Prog. Theor. Phys. Vol. 47 (1972), No. 3

Cohesive Force of Electron and Nambu's Mass-Formula*)

Daisuke ITÔ**)

Instituto de Física Teórica
Rua Pamplona, 145, São Paulo, Brazil

December 20, 1971

Many years ago, Poincaré[1] pointed out that the Lorentz electron cannot be stable unless a kind of cohesive force of non-electromagnetic nature exists. More recently, Sakata[2] and Pais succeeded to make the self-energy of electron finite, by assuming that the cohesive force is described by a neutral scalar field (Cohesive or C-meson), and it was through the critical analysis of Sakata's idea, applied to the scattering problem, that Tomonaga[3] developed his famous theory of renormalization.

Now, if such a cohesive force really exists and the electron is in a state of stable equilibrium, one could expect some kind of small vibrations around the equilibrium position. In this short note, we would like to examine such a possibility in a preliminary way, i.e., by assuming that the electron is a uniformly charged elastic sphere of radius a, the elastic and electrostatic energies being given by

$$U=\frac{\kappa}{2}a^2+\frac{3}{5}\frac{e^2}{a}, \qquad (1)$$

where only the radial vibration was considered.***) The Hamiltonian will be given by

$$H=\frac{P_\xi^2}{2m}+\frac{3e^2}{5(a+\xi)}+\frac{\kappa}{2}(a+\xi)^2+U_0, \qquad (2)$$

where ξ is a small departure from the equilibrium radius a, P_ξ the momentum conjugate to ξ, m a mass-parameter and U_0 an additive constant. By expanding the right-hand side of Eq. (2) in powers of ξ, we have

$$H=\frac{P_\xi^2}{2m}+U_0+\left(\frac{3}{5}\frac{e^2}{a}+\frac{\kappa}{2}a^2\right)+\left(\kappa a-\frac{3}{5}\frac{e^2}{a^2}\right)\xi+\left(\frac{3e^2}{5a^3}+\frac{\kappa}{2}\right)\xi^2+\cdots. \qquad (3)$$

Since a is the radius of equilibrium, the term linear in ξ must vanish. So we get

$$\kappa a=\frac{3e^2}{5a^2} \quad \text{or} \quad \kappa=\frac{3e^2}{5a^3}. \qquad (4)$$

Eliminating κ, Eq. (3) turns out to be

$$H=\frac{P_\xi^2}{2m}+U_0+\frac{9}{10}\frac{e^2}{a}+\frac{9}{10}\frac{e^2}{a^3}\xi^2. \qquad (5)$$

For simplicity, we shall assume that the mass-parameter m is equal to the self-mass, the third term of Eq. (5) divided by c^2:

$$m=\frac{9}{10}\frac{e^2}{ac^2} \quad \text{or} \quad a=\frac{9}{10}\frac{e^2}{mc^2}. \qquad (6)$$

Then Hamiltonian (5) will be written as

$$H=mc^2+\frac{P_\xi^2}{2m}+\left(\frac{10}{9}\right)^2\frac{(mc^2)^3}{e^4}\xi^2+U_0$$

$$=mc^2+\frac{P_\xi^2}{2m}+\frac{m\omega^2}{2}\xi^2+U^0, \qquad (7)$$

where ω is the proper frequency of the vibration and

$$\hbar\omega=\sqrt{2}\frac{10}{9}\frac{\hbar c}{e^2}mc^2. \qquad (8)$$

Now, the eigenvalue of Hamiltonian (7) is given by

$$E_n=mc^2\left(1+\frac{10\sqrt{2}}{9}\frac{n}{\alpha}\right), \quad n=0,1,2,\cdots, \qquad (9)$$

*) Work supported by the Fundação de Amparo à Pesquisa do Estado de São Paulo and Banco Nacional do Desenvolvimento Econômico, Brazil (Contract FUNTEC-125).

**) On leave of absence from the Physics Department of Saitama University, Urawa-city, Saitama.

***) Here, we shall not consider the rotational excitations, because it will lead to the well-known paradox $v/c \gtrsim \frac{1}{2}\hbar c/e^2$, where v is the equatorial surface velocity of electron.

if we fix the additive constant U_0 as $U_0 = -\hbar\omega/2$. Since α in Eq. (9) is the fine structure constant, Eq. (9) is essentially nothing but Nambu's empirical mass-formula.[5] If we tentatively assume that m is the observed mass of electron ($mc^2 = 0.511$ MeV), then the mass of the first excited state E_1 will be

$$E_1 = 216.29mc^2 = 110.52\ \text{MeV}\,, \quad (10)$$

which is very close to the observed mass of μ-meson:

$$m_\mu = 105.66\ \text{MeV}\,.$$

The numerical value obtained above should not, of course, be taken too seriously.*) However, it is to be noted that a small vibration around the equilibrium point of electrostatic and cohesive forces may result in such a large mass-splitting of the order mc^2/α, as suggested in Nambu's mass-formula.

The author would like to express his sincere appreciation to the President of The Fundação Instituto de Física Teórica, São Paulo, Dr. J. H. Leal Ferreira, and the Scientific Director, Professor Paulo Leal Ferreira, for their kind hospitality extended to him.

1) H. Poincaré, Rend. Pal. **21** (1906), 129.
2) S. Sakata, Prog. Theor. Phys. **2** (1947), 145.
S. Sakata and T. Inoue, Prog. Theor. Phys. **1** (1946), 143.
S. Sakata and O. Hara, Prog. Theor. Phys. **2** (1947), 30.
3) D. Itô, Z. Koba and S. Tomonaga, Prog. Theor. Phys. **3** (1948), 276, 325.
Z. Koba and S. Tomonaga, Prog. Theor. Phys. **3** (1948), 290.
T. Tati and S. Tomonaga, Prog. Theor. Phys. **3** (1948), 211.
4) Y. Nambu, Prog. Theor. Phys. **7** (1952), 595.

*) The first excited state could not be a model of μ-meson, unless the γ-transition to the ground state is excluded by some reasons.