Cognitive Relay Networks With Multiple Primary Transceivers Under Spectrum-Sharing

Trung Q. Duong, Phee Lep Yeoh, Vo Nguyen Quoc Bao, Maged Elkashlan, and Nan Yang

***Abstract*—We examine the impact of multiple primary transmitters and receivers (PU-TxRx) on the outage performance of cognitive decode-and-forward relay networks. In such a joint relaying/spectrum-sharing arrangement, we address fundamental questions concerning three key power constraints: 1) maximum transmit power at the secondary transmitter (SU-Tx), 2) peak interference power at the primary receivers (PU-Rx), and 3) interference power at SU-Rx caused by the primary transmitter (PU-Tx). Our answers to these are given in new analytical expressions for the exact and asymptotic outage probability of the secondary relay network. Based on our asymptotic expressions, important design insights into the impact of primary transceivers on the performance of cognitive relay networks is reached. We have shown that zero diversity order is attained when the peak interference power at the PU-Rx is independent of the maximum transmit power at the SU-TX.**

***Index Terms*—Cognitive radio, DF relays, spectrum-sharing.**

I. Introduction

Spectrum-sharing (SS) in cognitive networks (CNs) is emerging as a promising frontier to overcome the inefficient allocation and utilization of scarce radio frequency spectrum [1]–[3]. Recently, it was shown that introducing a relay in CNs can boost the outage performance relative to its direct link counterpart [4]. Assuming the secondary transmitter (SU-Tx) is not power limited (i.e., $\mathcal{P}_t \to \infty$), it was demonstrated in [5] that a relay network under SS preserves the full diversity order and significantly outperforms the direct link counterpart. The impact of the primary transmitter (PU-Tx) on the secondary network (SN) was examined in [6], where the interference power at secondary receiver (SU-Rx) caused by the PU-Tx transmission, P_I, and the peak interference power at the primary receiver (PU-Rx), $\mathcal{I}_p$, are jointly assessed. Previous works on cognitive relay networks have considered a single PU-Tx and/or a single PU-Rx. Very recently, the performance of cognitive relay network with multiple PU-Rxs has been addressed in [7]. However, the work in [7] has not considered the effect of interference at the SU-Rx P_I and has neglected the impact of $\mathcal{P}_t$.

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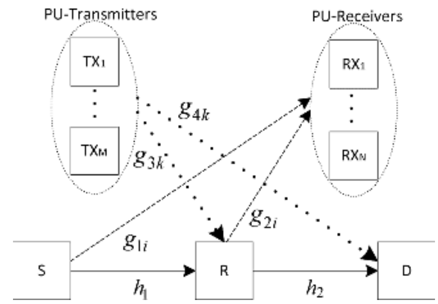

Fig. 1. A cognitive relay network with multiple PU-TxRx.

In this paper, we present a general framework for cognitive relay networks with multiple primary transceivers (PU-TxRx). This is a realistic consideration in large-scale CNs where the SUs transmit over long distances and may cause interference to multiple PUs at any given time [8]. For such an arrangement, we address fundamental questions surrounding the joint impact of three key power constraints on the SN performance, namely, 1) maximum transmit power at the SU-Tx, $\mathcal{P}_t$, 2) peak interference power at the PU-Rx, $\mathcal{I}_p$, and 3) interference power at the SU-Rx P_I incurred by the PU-Tx. In doing so, we derive new exact and asymptotic expressions that reveal important design insights into the behavior of the outage probability (OP). We confirm that the primary network (PN) has a major impact on the SN performance. Specifically, an error-floor OP exhibits in the high $\mathcal{P}_t$ regime. Finally, Monte-Carlo simulations are provided to validate our analysis.

II. System and Channel Models

Consider a CN as shown in Fig. 1 where the SN is allowed to share the same spectrum band licensed to the PN. The PN consists of M PU-Tx and N PU-Rxs, whereas the SN consists of a secondary source (S), a secondary decode-and-forward (DF) relay (R), and a secondary destination (D). The peak interference power constraint at any PU-Rx is denoted by $\mathcal{I}_p$, which is fixed as a predefined constant to guarantee that the secondary signals do not violate the PU-Rx.

The communication in the SN occurs over a dual-hop transmission consisting of two distinct phases. In the first phase, S transmits its data to R. Then, R decodes and forwards the resulting signal to D. We consider that the interference at the ℓ-th PU-Rx inflicted by S and R should not exceed a given maximum tolerable level $\mathcal{I}_p$. Further to satisfying $\mathcal{I}_p$, S and R are power-limited terminals such that the maximum allowable transmit power is $\mathcal{P}_t$. As such, the transmit powers at S and R are constrained according to [9]

$$P_{\mathsf{S}} = \min\left(\frac{\mathcal{I}_p}{\max\limits_{i=1,\dots,N} |g_{1i}|^2}, \mathcal{P}_t\right),$$

$$P_{\mathsf{R}} = \min\left(\frac{\mathcal{I}_p}{\max\limits_{i=1,\ldots,N} |g_{2i}|^2}, \mathcal{P}_t\right), \tag{1}$$

where g_{1i} and $g_{2i}, i \in \{1, \ldots N\}$, are the channel coefficients of the S $\rightarrow$PU-Rx and R $\rightarrow$ PU-Rx links, respectively. Due to the co-existence of transmissions from the PU-Tx to the PU-Rx, the received signals at R and D are impacted by interference from the PU-Tx denoted by P_I. As such, the instantaneous received signal-to-interference ratio (SIR)[1] at R and D are given by

$$\gamma_1 = \frac{P_{\mathsf{S}}|h_1|^2}{\sum_{k=1}^{M} \mathsf{P}_I|g_{3k}|^2}, \quad \gamma_2 = \frac{P_{\mathsf{R}}|h_2|^2}{\sum_{k=1}^{M} \mathsf{P}_I|g_{4k}|^2} \tag{2}$$

where h_1 and h_2 are the channel coefficients of the S $\rightarrow$ R and R $\rightarrow$ D links, respectively. The channel coefficients of the PU-Tx $\rightarrow$ R and PU-Tx $\rightarrow$ D links are denoted by g_{3k} and $g_{4k}, k \in \{1, \ldots, M\}$, respectively. Substituting (1) into (2), we rewrite the SIRs in a more compact form as

$$\gamma_1 = \min\left(\frac{\mathcal{I}_p}{Y_1}, \mathcal{P}_t\right)\frac{X_1}{Z_1}, \quad \gamma_2 = \min\left(\frac{\mathcal{I}_p}{Y_2}, \mathcal{P}_t\right)\frac{X_2}{Z_2} \tag{3}$$

where $X_1 = |h_1|^2$, $X_2 = |h_2|^2$, $Y_1 = \max\limits_{i=1,\ldots,N} |g_{1i}|^2$, $Y_2 = \max\limits_{i=1,\ldots,N} |g_{2i}|^2$, $Z_1 = \sum_{k=1}^{M} \mathsf{P}_I|g_{3k}|^2$, and $Z_2 = \sum_{k=1}^{M} \mathsf{P}_I|g_{4k}|^2$. We assume flat Rayleigh fading in all links such that $|h_1|^2$, $|h_2|^2$, $|g_{1i}|^2$, $|g_{2i}|^2$, $|g_{3k}|^2$, and $|g_{4k}|^2$ are exponentially distributed random variables (RVs) with parameters $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$, and β_4 respectively. Finally, the end-to-end SIR at the secondary destination is $\gamma_{\mathrm{S}} = \min(\gamma_1, \gamma_2)$.

III. Outage Probability

We derive the exact and asymptotic OP of the SN impacted by interference from multiple PU-TxRx. The outage probability, i.e., the probability that the end-to-end SIR γ_{S} falls below a certain threshold γ_{th}, is expressed as

$$P_{\mathsf{out}} = F_{\gamma 1}(\gamma_{\mathsf{th}}) + F_{\gamma 2}(\gamma_{\mathsf{th}}) - F_{\gamma 1}(\gamma_{\mathsf{th}})F_{\gamma 2}(\gamma_{\mathsf{th}}) \tag{4}$$

where $F_{\gamma_1}(\cdot)$ and $F_{\gamma_2}(\cdot)$ are the cumulative distribution functions (CDFs) of γ_1 and γ_2, respectively. As such, our aim is to derive the CDFs of γ_1 and γ_2 from (3), respectively.

A. Exact Analysis

Lemma 1: The exact CDF of γ_1 is shown in (5) at the bottom of the page, where $\mu = \mathcal{I}_p/\mathcal{P}_t, \epsilon_1 = F_{Y_1}(\mu) = (1 - e^{-\mu/\beta_1})^N$, and $Ei(\cdot)$ is the exponential integral function [10, Eq. (3.353.5)].

[1]In this work, we focused on the interference-limited scenario where the interference power from the primary user is dominant relative to the noise, and therefore noise effects can be neglected.

Proof: The CDF of γ_1 conditioned on Z_1, $F_{\gamma_1|Z_1}(\gamma) = \Pr(\gamma_1 < \gamma|Z_1)$ is written as

$$F_{\gamma 1|Z_1}(\gamma) = \underbrace{\Pr\left(\frac{\mathcal{I}_p X_1}{Y_1} < Z_1\gamma, Y_1 \geq \frac{\mathcal{I}_p}{\mathcal{P}_t}\right)}_{\mathcal{J}_1(Z_1)} + \underbrace{\Pr\left(\mathcal{P}_t X_1 < Z_1\gamma, Y_1 < \frac{\mathcal{I}_p}{\mathcal{P}_t}\right)}_{\mathcal{J}_2(Z_1)}. \tag{6}$$

The first summand in (6) is derived as

$$\mathcal{J}_1(Z_1) = \int_{\mu}^{\infty} f_{Y1}(y_1) \int_0^{\frac{Z_1\gamma}{\mathcal{I}_p}y_1} f_{X_1}(x_1)dx_1dy_1 = \int_{\mu}^{\infty} F_{X_1}\left(\frac{Z_1\gamma}{\mathcal{I}_p}y_1\right) f_{Y_1}(y_1)dy_1. \tag{7}$$

where $f_{Y_1}(\cdot)$ denotes the probability density function (PDF). Recall that Y_1 is the maximum of N exponentially distributed RVs. As such, the PDF and CDF of Y_1 are given by

$$F_{Y_1}(y_1) = \left(1 - e^{\frac{-y_1}{\beta_1}}\right)^N, \tag{8}$$

$$f_{Y_1}(y_1) = \frac{N}{\beta_1}\sum_{i=0}^{N-1}\binom{N-1}{i}(-1)^i \exp\left[-\left(\frac{i+1}{\beta_1}\right)y_1\right]. \tag{9}$$

Substituting (9) into (7) and applying straightforward mathematical operations, we derive $\mathcal{J}_1(Z_1)$ as

$$\mathcal{J}_1(Z_1) = 1 - F_{Y_1}(\mu) - \frac{N}{\beta_1}\sum_{i=0}^{N-1}\binom{N-1}{i}(-1)^i \times \exp\left[-\mu\left(\frac{i+1}{\beta_1} + \frac{\gamma Z_1}{\mathcal{I}_p\alpha_1}\right)\right]\left(\frac{i+1}{\beta_1} + \frac{\gamma Z_1}{\mathcal{I}_p\alpha_1}\right)^{-1}. \tag{10}$$

Next, the second summand in (6) is derived as

$$\mathcal{J}_2(Z_1) = F_{Y_1}(\mu)F_{X_1}\left(\frac{\gamma}{\mathcal{P}_t}Z_1\right) \tag{11}$$

due to the fact that X_1 and Y_1 are statistically independent.

The unconditional CDF of γ_1 is derived based on (10) and (11) by taking the expectation of (6) with respect to Z_1 according to

$$F_{\gamma_1}(\gamma) = \mathbb{E}_{Z_1}\left\{F_{\gamma_1|Z_1}(\gamma)\right\} = \mathcal{J}_3 + \mathcal{J}_4 \tag{12}$$

where $\mathcal{J}_3 = \mathbb{E}_{Z_1}\{\mathcal{J}_1(Z_1)\}$ and $\mathcal{J}_4 = \mathbb{E}_{Z_1}\{\mathcal{J}_2(Z_1)\}$. To calculate $\mathcal{J}_3$ and $\mathcal{J}_4$, the PDF of Z_1 is required. Since Z_1 is the sum

$$F_{\gamma 1}(\gamma) = 1 - \epsilon_1\left(1 + \frac{\mathsf{P}_I\beta_3\gamma}{\mathcal{P}_t\alpha_1}\right)^{-M} - \frac{\mathcal{I}_p N\alpha_1}{(\mathcal{I}_p\beta_3)^M\Gamma(M)\beta_1\gamma}\sum_{i=0}^{N-1}\binom{N-1}{i}(-1)^i e^{\left(-\frac{\mu(i+1)}{\beta_1}\right)}\left[(-1)^{M-2}\left(\frac{(i+1)\mathcal{I}_p\alpha_1}{\beta_1\gamma}\right)^{M-1}\right.$$
$$\left. \times e^{\left(\frac{(i+1)}{\beta_1}\left(\mu + \frac{\mathcal{I}_p\alpha_1}{\mathsf{P}_I\beta_3\gamma}\right)\right)} Ei\left(-\left(\frac{i+1}{\beta_1}\right)\left(\mu + \frac{\mathcal{I}_p\alpha_1}{\mathsf{P}_I\beta_3\gamma}\right)\right) + \sum_{k=1}^{M-1}\Gamma(k)\left(\frac{-(i+1)\mathcal{I}_p\alpha_1}{\beta_1\gamma}\right)^{M-1-k}\left(\frac{\mu\gamma}{\mathcal{I}_p\alpha_1} + \frac{1}{\mathsf{P}_I\beta_3}\right)^{-k}\right] \tag{5}$$

of M exponential RVs with parameter $\mathsf{P}_I\beta_3$, it is a chi-square RV with PDF given by

$$f_{Z_1}(z_1) = \frac{z_1^{M-1}\exp\left(-\frac{z_1}{\mathsf{P}_I\beta_3}\right)}{\Gamma(M)(\mathsf{P}_I\beta_3)^M}. \quad (13)$$

Utilizing (10) and (13), $\mathcal{J}_3$ is derived as

$$\mathcal{J}_3 = 1 - F_{Y_1}(\mu) - \frac{N_{\alpha_1}\mathcal{I}_p}{\Gamma(M)\beta_1(\mathcal{I}_p)^M} \times \sum_{i=0}^{N-1}\binom{N-1}{i}(-1)^i e^{-\frac{\mu(i+1)}{\beta_1}}\mathcal{J}_{31} \quad (14)$$

where $\mathcal{J}_{31}$ is an integral given by

$$\mathcal{J}_{31} = \int_0^\infty z_1^{M-1}e^{-\left(\frac{\mu\gamma}{\mathcal{I}_p\alpha_1}+\frac{1}{\mathsf{P}_I\beta_3}\right)z_1}\left[\frac{(i+1)\mathcal{I}_p\alpha_1}{\beta_1\gamma}+z_1\right]^{-1}dz_1. \quad (15)$$

An analytical solution for $\mathcal{J}_{31}$ is found by applying [10, Eq. (3.353.5)] as follows:

$$\mathcal{J}_{31} = \int_0^\infty z_1^{M-1}e^{-\phi z_1}(z_1+\psi)^{-1}dz_1 = (-1)^{M-2}\psi^{M-1}e^{\psi\phi} \times Ei(-\psi\phi) + \sum_{k=1}^{M-1}\Gamma(k)(-\psi)^{M-1-k}\phi^{-k} \quad (16)$$

where $\phi = (\frac{\mu\gamma}{\mathcal{I}_p\alpha_1}) + (\frac{1}{\mathsf{P}_I\beta_3})$ and $\psi = \frac{(i+1)\mathcal{I}_p\alpha_1}{\beta_1\gamma}$. To compute $\mathcal{I}_4$, we take the expectation of $\mathcal{J}_2(Z_1)$ in (11) with respect to Z_1 which results in

$$\mathcal{J}_4 = F_{Y_1}(\mu) - F_{Y_1}(\mu)\left(1+\frac{\mathsf{P}_I\beta_3\gamma}{\mathcal{P}_t\alpha_1}\right)^{-M}. \quad (17)$$

Substituting (16) into (14) and adding with (17) yields the final expression in (5), thus completing the proof. ■

The exact CDF of γ_2 is easily deduced from *Lemma 1* by substituting the parameters of γ_1 with their respective γ_2 counterparts, i.e., $\alpha_1 \to \alpha_2, \beta_1 \to \beta_2, \beta_3 \to \beta_4$, and $\epsilon_1 \to \epsilon_2$. Based on the CDFs of γ_1 and γ_2, the OP is easily evaluated according to (4).

B. Asymptotic Analysis

Lemma 2: The asymptotic CDF of γ_1 when $\mathcal{I}_p$ is fixed and independent of $\mathcal{P}_t$ is derived as

$$F_{\gamma_1}(\gamma) \overset{\mathcal{P}_t\to\infty}{\approx} \frac{MN\beta_1\beta_3}{\alpha_1\mu}\left[\sum_{i=0}^{N-1}\binom{N-1}{i}\frac{(-1)^i}{(i+1)^2}\right]\left(\frac{\mathsf{P}_{I\gamma}}{\mathcal{I}_p}\right) + \frac{M\epsilon_1\beta_3}{\alpha_1}\left(\frac{\mathsf{P}_{I\gamma}}{\mathcal{P}_t}\right). \quad (18)$$

Proof: By substituting $\mu = \mathcal{I}_p/\mathcal{P}_t$ into the lower limit of the integral in (7) and exchanging the variable $y_1 = (\mathcal{I}_p/\mathcal{P}_t)t$, we obtain

$$\mathcal{J}_1(Z_1) = \int_1^\infty \frac{\mathcal{I}_p}{\mathcal{P}_t}F_{X_1}\left(\frac{Z_1\gamma}{\mathcal{P}_t}t\right)f_{Y_1}\left(\frac{\mathcal{I}_p}{\mathcal{P}_t}t\right)dt. \quad (19)$$

Applying the Taylor series expansion, we approximate the CDF of X_1, i.e., $F_{X_1}(x_1) = 1 - e^{-(x_1/\alpha_1)}$, as $F_{X_1}(x_1) \overset{x_1\to 0}{\approx} = x_1/\alpha_1$, which results in

$$F_{X_1}\left(\frac{\gamma Z_1 t}{\mathcal{P}_t}\right) \overset{\mathcal{P}_t\to\infty}{\approx} \frac{\gamma Z_1 t}{\alpha_1\mathcal{P}_t}. \quad (20)$$

Then by plugging (20) into (7) and after simple mathematical manipulations, we have

$$\mathcal{J}_1(Z_1) \overset{\mathcal{P}_t\to\infty}{\approx} \frac{NZ_1\gamma}{\alpha_1\mathcal{P}_t}\left[\sum_{i=0}^{N-1}\binom{N-1}{i}(-1)^i\right] \times \int_1^\infty \frac{\mathcal{I}_p t}{\beta_1\mathcal{P}_t}\exp\left[-(i+1)\frac{\mathcal{I}_p t}{\beta_1\mathcal{P}_t}\right]dt \quad (21)$$

which then results in

$$\mathcal{J}_1(Z_1) \overset{\mathcal{P}_t\to\infty}{\approx} \frac{N\beta_1 Z_1\gamma}{\alpha_1\mathcal{I}_p}\left[\sum_{i=0}^{N-1}\binom{N-1}{i}\frac{(-1)^i}{(i+1)^2}\right] \times \left[1+\frac{(i+1)\mathcal{I}_p}{\beta_1\mathcal{P}_t}\right]\exp\left[-\frac{(i+1)\mathcal{I}_p}{\beta_1\mathcal{P}_t}\right] \quad (22)$$

The expression in (22) still contains the complicated forms, i.e., $(1+x)e^{-x}$. To further simplify (22), we expand the exponential function $e^{-\frac{(i+1)\mathcal{I}_p}{\beta_1\mathcal{P}_t}}$ into Taylor series and combine with $1+\frac{(i+1)\mathcal{I}_p}{\beta_1\mathcal{P}_t}$, which then yields

$$\mathcal{J}_1(Z_1) \overset{\mathcal{P}_t\to\infty}{\approx} \frac{N\beta_1 Z_1\gamma}{\alpha_1\mathcal{I}_p}\left[\sum_{i=0}^{N-1}\binom{N-1}{i}\frac{(-1)^i}{(i+1)^2}\right] \quad (23)$$

Next, applying (13) and (23), we can approximate $\mathcal{J}_3$ as

$$\mathcal{J}_3 = \mathbb{E}_{Z_1}\{\mathcal{J}_1(Z_1)\} \overset{\mathcal{P}_t\to\infty}{\approx} \frac{MN\beta_1\beta_3}{\alpha_1}\left(\frac{\mathsf{P}_I\gamma}{\mathcal{I}_p}\right)\sum_{i=0}^{N-1}\binom{N-1}{i}\frac{(-1)^i}{(i+1)^2} \quad (24)$$

In addition, by substituting (20) into (11) and using the fact that $\epsilon_1 = F_{Y_1}(\mu)$, $\mathcal{J}_2(Z_1)$ can be approximated as follows:

$$\mathcal{J}_2(Z_1) \overset{\mathcal{P}_t\to\infty}{\approx} \epsilon_1\frac{\gamma Z_1}{\mathcal{P}_t}, \quad (25)$$

which results in

$$\mathcal{J}_4 = \mathbb{E}_{Z_1}\{\mathcal{J}_2(Z_1)\} \overset{\mathcal{P}_t\to\infty}{\approx} \frac{M\epsilon_1\beta_3}{\alpha_1}\left(\frac{\mathsf{P}_I\gamma}{\mathcal{P}_t}\right). \quad (26)$$

Finally, by pulling (24) and (26) together, we get (18), which concludes the proof. ■

The asymptotic CDF of γ_2 then can be obtained in a similar fashion by exchanging the γ_1 parameters with their respective γ_2 counterparts, i.e., $\alpha_1 \to \alpha_2, \beta_1 \to \beta_2, \beta_3 \to \beta_4, \epsilon_1 \to \epsilon_2$. Based on the asymptotic CDFs of γ_1, γ_2 and from (4), after neglecting small terms, the OP of the considered system can be asymptotically approximated as

$$P_{\text{out}} \overset{\mathcal{P}_t\to\infty}{\approx} \Xi MN\left(\frac{\beta_1\beta_3}{\alpha_1}+\frac{\beta_2\beta_4}{\alpha_2}\right)\left(\frac{\mathsf{P}_I\gamma}{\mathcal{I}_p}\right) + M\left(\frac{\epsilon_1\beta_3}{\alpha_1}+\frac{\epsilon_2\beta_4}{\alpha_2}\right)\left(\frac{\mathsf{P}_I\gamma}{\mathcal{P}_t}\right) \quad (27)$$

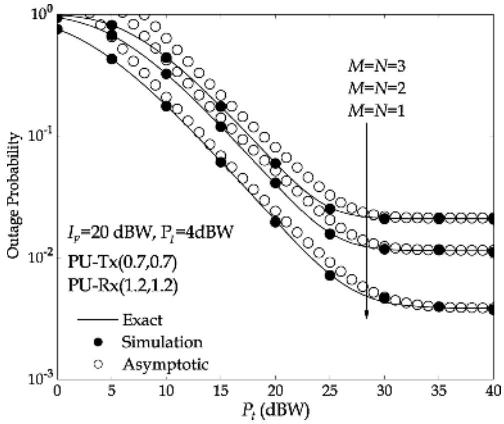

Fig. 2. OP of CNs: Varying the number of PU-TxRx.

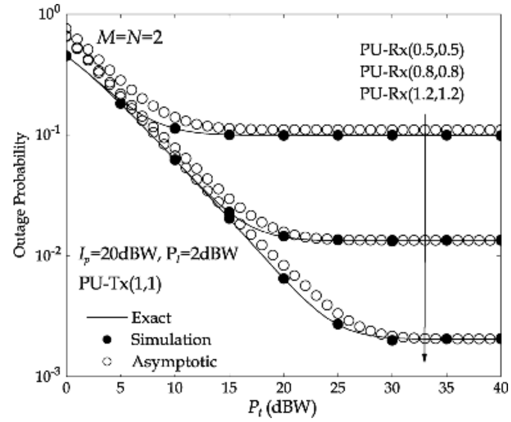

Fig. 3. OP of CNs: Varying the position of PU-Rx.

where $\Xi = \sum_{i=0}^{N-1} \binom{N-1}{i}((-1)^i/(i+1)^2)$.

As can be clearly observed from (27), when the peak interference power $\mathcal{I}_p$ is fixed and independent of the maximum transmit power $\mathcal{P}_t$, the SN exhibits the error-floor OP in the high regime of $\mathcal{P}_t$.

IV. Numerical Results

Numerical examples are provided to illustrate the impact of multiple PU-TxRx on the SN. We assume a two dimensional network topology, where the coordinate of a given node is defined as (x, y). We further assume an exponential decaying path loss, where the channel mean power is proportional to d^{-4} with d being the distance between the transceivers. The PN is assumed to be a cluster of nodes, where all PU-Tx terminals are closely located and as are all PU-Rx terminals. For the system setup, we investigate the co-linear topology for the SN, where the three nodes S, R, and D are placed along the x-axis. Without loss of generality, we assume that S is located at the origin [0,0] and the channel mean power of the link from S to D is normalized to unity. In addition, by assuming that the positions of R and D are respectively given as [1/2,0] and [1,0], we obtain $\alpha_1 = \alpha_2 = 16$. The channel mean powers for the links from PU-Tx and PU-Rx to the SN are defined by the locations, as $\alpha_1 = (\sqrt{d_{R_x}^2 + d_{R_y}^2})^{-4}$ and $\beta_3 = (\sqrt{d_{T_x}^2 + d_{T_y}^2})^{-4}$, where (d_{T_x}, d_{T_y}) and (d_{R_x}, d_{R_y}) are the coordinates of PU-TxRx terminals, respectively. We set the outage threshold $\gamma_{\rm th}$ as 3 dB. In all numerical examples, the maximal interference power $\mathcal{I}_p$ is fixed at 20 dBW.

Fig. 2 shows the impact of the number of PU-TxRx on the CN performance. In this scenario, we choose PU-Tx(0.5,0.5), PU-Rx(1,1), $M = N$, $\mathsf{P}_I = 2$ dBW, and the number of PU-TxRx nodes is varied from one to three. As can be seen from Fig. 2, increasing the number of PU-Tx significantly decreases system performance.

Fig. 3 illustrates the impact of PU-Rx's positions on the cognitive network performance. In this scenario, we set the network parameters as $M = N = 2$, PU-Tx(1,1), and varying the position of PU-Rx from (0.5,0.5) to (0.8,0.8), and (1.2,1.2). As can be observed from Fig. 3, the CN performance is degraded as the SN is closely located to PN and vice versa. In addition, as observed from these two figures, the outage curves result in the zero diversity in high $\mathcal{P}_t$ regime. Importantly, in all representative examples, a good agreement between our analytical results and simulations is shown to validate our derivations.

V. Conclusions

We derived new analytical expressions for the exact and asymptotic OP of the secondary cognitive DF relay network in the presence of multiple PU-Tx and PU-Rx terminals. Three power constraints were jointly considered, i.e., peak interference power on the PU-Rx $\mathcal{I}_p$, maximum transmit power at the SU-Tx $\mathcal{P}_t$, and interference at the SU-Rx P_I inflicted by the PU-Tx transmission. We highlighted several important insights on how the PN affects the performance of the SN under SS environment. For example, we verified that the cognitive network is only applicable in the low $\mathcal{P}_t$ regime since its outage performance yields zero diversity gain in the high $\mathcal{P}_t$ regime. Our observations provide useful guidelines for cognitive network designers to determine when to operate the SN such that the PN interference requirements are satisfied.

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