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Codistributive Elements of the Lattice of Semigroup Varieties

B. M. Vernikov[1*]

[1] *Ural State University, pr. Lenina 51, Ekaterinburg, 620000 Russia*

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Abstract—We prove that if a semigroup variety is a codistributive element of the lattice **SEM** of all semigroup varieties, then it either coincides with the variety of all semigroups or is a variety of semigroups with completely regular square. We completely classify strongly permutable varieties that are codistributive elements of **SEM**. We prove that a semigroup variety is a costandard element of the lattice **SEM** if and only if it is a neutral element of this lattice. In view of results obtained earlier, this gives a complete description of costandard elements of the lattice **SEM**.

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1. INTRODUCTION AND STATEMENTS OF THE MAIN RESULTS

The lattice of all semigroup varieties has been actively studied for more than four decades. The recent review [1] is dedicated to a systematic summary of the state-of-the-art in this area.

In the theory of lattices one pays much attention to the study of special elements of various types. Let us recall definitions of those of them which will be used below. An element x of a lattice $\langle L; \vee, \wedge\rangle$ is said to be *distributive*, if

$$\forall y, z \in L:\ x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z);$$

standard, if

$$\forall y, z \in L:\ (x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z);$$

modular, if

$$\forall y, z \in L:\ y \leq z \longrightarrow (x \vee y) \wedge z = (x \wedge z) \vee y;$$

upper-modular, if

$$\forall y, z \in L:\ y \leq x \longrightarrow x \wedge (y \vee z) = y \vee (x \wedge z);$$

neutral, if for any elements $y, z \in L$ elements x, y, and z generate a distributive sublattice in L.

Codistributive, *costandard*, and *lower-modular* elements are defined as dual to distributive, standard, and upper-modular ones, respectively.

See, for example, the monograph, [2], § III.2 for a wide information about [co]distributive, [co]standard, and neutral elements, which shows the natural character and importance of their study. It is evident that any neutral element is standard and costandard, each [co]standard element is modular, and each [co]distributive element is lower-modular [upper-modular]. It is also well known that each [co]standard element is [co]distributive (the proof of this fact can be found, for example, in [2], theorem III.2.3).

For brevity we will use adjectives that denote types of special elements of lattices for semigroup varieties representing elements of the corresponding types in the lattice of all semigroup varieties. Just

*E-mail: boris.vernikov@usu.ru.