Coherent Neutrino-Nucleus Scattering as a Probe
of the Weak Neutral Current

DANIEL Z. FREEDMAN
National Accelerator Laboratory, Batavia, Illinois   60439

and

Institute for Theoretical Physics, SUNY
Stony Brook, NY   11790

ABSTRACT

If there is a weak neutral current, then the elastic scattering process $\nu + A \rightarrow \nu + A$ should have a sharp coherent forward peak just as $e + A \rightarrow e + A$ does. Experiments to observe this peak can give important information on the isospin structure of the neutral current. The experiments are very difficult, although the estimated cross sections (about 10^{-38} cm^2 on carbon) are favorable. The coherent cross sections (in contrast to incoherent) are almost energy-independent. Therefore, energies as low as 100 MeV may be suitable.

There is recent experimental evidence[1] from CERN and NAL which

suggests the presence of a neutral current in neutrino-induced inter-

actions. A primary goal of future neutrino experiments is to confirm

the present findings and to investigate the properties of the weak neutral

current, for example, the space inversion and internal symmetry structure.

Our purpose here is to suggest a class of experiments which can

yield information on the isospin structure of the neutral current not obtain-

able elsewhere. The idea is very simple: if there is a weak neutral

current, elastic neutrino-nucleus scattering should exhibit a sharp

coherent forward peak characteristic of the size of the target just as

electron-nucleus elastic scattering does. In a sense we are talking

about measurements of the nuclear form factors of the weak neutral

current analogous to the measurements of the nuclear form factors

of the electromagnetic neutral current in elastic electron scattering exper-

iments.[2] In fact, for the same nucleus, these form factors should

have the same q^2 dependence. Therefore, the size of the cross

section or its extrapolated forward value gives information on the

structure of the weak current itself. In the simplest case, $S = 0$, $Z = N$

nuclei such as He^4 or C^{12}, the strength of the polar vector, isoscalar

component of the weak neutral current is measured directly.

Our suggestion may be an act of hubris, because the inevitable

constraints of interaction rate, resolution, and background pose grave

experimental difficulties for elastic neutrino-nucleus scattering. We

will discuss these problems at the end of this note but first wish to

present the theoretical ideas relevant to the experiments.

Although the weak neutral current finds a natural place in the

beautiful unified gauge theories[3], it is important to interpret experi-

mental results in a very broad theoretical framework.[4] We assume

a general current-current effective Lagrangian

$$\mathscr{L}_{eff} = \frac{1}{\sqrt{2}} \, G \, \ell^{\mu} \mathscr{J}_{\mu} \tag{1}$$

which is consistent with the early findings,[1] but far from established.

An intermediate neutral vector boson could be included here without

affecting the analysis of the low momentum transfer processes we are

interested in.

The currents will first be written in their fundamental form as

they would occur, for example, in particular unified gauge models of

the weak, electromagnetic, and strong interactions. We will then write

an expression which is essentially model independent and sufficiently

general to parameterize realistic experiments.

To begin with, we write the neutrino current as

$$\ell^{\prime}_{\mu} = \nu \gamma_{\mu} \, (1 - \alpha_{\nu} \gamma_5) \nu \tag{2}$$

where V-A coupling is not assumed. The hadronic current is assumed,

to begin with, to be a sum of components each corresponding to a

symmetry of strong interactions. For example, in a model with the

GIM mechanism,[5] one would have

$$\mathcal{J}'_\mu = b(J^B_\mu + \alpha_B A^B_\mu) + y(J^Y_\mu + \alpha_Y A^Y_\mu) + c(J^c_\mu + \alpha_c A^c_\mu)$$
$$+ t (J^{I=1, I=0}_\mu + \alpha_I^{I=1, I_3=0}) \qquad (3)$$

that is, a linear combination of baryon number, hyperchange, charm, and third component of isospin. We assume that the polar vector currents are conserved, and normalized (at zero momentum transfer) to the corresponding quantum numbers.

Realistic experiments are done with the left-handed neutrinos (and right-handed antineutrinos) from meson and muon decay. Because of chirality conservation, there is no loss in generality in writing

$$\ell^\mu = \bar{\nu} \gamma^\mu (1-\gamma_5) \nu \qquad (4)$$

and multiplying the hadronic current by the overall factor $\frac{1}{2}(1+\alpha_\nu)$. Further, with data from neutrino reactions involving nucleons or nuclei in the initial state, one cannot distinguish[6] among the three isoscalar components in (3), and it is sufficiently general to write

$$\mathcal{J}_\mu = a_0 (J^{(I=0)}_\mu + \alpha_0 A^{(I=0)}_\mu) + a_1(J^{(I=1)}_\mu + \alpha_1 A^{(I=1)}_\mu) \qquad (5)$$

where $J^{(I=0)}_\mu$ is a conserved vector current normalized to baryon number or hypercharge (which are identical for the reactions described above.)

The theoretical situation may be restated as follows: in any particular theoretical model with neutral current parameters α_ν, b, y, ...,

as in (2) and (3), the coefficients a_0, α_0, a_1, α_1 in (5) can be predicted

uniquely. Neutrino scattering data involving nucleonic or nuclear

targets can, in principle, tell us these four numbers, but the individual

components α_v, b, y, ..., can never be resolved. Thus (5) is a general

model-independent expression, whose parameters strongly constrain

any model. The key assumption here is current conservation, an

assumption which, as the reader will see, can be checked in part by

performing coherent scattering experiments on two (or more) nuclear

targets such as helium or carbon.

The coefficients a_0, α_0, a_1, α_1, are extremely important numbers,

indeed critical for theories of the weak and strong interactions. We

mention two models just for illustration. In the Weinberg model,

extended to hadrons[7] (either with or without GIM) $a_0 = -\sin^2\theta_w$,

$a_1 = 1 - 2\sin^2\theta_w$, while Sakurai[4] proposes $a_1 = 0$ with the entire

neutral current coupled to baryon number.

In experiments the coefficient values will be difficult to disentangle

from the matrix elements of the component currents. Experimental

determination of the coefficients a_0, α_0, a_1, α_1 is perhaps best done with

elastic transitions of nucleons and nuclei, where at least the vector

form factors are known. For spin zero nuclei, in particular, the axial

currents do not contribute, and, as discussed immediately below, the

vector form factors are entirely determined by Z, N and the r.m.s.

nucleus radius r. We have not been able to think of any other experimental

configuration, where the parameters a_0 and a_1 can be measured so cleanly,

We now analyze the case of neutrino scattering from an $S=0$, $Z=N=\frac{1}{2}A$ nucleus, where only $J_\mu^{(I=0)}$ contributes and we have the matrix element

$$< A(p') \left| \mathscr{J}_\mu \right| A(p)> = \frac{1}{(2\pi)^3} \frac{1}{\sqrt{2E2E'}} (p+p')_\mu a_0 F^{I=0}(q^2) .$$

$$(5)$$

The form factor $F^{I=0}(q^2)$ reflects the distribution of protons and neutrons in the nucleus and should have essentially the same shape as the nuclear electromagnetic form factors. For small q^2 it is sufficiently accurate to write

$$F^{I=0}(q^2) = A \, e^{-bq^2}$$

$$(7)$$

To make rate estimates we will use the electron scattering results, writing $b = \frac{1}{6}r^2$ relating the b parameters to r.m.s. nuclear radii.[8]

The differential cross section for $\nu + A \rightarrow \nu + A$ is

$$\frac{d\sigma}{dq^2} = \frac{G^2}{2\pi} a_0^2 A^2 e^{-2bq^2} \left[1 - q^2 \frac{2ME + M^2}{4M^2E^2} \right]$$

$$(8)$$

where E is the neutrino lab energy, q^2 the momentum transfer and M the target mass. For $q^2 \ll M^2$, a condition which is certainly satisfied over the first decade of fall-off from the forward peak in all nuclei, the equality $q^2 = q_R^2$ holds, where q_R is the laboratory recoil momentum of the nucleus. G is the conventional Fermi constant:

$$G = 1.01 \times 10^{-5} (M_{proton})^{-2}.$$

We estimate the expected observable partial cross section as follows. We assume, perhaps optimistically, that the recoil nucleus can be detected for $q_R > q_{min} = 100$ MeV/c, and that the steep decline of the nuclear form factors makes recoil momenta $q_R > q_{max} = 300$ MeV/c unlikely.

For a range of recoil momentum we integrate (8) and find

$$\sigma(q_{min} < q_R < q_{max}) = \frac{G^2}{2\pi} a_0^2 A^2 \left[f(q_{min}^2) - f(q_{max}^2) \right]$$

$$f(x) \equiv (2b)^{-1} e^{-2b\,x} \left[1 - (8 E^2 Mb)^{-1}(2E+M)(1+2bx) \right]. \quad (9)$$

This cross section is accurately energy independent for $E > 1$ GeV, and decreases slowly with energy for $E < 1$ GeV.

For helium, we have $r = 1.68 \times 10^{-13}$ cm, $2b = 24.2$ $(GeV/c)^{-2}$ and

$$\sigma(He^4,\ 100\ MeV/c < q_R < 300\ MeV/c) = a_0^2 \times 3.6 \times 10^{-39}\ cm^2 \quad E > 1\ GeV$$

$$= a_0^2 \times 2.5 \times 10^{-39}\ cm^2 \quad E = 200\ MeV$$

For carbon, $r = 2.42 \times 10^{-13}$ cm, $2b = 50.2$ $(GeV/c)^{-2}$ and

$$\sigma(C^{12},\ 100\ MeV/c < q_R < 300\ MeV/c) = a_0^2 \times 13.6 \times 10^{-39} cm^2 \quad E > 1\ GeV$$

$$= a_0^2 \times 11.2 \times 10^{-39} cm^2 \quad E = 200\ MeV$$

For heavier nuclei the approximate estimates should be scaled upward by $A^{4/3}$. In deuterium, the contribution from the polar vector current would

be about a factor of two below helium, but there are axial current

effects which are difficult to estimate.[9]

One possibly important effect which we have not considered here is

quasi-elastic neutrino scattering with the nucleus emerging in an excited

state. This process would add to the rate of observed recoil nuclei, but

may complicate the interpretation of results. If the quasi-elastic

processes could be observed, there would be very interesting implications.

For example, excitation of the low lying O^{+} states in light nuclei such

as O^{16} or Mg^{24} would provide a direct test of the conservation of the

polar vector part of $\mathscr{J}_{\mu}$. The transition form factors should vanish

as q^2 approaches zero if and only if the current is conserved.

Experimentally the most conspicuous and most difficult feature of

our process is that the only detectable reaction product is a recoil nucleus

of low momentum. Ideally the apparatus should have sufficient resolution

to identify and determine the momentum of the recoil nucleus and sufficient

mass to achieve a reasonable interaction rate. Neutron background is a

serious problem because elastic n+A cross sections are generally large.

Kinematics gives the relation

$$\cot \theta_L = \frac{1}{2} q_R \frac{E+M}{ME} \left[1 + \frac{q_R^2 (1 + \frac{2E}{M})}{4E^2} \right]^{-\frac{1}{2}} \tag{11}$$

between lab frame angle to the beam, recoil momentum and neutrino

energy. Under the conditions $q_R \ll M$, $q_R \ll E$ the recoil nucleus

emerges close to 90° to the beam. This can provide discrimination

against background, if the recoil angle can be measured.

Careful consideration of all constraints must be given before the feasibility of these experiments can be determined. This note will serve its purpose if our statement of the theoretical issues stimulates experimenters to give the consideration necessary. Our own naive thinking about the experimental possibilities has included deuterium and helium bubble chambers, mineral oil or liquid helium scintillator tanks and helium and neon streamer chambers.

There is another important point which may have bearing on the experimental possibilities and on our general picture of neutrino inter-actions. The coherent cross sections[9] are still quite large at E=200 MeV, whereas the conventional charge lepton production cross sections decrease rapidly with energy. Therefore, it may be advantageous to perform the elastic scattering experiments with muon neutrinos in the 100 MeV region (accessible at a "meson factor") where that part of the neutron background due to neutrino production is small.

Even at very low energy (few MeV) where the nucleus is point-like and the coherent cross sections decrease quadratically with energy, the effects of coherent amplification remain. Therefore, in a medium con-taining nuclei of mass number A, the total cross section for electron neutrinos will contain an inverse β decay cross section of order 1 in mass number and a coherent scattering cross section of order A^2 with large angle scattering processes present. There may be astro-physical implications here.

ACKNOWLEDGMENTS

References

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