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Collision-induced shape transformations of partially coherent solitons

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We present experimental results related to collisional properties of partially coherent solitons (PCSs). In experiments with a photorefractive crystal we show that collisions in nonlinear media cause the PCS comprised of few mutually incoherent soliton components to change its shape. [S1063-651X(99)08804-2]

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Partially coherent solitons are objects which have recently attracted a great deal of attention [1–10]. Focusing of the spatially partially coherent beams has been suggested as early as in 1967 [11] and studied subsequently in [12,13]. On the other hand, temporally incoherent solitons have been considered in the original works of Hasegawa [14–16], both for plasma waves and for nonlinear pulses in multimode fibers. However, the creation of incoherent solitons in optical fibers requires unrealistically high pulse energies. In contrast, photorefractive materials allow experimental studies at very low optical powers as they generally exhibit strong nonlinear self-action effects [17–21]. In fact, the first experiments with the partially coherent soliton have been conducted with photorefractive nonlinearity [1,2,6].

The interaction of partially coherent solitons (PCS) is an interesting area of research and it has only been addressed in the recent works [8,22]. In the present paper, we experimentally demonstrate new collisional properties of PCS. Namely, we have discovered that the PCS may have variable shape [23] and that the collision of the PCS leads to a transformation of the soliton profile. In order to explain these phenomena we will first briefly discuss the main features of partially coherent solitons considering the solvable model of a Kerr-like medium. However, the nonlinear susceptibility of a general nonlinear medium, such as a photorefractive crystal, is usually well approximated by a saturable nonlinearity. Therefore, we will comment on PCS in a saturable nonlinearity discussing experimental results.

It has been shown that incoherent solitons can be represented as N self-trapped mutually incoherent wave packets which represent modes of the self-induced waveguide [5,6]. From this point of view, incoherent solitons are analogous to higher-order vector solitons which can also be considered as multimoded self-induced waveguides [24]. One of the physical reasons which makes various modes phase independent (incoherent) is a slow response of the nonlinear medium. Usually in photorefractive media the response time is much slower than the random fluctuation of the phases of the partially coherent beam so its refractive index change is a function of the time-averaged light intensity. On the other hand, the light beam components can be initially incoherent in time leading to qualitatively the same phenomena. In either case, the evolution of the N self-trapped mutually incoherent wave packets in a medium with Kerr-like nonlinearity can be represented by a set of N coupled dimensionless equations

$$i\frac{\partial \psi_i}{\partial z}+\frac{1}{2}\frac{\partial^2 \psi_i}{\partial x^2}+\delta n(\psi_i)\psi_i=0, \tag{1}$$

ψ_i denotes the ith component of the beam, x is the transverse coordinate, z is the co-ordinate along the direction of propagation, and

$$\delta n(\psi_i)=f\left(\sum_{i=1}^{N}|\psi_i|^2\right)=f(I) \tag{2}$$

is the change in refractive index profile created by the partially coherent beam, which, because of the lack of temporal correlation between the modes, is a nonlinear function of just a sum of modal intensities.

For the sake of simplicity, we take here the function $f(I)=I$. As a result, the set of equations (1) is an integrable set of coupled nonlinear Schrödinger equations (NLSE). This simplifies the analysis in the sense that all solutions can be written in analytical form. Examples of the lowest-order symmetric solutions and their interactions have been presented in recent works [7,8]. However, those solutions contain only one free parameter. They are symmetric solutions of a given (sech) profile and have variable amplitude (and width) for any particular N.

More general analysis has been done in [22]. In particular, it has been shown that the actual PCS profiles are multiparameter families of asymmetric solutions and that their shape and amplitude may vary. Moreover, it has been shown [22] that PCS can be best understood using the principle of complementarity. Namely, we can think of PCS both as a

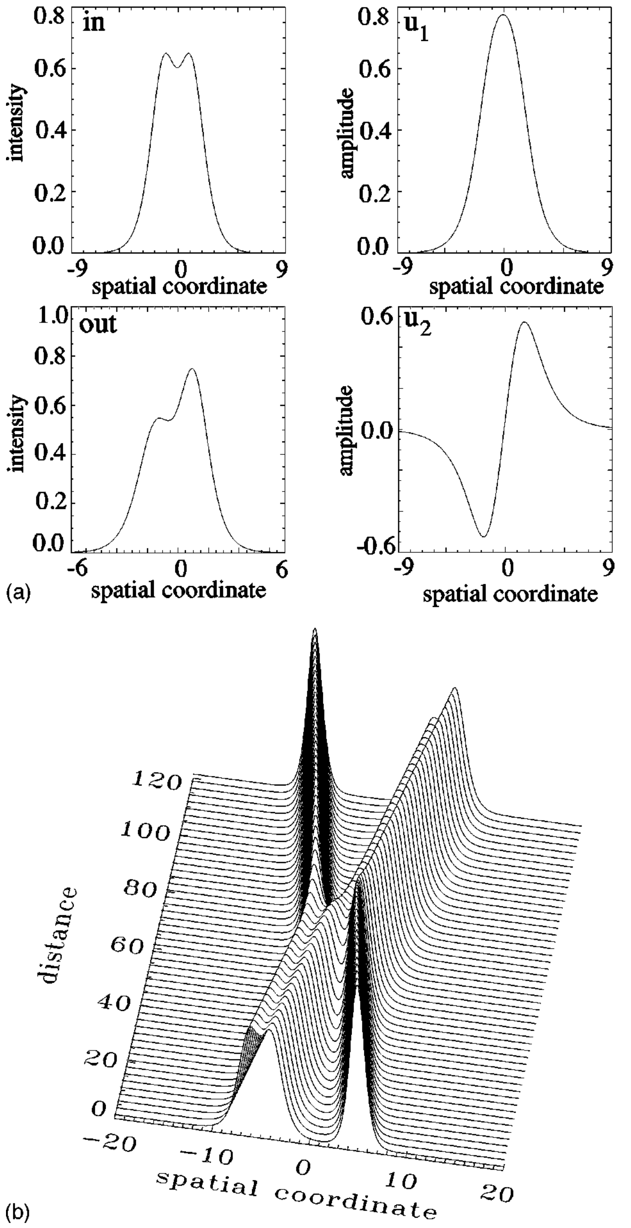

FIG. 1. Numerical simulation of the collision of a fundamental soliton ($\lambda=1$) with a partially coherent soliton formed by incoherent superposition of two modes. (a) Input and output intensity profile as well as amplitude profiles of the constituent components of PCS. (b) Surface plot illustrating the collision. In this case the corresponding eigenvalues for the components of PCS are $\lambda_1=1, \lambda_2=0.65$.

waveguide self-induced by its linear modes (characterized by the corresponding propagation constants λ) and also as a multisoliton complex consisting of a certain number of fundamental solitons. Interestingly enough, the number of linear modes in the waveguide coincides with the number of fundamental solitons N in the multisoliton complex which forms the PCS.

If the ability of PCS to take a variety of profiles is an unusual novel feature itself, then the collisional properties of these solitons are even more amazing. We will first illustrate the collisions of PCSs using numerical simulations. The results are shown in Figs. 1 and 2. In both cases collision involves standard coherent soliton and partially coherent soliton (soliton complex) consisting of two (mutually incoherent) modes of the self-induced waveguide. These simulations reveal that the collision induces a dramatic change of shape of the PCS. After collision, PCS remains a soliton but has an intensity profile different from the initial one. It is evident from Fig. 1 that an initially symmetric soliton becomes clearly asymmetric.

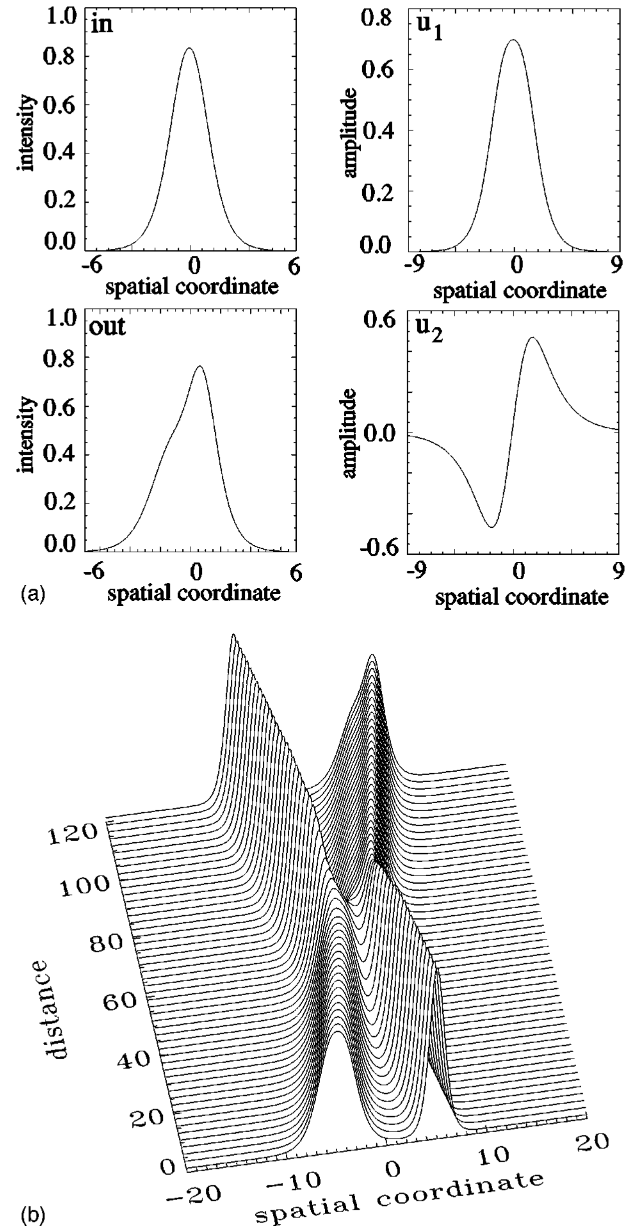

FIG. 2. The same as in Fig. 1 but for PCS with $\lambda_1=1$, $\lambda_2=0.5$.

In the case displayed in Fig. 2 the structural change of the PCS involves asymmetry and change in the separation of intensity peaks. Additional simulations show that after numerous collisions, N-component partially coherent solitons may be completely separated into N fundamental solitons. This is a remarkable feature of a PCS collision, and it differs drastically from a standard collision between two fundamental bright solitons.

As has been pointed out in the earlier work [22], to explain the reshaping phenomenon, we can think of the PCS as a multisoliton complex. Then the transformation of the shape of the PCS is caused by different lateral shifts experienced during the collision by each individual soliton forming this multisoliton complex. The complementary point of view is that the modes of the self-induced waveguide experience a different rate of refraction when intersecting another PCS. This also explains a change of shape of the PCS after collision. This simple physics is responsible for reshaping of PCS not only in Kerr media but also in media with other types of nonlinearity including photorefractive media. In the latter case, the effect of PCS formation can be studied by using the equation for solitons in media with saturable nonlinearity. These simulations give qualitatively similar results and show the existence of multiparameter families of asymmetric solutions. Numerical examples of asymmetric PCS in saturable media and their collision have been found recently in [25], while the experimental confirmation of this effect is given below. We show in this paper that partially coherent solitons in photorefractive media also reshape after collisions, although their subsequent dynamics might be more complicated than in a model with Kerr-like nonlinearity.

Collision of partially coherent spatial solitons has been studied experimentally with the screening solitons formed in a photorefractive strontium barium niobate crystal [19–21]. We used a setup similar to that described in our prevoius works [26]. A laser beam (@514.5 nm) was split into two parts. The first one was reflected from a PZT mounted mirror driven by an ac signal. The vibration of the mirror introduced a frequency shift into the beam. This frequency-shifted beam constituted the fundamental component of the input beam. A second, higher-order component was created by introducing a phase mask across part of the second beam to induce a π phase shift. Both beams were then recombined and focused by a set of cylindrical lenses to form a narrow (≈ 20 μm) stripe on the input face of the crystal. As the temporal response of the photorefractive crystal is slow, fast changes of the relative phase between both components (due to the frequency shift) effectively make them mutually incoherent when propagating together through the crystal (which responds to the sum of their intensities).

This composite beam then intersected inside the crystal with another one-dimensional, single-component stripe derived from the same laser. The optical path for this second beam exceeded that of the first one by a few coherence lengths so both beams were mutually incoherent. The propagation distance inside the crystal was 10 mm. The output face of the crystal was imaged onto a CCD camera and recorded by the computer. The crystal was biased with a 1.5 kV dc electric field applied along its c axis while all beams were extraordinarily polarized to make use of the largest r_{33} electro-optic coefficient. Additionally, the crystal was illuminated by the broad white light beam which provided the necessary background illumination used to control the degree of saturation of the nonlinearity which in our experiments was estimated to be of the order of unity [27].

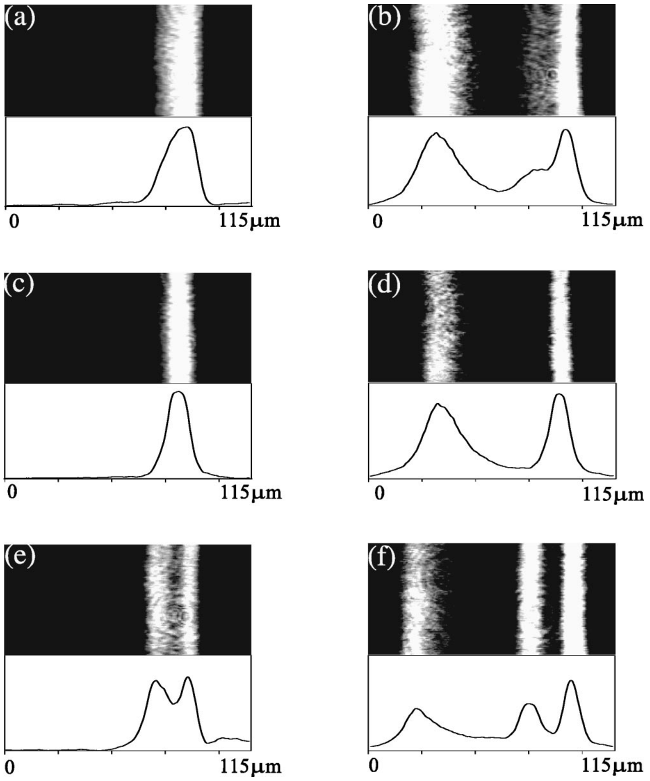

FIG. 3. Experimental results of the collision of 1D partially coherent solitons in photorefractive crystal. The plots show only the light intensity distribution on the exit face of the photorefractive crystal. These stripes represent one-dimensional solitary beams. In the left column we show free propagation of the PCS (no collision). The right column illustrates the effect of a collision with a single-component solitary beam. The angle of collision is $\approx 0.6°$. (a) and (b) Individual PCS and its collision with a single-component soliton. The power of the fundamental component is much larger than that of the first-order component. (c) and (d) A single-component soliton and its collision with another single-component soliton. The structure of the solitons remains unchanged. (e) and (f) Individual PCS with a strong contribution of the first-order component and its collision with a single-component soliton.

Initially we propagated each beam separately. We found that each of them (composite as well as fundamental) formed solitary beams. The total power of the single-component beam was 70 μW while that of the composite beam varied from case to case. Typical results are presented in Fig. 3, where we show the light intensity distribution on the exit face of the crystal. In the graph in Fig. 3(a) we show the composite (or partially coherent soliton) propagating alone in the crystal. To create this soliton the initial power ratio of the fundamental to first-order mode was set to 3 while the total power was 60 μW. This is a single-peaked soliton. The next graph, Fig. 3(b), shows the result of the collision between this composite soliton and the fundamental soliton. It is quite clear that after the collision the partially coherent soliton becomes strongly asymmetric, which is an indication of the partial separation of its constituent modes. For comparison, in Figs. 3(c) and 3(d) we show the collision between two fundamental solitons. The first graph [Fig. 3(c)] shows the fundamental soliton propagating alone while the graph in Fig. 3(d) illustrates the effect of the collision. Besides the usual lateral shift of the soliton, its profile remains symmetric. The two final graphs, Figs. 3(e) and 3(f), illustrate another example of the collision of the composite and fundamental solitons. This time the contribution of the first mode of the PCS is stronger than before (the initial power ratio of the fundamental to first-order mode equals 1.3 while the total power is 70 μW). The composite soliton exhibits clear double-peaked structure with the separation between the peaks of 24 μm [Fig. 3(e)]. When it collides with the fundamental soliton, the two-peaked structure becomes even more pronounced and the separation between peaks increases to 34 μm. This is qualitatively the same type of behavior as found in our numerical simulations (Fig. 2).

The above experimental results confirm the effect predicted both theoretically and in our numerical simulations. Namely, the composite solitons reshape during the collisions. However, for saturable media, the final state of each PCS is not necessarily a stationary beam. The reason is the following. In the case of the completely integrable Kerr-like model, there is no binding energy between constituent fundamental soliton components. Hence, after the collision the reshaped PCS propagates as a stationary beam. It turns out that in the case of a saturable medium such as photorefractive crystal, the deviation from integrability leads to the existence of binding energy among constituent soliton components. Nevertheless, the stationary asymmetric soliton shapes still do exist but their parametrization is different. While details of this problem will be published elsewhere, let us only notice here that the main consequence of the interaction between the constituent fundamental solitons is that after the collision the propagation of the PCS becomes nonstationary and exhibits strong mutual oscillations of the components. We illustrate this behavior in Fig. 4, where the collision of two identical PCS formed by incoherent superposition of the fundamental and first-order components is shown. The nonlinearity in this example is modeled as $\Delta n \propto I/(1+0.05I)$ and for the peak intensity $I_{\max}\approx 5$ corresponds to a mild saturation. It is evident from this graph that indeed collision leads to the shape transformation of both solitons. However, as the new shape does not correspond to the proper soliton profile, beams do not propagate as stationary solitons but exhibit mutual oscillation of the constituent components. In the inset of this figure we show both input and output intensity distributions. The asymmetry of the outgoing solitary beam is clearly visible. In the light of this simulation we can say that although experimental results shown in Fig. 3 show a tendency of partially coherent solitons to transform their shape upon the collision, the intensity distribution recorded on the exit face of the crystal reflects the stage of nonstationary oscillation of the constituent components.

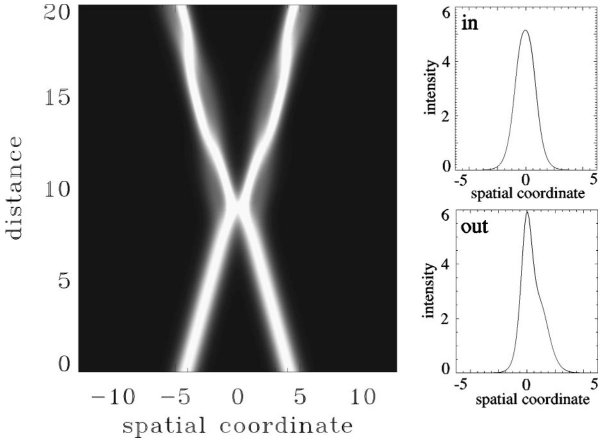

FIG. 4. Numerical simulation of collision of two-component PCS solitons in a saturable nonlinear medium with the nonlinearity $\Delta n \propto I/(1+0.05I)$.

In summary, we have studied experimentally the collisions of partially coherent solitons which are formed by incoherent superposition of the corresponding modes of the self-induced waveguide. We found that the collisions induce a transformation of the intensity profile of these solitons. This effect is caused by the different lateral shifts of the fundamental soliton components which constitute the partially coherent soliton. In Kerr-like media, the reshaped soliton propagates as a stationary beam. On the other hand, in the saturable nonlinear medium, collision also induces reshaping of the partially coherent soliton but further propagation of the soliton is no longer stationary with strong mutual oscillation of its constituent components.

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