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ON COAXIAL WIRE MEASUREMENTS OF THE LONGITUDINAL COUPLING IMPEDANCE

H. Hahn and F. Pedersen

April 1978

ACCELERATOR DEPARTMENT

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ABSTRACT

The measurement of the longitudinal coupling impedance of an accelerator component by coaxial wire methods is discussed. Potential errors intrinsic to this method are pointed out and analyzed. It is concluded that measurements using the transmission rather than the reflection coefficient are preferable and are expected to give adequate results in the limit of thin center conductors.

I. INTRODUCTION

The longitudinal coupling impedance of an accelerator component is usually measured by inserting a wire in the center of the beam pipe to form a coaxial transmission line. Under certain conditions the measurable circuit parameters are related to the wall impedance which couples to the beam. Either the reflected wave or the transmitted wave have been used to determine the coupling impedance.[1-8]

The relation between a single, lumped wall impedance and the scattering matrix describing the two-port is simple and transparent for both reflection and transmission coefficient.

In this report it will be shown that if two or more impedances are present it becomes advantageous to use the transmission-coefficient, as the sum of all impedances enters directly provided there is no change in cross section of the beam pipe.

As an example of a distributed coupling impedance associated with changes in cross section a cylindrical cavity, for which the coupling impedance is well known,[9,10] is considered. The impedance determined from the reflection coefficient may now be wrong by as much as an order of magnitude, and the error is independent of the choice of diameter of the center conductor. Again, better results are obtained

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by using the transmission coefficient and the correct result is obtained in the limit of an infinitely thin center conductor. But for the diameters used so far in practice the errors are still substantial.

In general, results from coaxial wire measurements should be interpreted with caution. Transmission measurements are less erroneous than reflection measurements, especially when the center conductor is chosen as thin as measurement accuracy permits.

II. SINGLE, LUMPED IMPEDANCE

We consider a single localized impedance Z_w associated with a short gap in the circular beam pipe, Fig. 1. Coupling impedance $Z_c = Z_w$ in this case.

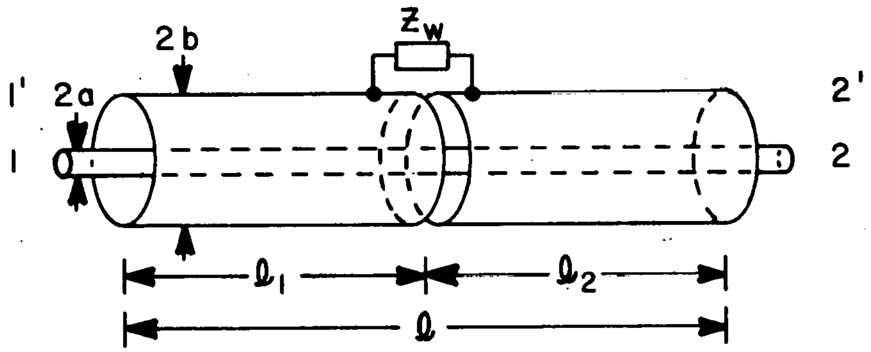

Fig. 1

The characteristics of a two-port in a transmission line environment is most conveniently described by the scattering matrix or the S-parameters as defined in appendix A.

The two-port above can be described as 3 cascaded two-ports, α, β, and γ (Fig. 2).

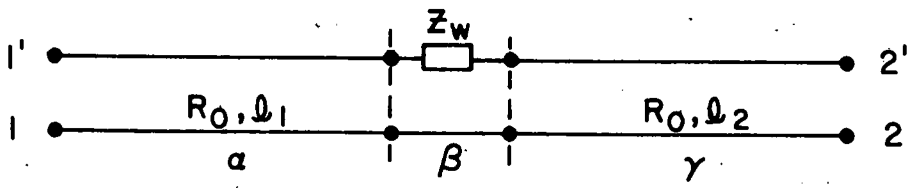

Fig. 2

where

$$R_o = \frac{Z_o}{2\pi} \ln \frac{b}{a} \tag{1}$$

is the characteristic impedance of the transmission line. Z_o is the impedance of free space, $Z_o = c\mu_o \simeq 377\ \Omega$. From the definitions of the S-parameters we get:

$$S_{11}^{\beta} = S_{22}^{\beta} = \frac{Z_w}{Z_w + 2R_o}$$

$$\simeq \frac{Z_w}{2R_o} \text{ for } Z_w \ll R_o$$

$$S_{12}^{\beta} = S_{21}^{\beta} = 1 - \frac{Z_w}{Z_w + 2R_o}$$

$$\simeq 1 - \frac{Z_w}{2R_o} \text{ for } Z_w \ll R_o$$

$$\{S^{\alpha}\} = \begin{Bmatrix} 0 & e^{-jk\ell_1} \\ e^{-jk\ell_1} & 0 \end{Bmatrix}, \{S^{\gamma}\} = \begin{Bmatrix} 0 & e^{-jk\ell_2} \\ e^{-jk\ell_2} & 0 \end{Bmatrix}$$

where

$$k = \omega/c \tag{2}$$

is the propagation constant for the transmission lines. From the cascading formulaes (appendix B) we get the total S-matrix:

$$S_{11}^{\alpha\beta\gamma} = S_{11}^{\beta} e^{-jk2\ell_1} = \frac{Z_w}{Z_w + 2R_o} e^{-jk2\ell_1} \simeq \frac{Z_w}{2R_o} e^{-jk2\ell_1} \tag{3}$$

$$S_{22}^{\alpha\beta\gamma} = S_{22}^{\beta} e^{-jk2\ell_2} = \frac{Z_w}{Z_w + 2R_o} e^{-jk2\ell_2} \simeq \frac{Z_w}{2R_o} e^{-jk2\ell_2} \tag{4}$$

$$S_{12}^{\alpha\beta\gamma} = S_{21}^{\alpha\beta\gamma} = S_{21}^{\beta} e^{-jk\ell} = \left(1 - \frac{Z_w}{Z_w + 2R_o}\right) e^{-jk\ell} \simeq \left(1 - \frac{Z_w}{2R_o}\right) e^{-jk\ell} \tag{5}$$

where $\ell = \ell_1 + \ell_2$ is the total length. It is seen that the S parameters do not change in magnitude along the transmission lines, only their phase is changed. The measured reflection-coefficient, $S_{11}^{\alpha\beta\gamma}$, must thus be corrected for the phase shift caused by $2\ell_1$,

$$S_{11}^{\beta} = S_{11}^{\alpha\beta\gamma} e^{jk2\ell_1}$$

and Z_w can be determined:

$$Z_w = \frac{2R_o S_{11}^{\beta}}{1 - S_{11}^{\beta}} \approx 2R_o S_{11}^{\beta} \text{ for } |Z_w| \ll R_o \quad . \tag{6}$$

Alternatively the transmission-coefficient, $S_{21}^{\alpha\beta\gamma}$, must be corrected for the phase shift caused by the total line length, ℓ:

$$S_{21}^{\beta} = S_{21}^{\alpha\beta\gamma} e^{jk\ell}$$

and Z_w can be determined:

$$Z_w = \frac{2R_o(1 - S_{21}^{\beta})}{S_{21}^{\beta}} \approx 2R_o(1 - S_{21}^{\beta}) \text{ for } |Z_w| \ll R_o \quad . \tag{7}$$

For a single localized impedance the two methods appear equally good except that the exact location of the gap must be known in case of the reflection-coefficient while only the total length has to be known in case of the transmission-coefficient as the transmission is independent of the location of the gap.

Transmission measurements may also be preferred if the impedance to be measured is large compared to R_o (accelerating cavity e.g.), as it is much easier to measure a transmission close to zero than a reflection close to unity.

III. TWO LUMPED IMPEDANCES. DISTRIBUTED IMPEDANCES.

For two localized impedances we have the circuit equivalent Fig. 3.

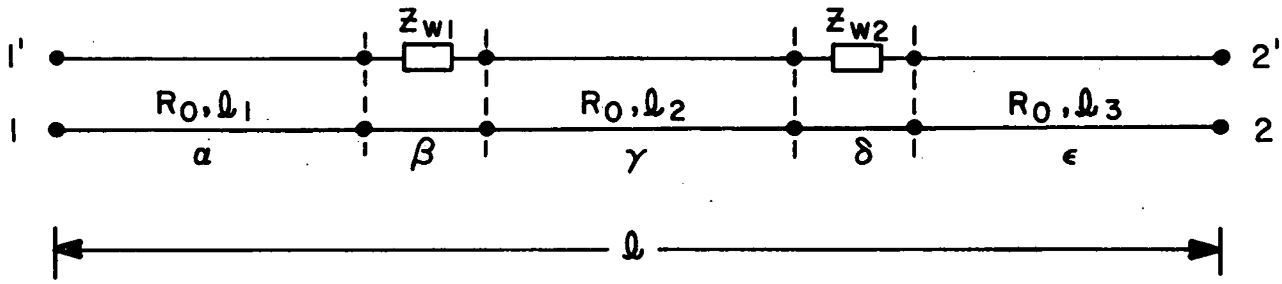

Fig. 3

The elements of the scattering matrices are:

$$S_{11}^{\beta} = S_{22}^{\beta} = \frac{Z_{w1}}{Z_{w1} + 2R_o} = \Gamma_1$$

$$S_{12}^{\beta} = S_{21}^{\beta} = 1 - \frac{Z_{w1}}{Z_{w1} + 2R_o} = 1 - \Gamma_1$$

$$S_{11}^{\delta} = S_{22}^{\delta} = \frac{Z_{w2}}{Z_{w2} + 2R_o} = \Gamma_2$$

$$S_{12}^{\delta} = S_{21}^{\delta} = 1 - \frac{Z_{w2}}{Z_{w2} + 2R_o} = 1 - \Gamma_2 \ .$$

For the combined S-matrix we find:

$$S_{11}^{\alpha\beta\gamma\delta\epsilon} = e^{-jk2\ell_1} \frac{\Gamma_1 + \Gamma_2 (1 - 2\Gamma_1) e^{-jk2\ell_2}}{1 - \Gamma_1\Gamma_2 e^{-jk2\ell_2}}$$

$$\left.\begin{aligned} &\simeq \Gamma_1 e^{-jk2\ell_1} + \Gamma_2 e^{-jk(2\ell_1+2\ell_2)} \\ &\simeq \frac{Z_{w1} e^{-jk2\ell_1} + Z_{w2} e^{-jk(2\ell_1+2\ell_2)}}{2R_o} \end{aligned}\right\} \quad \begin{aligned} |Z_{w1}| \ll R_o \\ |Z_{w2}| \ll R_o \end{aligned} \qquad (8)$$

$$S_{22}^{\alpha\beta\gamma\delta\epsilon} = e^{-jk2\ell_3} \frac{\Gamma_2 + \Gamma_1 (1 - 2\Gamma_2) e^{-jk2\ell_2}}{1 - \Gamma_1\Gamma_2 e^{-jk2\ell_2}}$$

$$\left.\begin{aligned} &\simeq \Gamma_2\, e^{-jk2\ell_3} + \Gamma_1\, e^{-jk(2\ell_2+2\ell_3)} \\ &\simeq \frac{Z_{w2}\, e^{-jk2\ell_3} + Z_{w1}\, e^{-jk(2\ell_2+2\ell_3)}}{2R_o} \end{aligned}\right\} \begin{aligned} |Z_{w1}| \ll R_o \\ |Z_{w2}| \ll R_o \end{aligned} \tag{9}$$

$$S_{12}^{\alpha\beta\gamma\delta\varepsilon} = S_{21}^{\alpha\beta\gamma\delta\varepsilon} = e^{-jk\ell}\, \frac{1 - (\Gamma_1 + \Gamma_2) + \Gamma_1\Gamma_2}{1 - \Gamma_1\Gamma_2\, e^{-jk2\ell_2}}$$

$$\left.\begin{aligned} &\simeq e^{-jk\ell}\, [1 - (\Gamma_1 + \Gamma_2)] \\ &\simeq e^{-jk\ell} \left(1 - \frac{Z_{w1} + Z_{w2}}{2R_o}\right) \end{aligned}\right\} \begin{aligned} |Z_{w1}| \ll R_o \\ |Z_{w2}| \ll R_o \end{aligned} \tag{10}$$

Even if the relative location of the two lumped impedances is known one cannot in general determine the impedances from the reflection coefficient as they enter with different phases. Only if, a priori, it is known that the two impedances are identical we get:

$$S_{11} \simeq e^{-jk2(\ell_1+\frac{1}{2}\ell_2)}\, \frac{2Z_{w1}\, \cos k\ell_2}{2R_o}\ , \text{ if } Z_{w1} = Z_{w2} \tag{11}$$

and the measuring frequency will have to be restricted ($k\ell_2 \ll 1$).

In the transmission-coefficient the two contributions simply add up independently of their location, and we get immediately the total impedance provided that each contribution is small compared to R_o.

This result can easily be extended to more than two impedances or to a general, nonuniform, distributed impedance:

$$\begin{aligned} &dZ_w = Z(x)\,dx \\ &Z_w = \int_o^{\ell} Z(x)\,dx \\ &S_{21} \simeq e^{-jk\ell} \left(1 - \frac{Z_w}{2R_o}\right),\ |Z_w| \ll R_o\ . \end{aligned} \tag{12}$$

By splitting the impedance into its real and imaginary parts,

$Z_w = R_w + jX_w$,

$$S_{21} \simeq e^{-jk\ell} \left(1 - \frac{R_w}{2R_o} - j \frac{X_w}{2R_o} \right)$$
$$\simeq e^{-jk\ell} \, e^{-j(X_w/2R_o)} \left(1 - \frac{R_w}{2R_o} \right) . \tag{13}$$

One can see that the imaginary part affects the phase while the real part affects the amplitude of the transmitted signal.

This result agrees with Faltens et al.[1] although they mainly emphasized the phase shift caused by the imaginary part. If the transmitted signal is compared with a reference signal from a transmission line with same delay, one should be able to measure small amplitude variations too and thus the real part.

It is, however, important to point out that this derivation assumes that the coupling impedances are not associated with any change in cross-section of the beam pipe.

III. A DOUBLE STEP CROSS SECTION CHANGE

As an example of a distributed coupling impedance associated with cross section changes a cylindrical cavity is considered.

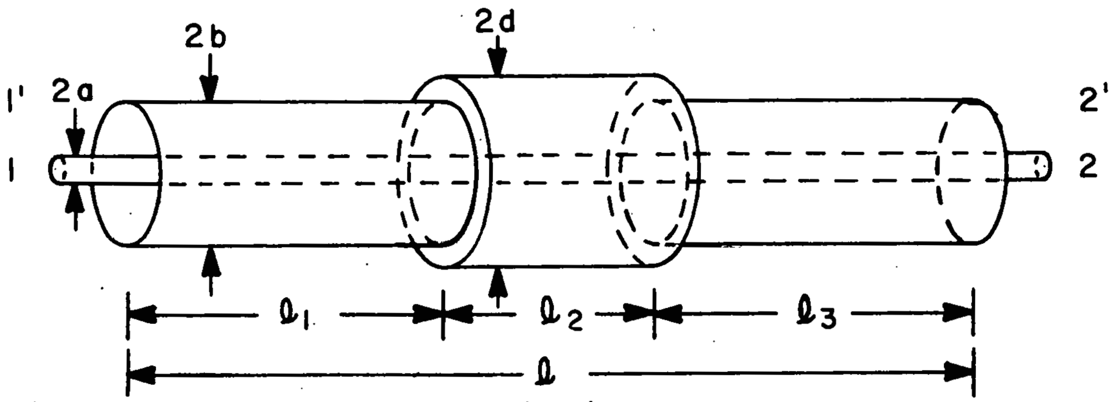

Fig. 4

For frequencies well below the cutoff frequency of the beam pipe the coupling impedance is purely inductive.[9,10] If the cavity length ℓ_2 is longer than the pipe radius the inductance is independent of ℓ_2 as there is no interaction between the two steps. The wall can, therefore, be characterized by a series inductor associated with each step, and we get the following circuit equivalent with the center conductor inserted, Fig. 5:

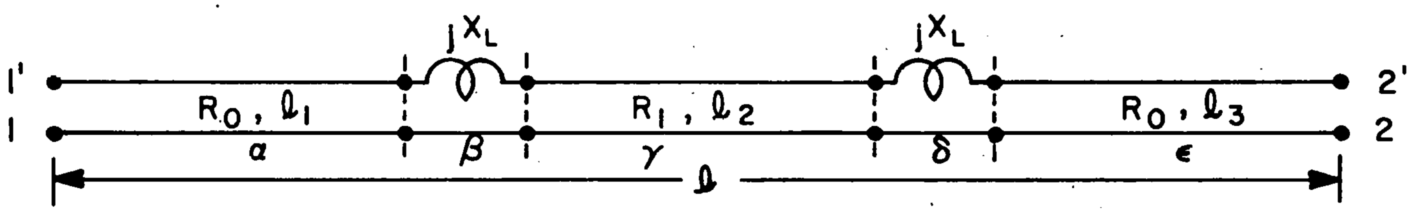

Fig. 5.

where $R_o = \frac{Z_o}{2\pi} \ell n \frac{b}{a}$ and $R_1 = \frac{Z_o}{2\pi} \ell n \frac{d}{a}$ are the characteristic impedances of the transmission lines involved.

For the two-ports β and δ we use reference impedances for the S-matrix equal to the characteristic impedances of the transmission lines attached to them as this gives simple S-matrices for the transmission lines. We get:

$$S_{11}^{\beta} = S_{22}^{\delta} = \frac{R_1 - R_o + jX_L}{R_1 + R_o + jX_L}$$

$$S_{22}^{\beta} = S_{11}^{\delta} = \frac{-(R_1 - R_o) + jX_L}{R_1 + R_o + jX_L}$$

$$S_{12}^{\beta} = S_{21}^{\beta} = S_{12}^{\delta} = S_{21}^{\delta} = \frac{2\sqrt{R_o R_1}}{R_1 + R_o + jX_L} \quad .$$

From the cascading formulaes (appendix B) we obtain after some manipulations:

$$S_{11}^{\alpha\beta\gamma\delta\epsilon} = \frac{e^{-jk(2\ell_1+\ell_2)}}{2R_o+\Delta R+jX_L}\left\{\frac{(\Delta R+jX_L)e^{jk\ell_2}-(\Delta R-jX_L)[1-2jX_L/(2R_o+\Delta R+jX_L)]e^{-jk\ell_2}}{1-[(\Delta R-jX_L)/(2R_o+\Delta R+jX)]^2 e^{-jk2\ell_2}}\right\} \tag{14}$$

$$\begin{aligned} S_{21}^{\alpha\beta\gamma\delta\epsilon} &= S_{12}^{\alpha\beta\gamma\delta\epsilon} \\ &= e^{-jk\ell}\left\{1 - \frac{4R_1 jX_L+(\Delta R-jX_L)^2\ (1-e^{-jk2\ell_2})}{(2R_o+\Delta R+jX)^2\ [1-(\Delta R-jX_L)^2/(2R_o+\Delta R+jX_L)^2 e^{-jk2\ell_2}]}\right\} \end{aligned} \tag{15}$$

where $\Delta R = R_1 - R_o$.

S_{22} is identical to S_{11} except that ℓ_1 is replaced by ℓ_3.

We will first consider the reflection coefficient. If the step height is relatively small, $\Delta R \ll R_o$ and $X_L \ll R_o$, and we retain only lowest order terms in $\Delta R/R_o$ and X_L/R_o:

$$\begin{aligned} S_{11}^{\alpha\beta\gamma\delta\epsilon} &\simeq \frac{e^{-jk(2\ell_1+\ell_2)}}{2R_o}\left\{2\Delta R\ \frac{e^{jk\ell_2} - e^{-jk\ell_2}}{2} + j2X_L\ \frac{e^{jk\ell_2} + e^{-jk\ell_2}}{2}\right\} \\ &= \frac{e^{-jk(2\ell_1+\ell_2)}}{2R_o}\left\{ j2\Delta R \sin k\ell_2 + j2X_L \cos k\ell_2 \right\} \end{aligned} \tag{16}$$

where the phase term $\exp[-jk(2\ell_1 + \ell_2)]$ corresponds to the center of the cavity. We get the beam coupling impedance term $j2X_L$ with the cosine factor as in Eq. (11) plus an apparent additional inductive term associated with the coaxial wire setup and caused by the change in characteristic impedance, ΔR. For low frequencies

$$k\ell_2 = \frac{\omega\ell_2}{c} \ll 1 \quad , \tag{17}$$

and Eq. (16) reduces further to

$$S_{11}^{\alpha\beta\gamma\delta\epsilon} \simeq \frac{e^{-jk(2\ell_1+\ell_2)}}{2R_o} \left\{ j\omega \left[\frac{2\Delta R\ell_2}{c} \right] + j\omega[2L_s] \right\} \tag{18}$$

from which it is seen that the additional measured "inductance" is proportional to the step separation ℓ_2.

From Keil and Zotter[9] we get the low frequency impedance of the double step ($\ell_2 \gg b$):

$$\frac{Z}{n} = \frac{j2X_L}{n} = j\,\frac{0.241\;Z_o}{R}\,\frac{(d-b)^2}{d+0.412b}$$

$$\simeq j\;0.171\,\frac{Z_o(d-b)^2}{Rb} \qquad \text{for} \quad d-b \ll b \quad .$$

If $n = \omega R/c$ is eliminated:

$$Z \simeq j\omega\,\mu_o 0.171\,\frac{(d-b)^2}{b}$$

and the inductance of two steps is thus:

$$2L_s \simeq \mu_o 0.171 \frac{(d - b)^2}{b} \quad . \tag{19}$$

The change in characteristic impedance is

$$\Delta R = R_1 - R_o = \frac{Z_o}{2\pi} \ln \frac{d}{b} \simeq \frac{Z_o}{2\pi} \frac{d - b}{b} \tag{20}$$

so the additional measured "inductance" L_r becomes

$$L_r = \frac{2\Delta R \ell_2}{c} = \mu_o \frac{\ell_2}{\pi} \ln \frac{d}{b} \simeq \mu_o \frac{(d - b)\ell_2}{\pi b}$$

which becomes equal to the step inductance $2L_s$ for

$$\ell_2 = 0.54 \times (d - b) \; . \tag{21}$$

It follows that we get a factor 2 error in the measured result already for a cavity length which is about half the step height.[11]

For the example of the ISABELLE prototype pickup and clearing electrode box with

$$\ell_2 = 31 \text{ cm}, \; d = 5.8 \text{ cm}, \; b = 3.6 \text{ cm}$$

the steps can be considered reasonably independent and the measured inductance as given by Eq. (18) will be 30.4 times larger than the double step inductance $2L_s$.

The much too large inductance obtained from "black-box" interpretation of the reflection coefficient agrees with measurements.[12] As the structure in this case is simple and well known, one can, however, in principle take the change in characteristic impedance properly into account.[12]

11. The formulae (19) for the double step inductance is not even valid for lengths that short.
12. S. Giordano, private communication.

But as the correction term in this example is about 30 times larger than the coupling impedance term this will drown in measurement inaccuracies. Furthermore, the exact formulae (14) will have to be used as the correction term must be precisely known. Finally, the reflection method cannot be used for more complex structures, for which the correction term is unknown.

From Eq. (16) the origin of this error is obvious. In addition to the reflections from the step inductance which couples to the beam there are two reflections with opposite signs except for a small phase shift $\exp(-jk2\ell_2)$. They originate from the change in impedance ΔR (which in the relativistic limit $\beta = 1$ does not couple to the beam). Added they appear shifted 90^o and proportional to frequency and look therefore exactly like an additional inductance. Only for very short cavities the correct result is expected as the cavity now can be considered as a lumped element associated with a short gap. For a given length ℓ_2 the relative error increases as the step height decreases because the undesired reflections are proportional to the step height, Eq. (20), while the step inductance is proportional to the step height squared, Eq. (19). Notice that the relative error for reflection measurements is <u>independent of the radius</u> of the central conductor.

Consider now the transmission, Eq. (15). If the step height is relatively small, $X_L \ll \Delta R \ll R_o$, and we retain only lowest order terms in $\Delta R/R_o$ and X_L/R_o:

$$S_{21}^{\alpha\beta\gamma\delta\varepsilon} \simeq e^{-jk\ell} \left\{ 1 - j \frac{2X_L}{2R_o} - \left(\frac{\Delta R}{2R_o} \right)^2 + \left(\frac{\Delta R}{2R_o} \right)^2 e^{-jk2\ell_2} \right\} . \qquad (22)$$

For low frequencies, the approximation of Eq. (17) is valid and we get:

$$1 - e^{-jk2\ell_2} \simeq j2k\ell_2$$

Thus Eq. (22) reduces to:

$$S_{21}^{\alpha\beta\gamma\delta\epsilon} \simeq e^{-jk\ell}\left\{1 - j\,\frac{2X_L}{2R_o} - j\,\frac{(\Delta R)^2 k\ell_2/R_o}{2R_o}\right\}$$

$$= e^{-jk\ell}\left\{1 - j\,\frac{\omega(2L_s)}{2R_o} - j\,\frac{\omega[(\Delta R)^2\ell_2/R_o c]}{2R_o}\right\}. \tag{23}$$

It is seen that the effect of the changes in characteristic impedance again shows up as an apparently increased inductance, although generally much smaller than in the reflection case.

The additional measured inductance L_t has the value

$$L_t \simeq \frac{(\Delta R)^2\ell_2}{R_o c} = \frac{\mu_o}{2\pi\ell n(b/a)}\,\frac{(d-b)^2\ell_2}{b^2} \tag{24}$$

which becomes equal to the step inductance $2L_s$ for

$$\ell_2 \simeq 1.07\ b\ \ell n(b/a) \tag{25}$$

which generally is much longer than ℓ_2 in Eq. (21). The error can in principle be made as small as desired by choosing a sufficiently thin wire as $L_t \rightarrow 0$ for $a \rightarrow 0$, but the error does not decrease very rapidly due to the logarithm function and there are limits to how thin a wire can be used.

For the ISABELLE pickup box previously considered and assuming:

$$b/a = 10^3,\ R_o = 60\ \ell n\ b/a = 415\ \Omega \tag{26}$$

the correction term is about equal to the step inductance; a substantial improvement over the factor 30 in the reflection case.

The assumption $X_L \ll \Delta R$ used for the approximation, Eq. (23), can be shown to be equivalent to:

$$\lambda = \frac{2\pi c}{\omega} \gg 3.36\ (d - b).$$

which always can be considered to be fulfilled for frequencies for which our model is valid.

The four terms in Eq. (22) can be explained as follows. The first term is the unperturbed transmission, the second is the transmission loss from the two step inductances, the third is the transmission loss from two changes in impedance, which is partially compensated by the fourth term, which represents the twice reflected signal and which, therefore, is shifted by $2k\ell_2$ in phase. As either two transmissions or two reflections are involved in the undesired terms, the relative change in impedance enters squared, and the undesired terms become second order terms, which can be made small by choosing a high characteristic impedance or a thin center wire.

It is worth pointing out that since both desired and undesired terms are proportional to the step height squared, the relative error is independent of the step height.

The importance of choosing the proper wire diameter was already pointed out by Sands and Rees[5], although their argumentation was different. Requiring that the transmitted pulse is only slightly modified relative to the incident pulse is the same as requiring that the transmission-coefficient does not deviate too much from unity within the frequency spectrum of the pulse; a requirement that was shown necessary for the validity of Eq. (12): $|Z_w| \ll R_o$. In the present case $X_L \ll R_o$ is a <u>necessary</u> condition for a small error, but not always a <u>sufficient</u> condition. The necessary condition can always be satisfied at low frequencies. The requirement

$$\ell_2 \ll b\ \ln b/a \qquad \text{or} \qquad a \ll b e^{-\ell_2/b} \tag{27}$$

is often a stricter limitation than $X_L \ll R_o$ for the choice of wire size. There are limitations to how thin the wire may be chosen.[5] First, there are limits to how thin a wire can be safely handled. Second, the losses may limit the wire dimension although a reference line could eliminate this to a large extent. But the most serious limitation is probably loss of measurement accuracy. As the undesired terms are reduced relative to the desired term by choosing a higher R_o and thus reducing the systematic error, the desired term is simultaneously reduced compared to unity [Eqs. (22) and (23)] and the small change in transmission may drown in inaccuracies and random errors in the measuring setup.

The low frequency requirement, Eq. (17), is not necessary when measuring transmissions. The perturbing terms, Eq. (22), will not appear purely inductive though, but if Eq. (27) is satisfied the perturbing terms will remain small.

For higher frequencies (above waveguide cutoff) the model used is no longer valid, and a field calculation and comparison between the conductor/pipe and beam pipe systems will be required. Notwithstanding, the transmission method is expected to be superior to the reflection method as indicated by the results at low frequencies.

Summarizing the above discussion, we suggest the transmission setup shown in Fig. 6 to measure the longitudinal coupling impedance of wave guide discontinuities and typical accelerator components.

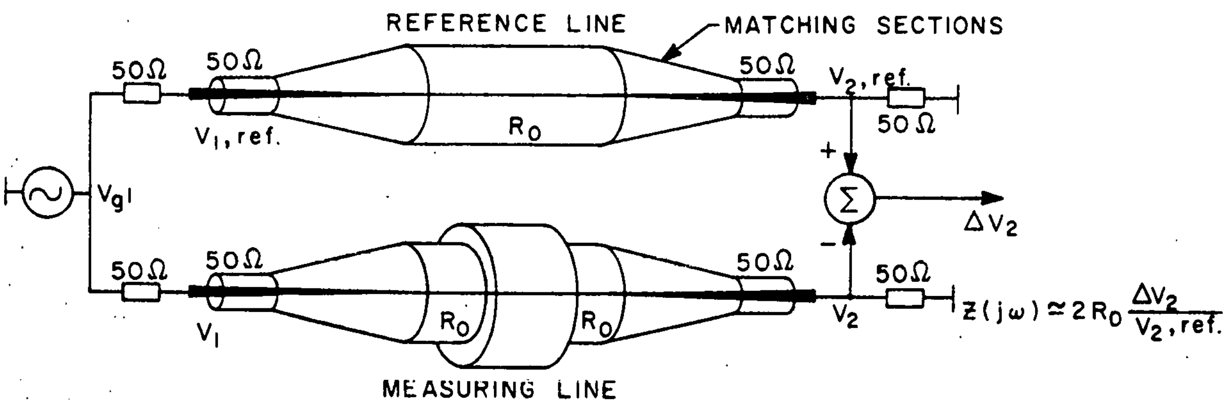

Fig. 6

Tapered matching sections are required to change the impedance from 50 Ω, for which standard high frequency components are available, to the much higher R_o required in the measuring line. Imperfections in the matching sections are cancelled out by identical matching sections in the reference line. The two transmitted signals are then subtracted in a hybrid junction, and the difference relative to the reference gives the impedance relative to $2R_o$. It is important that the two branches of the generator are well isolated, as the reflected signal often is much larger than the change in transmission, and the reflected signal must not perturb the reference signal.

IV. ACKNOWLEDGEMENTS

The authors acknowledge the many and fruitful discussions with S. Giordano. These have contributed substantially to the ideas presented in this report.

APPENDIX A

The Scattering Matrix - Definitions

The scattering matrix is a convenient way to describe a two-port in a transmission line environment.[13]

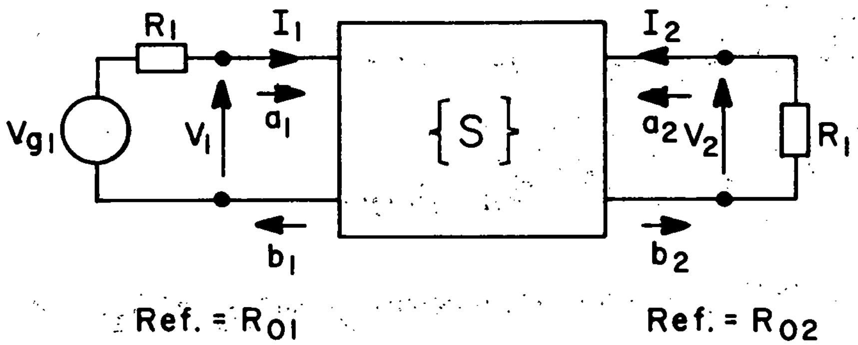

Fig. 7

Incident (a's) and reflected (b's) wave parameters are defined by:

$$a_1 = \tfrac{1}{2}\left(\frac{V_1}{\sqrt{R_{01}}} + \sqrt{R_{01}}\, I_1\right) \qquad b_1 = \tfrac{1}{2}\left(\frac{V_1}{\sqrt{R_{01}}} - \sqrt{R_{01}}\, I_1\right)$$

$$a_2 = \tfrac{1}{2}\left(\frac{V_2}{\sqrt{R_{02}}} + \sqrt{R_{02}}\, I_2\right) \qquad b_2 = \tfrac{1}{2}\left(\frac{V_2}{\sqrt{R_{02}}} - \sqrt{R_{02}}\, I_2\right) \qquad \text{(A1)}$$

where R_{01} and R_{02} are arbitrary, positive, real reference impedances. The scattering parameters S_{ij} are defined by:

$$\begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} = \begin{Bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{Bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} . \qquad \text{(A2)}$$

13. F.F. Kuo, Network Analysis and Synthesis, (John Wiley & Sons, New York, 1962).

The <u>input reflection coefficient</u> S_{11} is thus given by:

$$S_{11} = \frac{b_1}{a_1}\bigg|_{a_2=0} = \frac{Z_1 - R_{01}}{Z_1 + R_{01}}\bigg|_{R_2=R_{02}} \tag{A3}$$

where Z_1 is the input impedance with the output port terminated in the reference impedance.

The <u>forward transmission coefficient</u> S_{21} is given by:

$$S_{21} = \frac{b_2}{a_1}\bigg|_{a_2=0} = \frac{2V_2}{V_{g1}}\sqrt{\frac{R_{01}}{R_{02}}}\bigg|_{R_1=R_{01},R_2=R_{02}} \tag{A4}$$

where $R_1 = R_{01}$ gives a simple relationship between V_{g1} and a_1, and $R_2 = R_{02}$ implies $a_2 = 0$.

By reversing input and output ports we get the <u>output reflection coefficient</u>:

$$S_{22} = \frac{b_2}{a_2}\bigg|_{a_1=0} = \frac{Z_2 - R_{02}}{Z_2 + R_{02}}\bigg|_{R_1=R_{01}} \tag{A5}$$

and the <u>reverse transmission coefficient</u>:

$$S_{12} = \frac{b_1}{a_2}\bigg|_{a_1=0} = \frac{2V_1}{V_{g2}}\sqrt{\frac{R_{02}}{R_{01}}}\bigg|_{R_1=R_{01},R_2=R_{02}} . \tag{A6}$$

If the two-port is a lossless transmission line it is convenient to choose the reference impedances equal to the characteristic impedance: $R_{01} = R_{02} = Z_0$. The wave parameters can then be related to incident and reflected voltages. The scattering matrix is then:

$$\{S\} = \begin{Bmatrix} 0 & e^{-jk\ell} \\ e^{-jk\ell} & 0 \end{Bmatrix} \tag{A7}$$

where jk is the propagation constant and ℓ the line length.

APPENDIX B

The S-matrix for Cascaded Two-Ports

As the S-matrix gives reflected waves in terms of incident waves, the S-matrix for cascaded two-ports cannot be obtained by simple matrix multiplication.

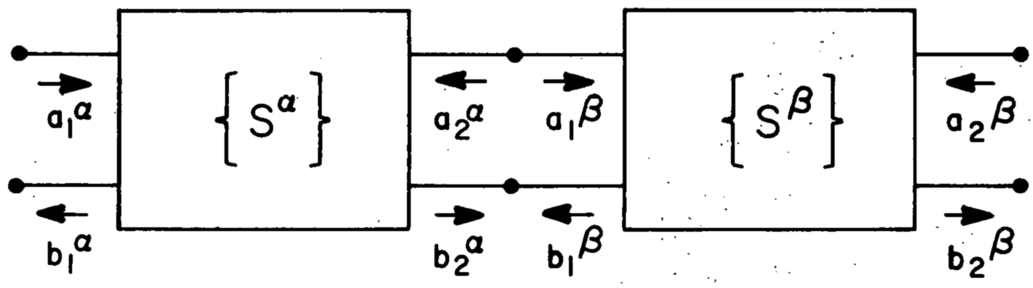

Fig. 8

$$\begin{Bmatrix} b_1^\alpha \\ b_2^\alpha \end{Bmatrix} = \begin{Bmatrix} S_{11}^\alpha & S_{12}^\alpha \\ S_{21}^\alpha & S_{22}^\alpha \end{Bmatrix} \begin{Bmatrix} a_1^\alpha \\ a_2^\alpha \end{Bmatrix} \tag{B1}$$

$$\begin{Bmatrix} b_1^\beta \\ b_2^\beta \end{Bmatrix} = \begin{Bmatrix} S_{11}^\beta & S_{12}^\beta \\ S_{21}^\beta & S_{22}^\beta \end{Bmatrix} \begin{Bmatrix} a_1^\beta \\ a_2^\beta \end{Bmatrix} \tag{B2}$$

If the reference impedances at the junction point are chosen identical, $R_{02}^\alpha = R_{01}^\beta$, we have

$$a_1^\beta = b_2^\alpha \quad \text{and} \quad a_2^\alpha = b_1^\beta ,$$

and these four parameters can be eliminated in (B1) and (B2):

$$b_1^\alpha = \left(S_{11}^\alpha + \frac{S_{12}^\alpha S_{11}^\beta S_{21}^\alpha}{1 - S_{11}^\beta S_{22}^\alpha} \right) a_1^\alpha + \frac{S_{12}^\alpha S_{12}^\beta}{1 - S_{11}^\beta S_{22}^\alpha} a_2^\beta$$

$$b_2^\beta = \frac{S_{21}^\beta S_{21}^\alpha}{1 - S_{11}^\beta S_{22}^\alpha} a_1^\alpha + \left(S_{22}^\beta + \frac{S_{21}^\beta S_{22}^\alpha S_{12}^\beta}{1 - S_{11}^\beta S_{22}^\alpha} \right) a_2^\beta$$

from which we get the S-parameters of the cascaded two-ports:

$$
\begin{aligned}
S_{11}^{\alpha\beta} &= S_{11}^{\alpha} + \frac{S_{12}^{\alpha} S_{11}^{\beta} S_{21}^{\alpha}}{1 - S_{11}^{\beta} S_{22}^{\alpha}} \\
S_{12}^{\alpha\beta} &= \frac{S_{12}^{\alpha} S_{12}^{\beta}}{1 - S_{11}^{\beta} S_{22}^{\alpha}} \\
S_{21}^{\alpha\beta} &= \frac{S_{21}^{\beta} S_{21}^{\alpha}}{1 - S_{11}^{\beta} S_{22}^{\alpha}} \\
S_{22}^{\alpha\beta} &= S_{22}^{\beta} + \frac{S_{21}^{\beta} S_{22}^{\alpha} S_{12}^{\beta}}{1 - S_{11}^{\beta} S_{22}^{\alpha}} .
\end{aligned}
\qquad \text{(B3)}
$$

Distribution: External