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Collective Modes of the π^0 Condensed Phase in Pure Neutron Matter*)

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Collective modes of the π^0 condensed phase in pure neutron matter are studied, within the framework of the Generator Coordinate Method, for an [ALS] structure of the nucleon system. A quantal description of collective modes, related to the quantal oscillations of the condensed pion field, is obtained and the importance of the correlation energy is discussed Conventional methods to evaluate the total mass of the system are also discussed.

§ 1. Introduction

The possibility of a new phase in nuclear medium characterized by a one-dimensional solid-like structure with a particular spin-isospin ordering has been recently recognized.[1),2)] The so-called Alternating-Layer-Spin [ALS] model was proposed to describe this structure, which provides the source for a π^0 condensate.[2)] The ground state energy of such a system, calculated by means of a simple variational technique, was found to lead to a remarkable energy gain above a certain critical value of the density.

Collective excitations of the π^0 condensed phase of pure neutron matter have also been considered[3)] within a very simple model and using a semiclassical approach equivalent to the Random Phase Approximation (R.P.A.).

In the present work a fully quantal method to study this problem is proposed making use of the Generator Coordinate Method (G.C.M.) (Refs. 4)~7) and references therein). Assuming an [ALS] structure for the nucleon field we apply the G.C.M. in its simplest form, by making Gaussian and harmonic approximations. This method, already applied to other models of pion condensation[8),9)] has the advantage of providing not only the frequencies of collective modes but also a correction to the Hartree-Fock ground state energy (the correlation energy), since it accounts for quantum fluctuations around the zero point.

A discussion of some of the conventional methods to compute the total mass of the system is also presented.

This study is restricted to π^0 condensate in pure neutron matter though the method might be easily extended to other kinds of condensates.

After a brief description of the model in § 2, the study of the collective modes

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of the system in terms of the G.C.M. is presented in § 3. In § 4 the calculation of the total mass of the system is performed and finally some concluding remarks are made in § 5.

§ 2. The [ALS] model

The [ALS] model, originally proposed to study the π^0 condensation,[2] has been later improved and successfully used to study different aspects of pion condensation.[10)~14)]

In the framework of this model the ground state energy of the π^0 condensed phase in pure neutron matter was found by looking for the minimum of the expectation value of the Hamiltonian in the class of wave functions:

$$|\psi\rangle = |\phi\rangle \otimes |\xi\rangle\,, \tag{2·1}$$

where $|\phi\rangle$ describes a system of neutrons localized in layers along the z direction, with the spin changing layer by layer, and $|\xi\rangle$ represents the condensed pion field (see Refs. 13) and 14)).

$|\phi\rangle$ is a Slater determinant of the individual wave functions:

$$|\varphi_i\rangle = n\sum_{\boldsymbol{p},\gamma} e^{-\lambda p_z^2} e^{ip_z z_{l_i}} \eta_\gamma{}^i C^+_{\boldsymbol{p},-1/2,\gamma} |\varphi_i\rangle \delta_{p_x p_{x_i}} \delta_{p_y p_{y_i}}\,, \tag{2·1a}$$

where n is a normalization constant, $\eta_\gamma{}^i$ the usual spin wave function, λ the parameter of the Gaussian function which describes the localization and l_i the index which labels the layers: $z_{l_i} = l_i b$, with b denoting the layer spacing; the momentum along the xy plane are subjected to the Fermi level momentum restriction $\sqrt{p_{x_i}^2 + p_{y_i}^2} \le p_F$.

$|\xi\rangle$ is a coherent state:

$$|\xi\rangle = \mathcal{N} \exp\Big(\sum_{\boldsymbol{k}} \xi_{\boldsymbol{k},0} A^+_{\boldsymbol{k},0}\Big)|\rangle\,. \tag{2·1b}$$

As for the Hamiltonian it was taken as a sum of the kinetic energy terms for pions and nucleons with the usual $\boldsymbol{\sigma}\cdot\boldsymbol{k}$ coupling, since only a π-N P-wave interaction was considered.

The ground state energy was found to be:

$$E_0 = N\left[\frac{1}{2M}\left(2\pi\rho b + \frac{1}{4\lambda}\right) - g^2 \rho e^{-\lambda k_c^2} \frac{k_c^2}{\omega_c^2}\right], \tag{2·2}$$

where N is the number of particles, ρ the density, g the coupling constant of the π-N P-wave interaction, $|k_c| = \pi/b$ the momentum of the lowest mode available for pions, $\omega_c = \sqrt{k_c^2 + m_\pi^2}$ and m_π and M denote the pion and nucleon mass respectively.

§ 3. The collective modes

a) *The method*

Collective excitations of the new phase described by the above wave functions are now to be considered in the framework of the Generator Coordinate Method.

The usual trial wave function of the G.C.M. is a linear combination of the generator wave functions $|\phi(\alpha)\rangle$:

$$|\Phi\rangle=\int|\phi(\alpha)\rangle f(\alpha)d\alpha\ , \tag{3·1}$$

where $|\phi(\alpha)\rangle$ is such that $|\phi(0)\rangle\equiv|\phi\rangle$, the ground state wave function given by Eqs. (2·1) and α generically denotes the variational parameters which describe collective deformations of the nucleon single particle potential. Firstly we consider two independent variational parameters, the Gaussian parameter λ and the layer spacing b. They are assumed to have the form: $\lambda=\lambda_0+\gamma$ and $b=b_0+\beta$, λ_0 and b_0 being the values that minimize the expectation value of the Hamiltonian and γ and β small variations. Since the pion amplitudes, $\xi_{k_z,0}$, determined through the minimization of the expectation value of the Hamiltonian depend on λ and b as

$$\xi_{k_z,0}=-\frac{g}{\sqrt{2V}}\sum_{p_{xi},p_{yi},l_i}e^{-(\lambda/2)k_z^2}e^{ik_zl_ib}\frac{k_z}{\omega_{k_z}^{3/2}} \tag{3·2}$$

with V being the normalization volume, one can understand that these deformations of the nucleon potential will induce quantum oscillations on the condensed pion field.

In order to solve the integral Hill-Wheeler equation,

$$\int[\langle\phi(\alpha)|H|\phi(\alpha')\rangle-E\langle\phi(\alpha)|\phi(\alpha')\rangle]f(\alpha')d\alpha'=0 \tag{3·3}$$

obtained through the condition that $|\Phi\rangle$ minimizes the energy, the Gaussian and harmonic approximations are used respectively for the overlap and energy kernels, $I(\alpha,\alpha')=\langle\phi(\alpha)|\phi(\alpha')\rangle$ and $K(\alpha,\alpha')=\langle\phi(\alpha)|H|\phi(\alpha')\rangle/\langle\phi(\alpha)|\phi(\alpha')\rangle$. Standard techniques[5),6)] are then applied and the integral equation is transformed into a differential one, leading finally to a Schrödinger type equation:

$$\hat{\mathcal{H}}\tilde{f}=E\tilde{f}\ . \tag{3·4}$$

The collective Hamiltonian $\hat{\mathcal{H}}$ has a diagonalized form:

$$\hat{\mathcal{H}}=E_{\rm HF}+E_c+\omega\theta^+\theta\ , \tag{3·5}$$

where $W=\sqrt{A^2-B^2}$ is the frequency of the collective modes, $E_c=\frac{1}{2}(W-A)$ is the correlation energy and θ^+, θ boson operators defined in a convenient way.[5),6)] A and B are defined by the relations:

$$SA=\left[\frac{\partial^2 K(\alpha,\alpha')}{\partial\alpha\partial\alpha'}\right]_{\alpha=\alpha'=0},$$

$$SB=\left[\frac{\partial^2 K(\alpha,\alpha')}{\partial\alpha^2}\right]_{\alpha=\alpha'=0} \tag{3·6}$$

with

$$S=\left[\frac{\partial^2 \ln I(\alpha,\alpha')}{\partial\alpha\partial\alpha'}\right]_{\alpha=\alpha'=0}.$$

The expressions obtained for A and B in the case of oscillations of the Gaussian parameter are:

$$A^{(\lambda)}=\frac{\frac{1}{16M\lambda_0^3}-g^2\frac{\rho}{2}e^{-\lambda_0 k_c^2}\frac{k_c^4}{\omega_c^2}\left(\frac{k_c^2}{4}-\frac{1}{\lambda_0}\right)}{\frac{1}{8\lambda_0^2}+g^2\frac{\rho}{4}e^{-\lambda_0 k_c^2}\frac{k_c^6}{\omega_c^3}}, \tag{3·7}$$

$$B^{(\lambda)}=\frac{\frac{1}{16M\lambda_0^3}-g^2\frac{\rho}{2}e^{-\lambda_0 k_c^2}\frac{k_c^4}{\omega_c^2}\left(3\frac{k_c^2}{4}+\frac{1}{\lambda_0}\right)}{\frac{1}{8\lambda_0^2}+g^2\frac{\rho}{4}e^{-\lambda_0 k_c^2}\frac{k_c^6}{\omega_c^3}}. \tag{3·8}$$

The collective mode with frequency $W^{(\lambda)}=\sqrt{A^{(\lambda)2}-B^{(\lambda)2}}$ can be proved to be related to the oscillations of the amplitude of the pion field.

The expressions obtained in the case of oscillations of the layer spacing are:

$$A^{(b)}=-B^{(b)}=\frac{\frac{1}{16M\lambda_0^2}+g^2\frac{\rho}{2}e^{-\lambda_0 k_c^2}\frac{k_c^4}{\omega_c^2}}{\frac{1}{4\lambda_0}+g^2\rho e^{-\lambda_0 k_c^2}\frac{k_c^4}{\omega_c^3}}. \tag{3·9}$$

This mode, which has frequency $W^{(b)}=0$, is related to the oscillations of the phase of the pion field. The fact that the frequency is zero is quite natural since we have done nothing but a translation of the system along the z-direction. In fact the correlation energy is the same as the one obtained with the Peierls-Yoocoz method[15)] as it will be seen in next section.

Expressions for the correlation energies corresponding to the zero point fluctuations of λ and b are also obtained from the expressions (3·7), (3·8) and (3·9) respectively. The importance of these contributions for the lowering of the ground state energy will be discussed later.

We have been considering, up to now, collective modes related only with two degrees of freedom of the system. However, collective modes related with other degrees of freedom exist and should be taken into account. Within our model it is easy to consider a number of collective modes equal to the number of layers, by allowing the variational parameters λ and b to change from layer to layer. Collective modes related with the degrees of freedom within each layer are difficult to describe and are not considered here.

The study of the collective modes using the same variational parameters for all particles in the same layer but different variational parameters from layer to layer is possible due to translational symmetry. The deformations of λ and b are assumed to be periodic functions of the layer positions and so we will use the following expressions for the corresponding oscillations around λ_0 and b_0: $\gamma_l=\gamma\cos\alpha_j l$, $\beta_l=\beta\cos\alpha_j l$ with $\alpha_j=2\pi(b_0/L)j$ and j, $l\in(0, L/b_0)$; L/b_0 is the number of layers. In both cases a specific mode with frequency $W_j=\sqrt{A_j^2-B_j^2}$ is associated to each value of j. The corresponding correlation energies are of the form: $\varepsilon_{cj}=\frac{1}{2}(W_j-A_j)$.

The expressions obtained for A_j and B_j, defined by expressions similar to (3·6), are the following:

$$A_j^{(\lambda)}=\frac{\frac{1}{16M\lambda_0^3}-g^2\frac{\rho}{2}\sum_{k_z,n}\frac{k_z^4e^{-\lambda_0k_z^2}}{2\omega_{k_z^2}}\left(\frac{k_z^2}{4}-\frac{1}{\lambda_0}\right)\delta(k_z+k_n)}{\frac{1}{8\lambda_0^3}+g^2\frac{\rho}{4}\sum_{k_z,n}\frac{k_z^6e^{-\lambda_0k_z^2}}{2\omega_{k_z}^3}\delta(k_z+k_n+\alpha_j/b_0)}, \tag{3·10}$$

$$B_j^{(\lambda)}=\frac{\frac{1}{6M\lambda_0^3}-g^2\frac{\rho}{2}\sum_{k_z,n}\frac{k_z^4e^{-\lambda_0k_z^2}}{2\omega_{k_z}^2}\left[\left(\frac{k_z^2}{4}+\frac{1}{\lambda_0}\right)\delta(k_z+k_n)+\frac{k_z^2}{2}\delta(k_z+k_n+\alpha_j/b_0)\right]}{\frac{1}{8\lambda_0^2}+g^2\frac{\rho}{4}\sum_{k_z,n}\frac{k_z^6e^{-\lambda_0k_z^2}}{2\omega_{k_z}^3}\delta(k_z+k_n+\alpha_j/b_0)} \tag{3·11}$$

for oscillations of the Gaussian parameter, and:

$$A_j^{(b)}=\frac{\frac{1}{16M\lambda_0^2}+g^2\frac{\rho}{2}\sum_{k_z,n}\frac{k_z^4e^{-\lambda_0k_z^2}}{2\omega_{k_z}^2}\delta(k_z+k_n)}{\frac{1}{4\lambda_0}+g^2\rho\sum_{k_z,n}\frac{k_z^4e^{-\lambda_0k_z^2}}{2\omega_{k_z}^3}\delta(k_z+k_n+\alpha_j/b_0)}, \tag{3·12}$$

$$B_j^{(b)}=\frac{-\frac{1}{16M\lambda_0^2}+g^2\frac{\rho}{2}\sum_{k_z,n}\frac{k_z^4e^{-\lambda_0k_z^2}}{2\omega_{k_z}^2}\left[\delta(k_z+k_n)-2\delta(k_z+k_n+\alpha_j/b_0)\right]}{\frac{1}{4\lambda_0}+g^2\rho\sum_{k_z,n}\frac{k_z^4e^{-\lambda_0k_z^2}}{2\omega_{k_z}^3}\delta(k_z+k_n+\alpha_j/b_0)} \tag{3·13}$$

for oscillations of the layer spacing.

b) *Numerical results and discussion*

The frequencies and correlation energies corresponding to L/b parameters depend on sums of the generic form: $\sum_{k_z,n}f(k_z,\lambda_0)\delta(k_z+k_n+\alpha_j/b_0)$. In order to obtain numerical results, an approximation should be made, which would take into account only the dominant contributions. The same problem appears in the calculation of the ground state energy, since the π-N interaction term depends on sums of a similar type (Cf. Eq. (3·2″) of Ref. 2)). In Ref. 2) the dominant contributions were taken as coming from $n=-1, 0$, leading to Eq. (2·2), from which λ_0 and b_0 were determined by minimization.

In the present case the choice of the dominant contributions is somewhat more complex since if only terms with $n=-1, 0$ are considered the results are not symmetric around $j/(L/b_0)=\frac{1}{2}$ as they should be. It can be seen from Table I that the lowest contributions to be chosen should be either the ones with $n=-2, -1, 0$ for all values of $j/(L/b_0)$, or those with $n=-1, 0$, for $j/(L/b_0)\leq\frac{1}{2}$, and $n=-2, -1$, for $j/(L/b_0)>\frac{1}{2}$. Results for the frequencies corresponding to the first choice are plotted in Figs. 1 and 2; though the second choice seems to be less consistent, we also plot the results for the frequencies in Figs. 3 and 4. The next step would be to include the next two values of n in order to see if convergence is achieved. However it is not correct to include in the frequencies and correla-

Table I.

$j/(L/b_0)$ \ n	-3	-2	-1	0	1
0	-5	-3	-1	1	3
1/4	$-9/2$	$-5/2$	$-1/2$	3/2	7/2
1/2	-4	-2	0	2	4
3/4	$-7/2$	$-3/2$	1/2	5/2	9/2
1	-3	-1	1	3	5

$$\left(k_z/\frac{\pi}{b_0}\right)=\left(2n+1+2j\Big/\frac{L}{b_0}\right)$$

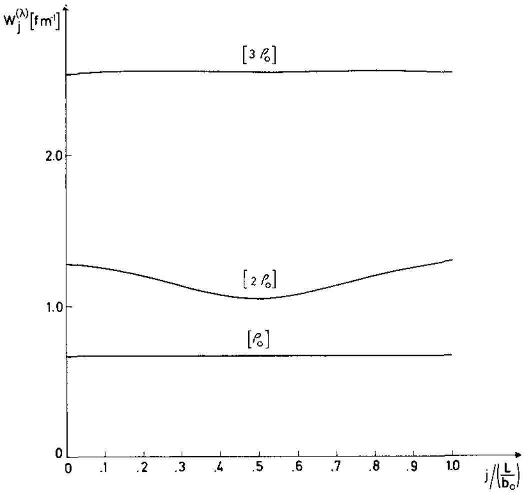
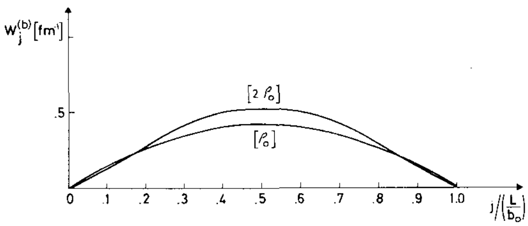


Fig. 1. Frequencies of collective modes due to oscillations of the Gaussian parameter, obtained with $n=-2, -1, 0$ and λ_0 and b_0 in columns 3 and 5 of Table II.

Fig. 2. Frequencies of collective modes due to oscillations of the layer spacing, obtained with the values of n, λ_0, b_0 of Fig. 1.



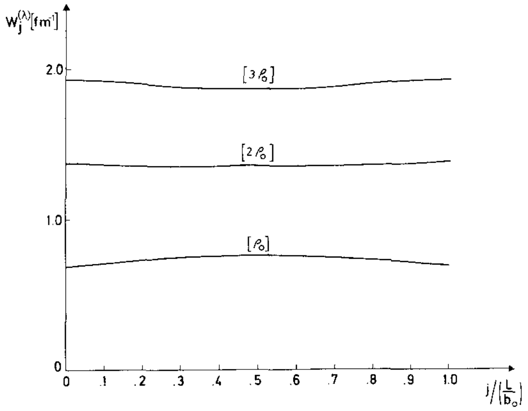


Fig. 3. Frequencies of collective modes due to oscillations of the Gaussian parameter, obtained with $n=-1$, 0 for $j/(L/b_0)\leq\frac{1}{2}$ and $n=-2$, -1 for $j/(L/b_0)>\frac{1}{2}$ and λ_0 and b_0 in columns 2 and 4 of Table II.

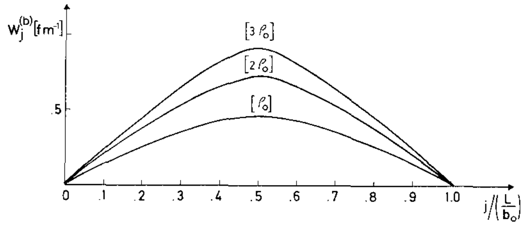


Fig. 4. Frequencies of collective modes due to oscillations of the layer spacing, obtained with the values of n, λ_0 and b_0 of Fig. 3.

tion energies terms corresponding to values of n such that $|k_n|>(\pi/b)$, if λ_0 and b_0 were obtained by minimizing the ground state energy including only these values of n. Such a problem also appeared when contributions with $n=-2$, -1, 0 were considered in $W_j(\lambda)$ and $W_j(b)$. Therefore the values of λ_0 and b_0 used were the ones obtained through the minimization of the ground state energy including the contributions depending on $|k_n|=3\pi/b$, besides those depending on $|k_n|=\pi/b$. For the densities ρ_0 and $2\rho_0$ ($\rho_0=18\,\mathrm{fm}^{-3}$ is the normal nuclear matter density) the values of λ_0 and b_0 (see Table II) are equal to the ones obtained only with terms on $|k_n|=\pi/b$ and it was verified that there is convergence. For $\rho=3\rho_0$ these values change strongly and convergence is not obtained. The values of $W_j(\lambda)$ at this density are compared to those of ρ_0 and $2\rho_0$ and the correlation energy is extremely small; $W_j(b)$ has imaginary values. However, since for $\rho\geq3\rho_0$ convergence is not obtained one cannot be sure that this imaginary value means a real physical instability. What we can conclude is that, in order to study the collective modes of the system for $\rho>3\rho_0$, a more realistic Hamiltonian should be

Table II. Values of λ_0 and b_0 that minimize the ground state energy, obtained with inclusion of terms depending on $|k_n|=\pi/b$ (columns 2 and 4) and $|k_n|=\pi/b$, $3\pi/b$ (columns 3 and 5).

ρ	λ_0		b_0	
ρ_0	.235	.235	1.647	1.647
$2\rho_0$	.125	.125	1.460	1.460
$3\rho_0$	.085	.055	1.300	1.950
(fm^{-3})	(fm^2)		(fm)	

used, namely including short-range repulsive nuclear forces.

Nevertheless, the results for $\rho < 3\rho_0$ are consistent, and we will make some considerations about them in connection with the work by Suzuki et al.[3)]

These authors described formally two types of collective modes of the π^0 condensate phase of pure neutron matter, by defining operators which account for quantum oscillations of the condensed pion field. One of the modes is interpreted as due to amplitude oscillations, and the other, with zero frequency, as due to phase oscillations. The treatment is equivalent to the R.P.A. In the present work the G.C.M. was applied to the [ALS] model, which gives a good description of the ground state of the condensed phase. Within this treatment the relation between collective deformations of the nucleon field and quantum oscillations of the condensed pion field becomes clear. The corresponding collective modes may be studied in detail and numerical results may be easily obtained.

Looking at Fig. 1 one can see that $W_j(\lambda)$ has a finite value for all values of the phonon momentum, a_j/b_0, and the variation is smooth. These modes correspond to the amplitude oscillation modes of Ref. 3). As for $W_j(b)$, (see Fig. 2) its variation is proportional to a_j/b_0 and seems to correspond, for $j=0$, to the phase oscillation modes of Ref. 3). The modes here described are phonon-like and the sound velocity may be determined through the derivative of the curves of Fig. 2.

A well-known property of the G.C.M. is that it accounts for quantum fluctuations around the zero point, leading to the correlation energy associated to this process. The correlation energies corresponding to oscillations of the Gaussian parameter and of the layer spacing are plotted in Figs. 5 and 6, respectively.

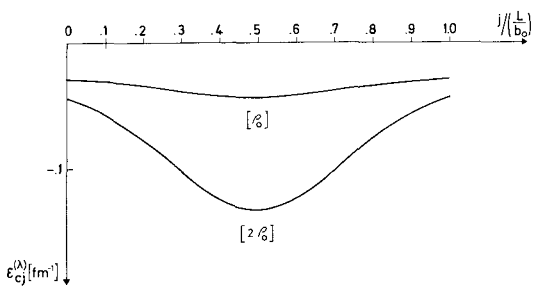


Fig. 5. Correlation energies due to zero-point fluctuations of the Gaussian parameter, obtained with the same values of n, λ_0, b_0 of Fig. 1.

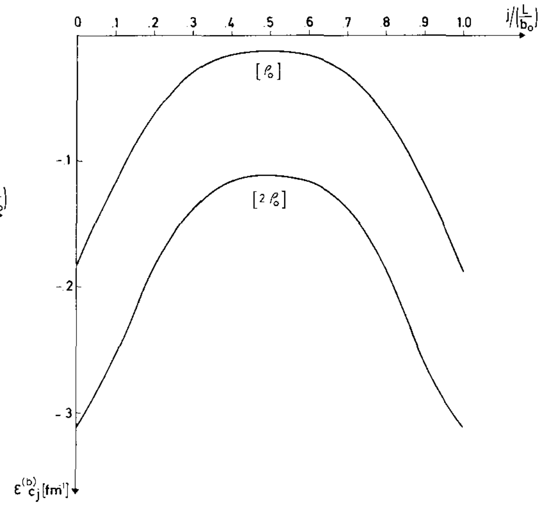


Fig. 6. Correlation energies due to zero-point fluctuations of the layer spacing, obtained with the same values of n, λ_0, b_0 of Fig. 1.

Here we have contributions proportional to the number of layers. This is a fraction of the total number of modes of the order $N^{-2/3}$. The zero point energy associated to these modes is therefore vanishingly small, so that it is meaningless to take it into account. Nevertheless, this calculation shows that there exists an appreciable correlation energy per mode which lowers the ground state energy. So one might expect that a correct treatment of the correlation energy would become important in the accurate determination of the critical density and discussions about the transition properties of pion condensation, especially for finite nucleon systems.

§ 4. The total mass of the system

It is of course of fundamental importance to determine the total mass of the system. Indeed the mass is a basic physical property, which is obviously modified by the presence of the pions. In order to calculate the total mass we will apply the Peierls-Yoocoz method and a constrained variational method. Both methods restore the translational symmetry which is broken by our trial wave function.

The Peierls-Yoocoz method[15] is a variant of the G.C.M. that projects out the wave function into a state of well defined momentum. However, although this method was devised to calculate the mass parameter, it leads generally to a not very correct expression. So, besides this calculation, another one has been performed which consists on the application of a constrained variational principle: The expectation value of the Hamiltonian is minimized subjected to a restriction on the z-component of the total momentum operator, that is,

$$\hat{P}_z = \sum_{p,\gamma} p_z c^+_{p,-1/2,\gamma} c_{p,-1/2,\gamma} + \sum_k k_z A^+_{k,0} A_{k,0} \,. \tag{4·1}$$

The present calculations follow the method described in Ref. 16).

In the case of the Peierls-Yoocoz projection we use a wave function:

$$|\Phi\rangle = \int dz\, f(z) e^{iz\hat{P}_z} |\psi\rangle \,, \tag{4·2}$$

where $|\psi\rangle$ is the wave function given by Eq. (2·1) and $f(z)$ a weight function of the form $f(z) = e^{iqz}$. The Hill-Wheeler equation is easily solved, leading to:

$$E(q) = E_0 + E_c + \frac{q^2}{2M_t} \,, \tag{4·3}$$

where

$$E_c = -\frac{1}{2} \frac{\dfrac{1}{16M\lambda_0{}^2} + g^2 \dfrac{\rho}{2} e^{-\lambda_0 k_c{}^2} \dfrac{k_c{}^4}{\omega_c{}^2}}{\dfrac{1}{4\lambda_0} + g^2 \rho e^{-\lambda_0 k_c{}^2} \dfrac{k_c{}^4}{\omega_c{}^3}} \tag{4·4}$$

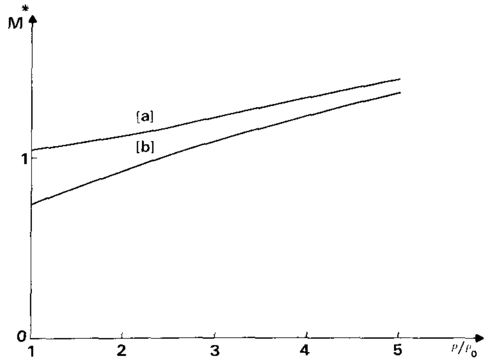


Fig. 7. Mass per nucleon of the system in units of the nucleon mass.
[a]-Peierls-Yoocoz method.
[b]-Constrained variational.

is the correlation energy which is equal to the one obtained from the expression (3·9) as it has already been mentioned, and

$$M_t = N\frac{\left(\frac{1}{4\lambda_0} + g^2\rho e^{-\lambda_0 k_c^2}\frac{k_c^2}{\omega_c^3}\right)^2}{\frac{1}{8M\lambda_0^2} + g^2\rho e^{-\lambda_0 k_c^2}\frac{k_c^4}{\omega_c^2}} \tag{4·5}$$

is the total mass of the system.

The calculation by means of the constrained variational principle makes use of a wave function $|\psi\rangle_q$ which is obtained from $|\psi\rangle$ by means of the canonical transformation: $p_z \to p_z - q$, and minimizes the expectation value of the Hamiltonian $\hat{H} - \mu\hat{P}_z$, μ being a Lagrange multiplier. The following expressions are finally obtained,

$$_q\langle\psi|\hat{P}_z|\psi\rangle_q = \mu N\left(M + 2g^2\rho e^{-\lambda_0 k_c^2}\frac{k_c^4}{\omega_c^4}\right) \tag{4·6}$$

and

$$_q\langle\psi|\hat{H}|\psi\rangle_q = E_0 + \frac{\mu^2}{2}N\left(M + 2g^2\rho e^{-\lambda_0 k_c^2}\frac{k_c^4}{\omega_c^4}\right). \tag{4·7}$$

Interpreting μ as the velocity of the system we readily obtain its mass:

$$M_t = N\left(M + 2g^2\rho e^{-\lambda_0 k_c^2}\frac{k_c^4}{\omega_c^4}\right). \tag{4·8}$$

The values of the total mass per nucleon, $M^* = M_t/N$, given by both methods are plotted in Fig. 7, as a function of the density. It can be seen that M^*grows with the density and, in the case of the second calculation, is always greater than the nucleon mass. This is natural since the critical density for the π^0 condensate in pure neutron matter is $\rho_c \simeq .85\rho_0$. So, at the densities plotted, a certain number of condensed pions is present and the mean number of pions, $\bar{\chi}$, is proportional to the density: $\bar{\chi}/N = g^2\rho e^{-\lambda_0 k_c^2}(k_c^2/\omega_c^3)$.

As usual, the Peierls-Yoocoz method gives a lower value for M^*, which, for densities near ρ_0, is even lower than the nucleon mass.

§ 5. Conclusions

A quantal description of collective modes of the π^0 condensed phase in pure neutron matter, provided by the Generator Coordinate Method has been presented.

The frequencies of the collective modes due to collective deformations of the Gaussian parameter and the layer spacing were obtained. The analysis of the results shows that these collective modes should be related with the quantum oscillations of the condensed pion field, namely oscillations of amplitude and phase.

Contributions to the correlation energy due to zero-point fluctuations of the mentioned parameters were also obtained. Although it is not meaningful to compare this contributions with the ground state energy since they are not proportional to the number of particles, they emphasize that the zero-point fluctuations may be important to evaluate accurately the ground state energy and the critical density.

Classical methods to compute the total mass of the system have also been discussed. As it was expected, the mass per particle has been found to increase with the density due to the presence of the condensed pions.

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