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COALITIONAL APPROACHES TO COLLUSIVE AGREEMENTS IN OLIGOPOLY GAMES

SERGIO CURRARINI AND MARCO A. MARINI

Abstract. In this paper we review a number of coalitional solution concepts for the analysis of the stability of cartels and mergers under oligopoly. We show that, although so far the industrial organization and the cooperative game-theoretic literature have proceeded somehow independently on this topic, the two approaches are highly inter-connected. We first consider the basic problem of the stability of the whole industry association of firms under oligopoly and, for this purpose, we introduce the concept of core in games with externalities. We show that different assumptions on the behaviour as well as on the timing of the coalitions of firms yield very different results on the set of allocations which are core-stable. We then consider the stability of associations of firms organized in coalition structures different from the grand coalition. To this end, various coalition formation games recently introduced by the so called *endogenous coalition formation* literature are critically reviewed. Again, different assumptions concerning the timing and the behaviout of firms are shown to yield a wide range of different results.

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1. Introduction

Since the seminal work by Salant *et al.* (1983) on merger profitability, there has been a large interest in the stability of collusive agreements between firms under oligopoly, as in the case of cartels and mergers (see, among the others, d'Aspremont *et al.* 1982, 1986, Deneckere et al. 1985, Donsimoni *et al.* 1986, Rajan, 1989 and Huck *et al.* 2005 for a survey). A relevant number of the initial works on this topic has mainly focussed on the conditions under which a collusive agreement within one group of firms can be viewed as stable when the remaining firms in the industry act either as price-takers (d'Aspremont et al., 1982, Donsimoni et al., 1986, among others) or as oligopolistic firms (see Shaffer, 1995). As in the traditional price-leadership model (Markham, 1951), in the above literature a group of dominant firms is assumed to behave as one Stackelberg leader, i.e., taking as given the reaction of the remaining firms in the fringe. Since in absence of synergies the cooperation within a cartel is formally equivalent to the outcome of a horizontal merger, many of the results of the horizontal merger literature (Salant *et al.*,1983, Deneckere and Davidson, 1985, Perry and Porter, 1985, Farrell and Shapiro, 1990, among others) also apply to the problem of cartel stability.[1] However, differently from the cartel literature, most of the works on horizontal mergers examines the profitability of mergers in (oligopolistic) markets in which a group of collusive firms and the fringe of competitors take their strategic decisions *simultaneously*.[2]

A common feature of both groups of contributions listed above is that the notion of stability usually adopted is one of *individual stability*: for a cartel (or merger) to be stable, no firm of the fringe must have an incentive to enter the cartel (*external stability*) and no firm of the cartel must possess an incentive to quit (*internal stability*). Recognizing the fact that this approach "...ignores the possibility that a group of players might jointly make themselves better off by leaving the cartel (Shaked, 1986)", later on some contributions have, in various ways, attempted to use a notion of *coalition stability* to approach the problem (see, for instance, d'Aspremont and Gabszewicz, 1986, Rajan, 1989, Zhao, 1997, Thoron, 1998). The major purpose of these works is mainly to check whether some imputations exist under which a collusive agreement signed by all firms in the industry is stable, that is, immune to deviations by every subcoalition of the firms in the industry. As in the horizontal mergers literature, the stability of an agreement is examined in a context in which a deviating coalition and the remaining firms of the industry act simultaneously. In such a literature, the *sequential* approach typical of the price-leadership model is thus lost.

It may be questioned if the defection of a group of firms from a cartel has to be viewed as happening before or at the same time the remaining firms take their action. Indeed, it is often the case that a coalition of firms deciding to leave the cartel and carry out its own collusive production, can choose such an action before its formation is publicly observed. In other terms, such a group can act as a Stackelberg leader with respect to the outside firms, that thus react to its action as followers. Clearly, the sequential structure is useful to describe only those situations in which a coalition of firms can precommit to a joint strategy

[1] This equivalence holds in particular if the firms in the cartel are assumed to sign a binding agreement on their joint prices or quantities.

[2] Other recent works on this topic also looks at the profitability of mergers under non linear demand (Fauli-Oller, 1997, Cheung, 1992), strategic delegation in mergers (Ziss, 2001, Gonzalez-Maestre et al. 2001), mergers under incomplete information (Amir et al. 2004), mergers and cartels with Stackelberg leaders and followers (Daughety, 1990, Huck et al., 2001, Escrihuela-Villar and Fauli-Oller, 2008).

expecting outside firms observing the effects of its action, and being left with no choice but optimally reacting to it.
In order to examine all these questions, we introduce in Section 2 the notion of *core of games with externalities* and thus apply it to check the stability of a merger or a cartel made by all firms in the industry. We show that while the *simultaneous* approach to the cartel formation described above corresponds to the gamma-core or delta-core of an oligopolistic game (see, for instance Chander & Tulkens, 1998), the *sequential* approach can be obtained by assuming a Stackelberg behaviour for all deviating coalitions (see Currarini & Marini, 2003). In this way, we are able to see that some classical results on merger stability contained, for instance, in Rajan (1989), can be easily extended. In particular, this author considers a linear and symmetric Cournot oligopoly with quadratic costs, and looks at the stability of cartels. In the case in which every deviation from a cartel implies that the remaining firms stick togheter, the author is able to prove that, for $n = 2$, the game is convex and the core is non empty, while for $n \geq 3$, the core is empty. Moreover, for $n = 3$ and $n = 4$, the only stable coalition structure is that in which every firms act as singletons. However, when the deviation of a firm from a cartel implies that remaining firms split up in singletons, for $n \geq 3$, the firms never chose to stay separate and for $n = 3$ and $n = 4$,the core is non empty. In the terminology of cooperative games, Rajan (1989) makes use of the γ-core. We will show here that the set of allocations in the γ-core strictly contains those included in a sequential solution concept (here denoted λ-core). The possibility that the remaining firms can observe the other firms deviating from an agreement represents in such case a refinement of the set of acceptable allocations of the joint surplus. Moreover, we prove that in the linear Cournot model, the λ-core comprehends a unique allocation.

Obviously, the formation of collusive structures which are different from the whole association of firms in the industry may also represent a serious options for firms in oligopoly. The recent developments in the theory of endogenous coalition formation have, in this respect, provided a new set of game-theoretic tools to study this problem (Hurt and Kurz, 1983, Bloch, 1995, Ray and Vohra, 1997, Shin and Yi, 1997 and also Yi, 2003, Bloch 1997, 2003, Marini, 2008, for surveys of this literature). In all these works, the cooperation (and, hence also the formation of an association of firms) is modelled as a two stage process: at the first stage players form coalitions, while at the second stage formed coalitions interact in a well defined strategic setting. This process is formally described by a coalition formation game, in which a given rule of coalition formation maps players' intentions to form coalitions into a well defined coalition structure, which, in turn, determines the equilibrium strategies chosen by players at the second stage. A basic difference among the various models lies in the timing assumed for the coalition formation game, which can either be simultaneous (Hurt & Kurz, 1983, Ray & Vohra,1997, Yi, 1997) or sequential (Bloch, 1994, Ray & Vohra, 1999). As far as the application to associations of firms is concerned, Bloch, 1995 shows that in a linear Cournot oligopoly firms may form in equilibrium an asymmetric association of firms, comprising aproximately three-quarters of the firms, while the remaining firms stay as singletotns. We will show that this result is related to the well known Salant's *et al.*, 1983 result on merger profitability. Finally, Ray and Vohra, 1997 show that there may also be a cyclical pattern in the formation of associations in a linear Cournot oligopoly. By using a recursive concept of solution - denoted *equilibrium binding agreement* - the authors prove that, for $n = 2$, there is a stable merger, while, for $3 \leq n \leq 8$, any merger is unstable. Finally, for $n = 9$, the grand coalition forms and is stable. In Section 3 we will review some of these models and show that, when applied to the formation of collusive agreements, their results vary extensively according to the different assumptions made on the timing and the behaviour of firms.

The paper is organized as follows. The next section will be devoted to introduce a basic quantity oligopoly game adopted as underlying strategic form game in all coalitional equilibrium concepts introduced later on in the paper. Section 3 is concerned with the formation of the grand coalition of firms and, for this reason, it reviews some classical coalitional concepts as the core, the strong Nash equilibrium and some variations of these two key solution concepts. Section 4 considers the stability of partial cartels and mergers and reviews some relevant approaches to the endogenous coalition formation problem. Section 5 briefly concludes.

2. A Quantity Oligopoly Game

Let the profit function of every firm $i \in N = \{1, 2, ..., n\}$ be defined as

$$\pi_i(y, y_i) = p(y)\, y_i - C_i(y_i),$$

where y_i is the output of each firm, $y = \sum_{i=1}^{n} y_i$ the total industry output, $p(y)$ the inverse demand function and $C_i(y_i)$ the cost function of every firm. Let also $C_i(.) = C_j(.)$, for every i,j in N. Thus, we can represent the Cournot oligopoly through the following strategic form game, $G = (\{Y_i, \pi_i\}_{i \in N}, \{Y_S\}_{S \subseteq N})$. In such a game the set of players is represented by the set of firms N and every firm's strategy set is defined as

$$Y_i = \{y_i \in R_+ : y_i \leq \overline{y}_i\}$$

where $\overline{y}_i$ is a capacity constraint. Let also players' preferences be linear in profit and, for every coalition of firms $S \subseteq N$, let the strategy set be represented by:

$$Y_S \equiv Y_S \times T_S$$

where $Y_S = \prod_S Y_i$ and $T_S = (t_1, ..., t_s)$ is a vector of transfers such that $\sum_{i \in S} t_i = 0$.[3]

In what follows we make the following standard assumptions:

A.1 The function $\pi_i(.)$ and $C_i(.)$ are twice continously differentiable for every $i = 1, .., n$;

A.2 For every $i \in N$, the capacity constraint $\overline{y}_i < \infty$ determines the maximum production level;

A.3 For every $i \in N$, $p''(.)\, y_i + p'(.) < 0$ and $p'(.) - C_i'' < 0$.

Definition 1. *A (Cournot) Nash equilibrium of G is a strategy profile y^* such that, for all $i \in N$, $y_i^* \in Y_i$ and, for all $y_i \in Y_i$, $\pi_i(y^*) \geq \pi_i\left(y_i, y_{-i}^*\right)$.*

Proposition 1. *There exists a unique (Nash) equilibrium of the game G.*

Proof. By assumptions A.1, A.2 and A.3 every player's payoff functions is continuous in the strategy profile y_N and strictly concave on y_i. Strategy sets are non empty, compact and convex ($y_i \leq \overline{y}_i < \infty$), so that existence of a Nash equilibrium follows. Uniqueness is proved as follows. By assumption A.3, the function $\Phi(y_i, y) \equiv p'(y)\, y_i + p(y) - C_i'(y_i)$ is decreasing

[3] Since we limit ourselves to consider game in transferable utility, we want every strategy profile to define exactly the payoff of a coalition of firms. To this purpose we include the transfer in the definition of every coalition of firms strategy set.

both in y_i and y. In fact, $\frac{\partial F(y_i,y)}{\partial y_i} = p' - C_i'' < 0$ and $\frac{\partial \Phi(y_i,y)}{\partial y} = p'' y_i + p' < 0$ Suppose now that there exist two Nash Equilibria $(y_1^1, ..., y_n^1)$ and $(y_1^2, ..., y_n^2)$ of G. Equilibrium conditions require that for each i

$$\Phi\left(y_i^1, y^1\right) = 0$$

and

$$\Phi\left(y_i^2, y^2\right) = 0$$

Thus, if $y_i^1 > y_i^2$, then $y^2 > y^1$. This in turns implies that $y_j^1 > y_j^2$ for all j, contradicting the fact that $y^2 > y^1$. Therefore, it must be that $y_i^1 = y_i^2$ for all $i \in N$. □

3. Grand Coalition Stability

In this section we introduce the concept of core in games with externalities in order to check the stability of collusive agreements among firms in an oligopolistic market.

3.1. **Cooperative Games with Externalities.** Since von Neumann and Morgenstern (1944), a wide number of works have developed solution concepts specific to games with coalitions of players. This literature, known as *cooperative games* literature, made initially a predominant use of the characteristic function to represent the worth of a coalition of players.

Definition 2. *A cooperative game with transferable utility (TU cooperative game) can be defined as a pair (N, v), where $N = \{1, 2, ..i, ..N\}$ is a finite set of players and $v : 2^N \to R_+$ is a mapping (characteristic function) assigning a value or worth to every feasible coalition $S \in 2^N$.*[4]

The value $v(S)$ can be interpreted as the maximal aggregate amount of utility members of coalition S can achieve by coordinating their strategies. However, in strategic environments players' payoffs are defined on the strategies of all players and the worth of a group of players cannot be defined independently of the groups (or coalitions) formed by external players $(N \backslash S)$.[5] Hence, to obtain $v(S)$ from a strategic situation we need first to define an underlying strategic form game. In our case, the strategic form game will be represented by a standard Cournot oligopoly game.

3.2. **α- and β-characteristic functions.** The concepts of α- and β- core, formally studied by Aumann (1967), are based on von Neumann and Morgenstern's (1944) early proposal of representing the worth of a coalition as the minmax or maxmin aggregate payoff that it can guarantee its members in the underlying strategic form game. Accordingly, the characteristic function $v(S)$ in games with externalities can be obtained assuming that outside firms act to minimize the payoff of every deviating coalition of firms $S \subset N$. In this minimax formulation, if members of S move second, the obtained characteristic function,

$$v_\beta(S) = \min_{y_{N \backslash S}} \max_{y_S} \textstyle\sum_{i \in S} u_i(y_S, y_{N \backslash S}), \tag{3.1}$$

[4]Here we mainly deal with games with transferable utility. In games without transferable utility, the worth of a coalition associates with each coalition a players' utility frontier (a vector of utilities).

[5]See also the discussion contained in Gambarelli (2007).

denoted β-characteristic function, represents what firms in S *cannot be prevented from getting*. Alternatively, if members of S move first, we have

$$v_\alpha(S) = \max_{y_S} \min_{y_{N\setminus S}} \textstyle\sum_{i\in S} u_i(y_S, y_{N\setminus S}) \tag{3.2}$$

denoted α-characteristic function, which represents what firms in S *can guarantee themselves*, when they expect a retaliatory behaviour from the complement coalition $N\setminus S$.[6]

When the underlying strategic form game G is zero-sum, (1) and (2) coincide. In non-zero sum games they can differ and, usually, $v_\alpha(S) < v_\beta(S)$ for all $S \subset N$.

However, and characteristic functions express an irrational behaviour of coalitions of firms, acting as if they expected their rivals to minimize their payoff. Although appealing because immune from any *ad hoc* assumption on the reaction of the outside firms (indeed, their minimizing behavior is here not meant to represent the expectation of S but rather as a mathematical way to determine the lower bound of S's aggregate payoff), still this approach has important drawbacks: deviating coalitions are too heavily penalized, while outside firms often end up bearing an extremely high cost in their attempt to hurt deviators. Moreover,the little profitability of coalitional objections usually yield very large set of solutions (e.g.,large cores).

3.3. **Simultaneous Interaction among Coalitions: the γ-characteristic Function.** Another way to define the characteristic function in games with externalities is to assume that in the event of a deviation from N, a coalition S plays à la Nash with the remaining firms.[7] Similarly to the Γ coalition formation game introduced by Hart and Kurz (1983), the γ-approach implicitly restricts the dynamic structure of deviations and reactions to the *coalition formation* stage, and treats the strategy choice stage as a simultaneous game *given* the coalition structure induced by the deviation. In other terms, in a first stage a coalition of firms forms and remaining firms react splitting up as singletons; in a second stage, optimal strategies are simultaneously chosen both by the deviating coalition of firms and by the fringe of excluded firms. Consequently, the strategy profile induced by the deviation of a coalition $S \subset N$ is precisely the Cournot equilibrium among S and each individual player in $N\setminus S$. The worth of a cartel of firms S under the γ assumption is thus its aggregate payoff in the Cournot equilibrium between S and the outside firms acting as singletons. This is the setup *implicitly* underlying papers like Salant et al (1983) and Rajan (1989) to analyse the profitability of firms' collusion. Thus, the characteristic function $v_\gamma(S)$ can be defined for all $S \subseteq N$ as:

$$v_\gamma(S) = \sum_{i\in S} \pi_i\left(y_S^*, \{y_j^*\}_{j\in N\setminus S}\right) \tag{3.3}$$

where

$$y_S^* = \underset{y_S\in Y_S}{\arg\max} \sum_{i\in S} \pi_i\left(y_S, \{y_j^*\}_{j\in N\setminus S}\right) \tag{3.4}$$

[6] Note that firms outside S are treated as one coalition, so the implicit assumption here is that firms in $N\setminus S$ stick together after S departure from the grand coalition N.

[7] This way to define the worth of a coalition in as a noncooperative equilibrium payoff of a game played between coalitions was firstly proposed by Ichiishi (1983).

and $\forall j \in N \backslash S$,

$$y_j^* = \underset{y_j \in Y_j}{\arg\max} \pi_j \left(y_S^*, \{y_k^*\}_{k \in (N \backslash S) \backslash \{j\}}, y_j \right). \tag{3.5}$$

where $y^* = (y_1^*, ..., y_n^*)$ is characterized by the following n first order conditions:

$$p\left(y^*\right) + p'\left(y^*\right) \sum_{i \in S} y_i = C_i'\left(y_i^*\right) \quad \text{for all } i \in S \tag{3.6}$$

$$p\left(y^*\right) + p'\left(y^*\right) y_j^*\left(y_S^*\right) = C_j'\left(y_j^*\right), \text{ for all } j \in N \backslash S. \tag{3.7}$$

Moreover,

$$v_\delta(S) = \sum_{i \in S} \pi_i \left(y_S^*, y_{N \backslash S}^* \right) \tag{3.8}$$

where,

$$y_S^* = \arg \max_{y_S \in Y_S} \sum_{i \in S} \pi_i \left(y_S, y_{N \backslash S}^* \right)$$
$$y_{N \backslash S}^* = \arg \max_{y_{N \backslash S} \in Y_{N \backslash S}} \sum_{j \in N \backslash S} \pi_j \left(y_S^*, y_{N \backslash S} \right).$$

In both cases, for (3.3) and (3.8) to be well defined, the Nash equilibrium of the strategic form game played among coalitions must be unique. Usually, $v_\alpha(S) < v_\beta(S) < v_\delta(S)$ for all $S \subset N$.

3.4. **Sequential Interactions among Coalitions: the λ-characteristic Function.** It is also conceivable to modify the γ- or δ-assumption (coalitions playing simultaneously *à la* Nash in the event of a deviation from the grand coalition) reintroducing the temporal structure typical of the α and β-assumptions.[8] When a deviating coalition S moves first under the γ-assumption, the members of S choose a coordinated strategy as leaders, thus anticipating the reaction of the players in $N \backslash S$, who simultaneously choose their best response as singletons. The strategy profile associated with the deviation of a coalition S is thus the Stackelberg equilibrium of the game in which S is the leader and the players in $N \backslash S$ are, individually, the followers. We denote this strategy profile as a *Stackelberg equilibrium with respect to* S. Formally, this is the strategy profile $\widetilde{y}\left(S\right) = (\widetilde{y}_S, y_j(\widetilde{y}_S))$ such that

$$\widetilde{y}_S = \underset{\widetilde{y} \in Y_S}{\arg\max} \sum_{i \in S} \pi_i \left(y_S, \{y_j(y_S)\}_{j \in N \backslash S} \right) \tag{3.9}$$

and, $\forall j \in N \backslash S$,

$$y_j(y_S) = \underset{y_j \in Y_j}{\arg\max}\, \pi_j \left(y_S, \{y_k(y_S)\}_{k \in (N \backslash S) \backslash \{j\}},\ y_j \right). \tag{3.10}$$

We now establish conditions under which there exists a Stackelberg equilibrium with respect to S. For every coalition of firms $S \subset N$ and strategy profile $y_S \in Y_S$, let $G\left(N \backslash S, y_S\right)$ denote the restriction of the game G to the set of firms $N \backslash S$, given the strategy profile y_S.

Proposition 2. *For every coalition of firms $S \subset N$ there exists a Stackelberg equilibrium with respect to S.*

[8] See Currarini and Marini (2003, 2004) for more details.

Proof. By condition (3.10) and proposition 1, the strategy profile $\{y_j(y_S)\}_{j\in N\setminus S}$ is the unique Nash equilibrium of $G(N\setminus S, y_S)$. By the closedness of the Nash equilibrium correspondence (see, for instance, Fudenberg and Tirole (1991), pag.30), members of S maximize a continuous function over a compact set (assumption 2); thus, by Weiestrass Theorem, a maximum exists. $\square$

Note that condition (3.9) implies that in every Stackelberg equilibrium with respect to S the aggregate payoff of S is the same. We thus able to define the joint payoff (or worth) of every coalition of firms $v_\lambda(S)$ in the sequential case as uniquely defined as follows:

$$v_\lambda(S) = \sum_{i\in S} \pi_i \left(\widetilde{y}_S, \{y_j(\widetilde{y}_S)\}_{j\in N\setminus S} \right) \tag{3.11}$$

where $(\widetilde{y}_S, y_j(\widetilde{y}_S))$ is a Stackelberg equilibrium with respect to S and the vector $(\tilde{y}_1, ..., \tilde{y}_n)$ is fully characterized by the following n first order conditions:

$$p(\tilde{y}) + p'(\tilde{y})\left(1 + (n-s)g(\tilde{y}_S)\right)\sum_{i\in S}\tilde{y}_i = C_i'(\tilde{y}_i) \quad \text{for all } i \in S \tag{3.12}$$

$$p(\tilde{y}) + p'(\tilde{y})\, y_j(\tilde{y}_S) = C_j'(y_j(\tilde{y}_S)), \quad \text{for all } j \in N\setminus S. \tag{3.13}$$

where $g(\sum_{i\in S}\tilde{y}_i)$ is the Cournot Equilibrium strategy of each player in the game $\Gamma_c(N\setminus S, \tilde{y}_S)$.

Obviously, $v_\lambda(S) \geq v_\gamma(S)$. In a similar way, the γ-assumption can be modified by assuming that a deviating coalition S plays as follower against all remaining players in $N\setminus S$ acting as singleton leaders. Obviously, the same can be done under the δ-assumption.

3.5. **The Core in Games with Externalities.** We can test the various conversions of $v(S)$ introduced above by examining the different predictions obtained using the *core* of (N, v).

We first define an imputation for (N, v) as a vector $\mathbf{z} \in \mathcal{R}^n_+$ such that $\sum_{i\in N} z_i \leq v(N)$ (feasibility) and $z_i \geq v(\{i\})$ (individual rationality) for all $i \in N$.

Definition 3. *The core of a TU cooperative game (N, v) is the set of all imputations $\mathbf{z} \in R^n_+$ such that $\sum_{i\in S} z_i \geq v(S)$ for all $S \subseteq N$.*

3.6. **Some Results in a Linear Oligopoly.** We first introduce a linear oligopoly, *i.e.*, the case in which $p(y) = a - by$ and, for every firm $i \in N$, $C_i(y_i) = cy_i$. For every firm $i \in N$, let the cost function be:

$$C_i(y_i) = cy_i$$

The constraints on the parameters are:

$$a > c \geq 0 \text{ and } b > 0.$$

3.6.1. *α- and β-core.* Under the α- and β-assumptions, if either one single firm or a group of firms leave the grand coalition N, the remaining firms will play a minimizing strategy in such a way that, for every $S \subset N$, $v_\alpha(S) = v_\beta(S) = 0$. In this case, the core coincides with all Pareto-efficient imputations. The predictive power of the α- and β-core is thus minimal for the oligopoly games.

3.6.2. *The γ-core.* According to definition (3.3) the worth of a group of firms S is given by:

$$v_\gamma(S) = \sum_{i\in S}\left[p\left(y_S^*, y_{-S}^*\right)y_i^* - C_i\left(y_i^*\right)\right]. \tag{3.14}$$

In the linear case introduced above, this is equivalent to:

$$v_\gamma(S) = \max_{y_S} \pi_S\left(y_S, (n-s)y_j\right) = (a - by)\, y_S - c y_S$$

where, by the symmetry of firms, $y = sy_i + (n-s)y_j$.

The F.O.C. for coalition S is:

$$a - 2by_S - b(n-s)y_j - c = 0$$

from which, the best-reply function is:

$$y_S((n-s)y_j) = \frac{a - c - b(n-s)y_j}{2b}. \tag{3.15}$$

Note that, if we consider separately the FOC for every $i \in S$, we obtain the following best-reply function:

$$y_i\left((n-s)y_j\right) = \frac{a - c - b(n-s)y_j}{2bs} \tag{3.16}$$

and the analysis proceeds as shown below by summing up every i 's best-reply.

Every $j \in N\backslash S$ aims at maximizing:

$$\pi_j\left(y_j, (n-s-1)\underset{r\in(N\backslash S)\backslash j}{y_r}, y_S\right) = (a - by_j - by_S - b(n-s-1)y_r)y_j - cy_j$$

with F.O.Cs, for every $j \in N\backslash S$,

$$a - 2by_j - b\left(n-s-1\right)\underset{\substack{r\neq j\\ r\in N\backslash S}}{y_r} - by_s - c = 0.$$

By symmetry, every j's best -reply can be written as:

$$y_j\left(y_S\right) = \frac{a - by_S - c}{b\left(n-s+1\right)} \tag{3.17}$$

From the two best-replies (3.15) and (3.16) we get:

$$y_j^* = \frac{a-c}{b(n-s+2)}$$

and, similarly:

$$y_S^* = \frac{a-c}{b\left(n-s+2\right)}.$$

Now, in order to obtain $v_\gamma(S)$, we first compute the equilibrium price:

$$p\left(y^{*}\right)=a-by_{S}^{*}-b(n-s)y_{j}^{*},$$

that is,

$$p\left(y^{*}\right)=\frac{a+\left(n-s+1\right)c}{\left(n-s+2\right)},$$

and then,

$$v_{\gamma}\left(S\right)=\sum_{i\in S}\pi_{i}\left(y^{*}\right)=\pi_{S}=p\left(y^{*}\right)y_{S}^{*}-cy_{S}^{*}$$

that can be written as:

$$v_{\gamma}\left(S\right)=\frac{\left(a-c\right)^{2}}{b(n-s+2)^{2}}.$$

Note that, for $s=n$,

$$v_{\gamma}\left(N\right)=\frac{\left(a-c\right)^{2}}{4b}.$$

Proposition 3. *Under the linear quantity oligopoly game, the γ-core is non empty and strictly includes the equal split allocation.*

Proof. We know from (3.3) that

$$v_{\gamma}\left(N\right)=\frac{\left(a-c\right)^{2}}{4b}$$

and

$$v_{\gamma}\left(S\right)=\frac{\left(a-c\right)^{2}}{b\left(n-s+2\right)^{2}}.$$

Without loss of generality let us normalize $\frac{(a-c)^2}{b}=1$, so that the equal-split allocation gives to each player in N a payoff of $\frac{v_\gamma(N)}{|N|}=\frac{1}{4n}$ and $v_{\gamma}\left(S\right)=\frac{1}{(n-s+2)^2}$.Consider now the equal split allocation for a coalition of firms S, $\frac{v_\gamma(S)}{|S|}=\frac{1}{s(n-s+2)^2}$. Whatever distribution of the worth $v_{\gamma}\left(S\right)$ may be chosen by S, at least one player in S must get at most a payoff equal to $\frac{1}{s(n-s+2)^2}$. This implies that coalition S improves upon the equal split allocation for N if and only if:

$$\frac{1}{s\left(n-s+2\right)^{2}}>\frac{1}{4n}.$$

Straightforward calculations show that the above inequality is satisfied respectively for:

$$\begin{aligned}
&s>n\\
&s<2+\frac{n-\sqrt{n^{2}+8n}}{2}<1\\
&s>2+\frac{n+\sqrt{n^{2}+8n}}{2}>n
\end{aligned}$$

and hence, it is never satisfied for $1<s\leq n$. It follows that the equal-split allocation for N characterized by the strategy vectors $\left(y^{*},t^{*}\right)$, where $t^{*}=\left(0,0,...,0\right)$, belongs to the γ-core. To see that this allocation is strictly included in the γ-core, note that, since individual deviations assign to a player just $v_{\gamma}\left(\{i\}\right)=\frac{1}{(n+1)^2}<\frac{v_\gamma(N)}{|N|}=\frac{1}{4n}$, different and unequal allocations belong as well to the γ-core. In particular, any allocation giving to a player i his worth $v_\gamma(\{i\})$, and $\frac{v_\gamma(N)-v_\gamma(\{i\})}{|N-1|}$ to any remaining player in N, it is not objectable.

□

3.6.3. *The δ-core.* Using the same linear setup introduced above, the following result can be easily proved.

Proposition 4. *Under the linear quantity oligopoly, the δ-core is empty.*

Proof. Under the δ-assumption, when a single firm leaves the grand-coalition of firms $\{N\}$, a simultaneous duopoly game is played between the firm $\{i\}$ and the remaining firms $N\setminus\{i\}$ acting as a single coalition. As a result, $v(\{i\}) = \frac{(a-c)^2}{9b}$, which is greater than $\frac{v_\delta(N)}{n} = \frac{(a-c)^2}{4nb}$ for $n > 2$, the maximum payoff that at least one firm will obtain inside the grand coalition. Therefore, the core is empty. □

3.6.4. *The sequential case and the λ-core.* According to (3.11), the worth of a coalition S in this case can be defined as:

$$v_\lambda(S) = \sum_{i\in S}\left[p\left(\tilde{y}_S, \{y_j(\tilde{y}_S)\}_{j\in N\setminus S}\right)\tilde{y}_i - C_i(\tilde{y}_S)\right] \tag{3.18}$$

As before, every $j \in N\setminus S$ maximizes π_j,for a given y_S, and its best-reply is:

$$y_j(y_S) = \frac{a - by_S - c}{b(n-s+1)}. \tag{3.19}$$

The coalition S acts as leader and maximizes:

$$\sum_{i\in S}\pi_i\left(y_i, (s-1)\underset{\substack{h\neq i\\ h\in S}}{y_h}, (n-s)y_j(y_S)\right) = \pi_S((n-s)y_j(y_S)).$$

This is equivalent to:

$$\pi_S((n-s)y_j(y_S)) = \left(a - by_S - b(n-s)\frac{a-by_S-c}{b(n-s+1)}\right)y_S - cy_S.$$

The F.O.C. of this problem is:

$$a - 2by_S - (n-s)\frac{a-c-2by_S}{(n-s+1)} - c = 0,$$

from which:

$$\widetilde{y}_S = \frac{a-c}{2b} \tag{3.20}$$

and

$$y_j(\widetilde{y}_S) = \frac{a-c}{2b(n-s+1)}.$$

Therefore, in order to obtain $v_\lambda(S)$, we first compute the equilibrium price:

$$p(\widetilde{y}) = a - b\widetilde{y}_S - b(n-s)y_j(\widetilde{y}_S),$$

as:

$$p\left(\widetilde{y}\right)=\frac{a+2\left(n-s\right)c+c}{2\left(n-s+1\right)}.$$

Finally,

$$v_{\lambda}\left(S\right)=\pi_{S}\left(\widetilde{y}_{S},(n-s)y_{j}\left(\widetilde{y}_{S}\right)\right)=p\left(\widetilde{y}\right)\widetilde{y}_{S}-c\widetilde{y}_{S}$$

that is,

$$v_{\lambda}\left(S\right)=\frac{\left(a-c\right)^{2}}{4b(n-s+1)}. \tag{3.21}$$

Again, the worth of the grand coalition ($n=s$) can be written as:

$$v_{\lambda}\left(N\right)=\frac{\left(a-c\right)^{2}}{4b}. \tag{3.22}$$

Proposition 5. *For the linear quantity oligopoly, the equal split efficient allocation is the unique element of the λ-core.*

Proof. Without loss of generality let us normalize $\frac{(a-c)^2}{b}=1$, so that the equal split allocation gives to each player in N a payoff of $\frac{v_\lambda(N)}{n}=\frac{1}{4n}$ and $v_{\lambda}\left(S\right)=\frac{1}{4(n-s+1)}$, where $s=|S|$ and $n=|N|$. We first show that the equal split allocation belongs to the core. Consider the value $\frac{v_\lambda(S)}{s}$ for an arbitrary coalition S. We have that for all S such that $s\leq n$

$$\frac{v_{\lambda}\left(S\right)}{s}=\frac{1}{4s\left(n-s+1\right)}\leq\frac{1}{4n}=\frac{v_{\lambda}\left(N\right)}{n}. \tag{3.23}$$

In fact, the above inequality reduces to

$$\left(n-s+1\right)\geq n \tag{3.24}$$

which is satisfied for $n\geq s$. It follows that if coalition S forms, at least one player gets a payoff less than or equal to $\frac{v_\lambda(S)}{s}$, and therefore less than or equal to $\frac{v_\lambda(N)}{n}$. This implies that the equal split allocation is in the λ-core. To see that the equal-split is the unique allocation in the λ-core, note that (3.24) is satisfied with equality for $s=n$ and for $s=1$. This means that $v_{\lambda}\left(\{i\}\right)=\frac{v_\lambda(N)}{n}$ for all $i\in N$. Thus, consider the allocation z' different from the equal split allocation; in z', some player j receives a payoff $v_j<\frac{v_\lambda(N)}{n}$. Player j can thus improve upon z' by getting $v_{\lambda}\left(\{i\}\right)=\frac{v_\lambda(N)}{n}$, which implies that z' is not in the λ-core. □

The λ-core is non-empty and selects a unique symmetric allocation out of the γ-core, that includes instead a *continuum* of other asymmetric allocations. The λ-core can be therefore viewed as a refinement of the γ-core, one that selects out of the latter the most "reasonable" allocation for the symmetric Cournot setting.

3.7. **The case of linear demand and quadratic cost.** We can now consider also the case with a quadratic cost function. As indicated above, we know from Rajan (1989), that for $n=2$, $n=3$ and $n=4$, the γ-core is non empty. We now show that this result does not hold under the λ-core assumption.

Let $C_i\left(y_i\right)=\frac{y_i^2}{2}$. Let also for simplicity $p\left(y\right)=a-y$.

Proposition 6. *Under linear demand and quadratic cost quantity oligopoly, the λ-core can be empty.*

Proof. From first order conditions, it is obtained that:

$$v_\lambda(N) = \frac{a^2 n^2}{(1+2n)^2}$$

and

$$v_\lambda(\{i\}) = \frac{a^2(a^2+5n-1)}{(n+1)(n+5)^2}.$$

Simple calculations show that, for every $i \in N$, and for $n \geq 2$, $v_\lambda(\{i\}) > \frac{v_\lambda(N)}{|N|}$. By the efficiency of the equal-split solution, in any other efficient allocation at least one player would receive a lower utility. This fact together with the above result that any player can improve upon the equal-split allocation by deviating as singleton, imply that any efficient allocation can be objected by a deviation of a single player. This, in turn, implies that the λ-core is empty. □

3.8. **Coalitional Equilibria in Strategic Form Games.**

3.8.1. *Strong Nash Equilibrium.* In the 'core approach' described above, players can sign binding agreements. When this assumption is relaxed, a Nash approach to coalitional deviations becomes more appropriate. The concept of equilibrium proposed by Aumann (1959), denoted strong Nash equilibrium, extends the Nash equilibrium to every coalitional deviation. Accordingly, a strong Nash equilibrium is defined as a strategy profile that no group of players can profitably object, given that remaining players are expected not to change their strategies.

A strategy profile $\widehat{x} \in X_N$ for G is a strong Nash equilibrium **(SNE)** if there exists no $S \subset N$ and $x_S \in X_S$ such that

$$u_i\left(x_S, \widehat{x}_{N\setminus S}\right) \geq u_i(\widehat{x}) \;\; \forall i \in S$$
$$u_h\left(x_S, \widehat{x}_{N\setminus S}\right) > u_h(\widehat{x}) \;\text{ for some } h \in S.$$

Obviously, all SNE of G are both Nash Equilibria and Pareto Efficient; in addition they satisfy the Nash stability requirement for each possible coalition. As a result, SNE fails to exist in many economic problems, and in particular, whenever Nash Equilibria fail to be Pareto Efficient.

Proposition 7. *For the linear oligopoly, the set of strong Nash equilibrium is empty.*

Proof. The symmetric strategy profile $y = \left(\frac{a-c}{2nb}, \frac{a-c}{2nb}, \frac{a-c}{2nb}\right)$, associated with the Pareto-efficient allocation, is not a Nash equilibrium, and the result follows. □

4. Stable Associations of Firms

4.1. **Cooperative Games with Coalition Structures.** According to the original spirit of von Neumann and Morgenstern (1944), "the purpose of game theory is to determine everything can be said about coalitions between players, compensation between partners in every coalition, mergers or fight between coalitions" (p.240). To introduce the topic of competition among coalitions, a framework different from which used by traditional cooperative games is required. The first required step is to extend the game (N, v) to a game with a coalition structure $P = (S_1, S_2; ...., S_m)$, i.e., a partition of players N such that for all $S_h, S_j \in P$, $S_h \cap S_j = \varnothing$ and $\bigcup_{k=1,2,..m} S_k = N$. The second step is to define the worth to every coalition belonging to a given coalition structure. Finally, a relevant issue is which coalition structure can be considered stable.

In their seminal contribution, Aumann and Drèze (1974) extend the solution concepts of cooperative game theory to games with exogenous coalition structures. In every $P \in \mathcal{P}(N)$, the set of all partitions of the N players, each coalition is allowed to distribute its members only its own worth $v(S_k)$, here assumed equal to the Shapley value defined for every given coalition structure $P \in \mathcal{P}$.[9] However, the above restriction has been criticized as inadequate for all models in which "the *raison d'etre* for a coalition S to form is that its members try to receive more than $v(S)$ - the worth of S." (Greenberg, 1994, p.1313). A part from this criticism, the most commonly used stability concept within this framework is the coalition structure core.

Definition 4. *Let (N, v) be a cooperative game. The coalition structure $P \in \mathcal{P}(N)$ is stable if its core is nonempty, i.e., if there exists a feasible payoff $z \in Z(P)$ such that, for every $S_k \in P$, $z_k \geq v(S_k)$. The game (N, v) has a coalition structure core if there exists at least one partition that is stable.*

4.1.1. *The Partition Function Approach.* The presence of externalities among coalitions of players calls for a more encompassing approach than that offered by a cooperative games in characteristic function form. For this purpose, in a seminal paper Thrall and Lucas (1963) introduce the games in *partition function form*.

Definition 5. *A TU game in partition function form can be defined as a triple $(N; P, w)$; where $P = (S_1, S_2; ...., S_m)$ is a partition of players N and $w(S; P) : 2^N \times \mathcal{P} \rightarrow \mathcal{R}$ is a mapping that assigns to each coalition S embedded in a given partition $P \in \mathcal{P}(N)$ a real number (a worth).*

In this way, the authors can define the value of every non-empty coalition S of N as

$$v(S) = \min_{\{P|S \in P\}} w(S, P),$$

where this minimum is over all partitions π which contain S as a distinct coalition. This approach constitutes a generalization of the cooperative game $(N; v)$ and the two games coincides when the worth of a coalition is independent of the coalitions formed by the other players. When coaltions.payoffs are not independent, some assumptions are still required to model the behaviour of coalitions with respect to rival.coalitions. Since Ichiishi (1983), the modern theory of coalition formation adopts the view that coalitions cooperate inside and compete *à la* Nash with the other coalitions.

4.1.2. *The Valution Approach.* Since the games in partition function are hard to handle and often pose technical difficulties, many recent contributions have imposed a *fixed allocation rule* distributing the worth of a coalition to all its members. Such a fixed sharing rule gives rise to a per-member payoff (*valuation*) mapping coalition structures into vectors of individual payoffs.

[9] The Shapley value is defined as $\phi(N, v) = \sum_{S \subset N} q(s)\Delta_i(s)$, where $q(s) = \frac{(s-1)!(n-s)!}{n!}$, and $\Delta_i(s) = v(S) - v(S \setminus \{i\})$ is the marginal contribution of player i to any coalition S in the game $(N; v)$: Therefore, the Shapley value of player i represents the weighted sum of his marginal contribution to all coalitions he can join.

Definition 6. *A game in valuation form can be defined as a triple* (N, P, v_i), *where* $P = (S_1, S_2; ...., S_m)$ *is a partition of players* N *and* $v_i(S) : 2^N \times \mathcal{P} \to \mathbf{R}^{|S|}$ *is a mapping that assigns to each individual belonging to a coalition* S *embedded in a given partition* $P \in \mathcal{P}$ *(the set of all feasible partitions) a real number (a valuation).*

Definition 7. *A coalition structure is core stable if there not exists a coalition* S *and a coalition structure* P' *such that for* $S \in P'$ *and for all* $i \in S$, $v_i(S, P') > v_i(S, P)$.

Analogous concepts of α, β, γ, δ, λ-core stability can be defined for games in valuation form. See the proposition that follows.

Proposition 8. *Under the linear oligopoly game, the grand coalition is a core-stable coalition structure under the valuations* v_i^α, v_i^β, v_i^γ *and* v_i^λ. *It is not core-stable under the valuation* v_i^δ. *Moreover, under the valuation* v_i^λ, *the grand coalition is the only core-stable coalition structure.*

Proof. It follows straightforwardly by propositions 3, 4 and 5. □

4.2. **Noncooperative Games of Coalition Formation.** Most recent approaches have looked at the process of coalition formation as a strategy in a well defined game of coalition formation (see Bloch, 1997, 2003 and Yi, 2003 for surveys). Within this new stream of literature, usually indicated as noncooperative theory of coalition formation (or endogenous coalition formation), the work by Hurt and Kurz (1985) represents a seminal contribution. Most recent contributions along these lines include Bloch (1995, 1996), Ray and Vohra (1997, 1999) and Yi (1997). In all these works, cooperation is modelled as a two stage process: at the first stage players form coalitions, while at the second stage formed coalitions interact in a well defined strategic setting. This process is formally described by a coalition formation game, in which a given rule of coalition formation maps players' announcements of coalitions into a well defined coalition structure, which in turns determines the equilibrium strategies chosen by players at the second stage. A basic difference among the various models lies on the timing assumed for the coalition formation game, which can either be simultaneous (Hurt & Kurz (1982), Ray & Vohra (1994), Yi (1997)) or sequential (Bloch (1994), Ray & Vohra (1995)).

4.2.1. *Hurt & Kurz's Games of Coalition Formation.* Hurt and Kurz (1983) were among the first to study games of coalition formation with a valuation in order to identify stable coalition structures.[10] As valuation, Hurt & Kurz adopt a general version of Owen value for TU games (Owen, 1977), i.e. a Shapley value with prior coalition structures, that they call Coalitional Shapley value, assigning to every coalition structure a payoff vector $\varphi_i(P)$ in $\mathcal{R}^N$, such that (by the efficiency axiom) $\sum_{i \in N} \varphi_i(P) = v(N)$. Given this valuation, the game of coalition formation is modelled as a game in which each player $i \in N$ announces a coalition $S \ni i$ to which he would like to belong; for each profile $\sigma = (S_1, S_2, ..., S_n)$ of announcements, a partition $P(\sigma)$ of N is assumed to be induced on the system. The rule according to which $P(\sigma)$ originates from σ is obviously a crucial issue for the prediction of which coalitions will emerge in equilibrium. Hurt and Kurz's game Γ predicts that a coalition emerges if and only if all its members have declared it (from which the name of "unanimity rule"also used to describe this game).

[10] Another seminal contribution is Shenoy (1979).

Formally:

$$P(\sigma) = \{S_i(\sigma) : i \in N\}$$

where

$$S_i(\sigma) = \begin{cases} S_i \text{ if } S_i = S_j \text{ for all } j \in S_i \\ \{i\} \text{ otherwise.} \end{cases}$$

Their game Δ predicts instead that a coalition emerges if and only if all its members have declare the same coalition S (which may, in general, differs from S). Formally:

$$P(\sigma) = \{S \subset N : i, j \in S \text{ if and only if } S_i = S_j\}.$$

Note that the two rules of formation of coalitions are "exclusive" in the sense that each player of a forming coalition has announced a list of its members. Moreover, in the gamma-game this list has to be approved unanimously by all coalition members. Once introduced these two games of coalition formation, a stable coalition structure for the game Γ (Δ) can be defined as a partition induced by a Strong Nash Equilibrium strategy profile of these games.

Definition 8. *The partition π is a γ-stable (δ -stable) coalition structure if $\pi = \pi(\sigma^*)$ for some σ^* with the following property: there exists no $S \subset N$ and $\sigma_S \in \Sigma_S$ such that*

$$v_i(\sigma_S, \sigma^*_{N \backslash S}) \geq v_i(\sigma^*) \textit{ for all } i \in S$$

and

$$v_h(\sigma_S, \sigma^*_{N \backslash S}) > v_h(\sigma^*) \textit{ for at least one } h \in S.$$

It can be seen that the two rules generate different partitions after a deviation by a coalition: in the Γ-game, remaining players split up in singletons; in the Δ-game, they stick together.

In the recent literature on endogenous coalition formation, the coalition formation game by Hurt and Kurz is usually modelled as a first stage of a game in which, at the second stage formed coalitions interact in some underlying strategic setting. The coalition formation rules are used to derive a valuation v_i mapping from the set of all players' announcements Σ into the set of real numbers. The payoff functions v_i are obtained by associating with each partition $P = \{S_1, S_2, ..., S_m\}$ a game in strategic form played by coalitions

$$G(P) = (\{1, 2, ..., m\}, (Y_{S_1}, Y_{S_2}, ..., Y_{S_m}), (\pi_{S_1}, \pi_{S_2}, ..., \pi_{S_m})),$$

in which Y_{S_k} is the strategy set of coalition S_k and $\pi_{S_k} : \Pi_{k=1}^m Y_{S_k} \rightarrow R_+$ is the payoff function of coalition S_k, for all $k = 1, 2, ..., m$. The game $G(P)$ describes the interaction of coalitions after P has formed as a result of players announcements in Γ.or Δ-coalition formation games. The Nash equilibrium of the game $G(P)$ (assumed unique) gives the payoff of each coalition in P; within coalitions, a fix distribution rule yields the payoffs of individual members.

Following our previous assumptions (see section 1.2) we can derived the game $G(P)$ from the the strategic form game G by assuming that $Y_{S_k} = \prod_{i \in S_k} Y_i$ and $\pi_{S_k} = \sum_{i \in S_k} \pi_i$, for every coalition $S_k \in P$. We can also assume $\pi_i = \frac{\pi_{S_k}}{|S_k|}$ as the per capita payoff function of members of S_k. Therefore, using the linear Cournot example for the Γ-game we know that the payoff of each firm $i \in S \subset N$ when all remaining firms split up in singletons, is given by:

$$\pi_i^\gamma(y(P(\sigma'))) = \frac{(a-c)^2}{s(n-s+2)^2}$$

where $n \equiv |N|$, $s \equiv |S|$ and $\sigma' = \left(\{S\}_{i \in S}, \{N\}_{i \in N \setminus S}\right)$. We can thus present the following proposition.

Proposition 9. *Under the linear oligopoly, the grand coalition induced by the profile $\sigma^* = \left(\{N\}_{i \in N}\right)$, is a stable coalition structure in the Γ-game of coalition formation.*

Proof. it can be easily verified that the condition

$$\pi_i^\gamma \left(y\left(P\left(\sigma^*\right)\right)\right) = \frac{(a-c)^2}{4n} \geq \pi_i^\gamma \left(y\left(P\left(\sigma'\right)\right)\right) = \frac{(a-c)^2}{s(n-s+2)^2}.$$

holds for every $s \leq n$ and, therefore, the stability of the whole industry agreement holds under the linear oligopoly. □

4.2.2. *Sequential Games of Coalition Formation.* Bloch (1996,1997) introduces a sequential coalition-formation game with infinite horizon in which, as in Hurt and Kurz's (1988) Γ-game, a coalition forms if and only if all its members have agreed to form the same coalition. The sequence of moves of the coalition formation game is organized as follows. At the beginning, the first player (according to a given ordering) makes a proposal for a coalition to form. Then, the player on his list with the smallest index accepts or rejects his proposal. If he accepts, it is the turn of the following player on the list to accept or reject. If all players on the list accept the first player's proposal, the coalition is formed and the remaining players continue the coalition formation game, starting with the player with the smallest index who thus makes a proposal to remaining players. If any of the players has rejected first player's proposal, the player who first rejected the proposal starts proposing another coalition. Once a coalition forms it cannot break apart or merge with another player or a coalition of players. Bloch (1996) shows that this game yields the same stationary subgame perfect equilibrium coalition structure as a much simpler "size-announcement game", in which the first player announces the size of his coalition and the first s_1 players accept; then player i_{s_1+1} proposes a size s_2 coalition and this is formed and so on, until the last player is reached. This equivalence is basically due to the *ex ante* symmetry of players. It can also shown that this size-announcement game possesses a generically unique subgame perfect equilibrium coalition structure.

If we the linear oligopoly with $n > 2$ firms, the unique subgame perfect equilibrium coalition structure of Bloch's (1996) sequential game of coalition formation is a coalition structure $P = (\{S\}, \{j\}_{j \in N \setminus S})$, with $s = |S|$ equal to the first integer following $\left(2n + 3 - \sqrt{4n+5}\right)/2$.[11] The explanation is as follows. We know that when a merger of size s is formed in a Cournot market, the equal-split payoff of each firm $i \in S$ in the merger is $\pi_i(y^*(\{S\}, \{j\}_{j \in N \setminus S})) = (a-c)^2/s\left(n-s+2\right)^2$ which is greater than the usual Cournot profit $\pi_i(y^*(\{i\}_{i \in N})) = (a-c)^2/s\left(n-s+1\right)^2$ only for $s > \left(2n+3-\sqrt{4n+5}\right)/2$. When a merger of size s is in place, each independent firm outside the merger earns a higher profit than that of the members of the merger, equal to $\pi_j(y^*(\{S\}, \{j\}_{j \in N \setminus S})) = (a-c)^2/\left(n-s+2\right)^2$. Therefore, in the sequential game of coalition formation, the first firms choose to remain independent and free-ride on the merger formed by subsequent firms. When the number of remaining firms is exactly equal to the Salant *et al.*, 1983 minimal profitable merger size $s = \left(2n+3-\sqrt{4n+5}\right)/2$, they will choose to merge, as it is no longer profitable to remain independent.

[11] We know (Salant et al.,1983) that $\left(2n+3-\sqrt{4n+5}\right)/2.\simeq 0.8n$.

4.2.3. *Equilibrium Binding Agreement.* Ray and Vohra (1997) propose a different stability concept. In this solution concept, players start from some coalition structure and are only allowed to break coalitions to smaller ones. The deviations can be unilateral or multilateral (i.e., several players can deviate together). The deviators take into account future deviations, both by members of their own coalitions and by members of other coalitions. Deviations to finer partitions must be credible, i.e. stable themselves, and therefore the nature of the definition is recursive. We can start with a partition P and we can denote by $B(P)$ all coalition structures that are finer than P. A coalition $P' \in B(P)$ can be induced from P if P' is formed by breaking a coalition in P. A coalition S is a perpetrator if it can induce $P' \in B(P)$ from P. Obviously, S is a subcoalition of a coalition in P. Denote the finest coalition structure, such that $|S| = 1$ for all S, by P_0. There are no deviations allowed from P_0 and therefore P_0 is by definition stable. Recursively, suppose that for some P, all stable coalitions were defined for all $P' \in B(P)$, i.e., for all coalition structures finer than P. Now, we can say that a strategy profile (say a quantity profile of our oligopoly game) associated to a coalition structure $y(P)$ is sequentially blocked by $y(P')$ for $P' \in B(P)$ if i) there exists a sequence $\{y(P_1), y(P_2), ..., y(P_m)\}$ with $y(P_1) = y(P)$ and $y(P') = y(P_m)$; ii) for every $j = 2, ..., m$, there is a deviator S_j that induces P_j from P_{j-1}; iii) $y(P')$ is stable; iv) P_j is not stable for any $y(P_j)$ and $1 < j < m$; v) $\pi_i(y(P_0)) > \pi_i(y(P_{j-1}))$ for all $i \in S_j$ and $j = 2, ..., m$.

Definition 9. *$y(P)$ is an equilibrium binding agreement if there is no $y(P')$ for $P' \in B(P)$ that sequentially blocks $y(P)$.*

Applying the Equilibrium Binding Agreement to the linear oligopoly game with three firms, we obtain that, beside $y(P_0)$, with $P_0 = (\{1\} . \{2\} , \{3\})$, which is by definition stable, also the grand coalition strategy profile $y(P)$ with $P = (\{1.2, 3\}$ is an equilibrium binding agreement. For the n-firm merger game, Ray and Vohra's show that there is a cyclical pattern, in which, depending on n, the grand coalition can or not be a stable coalition structure. For $n = 3, 4, 5$ it is stable, but not for $n = 6, 7, 8$. For $n = 9$ is again stable and so on, with a rather unpredictable pattern. "The grand coalition survives if there exist 'large zones of instability in intermediate coalition structures." (Ray & Vohra, 1997, p.73).

5. Concluding Remarks

This paper has quickly reviewed a number of coalitional solution concepts for the analysis of both partial and full collusive agreements in oligopolistic markets. A number of illustrative results were presented to show that numerous connections exist between the Industrial Organization and Game Theory approaches on the subject, which may prove highly significant and instructive for the future research agenda of both disciplines.

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