	19
$\{\{a_1, a_4\}\{a_2\}\{a_3\}\}$	12		

We show a method to use *CL*. We enumerated all coalition structures in case that the number of agents is 4 in Table II. And we calculated the social surpluses of all coalition structures by using Table I. The greatest value of the social surplus is 19 in Table II. Therefore the coalition structure $\{\{a_1, a_2, a_3, a_4\}\}$ is the optimal and 0-robust coalition structure in RCSG problem and the optimal coalition structure in CSG problem. We find the optimal and 1-robust coalition structure. In Fig. 3, the coalitions with the robustness that is greater than 1 are $\{a_1, a_2\}$, $\{a_1, a_3\}$, $\{a_3, a_4\}$, $\{a_1\}$, $\{a_2\}$, $\{a_3\}$, and $\{a_4\}$. We can use these coalitions to find the optimal and 1-robust coalition structure, then we check the social surpluses of coalition structures that are $\{\{a_1\}\{a_2\}\{a_3\}\{a_4\}\}$, $\{\{a_1, a_2\}\{a_3\}\{a_4\}\}$, $\{\{a_1, a_2\}\{a_3, a_4\}\}$, $\{\{a_1\}\{a_2\}\{a_3, a_4\}\}$, $\{\{a_1, a_3\}\{a_2\}\{a_4\}\}$. The social surplus of $\{\{a_1, a_2\}\{a_3, a_4\}\}$ is greatest, and the coalition structure $\{\{a_1, a_2\}\{a_3, a_4\}\}$ is the optimal and 1-robust coalition structure. The coalition structure is also the optimal and 2-robust coalition structure because the robustness of $\{a_1, a_2\}$, $\{a_1, a_3\}$, $\{a_3, a_4\}$, $\{a_1\}$, $\{a_2\}$, $\{a_3\}$, and $\{a_4\}$ are greater than two.

In RCSG, our goal is to find Pareto optimal and k-robust coalition structures ($0 \le k \le |A| - 2$). Let CS_0 be the 0-robust coalition structure, i.e. $\{\{a_1, a_2, a_3, a_4\}\}$. And let CS_1 be the 1-robust coalition structure, i.e. $\{\{a_1, a_2\}\{a_3, a_4\}\}$. And let CS_2 be the 2-robust coalition structure, i.e.$\{\{a_1, a_2\}\{a_3, a_4\}\}$. We check the domination of them by comparing $V(CS_0)$, $V(CS_1)$, and $V(CS_2)$. CS_0 is the Pareto optimal and 0-robust coalition structure because no coalition structures dominated CS_0. $V(CS_2)$ is equal to or more than $V(CS_1)$. CS_1 is not the Pareto optimal and 1-robust coalition structure because CS_2 dominates CS_1. CS_2 is the Pareto optimal and 2-robust coalition structure because no coalition structure dominated CS_2. In RCSG, We can find Pareto and k-robust coalition structure by using *CL*.

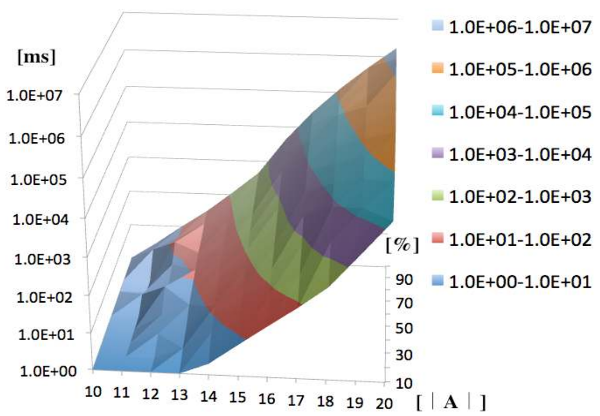

Fig. 5. Execution time of constructing *CL*

6. Evaluation

We inspected the execution time for constructing *CL* by using Algorithm 1 and Algorithm 2. We gathered the processing time to construct *CL* depending on the number of the agents and the non-linear degree.

The result is described in Fig. 5. The range of $|A|$ is from 10 to 20. The range of the non-linear degree *deg* is from 10% to 100%. Each coalition is classified into positive or negative coalitions when *deg* is 100%. When $|A|$ is less than 15, Algorithm 1 calculated in around one second. However, when $|A|$ is 20 and *deg* is 100%, the calculation took about 29 minutes. It is a matter of course, but the execution time becomes enormous when there are a lot of coalitions. In our research, the number of coalitions to be considered will be reduced by defining the sparse characteristic function based on partial linear model. If the non-linear degree of the sparse characteristic function is small enough, Algorithm 1 constructs *CL* in around 30 minutes in the case that $|A|$ is more than 30.

The number of possible coalition structures increases exponentially with the number of agents. CSG problem is NP-hard. In RCSG, we typically need to calculate all cases that k agents leave the coalition structure to distinguish whether the coalition structure is the k-robust coalition structure or not. Finding all Pareto optimal and k-robust coalition structures takes an enormous amount of time. We typically have to calculate NP problem numerous times to solve RCSG problem, i.e. the computational complexity will be NP^{NP}.

7. Conclusion

Robust coalition structure generation (RCSG) problem is a two-objective combinatorial optimization problem for multiagent systems considering leaving coalitions. An RCSG problem solver should solve an exponential number of CSG problems. We developed on a new date structure and algorithm to solve the problem efficiently.

Our contribution is the coalition lattice, a novel data structure for translating an RCSG problem to a CSG. It means that an RCSG problem is solvable by using an existing CSG algorithm. Our evaluation showed that our method is feasible because the construction time of the coalition lattice for 20 agents is about 29 minutes at worst.

References

(1) T. Sandholm, K. Larson, M. Andersson, O. Shehory, F. Tohmé, ”Coalition structure generation with worst case guarantees,” Artificial Intelligence, 111(1-2): 209–238, 1999.

(2) Y. Bachrach, P. Kohli, V. Kolmogorov, and M. Zadimoghaddam, “Optimal Coalition Structure Generation in Cooperative Graph Games,” In AAAI, pp.81-87, 2013.

(3) D. Y. Yeh, “A dynamic programming approach to the complete set partitioning problem,” BIT Numerical Mathematics Volume 26, Issue 4, pp.467-474, 1986.

(4) T. Rahwan, S. D. Ramchurn, V. D. Dang, A. Giovannucci, and N. R. Jennings, “Anytime Optimal Coalition Structure Generation,” In Proceedings of the 22nd National Conference on Artificial Intelligence, 2007.

(5) T. Rahwan and N. R. Jennings, “Coalition Structure Generation: Dynamic Programming Meets Anytime Optimization,” In AAAI, pp.156-161, 2008.

(6) T. Michalak, J. Sroka, T. Rahwan, M. Wooldridge, P. McBurney, and N. R. Jennings, “A Distributed Algorithm for Any Coalition Structure Generation,” In AAMAS, pp.1007-1014.

(7) T. Okimoto, N. Schwind, M. Clement, and K. Inoue, “Robust Team Formation Problem,” The Proceedings of Joint Agent Workshops and Symposium, pp.341-344, 2014 (in Japanese).

(8) T. Okimoto, N. Schwind, and K. Inoue, “A Study for Robust Coalition Structure Generation Problem,” Japanese Society for Artificial Intelligence, 3A3-3, 2014 (in Japanese).

(9) N. Ohta, V. Conitzer, R. Ichimura, Y. Sakurai, A. Iwasaki, and M. Yokoo, “Coalition Structure Generation Utilizing Compact Characteristic Function Representations,” Principles and Practice of Constraint Programming, pp.623-638, 2009.