Cole Electrical Impedance Model—A Critique and an Alternative

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***Abstract*—The Cole single-dispersion impedance model is based upon a constant phase element (CPE), a conductance parameter as a dependent parameter and a characteristic time constant as an independent parameter. Usually however, the time constant of tissue or cell suspensions is conductance dependent, and so the Cole model is incompatible with general relaxation theory and not a model of first choice. An alternative model with conductance as a free parameter influencing the characteristic time constant of the biomaterial has been analyzed. With this free-conductance model it is possible to separately follow CPE and conductive processes, and the nominal time constant no longer corresponds to the apex of the circular arc in the complex plane.**

***Index Terms*—Bioimpedance theory, biomaterial, cell suspension model, Cole model, constant phase element (CPE), dielectric, electrical network theory, electrical relaxation theory, tissue model.**

I. Introduction

The Cole impedance model was introduced in its final form by Kenneth Cole in 1940 [1]. It is based upon the replacement of the ideal capacitor in the Debye model [2] with a more general constant phase element (CPE) as shown in the equivalent circuit diagrams, Fig. 1. The idea of introducing a CPE came after a number of findings, both in electrochemistry and with tissue and cell suspensions, that measured impedance loci in the complex plane were in the form of circular arcs with depressed circle centers, and that such arcs were best modeled with a CPE.

The CPE together with a circular arc complex impedance analysis were introduced by Cole as early as 1928 [3]. A Cole impedance model consists however of three parts: an equation, an equivalent circuit, and a complex impedance circular arc locus. The impedance model developed gradually in the 1930s until completed with the mathematical equation [1]. In 1941 the Cole brothers introduced a similar model for dielectric permittivity, the Cole-Cole model [4]. A Cole model is, therefore, an impedance model, a Cole-Cole model a permittivity model.

The popularity of the models is due to the condensed complex form and the elegant and simple mathematical appearance, but as we shall see this is unfortunately at the expense of general applicability. It is the purpose of this paper to present a critique of the Cole model, and to analyze an alternative model with the time constant as a conductance dependent parameter. Such a model is in general better suited for tissue and cell suspensions, and actually this model often implicitly has been used. Only the basic Cole impedance model will be treated describing a single ideal dispersion. Often a biomaterial shows multiple dispersions, but it is not the purpose of this paper to analyze such systems, a more detailed description also of multiple-dispersion systems can be found in [5].

II. Debye and Cole Models

The Debye circuit [2] is shown on Fig. 1(a). The impedance of the circuit is

$$\boldsymbol{Z} = R_\infty + \frac{1}{G_{\rm var} + G_{\rm var} j\omega\tau_Z} \quad \tau_Z = C/G_{\rm var} \tag{1}$$

Manuscript received June 6, 2003; revised May 9, 2004. *Asterisk inidcates corresponding author.*

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Digital Object Identifier 10.1109/TBME.2004.836499

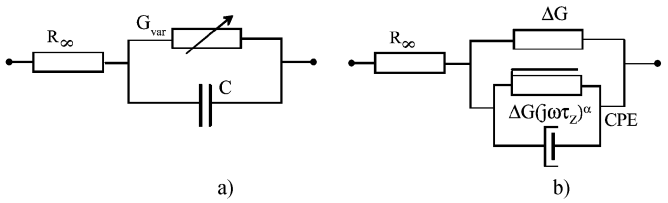

Fig. 1. Single-dispersion equivalent circuits. (a) Debye circuit with ideal components, (1). (b) Cole circuit with the ideal capacitor replaced by a CPE with frequency-dependent components, (2) and (4).

where $\boldsymbol{Z}$ is complex impedance $[\Omega]$, R_∞ is resistance $[\Omega]$ at very high frequencies, j is the imaginary unit, ω is the angular frequency [1/s], τ_Z is the characteristic relaxation time constant [s] of the circuit corresponding to a characteristic angular frequency $\omega_Z = 1/\tau_Z$, C is the parallel capacitance [farad, F], and $G_{\rm var}$ is an independent parameter conductance [siemens, S].

The Cole empirical equation for the frequency dependence of tissue or cell suspension complex impedance is [1]

$$\boldsymbol{Z} = R_\infty + \frac{R_0 - R_\infty}{1 + (j\omega\tau_Z)^\alpha} \tag{2}$$

where R_0 is the resistance at very low frequencies, τ_Z is the characteristic time constant of the system corresponding to a characteristic angular frequency $\omega_Z = 1/\tau_Z$, and α is an exponent [dimensionless].

The product $\omega\tau_Z$ is dimensionless, and $(j\omega\tau_Z)^\alpha$ represents a CPE as long as α is constant because

$$j^\alpha = \cos(\alpha\pi/2) + j\sin(\alpha\pi/2) \tag{3}$$

where $\alpha = \varphi_{\rm CPE}/90°$.

The equivalent circuit of a CPE consists of a resistor and a capacitor, both frequency dependent (nonideal) so that the phase $\varphi_{\rm CPE}$ becomes frequency independent. Equations (1) and (2) describe and define one ideal dispersion. A more detailed description can be found in [5].

By introducing $R_0 - R_\infty = 1/\Delta G$ the Cole equation becomes

$$\boldsymbol{Z} = R_\infty + \frac{1}{\Delta G + \Delta G(j\omega\tau_Z)^\alpha}. \tag{4}$$

From (4), the capacitance $C_{\rm CPE}$ of the CPE capacitor is found to be

$$C_{\rm CPE} = \frac{\Delta G}{\omega}(\omega\tau_Z)^\alpha \sin(\alpha\pi/2). \tag{5}$$

The dimension of G/ω and, therefore, $C_{\rm CPE}$, is [S s] or [F]. An equivalent circuit is shown in Fig. 1(b).

As described in [5], (4) reveals that the Cole model presupposes a CPE element in parallel with an ideal conductance ΔG. However, the CPE admittance $\Delta G(j\omega\tau_z)^\alpha$ is proportional to that same conductance ΔG. *Thus, a parallel conductance is not an independent parameter in the Cole model*. Equation (5) shows that $C_{\rm CPE}$ is proportional to ΔG with the consequence that τ_z is *independent* of ΔG, just as τ_Z in (1) is constant if C and $G_{\rm var}$ vary by the same factor.

α is related to the constant phase of the CPE according to (3), as well as to the frequency exponent in the term ω^α. *This double influence of α reveals that the Cole model is based upon a Fricke compatible system*. According to Fricke's law [5], [6], the phase angle φ and the frequency exponent m of the impedance in many electrolytic systems are related so that

$$\varphi = m \cdot 90° \quad \text{Fricke's law.} \tag{6}$$

In such cases it is common practice to set $m = \alpha$.

An admittance version of the Cole equation [5] is shown in Fig. 2

$$\boldsymbol{Y} = G_0 + \frac{G_\infty - G_0}{1 + (j\omega\tau_Y)^{-\alpha}} \tag{7}$$

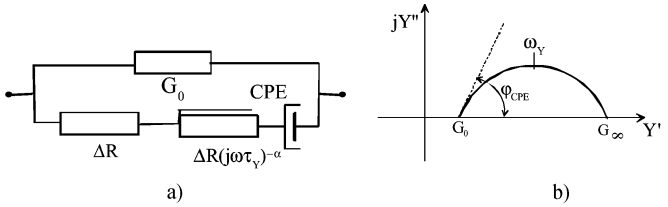

Fig. 2. Admittance Cole model. (a) Equivalent circuit according to (7) with $G_\infty - G_0 = 1/\Delta R$. (b) Complex plane Y-plot. ω_Y is the characteristic frequency, from which τ_Y is found: $\tau_Y = 1/\omega_Y$.

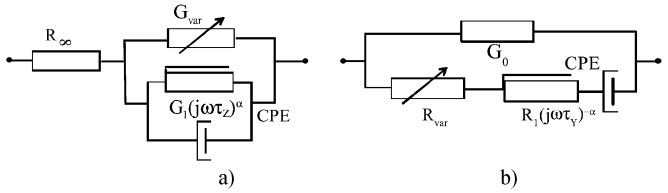

Fig. 3. General equivalent circuits for (9) with a free-conductance/resistance parameter and a dependent τ parameter. (a) Z-version, (b) Y-version.

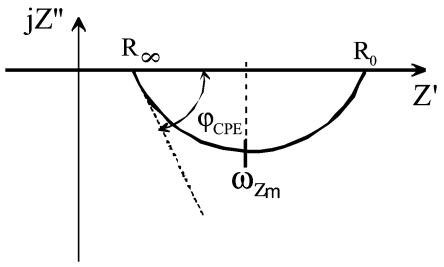

Fig. 4. Free conductance model, a Z-plot example. Three parameters $(R_0, R_\infty, \varphi_{\rm CPE})$ define a circular arc locus. Two additional parameters together with ω^α defines the frequency scale along the arc; for the Z-version of (9) it is the product $G_1\tau_Z^\alpha$.

where $\boldsymbol{Y}$ is the complex admittance ($\boldsymbol{Y} = 1/\boldsymbol{Z}$) [S], G_∞ is the conductance at very high frequencies, and G_0 is the conductance at very low frequencies.

Immittance values can be plotted in the complex (Argand or Wessel [5]) plane, Fig. 2(b) shows an admittance and Fig. 4 an impedance example. The immittance loci for all the equations given in this paper have the form of a circular arc.

The Cole equations are compatible with the Kramers–Kronig transforms [5]. A control of both Fricke [7] and Kramers–Kronig compatibility can be used for data quality evaluation.

III. Non-Cole Models

A general equation without any constraints of Fricke compatibility is

$$\boldsymbol{Z} = R_\infty + \frac{1}{G_{\rm var} + K j^\alpha \omega^m \tau_Z^\alpha} \tag{8}$$

where K is a real proportionality factor [S $s^{m-\alpha}$] for the CPE admittance.

This equation is compatible neither with the Cole model, nor the Fricke law, nor Kramers–Kronig transforms. It is not a very attractive alternative, but shown here for the sake of completeness.

It is possible to stick to the condition that the system shall be Fricke compatible and, therefore, also Kramers–Kronig compatible by introducing an *independent* ideal conductance $G_{\rm var}$ in parallel with the CPE (in the case of admittance: ideal resistance $R_{\rm var}$ in series). These earlier proposed [5] equations are

$$\begin{aligned} \boldsymbol{Z} &= R_\infty + \frac{1}{G_{\rm var} + G_1(j\omega\tau_Z)^\alpha} \\ \boldsymbol{Y} &= G_0 + \frac{1}{R_{\rm var} + R_1(j\omega\tau_Y)^{-\alpha}}. \end{aligned} \tag{9}$$

Fig. 3 shows the most adequate equivalent circuits.

To analyze (9) we use the Z-version as an example in the following. The complex admittance of the CPE element is: $\mathbf{Y}_{\rm CPE} = G_1(j\omega\tau_Z)^\alpha$. When (9) is plotted in the Wessel plane (Fig. 4), the (angular) frequency corresponding to the apex of an arc is the characteristic frequency.

From Fig. 4, we see that then the real impedance value is equal to $Z' = R_\infty + 1/2(R_0 - R_\infty)$. With that expression put into (9) the measured characteristic frequency at the apex of the arc ω_{Zm} and the measured relaxation time τ_{Zm} is

$$\tau_{Zm} = \frac{1}{\omega_{Zm}} = \tau_Z \left(\frac{G_1}{G_{\rm var}}\right)^{1/\alpha}. \tag{10}$$

The characteristic frequency corresponding to $1/\tau_Z$ is ω_Z. The τ_Z parameter in (9) corresponds to the apex of the arc only when $G_1 = G_{\rm var}$: then $\omega_{Zm} = \omega_Z = 1/\tau_Z$ and that is the Cole case. However, with $G_1 \neq G_{\rm var}\tau_Z$ no longer corresponds to the apex of the arc, ω_Z has become just a nominal characteristic frequency. The new time constant τ_{Zm} given in (10) shows that a change in $G_{\rm var}$ now indeed influences the time constant in accordance with relaxation theory: a higher value of $G_{\rm var}$ results in a shorter time constant τ_{Zm}. The true time constant $\tau_{Zm} = 1/\omega_{Zm}$ does not appear in (9) but is given by (10). With (10) it is found that ω_{Zm} is the crossover frequency when the CPE admittance $Y_{\rm CPE} = G_1(\omega\tau_Z)^\alpha$ becomes equal to $G_{\rm var}$.

IV. Complex Plane Z-Plot

The Cole (2) has four parameters: R_∞, R_0, τ_Z, and α; and ω as the variable. Equation (9) has five parameters, for the Z-version the parameters are: $R_\infty, G_{\rm var}, G_1, \tau_Z$, and α. The five parameters are independent parameters, τ_{Zm} in (10) is a derived, dependent but measured parameter.

A. Forward Analysis

Setting up a Cole model with the four parameters given is straightforward, so is setting up a model according to (9) with the five parameters given. In the latter case the geometry of the Z-arc is determined by the three parameters $R_\infty, G_{\rm var}$, and α. This is illustrated on Fig. 4 with $R_0 = 1/G_{\rm var} + R_\infty$ and $\varphi_{\rm CPE} = \alpha \cdot 90°$. Two additional parameters together with ω^α define the frequency scale along a given arc: G_1 and τ_Z. The true relaxation time τ_{Zm} is found from (10).

B. Inverse Analysis

The inverse problem of determining the four parameters of the Cole equation (2) from measurement results is simple; the parameters are read directly from the Wessel plot. For the Z case: R_0, R_∞, and $\varphi_{\rm CPE}$ determine the arc geometry, and the characteristic time constant $\tau_Z = 1/\omega_Z$ is found from the measured frequency at the apex of the arc. However, it may be impossible to interpret the results correctly if the experiment continues and new sets of data are sampled. Let us take the example that only ω_Z changes during an experiment. According to (4) that cannot be due to a change in the ideal conductance ΔG, a conclusion which may be seriously wrong.

Determining all five parameters in (9) from measured data is not possible. A circular arc is first fitted to the data in a Wessel plot. The

three geometrical parameters R_∞, $G_{\rm var}$, and α are found from the arc geometry. For the frequency scale on the arc, (10) is rearranged

$$G_1\tau_Z^\alpha = \frac{G_{\rm var}}{\omega_{Zm}^\alpha}. \tag{11}$$

Here the three parameters of the right hand side are found from the Wessel plot: $G_{\rm var}, \omega_{Zm}$, and α. Accordingly, the product $G_1\tau_Z$ is known, but not the individual values. As long as the product $G_1\tau_Z$ and all other parameters are constant, arcs with different $G_1 - \tau_Z$ combinations are indistinguishable from each other. A fixed nominal value for τ_Z together with G_1 as a free parameter is then a preferred alternative. A fixed nominal value for G_1 together with τ_Z as a free parameter is not preferred since τ_{Zm} should be the dependent variable, not τ_Z, (10). G_1 can then be selectively followed during an experiment with a fixed nominal value for τ_Z. From (10) we have

$$G_1 = G_{\rm var}\frac{1}{(\tau_Z\omega_{Zm})^\alpha}. \tag{12}$$

The nominal time constant τ_Z is not the true characteristic time constant τ_{Zm} as found by measurement. In practice (10) is used in forward analysis to find τ_{Zm} with a given τ_Z value. Equation (12) is used to follow G_1 values with a fixed nominal τ_Z value.

In conclusion, the inverse analysis has shown that the four parameters of the Cole model can be determined directly from the measurement results, but the interpretation of the results may be seriously misleading. In the new model four parameters can also be determined; in addition a fifth parameter G_1 can be calculated and selectively followed during an experiment.

C. An Example

Four skin impedance spectrographic complex data sets were obtained with three pregelled electrocardiogram-electrodes positioned on the skin of a human underarm. The effective skin-wetted area was 3 cm^2 and electrode center distances 4 cm. The first data set was taken 3 min after electrode onset to dry skin (Fig. 5 and Tables I and II), and the subsequent three sets after 6, 9, and 16 min (Tables I and II). Each spectrographic data set was measured in the frequency range 1 Hz–1 MHz with a Solartron model 1260/1294 setup and a measuring voltage amplitude of 10 mV(rms). The four circular arcs were determined with a ZView software package (Scribner Associates), Fig. 5 shows as an example the arc found with the first data set (continuous line). With the non-Cole model the G_1 values were calculated with a nominal $\tau_Z = 1$ [s].

With the non-Cole model (Table II) the samples showed a clear correlation with time between the α and G_1 (or $G_1\tau_Z^\alpha$) parameters. This is in agreement with the commonly accepted electrical model of human skin: the stratum corneum can be modeled by a CPE, and the sweat duct filling by a dc conductance in parallel [9], [17]. The postulated process is, therefore: First, the electrolyte penetrates the most superficial and electrically dominating layers of the stratum corneum resulting in a rapid increase of the admittance G_1 of the CPE. Then, gradually, the sweat ducts are filled with contact electrolyte increasing the conductance path $(G_{\rm var})$ in parallel with the CPE. With the Cole model (Table I) the four data sets showed no clear correlations between the τ_Z, α, and ΔG parameters with time, making it impossible to postulate any particular process in the skin.

V. Discussion

The Cole equation (2) or (4) is with the time constant τ as a conductance-independent parameter. In relaxation theory [7] the time constant is the time necessary to discharge a resistance–capacitance network, so that a large capacitance and resistance result in long time constants. The Cole equation is, therefore, incompatible with general relaxation theory. The alternative free-conductance model of (9) and (10) is in agreement because $G_{\rm var}$ influences the discharge process of the CPE capacitor so that higher conductance leads to higher characteristic frequency. This is true also if τ_Z^α is regarded as a parameter describing a distribution of relaxation time constants [7]. In biomaterials the capacitance values are usually much more stable than the resistance/conductance values. When that is the case, the Cole model with the time constant *independent* of conductance is not a logical starting point for model selection.

When data show that the time constant or characteristic frequency indeed varies [8], this may be due to a variation in an independent conductance according to the alternative model of (9), or a change in the nominal time constant independently of conductance changes according to the Cole (2).

Some authors have explicitly used a model similar to (9). Yamamoto *et al.* [9], for instance, called the independent parallel resistor R_2 and had it connected directly in parallel with the CPE as in Fig. 3(a). They measured the skin impedance of about 60 persons on four different occasions, and found low correlation between R_2 and the CPE capacitance, but a high correlation between R_2 and τ, in agreement with our argumentation. As long as many published varying time constant results are due to changing conductance, and the authors do not discuss the use of Cole models based upon a time constant as an independent parameter, many of them implicitly actually may have used the free-conductance model of (9).

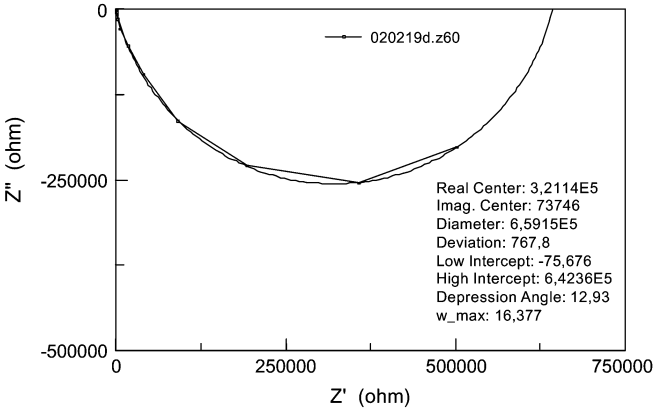

Fig. 5. Skin impedance plotted in the complex plane (broken line), human underarm. First spectrographic dataset as measured 3 min after electrode positioning on dry skin. Best circular arc found by regression analysis (continuous line) by the ZView program (Scribner Associates), calculated arc data given in the right lower corner.

TABLE I
COLE MODEL APPLIED TO THE MEASURED DATASETS

time [min]	ω_c *[rad/s]*	τ_Z *[ms]*	α	ΔG *[μS]*
3	16.4	61	0.86	1.6
6	13.3	75	0.81	1.6
9	16.3	61	0.82	2.0
16	24.0	42	0.78	3.1

TABLE II
FREE CONDUCTANCE MODEL APPLIED TO THE SAME MEASURED DATASETS AS IN TABLE I. $G_1\tau_Z^\alpha$ WAS CALCULATED WITH (11) AND G_1 WITH (12)

time [min]	ω_{Zm} *[rad/s]*	α	G_{var} *[μS]*	$G_1\tau_Z^\alpha$ *[Ss$^\alpha$ 10^{-6}]*	G_1 *[μS] (τ_Z=1s)*
3	16.4	0.86	1.6	0.14	0.14
6	13.3	0.81	1.6	0.20	0.20
9	16.3	0.82	2.0	0.20	0.20
16	24.0	0.78	3.1	0.26	0.26

An immittance equation should be in accordance with the theory of physics and electrical networks [10]. The problem of dimensions is already apparent in Frickes classical paper [6]. There, he presented the formula for his frequency-dependent capacitance as $C_p = \text{const} \cdot \omega^{-m}$, which implies that either the constant cannot have the dimension of capacitance or that the formula is dimensionally heterogeneous. Many authors have used such formulas, for instance [11]–[15]. However, network theory requires an angle in immittance equations, and the problem of dimensions is solved if ω (angular frequency) is replaced by $\omega\tau$ (angle). τ may be seen as a simple frequency scaling factor, and thereby looses some of its value as a biomaterial characteristic constant. An alternative to Frickes formula is then $C_p = C_1\,(\omega\tau)^{-m}$ where C_1 is a capacitance with the value corresponding to $\omega\tau = 1$, compare with (5). Several authors have also used the ω^α term alone but with τ^α hidden into a "constant" [9], [16]–[18], so that their equations actually are homogeneous. This is in accordance with the frequency scale being determined by the product $G_1\tau_Z^\alpha$. Commercial software packages do it the same way, e.g., in ZView (Scribner Associates) with the parameter CPE1-T being equal to G_1 [S], and the value of τ_Z chosen to be $\tau_Z = 1$ [s].

Equations (1), (2), (4), and (7)–(9), in both impedance and admittance versions, have perfect circular arc loci in the immittance plane and may, therefore, be equally preferable candidates for a fit to experimental data. Having the same descriptive power, a choice has to be made on the basis of knowledge about the biomaterial examined and the desired properties to be modeled. For instance, if the biomaterial is the human skin *in vivo*, the impedance model of Fig. 3(a) and (9) is a good starting point with the stratum corneum modeled by the CPE, and the independent filling of the sweat ducts by G_{var}, cf. the example given in Section V. The Cole model of Fig. 1(b) is not a preferred model because it does not have such an independent conductance. If the biomaterial is a cell suspension, the Y equivalent circuit of Fig. 3(b) and (9) may be a good starting point. Then, R_{var} models the cell interior, and the CPE models the cell membranes. ΔR as a dependent parameter according to the Cole Y-model (Fig. 2) is less preferable.

In this paper, we have limited our treatment to the CPE as an adequate model element for biomaterial dispersion analysis, without discussing its physical meaning. Cole himself was very well aware of the poor explanatory power of his models, [1], [4]. Cole apparently with time became less comfortable with his own model, and did not focus on it in his summing-up book [19]. Fricke did not use the Cole equations.

VI. Conclusion

The concept of dispersion is very fundamental to the field of bioimpedance, and it is important to know the properties and limitations of the models describing the immittance spectra usually found. The existence of a dependent conductance but an independent time constant parameter in the Cole model makes it incompatible with general relaxation theory and a more specialized model than usually realized. The analysis of the alternative free-conductance model with the relaxation time as a dependent parameter has shown that the model is preferable both because it is in accordance with general relaxation theory and actually measured data. Our results show that the time constant in the non-Cole equations no longer corresponds to the apex of the circular arc, and that the G_1 and τ_Z parameters cannot be determined separately from measured data, only the product $G_1\tau_Z$. However, it is possible to follow the relative changes in G_1 selectively during an experiment.

We find that dispersion analysis by circular arc plots in the complex plane together with equivalent circuit modeling is still an important part of the analysis of bioimmittance data.

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