Coherent photon transport from spontaneous emission in one-dimensional waveguides

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A two-level system coupled to a one-dimensional continuum is investigated. By using a real-space model Hamiltonian, we show that spontaneous emission can coherently interfere with the continuum modes and gives interesting transport properties. The technique is applied to various related problems with different configurations, and analytical solutions are given. © 2005 Optical Society of America

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Spontaneous emission is fundamental in the interactions of electromagnetic fields with atoms. Two regimes of spontaneous emission have been extensively explored. In a weak coupling regime, for instance, an excited atom in free space, the excited atom decays exponentially due to the ample photon phase space to which the atom couples. The spontaneous emission is generally treated as a loss, and a decoherence mechanism and is included as part of the absorption coefficient of the system. On the other hand, in the strong coupling regime, as is the case when an excited two-level system is placed in a microcavity,[1,2] the atom undergoes Rabi oscillation, since the photonic mode spectrum is now discrete.[3]

Here we explore a different regime of spontaneous emission. We consider two-level atoms coupled to a one-dimensional continuum. Such a continuum can act as a line defect waveguide in a complete photonic bandgap crystal (Fig. 1). The atom can either be in the waveguide or side coupled to the waveguide. In this case, the excited two-level system will decay exponentially. However, in the reduced dimensionality, when a single photon is incident upon the two-level system with a frequency on resonance, the wave function of the spontaneously emitted photon inevitably interferes coherently with that of the incident wave, because the forward and backward directions are the only directions in phase space. Such interference can result in the photon's being completely reflected with no loss. This occurs in spite of the fact that the physical dimension of the two-level system is typically far smaller than the wavelength of light. Thus spontaneous emission can be exploited to influence the coherent transport properties of a single photon. Interesting transport properties result from this coherent interference and can be utilized in the design of various quantum optoelectronic devices, such as ultranarrow bandwidth filters and nanomirrors.

The interaction between the photons and the two-level atoms is described by a Dicke Hamiltonian[4]:

$$H = \sum_{\mathbf{k}} \hbar\omega_k a_k^\dagger a_k + \frac{1}{2}\hbar\Omega S_z + \sum_{\mathbf{k}} V_k(a_k^\dagger + a_k)(S_+ + S_-),$$

(1)

where ω_k is the frequency of the mode of the radiation field corresponding to wave vector k (i.e., the dispersion relation), $a_k^\dagger$ (a_k) is the creation (annihilation) operator of the photon, Ω is the resonance energy of the atom, $V_k = (2\pi\hbar/\omega_k)^{1/2}\,\Omega\mathbf{D}\cdot\mathbf{e}_k$ is the coupling constant, $\mathbf{D}$ is the dipole moment of the atom, $\mathbf{e}_k$ is the polarization unit vector of the photon, $S_+ = a_e^\dagger a_g$ ($S_- = a_g^\dagger a_e$) is the creation (annihilation) operator of the atomic excited state, and $a_g^\dagger$ ($a_e^\dagger$) is the creation operator of the ground (excited) state of the electron.

In one dimension, when the resonance energy of the atom is away from the cutoff frequency of the dispersion relation, we rewrite the Hamiltonian of the system in real space as

$$H = \int dx \left\{ -iv_g c_R^\dagger(x)\frac{\partial}{\partial x}c_R(x) + iv_g c_L^\dagger(x)\frac{\partial}{\partial x}c_L(x) \right.$$
$$\left. + V\delta(x)[c_R^\dagger(x)S_- + c_R(x)S_+ + c_L^\dagger(x)S_- + c_L(x)S_+] \right\}$$
$$+ E_e a_e^\dagger a_e + E_g a_g^\dagger a_e,$$

(2)

where v_g is the group velocity of the photons and $c_R^\dagger(x)$ [$c_L^\dagger(x)$] is a bosonic operator creating a right-going (left-going) photon at x. $E_e - E_g(\equiv\Omega)$ is the energy difference between the atomic excited state and the ground state. In deriving Eq. (2), we assume that the dispersion relations are nondegenerate, linearize the dispersion relation of the photons in the waveguide, and replace V_k with a constant V.
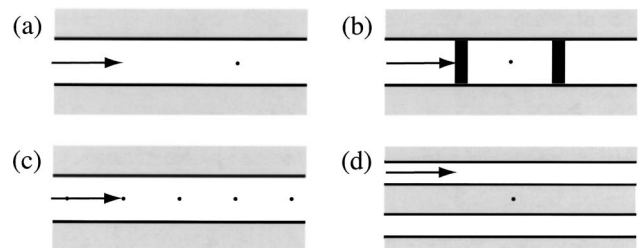

Fig. 1. Schematics of the systems: (a) An atom embedded in a one-dimensional waveguide. (b) An atom surrounded by two partial reflectors denoted by black vertical bars. (c) A chain of atoms. (d) An atom couples to two parallel waveguides. The waveguide is denoted by two horizontal black lines. The light shaded region denotes the photonic bandgap crystal. The atom is indicated by the black dot. The arrow indicates the direction of the input light.

This Hamiltonian is similar to the *s-d* model (Anderson model) in condensed matter physics, which describes the *S*-wave scattering of electrons off a magnetic impurity in three dimensions.[5,6] A similar one-mode Hamiltonian has also been investigated.[7] Here, we apply this Hamiltonian to the photon systems shown in Fig. 1.

Case (1): *A single atom in a waveguide.* Consider an optical system consisting of an atom embedded in a one-dimensional waveguide, as shown in Fig. 1(a). Assume that a photon is coming from the left with energy $E_k = v_g k$. The stationary state of the system is

$$|E_k\rangle = \int dx [\phi_{k,R}^+(x) c_R^{\dagger}(x) + \phi_{k,L}^+(x) c_L^{\dagger}(x)]|0,-\rangle$$
$$+ e_k a_e^{\dagger} a_g |0,-\rangle, \tag{3}$$

where $|0,-\rangle$ is the vacuum state with zero photons and the atom being unexcited and e_k is the probability amplitude of the atom in the excited state. Equation (3) represents a complete basis for the system. For a photon incident from the left, $\phi_{k,R}^+(x)$ and $\phi_{k,L}^+(x)$ take the forms

$$\phi_{k,R}^+(x) \equiv [\exp(ikx)\theta(-x) + t\exp(ikx)\theta(x)],$$

$$\phi_{k,L}^+(x) \equiv r\exp(-ikx)\theta(-x), \tag{4}$$

where t and r are the transmission and reflection amplitude, respectively.

From the eigenvalue equation $H|E_k\rangle = E_k|E_k\rangle$, together with the usual commutation relations between the creation and annihilation operators, we obtain

$$t = \cos b e^{ib}, \qquad r = i\sin b e^{ib}, \qquad e_k = -\frac{v_g}{V}\sin b e^{ib}, \tag{5}$$

where the phase shift is $b = \arctan\{V^2/[v_g(\Omega - E_k)]\}$.

The reflection coefficient is given by

$$R \equiv |r|^2 = \sin^2\left\{\arctan\left[\frac{V^2}{v_g(\Omega - E_k)}\right]\right\}$$
$$= \frac{\left(\frac{V^2}{v_g}\right)^2}{(\Omega - E_k)^2 + \left(\frac{V^2}{v_g}\right)^2}, \tag{6}$$

which shows a Lorentzian line shape.

From Eq. (5) it follows that the transfer matrix for one atom has the following form:

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{bmatrix} 1 - \dfrac{iV^2}{v_g(\Omega - E_k)} & -\dfrac{iV^2}{v_g(\Omega - E_k)} \\ +\dfrac{iV^2}{v_g(\Omega - E_k)} & 1 + \dfrac{iV^2}{v_g(\Omega - E_k)} \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \tag{7}$$

The transfer matrix relates incoming and outgoing wave amplitudes a and b on one side of the atom to outgoing and incoming wave amplitudes a' and b' on the other side.

Figure 2 shows typical transmission and reflection spectra. At resonance, the photon is completely reflected. While the form of the transfer matrix is the same as when the waveguide is side coupled to a single-mode cavity,[8–10] we note that here a single atom behaves as a mirror, and the size of the atom is at least 4 orders of magnitude smaller than the smallest microcavity structure. The width of reflection peak is proportional to $V^2/v_g \approx 2\pi\hbar\Omega e^2/(\hbar v_g)(a/\lambda)^2$, where a is the size of the atom and λ is the wavelength of light with energy equal to Ω. For a quantum dot with a size of 10 nm, the width is estimated to be of the order of 0.04 meV at incident light of 10^{15} Hz, which can be very narrow in the weak coupling regime. Within this limit, the atom serves as an ultranarrow filter. The notable feature of this result is that the spontaneous emission directly gives rise to the reflection, rather than to losses that degrade performance.

Case (2): *Fano interference.* A more general Fano line shape can be created if there are partial reflections in the waveguide such that the photon modes are no longer purely forward or backward propagating. As an example, consider a case where the atom is surrounded by a pair of thin dielectric slabs [Fig. 1(b)]. The response function of the system can be calculated by combining the transfer matrix of each individual element in the system. For the partially reflecting dielectric slab, the transfer matrix can be described as[10,11]

$$T_p = \frac{1}{i\sqrt{1-r^2}}\begin{bmatrix} -1 & -r \\ r & 1 \end{bmatrix}, \tag{8}$$

where r is the amplitude reflectivity of the slab. The shape of the transmission spectrum strongly depends on the value of r. Figure 3 shows the transmission spectrum of the total optical system for different r. For smaller r (=0.4), the spectrum consists of resonant features superimposed upon a background defined by the Fabry–Perot oscillations [Fig. 3(a)]. When r is increased to 0.9, the transmission spectrum shows tunneling peaks [Fig. 3(b)]. The presence
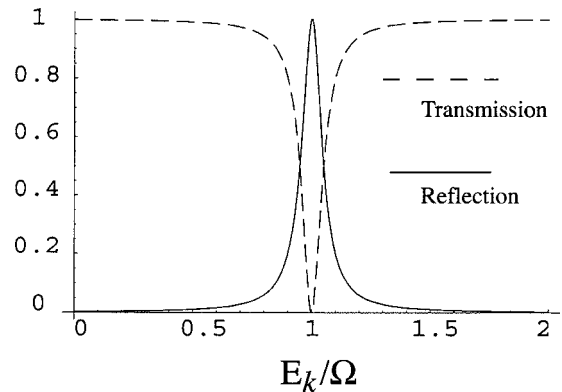

Fig. 2. Transmission spectrum (solid curve) and the reflection spectrum (dashed curve) for an atom in a one-dimensional waveguide as shown in Fig. 1(a). $V^2/\Omega v_g = 0.05$.
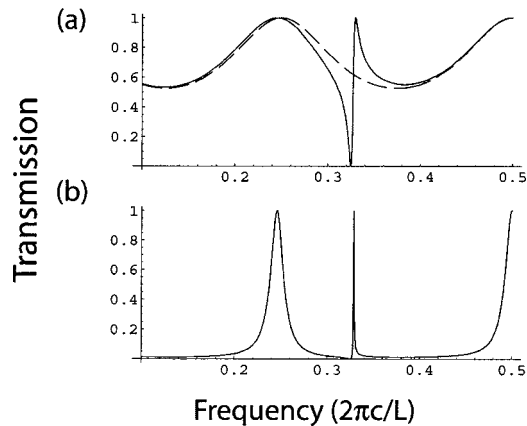
Fig. 3.   Transmission spectra through the optical system as shown in Fig. 1(b). $V^2/v_g = 0.002(2\pi c/L)$. The resonance energy is $\Omega = 0.325(2\pi c/L)$. $2L$ is the distance between the two partially reflecting elements. (a) $r=0.4$. The dashed curve represents the transmission spectrum through the two partially reflecting elements, without the presence of the atom. (b) $r=0.9$.
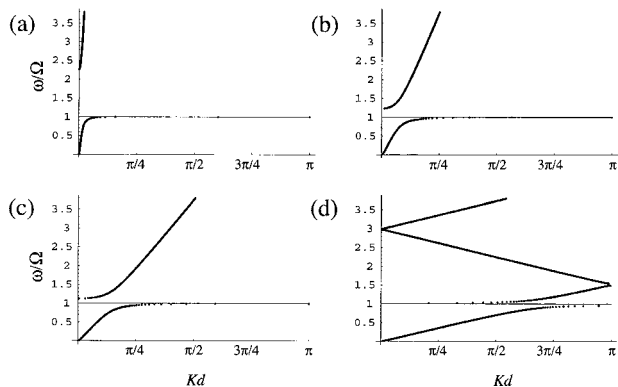

Fig. 4.   Dispersion relation between frequency ω and Bloch wave vector K. $\lambda_r \equiv 2\pi v_g/\Omega$ is the wavelength at resonance for a single atom. $v_g = 1$. (a) $d/\lambda_r = 3.4 \times 10^{-3}$, (b) $d/\lambda_r = 3.4 \times 10^{-2}$, (c) $d/\lambda_r = 6.8 \times 10^{-2}$, (d) $d/\lambda_r = 3.4 \times 10^{-1}$. The line at $\omega/\Omega = 1$ is a visual aid.

of the partially reflecting elements, as dielectric slabs in this case, introduces a background phase shift, in addition to the phase shift experienced by the photon due to the atom. These two phase shifts correspond to the direct and the resonance-assisted indirect pathways, respectively.[12] The Fano line shape of the transmission spectrum results from the interference between these two phase shifts.[13]

Case (3): *A linear chain of atoms in a waveguide* [Fig. 1(c)]. By cascading the transfer matrices, we can study the transport properties of a chain of any length of resonant atoms, periodic or disordered. Figure 4 shows the dispersion relations between frequency ω and Bloch wave vector K for an infinite periodic chain. For any value of lattice constant d, an energy gap always opens up around resonance energy Ω. The size of the gap decreases with increasing d. Note that a very large bandgap ($\gg V^2/v_g$) emerges even when the spacing between the atoms is an order of magnitude smaller than the resonance wavelength.

The appearance of a large bandgap is rather different from the case where the waveguide is side coupled to a chain of optical resonators. In that case, because the resonators are of the size of the optical wavelength, the resonance-induced bandgap has a size comparable to the coupling constant.[14] In contrast, in our case, since the atoms are small, many atoms can be accommodated within a wavelength, and the chain of atoms can be considered as continuous with a linear density distribution. The amplitudes of the photon wave at two adjacent atoms are essentially in phase and interfere constructively to give a large bandgap. We have also simulated a disordered chain of atoms where the distances between two adjacent atoms fluctuate by a few percent, and the presence of such a large gap appears robust against such subwavelength disorders.

Case (4): *An atom couples to two waveguides*. As a final example, we consider two parallel waveguides coupled to each other through an atom between them, as shown in Fig. 1(d). A calculation shows that, when the atom couples equally to the two waveguides, the forward and backward transmission in each waveguide is 1/4 at resonance. This situation can be shown to be mathematically equivalent to a case where there is only one waveguide present but with two degenerated modes, for example, two orthogonal polarizations.

As a final remark, we note that the proposed real-space Hamiltonian also applies to multiport systems and can be used to study the time evolution of the one-photon wave function. The study of one-photon dynamics would find applications in fields such as quantum communication and quantum computing.

References

1. S. John, Phys. Rev. Lett. **58**, 2486 (1987).
2. E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
3. T. Yoshie, A. Scherer, J. Hendrickson, G. Khitrova, H. M. Gibbs, G. Rupper, C. Ell, O. B. Shchekin, and D. G. Deppe, Nature **432**, 200 (2004).
4. R. H. Dicke, Phys. Rev. **93**, 99 (1953).
5. P. W. Anderson, Phys. Rev. **124**, 41 (1961).
6. P. B. Wiegmann and A. M. Tsvelick, J. Phys. C **16**, 2281 (1983).
7. V. I. Rupasov and V. I. Yudson, Sov. Phys. JETP **60**, 927 (1984).
8. H. A. Haus and Y. Lai, J. Lightwave Technol. **9**, 754 (1991).
9. Y. Xu, Y. Li, R. K. Lee, and A. Yariv, Phys. Rev. E **62**, 7389 (2000).
10. S. Fan, Appl. Phys. Lett. **80**, 908 (2002).
11. H. A. Haus, *Waves and Fields in Optoelectronics* (Prentice-Hall, 1984).
12. S. Fan, W. Suh, and J. D. Joannopoulos, J. Opt. Soc. Am. A **20**, 569 (2003).
13. U. Fano, Phys. Rev. **124**, 1866 (1961).
14. A. Yariv, Y. Xu, R. K. Lee, and A. Scherer, Opt. Lett. **24**, 711 (1999).