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Collisionless Microinstabilities in Configurations With Periodic Magnetic Curvature

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ABSTRACT

In view of investigating the stability of a closed (toroidal) configuration in the high temperature collisionless regime, a one-dimensional model simulating the effects of magnetic curvature variation, magnetic shear, and particle trapping is adopted. Use is made of the Vlasov equation including finite Larmor radius and wave-particle resonance effects. Low frequency electrostatic modes are considered. Then two types of wave having the same periodicity L as the magnetic curvature, or localized in a region where curvature is unfavorable, are found. One has the frequency of the known drift wave, and the other (the flute-"gravitational" wave) has frequency determined by the average favorable curvature along the lines of force. The latter wave is stabilized by imposing that L be sufficiently short as to ensure good ion communication making ion Landau damping effective. The former one by imposing that L make the effects of longitudinal ion sound wave prevail over the effects of ion inertia on their transverse motion. If the lines of force are not closed or if they are closed but their length is much larger than L , drift waves with wavelength

larger than L have to be considered. In the first case they can be stabilized by shear, in the second case waves with transverse wavelengths short enough as to make the effects of transverse inertia prevail over those of longitudinal ion inertia remain unstable. The influence of trapped particles is investigated finding that it contributes to reducing growth rates. Stability conditions are given for the most significant cases observing that, for non-hydromagnetic types of mode, they are easier than those obtained for the collisional regime. It is recalled that while no wave localized in a region of unfavorable curvature was found in the high temperature collisional regime, a wave driven by the known drift mechanism but localized over distances shorter than L can be found in the collisionless regime.

INTRODUCTION

It has been shown previously[1] that configurations which have favorable average magnetic curvature (e.g., are $\oint d\ell/B$ stable) but have no shear can be unstable in the presence of electron-ion collisions. The aim of the present work is to give an analysis for a model of this type of configuration considering the collisionless regime where wave-particle resonances replace the effects of electron-ion collisions. We limit ourselves to investigating electrostatic modes, considering the low β limit. Then the paper is structured as follows: In Sec. A we define the equilibrium and discuss the analytical aspects of the stability problem. In Sec. B we recognize the relevance of waves having the same periodicity as the magnetic curvature and study one of them, the "gravitational" wave, having its frequency determined by the average favorable magnetic curvature. In Sec. C we analyze the possibility of shear stabilization of this wave. In Sec. D, the drift wave is investigated showing that it can be stabilized either when the magnetic field lines close, and the wavelength along them has an upper bound, or by magnetic shear. Section E is devoted to the possible modifications of the results if the resonant particles responsible for the instabilities are trapped between local magnetic mirrors. The conclusions are finally given.

SECTION A

1. The Equilibrium

As a model of a system possessing shear and variable magnetic curvature with negative V'', a plasma layer is considered, perpendicular to the x direction, where the magnetic field is represented by:

$$\underset{\sim}{B} = B_o \left(1 - \frac{x}{R_o(\zeta)}\right) \underset{\sim}{e}_z + B_o \frac{x}{L_s} \underset{\sim}{e}_y \qquad (1.A)$$

Since we treat a low β system, we neglect diamagnetic effects and imagine these field changes, so simulating the vacuum magnetic field of external windings. Moreover, $B_o\, x/L_s$ represents the "shear" and $B_o\,[1 - x/R_o(\zeta)]$ the main magnetic field and its curvature variation, ζ being a linear coordinate along the lines of force.

The $\oint d\ell/B$ stable configuration is represented by the curvature profile

$$\frac{1}{R_o(\zeta)} = \frac{1}{R_c}\left[\cos\frac{2\zeta}{L} - h\right]$$

where $0 < h < 1$. The scale lengths here introduced will be considered in the ordering $x \lesssim r < L < R_c < L_s$, where $r \equiv -(d\ell n\, n_o/dx)^{-1}$ n_o being the particle density. Later on we shall introduce an additional modulation of B_o to take into account the effects of trapped particles.

To deal with the collisionless regime we adopt the Vlasov equation, so that the equilibrium is described by:

$$\underset{\sim}{v} \cdot \nabla f_{oj} + \frac{e_j}{m_j} (\underset{\sim}{E}_o + \underset{\sim}{v} \times \underset{\sim}{B}_o) \cdot \nabla_v f_{oj} = 0 \qquad (2.A)$$

We assume that no electric field exists, $E_o = 0$, and choose as solution of Eq. (2.A) a suitable f_o close to thermal equilibrium. Then

$$f_{oj} = \left(\frac{\alpha_j}{\pi}\right)^{3/2} n_o\, e^{-\alpha_j v^2} \left[1 - \frac{1}{r^*}\left(x + \frac{v_y}{\Omega_j}\right) + \frac{2x}{R_c}\right] \tag{3.A}$$

where only the largest terms of the expansion in x/r^* are included, $\alpha_i \equiv (v_{thi})^{-2}$ and $r^{-1} = r^{*-1} - 2R_c^{-1}$. The diamagnetic velocities are $v_{di} = -(2\,\alpha_i\,\Omega_i\,r^*)^{-1}$ and $v_{de} = (2\,\alpha_e\,\Omega_e\,r^*)^{-1}$.

2. The Stability

Dealing with a system having low β, we limit consideration to electrostatic perturbations from the equilibrium, having phase velocity less than the Alfvén velocity.

Then, linearizing the Vlasov equation and integrating along particle trajectories, we have:

$$\hat{f}_j = \frac{e_j}{m_j}\int_{-\infty}^{t} dt'\, \nabla\hat{\phi}\cdot\nabla_v f_{oj} \quad ; \quad \nabla^2\hat{\phi} = \sum_j e_j \int \hat{f}_j\, d^3v$$

If a_j represents the gyration radius, for $a_i < r \ll L_s$, the lowest order expression for the particle orbits around $x = 0$ is

$$\begin{cases} x(t') - x = -\dfrac{v_\perp}{\Omega}\left[\sin(\theta - \Omega t') + \sin\theta\right] \\ y(t') - y = \dfrac{v_\perp}{\Omega}\left[\cos(\theta - \Omega t') - \cos\theta\right] + \dfrac{h\,t'}{\alpha R_c \Omega} \\ \qquad\qquad - \dfrac{1}{2}\,\dfrac{L}{\alpha R_c\,\Omega\, v_\parallel}\left[\sin\left(\dfrac{2}{L}\zeta(t')\right) - \sin\left(\dfrac{2}{L}\zeta\right)\right] \\ \zeta(t') - \zeta = v_\parallel\, t' \end{cases}$$

where $\zeta = z - (x/L_s)\, y \approx z'$ In consistence with our previous ordering of the scale length distances, we have taken into account only the curvature drift and neglected its velocity dependence. This will limit us to consider instabilities with frequency:

$$\omega >> \frac{k}{\alpha_j \Omega_j R_c} \sim k a_i \frac{v_{thi}}{R_c}$$

with the reasonable assumption that resonances with this drift frequency do not affect in an essential way the conclusions we shall derive.

3. Normal Mode Equations in the Absence of Shear

To represent this case we take $L_s = \infty$. Moreover we neglect for simplicity the Debye distance in comparison with the Larmor radii. Then, we consider modes of the form

$$\hat{\phi} = \tilde{\phi}(x, z)\; e^{i(k_y + \omega t)} \tag{4.A}$$

assuming that they are localized in the x direction and $k >> \partial/\partial x$. This leads us to consider $\tilde{\phi}(x, z) \approx \tilde{\phi}(z)$ and n and dn/dx as constants in lowest order. The dispersion relation is obtained by setting $\tilde{n}_i(z) = \tilde{n}_e(z)$. In particular, defining $\tau \equiv T_e/T_i$, we obtain after carrying out standard integrations of the orbit method[2]:

$$\tilde{n}_i = -\frac{\tau}{e\lambda_D^2}\left\{\tilde{\phi}(z) - i(\omega - k v_d)\left(\frac{\alpha_i}{\pi}\right)^{1/2} I_o(b)\, e^{-b}\int_{-\infty}^{+\infty} d v_{\parallel}\, e^{-\alpha v_{\parallel}^2} \times \right.$$
$$\left. \times \int_{-\infty}^{o} dt'\, \tilde{\phi}(z')\, e^{i\omega_i t' - i\frac{k v_g L}{2 v_{\parallel}} - (\sin 2z'/L - \sin 2z/L)}\right\} \tag{5.A}$$

$$\tilde{n}_e = \frac{1}{e\lambda_D^2}\left\{\tilde{\phi}(z) - i(\omega + kv_d\tau)\left(\frac{\alpha_e}{\pi}\right)^{1/2}\int_{-\infty}^{+\infty} dv_{\parallel}\, e^{-\alpha_e v_{\parallel}^2} \times \right.$$

$$\left. \times \int_{-\infty}^{0} dt'\, \tilde{\phi}(z')\, e^{i\omega_e t' + ikv_g\tau/2v_{\parallel}(\sin 2z'/L - \sin 2z/L)}\right\} \qquad (6.A)$$

where $v_d \equiv |v_{di}|$, $b \equiv \frac{1}{2}(ka_i)^2$, $v_g \equiv a_i/R_c\ v_{thi}$ (the ion curvature drift), $\omega_i \equiv \omega + khv_g$, $\omega_e \equiv \omega - khv_g\tau$, and $z' \equiv z + v_{\parallel}t'$. The electron Larmor radius has been taken as negligible.

We can carry out the integration over t' by expanding

$$e^{\pm ikv_g L/2v_{\parallel}(\sin 2z'/L - \sin 2z/L)}$$

in series of Bessel function, as was done before integration over $v_{\perp}$ for the gyration part of the orbit,[2] and considering a solution $\tilde{\phi}(z) = \Sigma_n \tilde{\phi}_n e^{i2n\, z/L}$ as suggested by the form of Eqs. (5.A) and (6.A) in analogy to the solution for Hill's types of equation. Then we obtain:

$$0 = (1+\tau)\tilde{\phi}_n - \sum_r \tilde{\phi}_r\left\{\tau(\omega - kv_d)I_o(b)\, e^{-b}\left(\frac{\alpha_i}{\pi}\right)^{1/2}\int_{-\infty}^{+\infty} dv_{\parallel}\, e^{-\alpha_i v_{\parallel}^2}\right.$$

$$\sum_m \frac{J_m(kv_g L/2v_{\parallel})\, J_{m+n-r}(kv_g L/2v_{\parallel})}{\omega_i + (m+n)\, 2v_{\parallel}/L} + (\omega + kv_d\tau)\left(\frac{\alpha_e}{\pi}\right)^{1/2} \times$$

$$\left. \times \int_{-\infty}^{+\infty} dv_{\parallel}\, e^{-\alpha_e v_{\parallel}^2}\sum_m \frac{J_m(-kv_g\tau L/2v_{\parallel})\, J_{m+n-r}(-kv_g\tau L/2v_{\parallel})}{\omega_e + (m+n)\, 2v_{\parallel}/L}\right\}$$

(7.A)

The dispersion relation results from setting equal to zero the Hill determinant resulting from Eqs. (7.A).

SECTION B

1. Modes with Bad Ion Communication

It is most interesting, as we shall also prove in Appendix 1, to consider modes with frequency $v_{thi}/L < \omega_R \lesssim v_{the}/L$, where $\omega = \omega_R - i\gamma$ and $\gamma << \omega_R$. Modes of this type are in fact characterized by poor ion communication over the distance L , while the electron Landau damping is expected to allow the instability. Since we have assumed $k v_g < \omega$ in order to neglect the velocity (spread) dependence of the curvature drift, we have for consistency $v_{the}/L > k v_g \tau$. Therefore, for the last term of Eq. (7.A) we can take the small argument expansion of the Bessel functions there contained or, more simply, expand the corresponding exponential in Eq. (6.A). We will be allowed to make a similar expansion for the ion term if $v_{thi}/L > k v_g$, implying $ka_i < R_c/L$, a reasonable assumption. As a consequence, we notice that if $\omega \approx k v_d \tau$ as it is found for the usual drift (universal) instability, we shall not be allowed to consider $v_d \tau \sim v_g$ corresponding to $T_e/T_i \sim h r/R_c$, the limit where the drift instability is stabilized by the effects of good curvature.[3] On the other hand, this limit is difficult to achieve in practice, so that we can be justified in not devoting special attention to it.

Limiting the expansion of the Bessel functions in Eqs. (7.A) to the terms of first order in their argument, the Hill determinant reduces to one with three nonzero elements per line, centered around the diagonal. If we consider modes with $\gamma << \omega_R$ and study the order of magnitude of the various terms recalling our assumptions on ka_i and r/R_c we are led to recognize two types of wave:

a drift wave with $\omega_R \cong - kv_d\tau$, which corresponds to the vanishing in lowest order of the coefficients of $\tilde{\phi}_n$ with $1 \leq |n| < kv_d\tau/(2v_{thi}/L)$

a gravitational wave, which corresponds to the vanishing in lowest order of the coefficient of $\tilde{\phi}_o$. This leads us to exclude consideration of instabilities localized over a distance less than L along the magnetic field lines. To this purpose we should in fact construct wave packets, utilizing such high values of n that the influence of the ion inertia along the field or the ion Landau damping would be sufficient to eliminate the instability. This general picture is not changed when we introduce the effects of the magnetic curvature component lying within the magnetic surface.[4]

2. Nonresonant Gravitational Wave

We shall consider at first the latest type of wave. Then in lowest order we have, for the real part of the frequency:

$$\omega (\omega - kv_d)(1 - I_o(b)\, e^{-b}) - h(1 + \tau)\, k^2 v_g v_d = 0 \qquad (1.B)$$

Choosing at first the smaller (negative) root we see that, in order to obtain $\omega > kv_g$, we have to take $b << 1$, and then

$$\omega = -\omega_o = \left(\sqrt{\frac{b}{8} + h(1+\tau)\frac{r}{R_c}} - \sqrt{\frac{b}{8}}\right)\frac{v_{thi}}{r} \qquad (2.B)$$

In order to have bad ion communication, i.e., $\omega > 2v_{thi}/L$, Eq. (2.B) demands

$$h(1+\tau)\frac{L^2}{4rR_c} > 1 + \frac{L}{2r}\sqrt{\frac{b}{2}} \tag{3.B}$$

Therefore, the system can be considered as stable against these waves, when

$$h(1+\tau)\frac{L^2}{rR_c} < 1$$

and good communication exists (Appendix 1).

If we assume $h(1+\tau)\;L^2/rR_c > 1$ we are led to consider

$$b < 2h^2(1+\tau)^2\frac{L^2}{R_c^2} \qquad \text{i.e.,}\quad ka_i < 2h(1+\tau)\;L/R_c$$

If we analyze the order of magnitude of the coefficients of the expanded Eqs. (7.A), we can see that the diagonal ones are much larger than the off diagonal ones. In fact, we have, for $n = 0$ and $\omega = -\omega_o + \delta\omega$

$$\left[b(2\omega_o + kv_d)\delta\omega + i\sqrt{\pi}\,\frac{L\tau}{2v_{the}}\,\frac{k^2v_g^2}{2}(kv_d\tau - \omega_o)\right]\tilde{\phi}_o$$

$$\frac{1}{2}kv_g\left[kv_d + \omega_o - (kv_d\tau - \omega_o)\,i\sqrt{\pi}\,\frac{\omega_o L}{2v_{the}}\right](\tilde{\phi}_{-1} + \tilde{\phi}_1) = 0 \tag{4.B}$$

and, for $n = 1$,

$$\left[b(2\omega_o + kv_d)\,\delta\omega - \left(1 + i\sqrt{\pi}\,\frac{\omega_o L}{2 v_{the}}\right)\omega_o\left(\frac{\omega_o}{\tau} - kv_d\right) + \frac{2v_{thi}^2}{L^2}\,\frac{\omega_o + kv_d}{\omega_o}\right]\tilde{\phi}_1$$

$$- \frac{1}{2} k v_g \left[k v_d + \omega_o - (k v_d \tau - \omega_o)\; i\sqrt{\pi}\,\frac{\omega_o L}{2 v_{the}}\right](\tilde{\phi}_o + \tilde{\phi}_2) = 0$$

A similar equation is obtained for $n = -1$. We can verify that the relative order of magnitude of the coefficients does not change for higher values of n. In this regard, we notice that for $b > (r/R_c)\, h$, $\omega \approx (k v_g/b)\, h$ and that the diagonal terms are of order $k v_g/b\ (\omega_o/\tau - k v_d)$ as compared with $k v_g\ (k v_d + \omega_o)$, the order of the diagonal ones.

Then, in lowest order, the Hill determinant can be reduced to that of a 3×3 matrix formed by the central elements. This leads to the dispersion relation,

$$\left[b(2\omega_o + kv_d)\delta\omega + \frac{i\sqrt{\pi}}{4}\,\frac{L\tau}{v_{the}}\,k^2 v_g^2\,(k v_d \tau - \omega_o)\right] \times$$

$$\times\left[b(2\omega_o + kv_d)\,\delta\omega + \frac{2 v_{thi}^2}{L^2}\,\frac{\omega_o + k v_d}{\omega_o} - \frac{\omega_o}{\tau}(\omega_o - k v_d \tau)\left(1 + \frac{i\sqrt{\pi}\,\omega_o L}{2 v_{the}}\right)\right]$$

$$= \frac{k^2 v_g^2}{2}(k v_d + \omega_o)\left[k v_d + \omega_o + 2 i\sqrt{\pi}\,\frac{\omega_o L}{2 v_{the}}\,(\omega_o - k v_d \tau)\right] \tag{5.B}$$

If $\omega_o \neq k v_d$, we have in lowest order

$$\delta\omega = \frac{k^2 v_g^2 \tau \left[(\omega_o + k v_d)^2 - \frac{i\sqrt{\pi}}{2}\,\frac{L\omega_o}{v_{the}}\,k^2 v_d^2 (1+\tau)^2\right]}{2 b \omega_o (2\omega_o + k v_d)(k v_d \tau - \omega_o)}\,, \tag{6.B}$$

and see that instability occurs for $\omega_o < k v_d \tau$, equivalent to

$$b > \frac{2h}{\tau}\,\frac{r}{R_c}\,, \tag{7.B}$$

where $\mathrm{Im}(\omega) < 0$. A very similar circumstance was found in the collisional regime[1] where the unstable gravitational wave was associated with the effects of resistivity.

We notice that now the order of magnitude of the growth rate is

$$\gamma \sim \tau \frac{k^2 v_g^2 L}{b\, v_{the}} = \frac{v_{thi}^2}{R_c^2} \frac{L}{v_{the}} , \tag{8.B}$$

and therefore rather small.

If we utilize condition (7.B) to evaluate the limit in which good ion communication occurs, we have

$$\frac{h L^2}{r R_c} < \frac{4}{\tau} . \tag{9.B}$$

We have verified that the larger root of Eq. (1.B) is stable.

3. The Resonant Case

Equation (6.B) shows that a resonance occurs for the growth rate when $\omega_o \approx k v_d \tau$, for $b\tau \approx 2hr/R_c$, which is the frequency of the drift wave. This is also a circumstance which is common to the resistive case.

Taking Eq. (5.B) we now obtain in lowest order

$$\left[b(2\omega_o + k v_d)\delta\omega\right]\left[2 \frac{v_{thi}^2}{L^2} \frac{\omega_o + k v_d}{\omega_o} + \frac{\omega_o}{\tau}\left(1 + i\sqrt{\pi}\, \frac{\omega_o L}{v_{the}}\right)(\delta\omega + \Delta\omega_o)\right]$$

$$= \frac{1}{2}\left[k v_g (\omega_o + k v_d)\right]^2 \tag{10.B}$$

where $\Delta\omega_o = k v_d \tau - \omega_o$.

We notice that, in order to reduce consistently the Hill determinant to the 3×3 one corresponding to Eq. (10. B), we need impose that the diagonal elements are larger than the off diagonal ones. For the lowest values of n, this implies $v_{thi}^2/L^2 \; k v_d/\omega_o > k^2 v_g v_d$ and since $\omega_o \approx h k v_g/b$, $R_c^2 > L^2 h$, which is an acceptable condition. Now, writing Eq. (10. B) in dimensionless form:

$$(1 + i\rho)(\delta\bar{\omega} + \Delta\bar{\omega}_o)\,\delta\bar{\omega} + I\;\delta\bar{\omega} = C\,,$$

we have $\delta\bar{\omega} \equiv \delta\omega/k v_d \tau$, $I \equiv 1/b\,(2r/L\tau)^2\,(1+\tau)$,

$$C \equiv \frac{2}{b}\left(\frac{r}{R_c}\right)^2\left(\frac{1+\tau}{\tau}\right)^2\frac{1}{1+2\tau}\,, \text{ and } \quad \rho = \frac{\sqrt{\pi}}{2}\,\frac{k v_d \tau L}{v_{the}}$$

For $\delta\bar{\omega} = \delta\bar{\omega}_R - i\bar{\gamma}$, we have

$$\delta\bar{\omega}_R\left[I + \Delta\bar{\omega}_o + \delta\bar{\omega}_R\right] = C \quad ,$$

and

$$\bar{\gamma}/\rho = \frac{C - I\,\delta\bar{\omega}_R}{I + \Delta\bar{\omega}_o + 2\,\delta\bar{\omega}_R}$$

with

$$2\,\delta\bar{\omega}_R = -\,(I + \Delta\bar{\omega}_o) \pm \sqrt{4C^2 + (I + \Delta\omega_o)^2}$$

The negative root is damped ($\bar{\gamma} < 0$) for all $\Delta\bar{\omega}_o$. On the other hand, the positive one:

$$\bar{\gamma}/\rho = \frac{C + 1/2\,I\,(I + \Delta\bar{\omega}_o)}{2\sqrt{C^2 + (\Delta\bar{\omega}_o + I)^2/4}} - \frac{I}{2}$$

is unstable for $\Delta \bar{\omega}_o >> - C/I$. We can recover as a special case, the result of (6.B) in the limit where $\Delta \bar{\omega}_o > I > C$. In particular, the maximum growth rate corresponds to $\Delta \bar{\omega}_o = I$, where

$$\bar{\gamma} = \frac{1}{2} \rho \left(\sqrt{I^2 + C} - I \right)$$

In the further limit where $I^2 > C$, i.e., $L^4/r R_c^3 < 4\tau (1 + \tau)/h$, we have:

$$\gamma = \frac{\sqrt{\pi}}{16} \frac{(1 + \tau) \tau^{1/2}}{1 + 2\tau} \left(\frac{m}{M} \right)^{1/2} h \left(\frac{L}{R_c} \right)^3 \frac{v_{thi}}{r}$$

SECTION C

Influence of Magnetic Shear on the Gravitational Wave

We represent the magnetic shear by introducing in the equilibrium the field component $(x/L_s)\,B_o\,\underset{\sim}{e}_y$, with $L_s >> L$. Then, if we consider the rotating coordinates $\zeta = z + (x/L_s)y$ following the magnetic lines and $\chi = y - xz/L_s$ perpendicular to them, we look for "quasi-modes" of the form $\tilde{\phi}(\zeta)\,e^{ik\chi}$. Defining $\bar{\zeta} = 2\zeta/L$, the equation for the perturbed ion density acquires the form

$$- \frac{1}{\tau e \lambda_D^2}\,\tilde{n}_i(\bar{\zeta}) = \tilde{\phi}(\zeta) - i(\omega - k v_d)\left(\frac{\alpha_i}{\pi}\right)^{1/2} \int_{-\infty}^{+\infty} d v_{\parallel}\, e^{-\alpha_i v_{\parallel}^2} \times$$

$$\int_{-\infty}^{0} dt'\,\tilde{\phi}(\bar{\zeta}')\,e^{i\omega_i t' - \frac{ikv_g L}{2v_{\parallel}}(\sin\bar{\zeta}' - \sin\bar{\zeta})} \left\{ I_o(b)e^{-b} - \frac{L^2}{4L_s^2}\,\bar{\zeta}'^2\, b\left[I_o(b) - I_1(b)\right]e^{-b} \right\} \tag{1.C}$$

where $\bar{\zeta}' = \bar{\zeta} + 2v_{\parallel}t'/L$ and $L^2/4L_s^2 << 1$.

In order to see the influence of shear on the modes found in its absence and represented by Eqs. (1.B), we consider wave packets of the form[1]

$$\tilde{\phi}(\bar{\zeta}) = \sum_n \int \bar{\phi}(n+\kappa)\,e^{i(n+k)\bar{\zeta}}\,d\kappa\ , \tag{2.C}$$

where clearly $\kappa \to 0$ for $L/L_s \to 0$.

Therefore, we take the Fóurier transform of Eq. (1.C) and the corresponding one for $\tilde{n}_e$, set $\tilde{n}_i(\kappa) = \tilde{n}_e(\kappa)$ and repeat the procedure which led to Eqs. (7.A).

Then we obtain, in the small b limit,

$$0 = (1+\frac{1}{\tau})\,\tilde{\phi}(n+\kappa) - \sum_r \left\{ (\omega - k v_d)\left[1 - b\left(1 - \Sigma \frac{d^2}{d\kappa^2}\right)\right] \times \right.$$

$$\times \left(\frac{\alpha_i}{\pi}\right)^{1/2} \int_{-\infty}^{+\infty} d v_{\parallel}\, e^{-\alpha_i v_{\parallel}^2} \sum_m \frac{J_m(\upsilon)\, J_{m-r+n}(\upsilon)}{\omega_i + (m+n+\kappa)\, 2 v_{\parallel}/L}$$

$$\left. + \left(\frac{\omega}{\tau} + k v_d\right)\left(\frac{\alpha_e}{\pi}\right)^{1/2} \int_{-\infty}^{+\infty} d v_{\parallel}\, e^{-\alpha_e v_{\parallel}^2} \sum_m \frac{J_m(-\upsilon\tau)\, J_{m-r+n}(-\upsilon\tau)}{\omega_e + (m+n+\kappa)\, 2 v_{\parallel}/L} \right\} \tilde{\phi}\,(r+\kappa) \tag{3.C}$$

where

$$\upsilon \equiv k v_g L/2 v_{\parallel} \quad \text{and} \quad \Sigma = L^2/4 L_s^2$$

As in the previous sections we consider modes with $2 v_{thi}/L < \omega < 2_{the}/L$ and suppose that shear is so small that $\omega > 2\kappa\, v_{the}/L$, for $\kappa \ll 1$. Then, with the intention of examining the gravitational wave, we obtain for $n = 0$

$$\left\{ -\omega(\omega - k v_d)\, b\left(1 - \Sigma \frac{d^2}{d\kappa^2}\right) + k^2 v_g v_d h(1+\tau) + \omega\left(\frac{\omega}{\tau} + k v_d\right) \frac{2 v_{the}^2}{L^2 \omega^2}\, \kappa^2 \right.$$

$$\left. + \frac{i}{2}\rho \frac{\tau}{\omega} k^2 v_g^2 (k v_d \tau + \omega) \right\} \bar{\phi}(\kappa) = \frac{1}{2} k v_g \left[k v_d - \omega - (k v_d \tau + \omega)\, i\rho\right]\left[\bar{\phi}(1) + \bar{\phi}(-1)\right] , \tag{4.C}$$

where $\rho = -\sqrt{\pi}\ \omega L/2 v_{the}$. For $n = 1$ we have, to lowest order,

$$\left\{ -\omega(\omega - k v_d)\, b\left(1 - \Sigma \frac{d^2}{d\kappa^2}\right) + k^2 v_g v_d h(1+\tau) + \omega(\omega - k v_d) \frac{2 v_{thi}^2}{L^2 \omega^2} \right.$$

$$\left. - \omega\left(\frac{\omega}{\tau} + k v_d\right)(1 + i\rho) \right\} \bar{\phi}\ (1) = \frac{1}{2} k v_g \left[k v_d - \omega - (k v_d \tau + \omega) i\rho\right]\left[\bar{\phi}(\kappa) + \bar{\phi}(2)\right] , \tag{5.C}$$

and an analogous equation for $n = -1$.

If we consider the operator acting upon $\overline{\phi}(1)$ in Eq. (5.C), for the modes we study, the prevailing terms are: $\omega(\omega - kv_d)\, 2v_{thi}^2/L^2\omega^2 - \omega(\omega/\tau + kv_d)(1 + i\rho)$. The latter term is the more important when we do not consider the resonant condition, i.e., $|\omega| < kv_d\tau$. We are then led to the equation, defining $\rho = -1/2\sqrt{\pi}\,\omega L/v_{the}$,

$$\Bigg\{ b\,\Sigma\,\omega(\omega - kv_d)\frac{d^2}{d\kappa^2} + 2\left(\frac{\omega}{\tau} + kv_d\right)\frac{v_{the}^2}{L^2\omega^2}\kappa^2 - \omega(\omega - kv_d)\,b + k^2 v_g v_d h(1+\tau) + \frac{i}{2}\rho\, k^2 v_g^2\left(1 + \frac{kv_d\tau}{\omega}\right) - \frac{1}{2}k^2 v_g^2 (kv_d - \omega)\frac{kv_d - \omega - 2i\rho(\omega + kv_d\tau)}{\omega(\omega - kv_d)\,2v_{the}^2/L^2\omega^2 - \omega(\omega/\tau + kv_d)(1+i\rho)}\Bigg\}\overline{\phi}(\kappa) = 0, \tag{6.C}$$

which is of the Weber type. We consider the lowest eigensolution $e^{-\sigma\kappa^2/2}$ In order to have localized solutions we require $\mathrm{Re}\,\sigma > 0$ and for the validity of the equation we require $k_x^2 a_i^2/2 < 1$ which corresponds to $b\sigma\Sigma < 1$ and $\omega > 2\kappa v_{the}/L$. Then, we obtain:

$$\sigma^2\Sigma = -\frac{2}{b}\,\frac{\omega + kv_d\tau}{\omega^2(\omega - kv_d)\tau}\left(\frac{v_{the}}{L}\right)^2, \tag{7.C}$$

$$\sigma\,\Sigma = \frac{k^2 v_g v_d h(1+\tau) - \omega(\omega - kv_d)\,b - G}{\omega(\omega - kv_d)\,b}, \tag{8.C}$$

where G represents the terms due to the unfavorable periodic curvature. Then we shall distinguish three cases:

1) $b > (2h/\tau)(r/R_c)$, corresponding to condition (7.B) for instability of the gravitational wave due to the unfavorable curvature. From the dispersion

relation resulting from Eqs. (7.C) and (8.C), with $G \equiv 0$ we can verify that no unstable localized solution can be found. Defining $S \equiv (M/m)(r/L_s)^2$ and $\bar{\omega} = \omega/kv_d$ we have, in fact,

$$\frac{S}{b^2} = \frac{[\bar{\omega}(\bar{\omega} - 1) - (2h/b)(1 + \tau)\, r/R_c]^2}{(\bar{\omega} + \tau)(1 - \bar{\omega})} \qquad (9.C)$$

and can consider the solutions of the dispersion relation as intersections of two curves, indicated in Fig. 1.

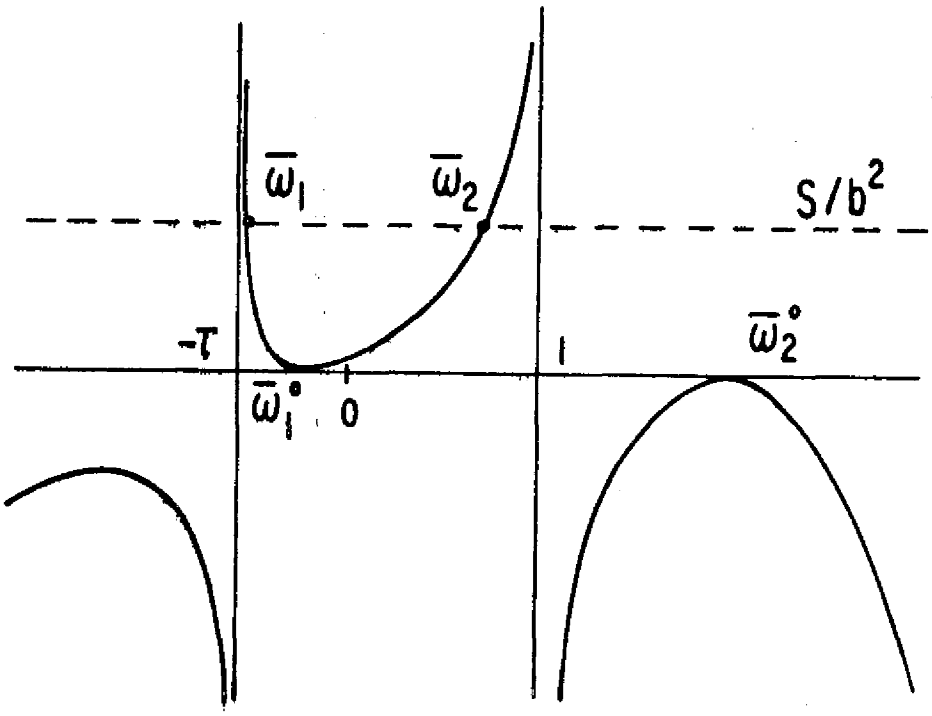

Fig. 1

There are four roots: two real and two complex. One of the real roots is never localized as $-\tau < \bar{\omega}_1 < \bar{\omega}_1^o$. The second real root is localized only if $-\tau < \bar{\omega}_2 < \bar{\omega}_1^o$, corresponding to $S^{1/2} < 2h(1 + \tau)/\tau^{1/2}\ r/R_c$. The remaining two complex roots are always nonlocalized. If now we take into account the unfavorable curvature terms, $G \neq 0$, we have to distinguish two subcases:

a) $$S^{1/2} < \frac{2h(1+\tau)}{\tau^{1/2}} \frac{r}{R_c}$$

Then if we set $\overline{\omega} = \overline{\omega}_1^o + \delta\overline{\omega}_1^o + \delta\overline{\omega}$, with $\delta\overline{\omega}$ due to the unfavorable curvature terms and representing the growth rate, we have

$$\delta\overline{\omega}_1^o = \pm \frac{S^{1/2}}{b} \frac{[(\overline{\omega}_i + \tau)(1 - \overline{\omega}_i)]^{1/2}}{(2\overline{\omega}_i - 1)}$$

the only localized root is the positive one. The shear does not stabilize this root and introduces only a shift in the real part of the frequency ω_1^o. Then, we verify the conditions of validity of Eq. (9.C). One is satisfied as

$$b\,\Sigma\,\sigma = \frac{(1 - 2\overline{\omega}_i)\, b\, \delta\overline{\omega}}{\overline{\omega}_1^o(\overline{\omega}_1^o - 1)} < 1$$

and the other, $\omega > 2\kappa v_{the}/L$, reads

$$\frac{2h(1+\tau)}{b\tau^{1/2}} \frac{r}{R_c} > S^{1/2}$$

and is also satisfied.

b) Increase the shear to a value such that $S^{1/2} > 2h(1+\tau)/\tau^{1/2}\,(r/R_c)$. In this case, neglecting the unfavorable curvature terms, the roots of the dispersion relation are all nonlocalized. If we include these small terms, the roots remain nonlocalized. Consequently, we can consider the condition $S^{1/2} > 2h(1+\tau)/\tau^{1/2}\,(r/R_c)$ as a condition of stability of the present mode.

We recall that this condition is of the same type as the one given in Ref. 1, for the inertial mode due to unfavorable curvature.

2) $b < (2h/\tau)(r/R_c)$. Then, referring to Eq. (9.C), the relevant roots of the dispersion relation without the unfavorable curvature terms are represented in Fig. 2.

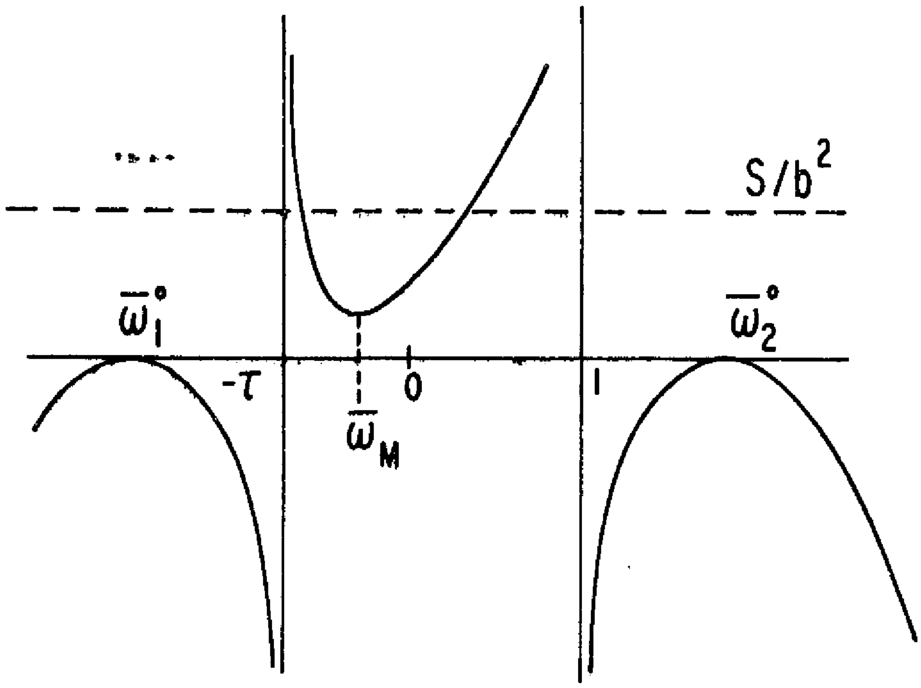

Fig. 2

We can see that if the shear is sufficiently small an unstable solution can be found. For this, we need to prove that the corresponding mode is localized. In particular, considering the condition of marginal stability, where the horizontal line is tangent in $\bar{\omega}_M$, we find $\bar{\omega}_M < 0$ if $b < h(r/R_c)(1-\tau)/\tau$, which is compatible with the condition posed above. Then we can give as a sufficient condition for stability for any value of b,

$$S > \left(\frac{2h(1+\tau)}{\tau^{1/2}} \frac{r}{R_c} \right)^2 \qquad (10.C)$$

which can be easily satisfied.

When this condition is not satisfied the relevant instability is of the same type as the one associated with the drift wave when, for $b\tau < 2hr/R_c$, the growth is related with the term due to periodic curvature.

When this condition is satisfied, we have to study the influence of the periodic curvature terms. Since $S^{1/2} > 2h(1+\tau)/\tau^{1/2}(r/R_c)$ the root of the dispersion relation will be close to $-\tau$. We can use this information to keep only the most important terms in Eq. (7.C). More precisely, we take:

$$G = \frac{1}{2} \frac{k^2 v_g^2 (k v_d - \omega)^2}{\omega(\omega - k v_d)\, 2 v_{thi}^2/L^2 \omega^2 - \omega(\omega/\tau + k v_d)(1 + i\rho)}$$

Then, if we set $\delta\bar{\omega} = (\omega + k v \tau)/k v \tau$, we get the dispersion relation

$$\delta\bar{\omega} = \frac{1+\tau}{S\tau} \left\{ \frac{2hr}{R_c} + \frac{C'}{I' + \delta\bar{\omega}(1 + i\rho)} \right\}$$

with $C' = 2(1+\tau)/\tau\,(r/R_c)^2$; $I' = (1+\tau)(2r/L\tau)^2\, 1/b$, and obtain the root:

$$\delta\bar{\omega}_R = \frac{1+\tau}{S\tau} \left[\frac{2hr}{R_c} \right]^2 < 1 \;.$$

Correspondingly, we obtain the growth rate

$$\bar{\gamma} = \rho \frac{4(1+\tau)}{S\tau} \frac{h r}{R_c} C' \frac{\delta \bar{\omega}_R}{[(I')^2 + \delta \bar{\omega}_R]}$$

Now we verify the conditions of validity of our derivation and precisely, $\sigma b \Sigma \simeq 2hr/R_c << 1$, which is always satisfied, and $\omega > 2\kappa v_{the}/L$ which gives:

$$S < \frac{h r}{R_c} . \quad (11.C)$$

In conclusion, we see that for the present mode we do not have a condition for stability but one for the breakdown of the asymptotic approximation when $r/L_s > [hr/R_c (m/M)]^{1/2}$. This condition is more severe than (10.C) previously obtained, but is still rather mild. To have an estimate for the order of magnitude of the growth rate, we take

$$S \sim \frac{h r}{R_c} , \quad b \sim \frac{2 h r}{R_c} , \quad \text{and } h L < R_c ,$$

obtaining

$$\gamma = 2\sqrt{\pi} (1+\tau) \left[\frac{L^2 h}{r R_c}\right]^3 \frac{r^2}{R_c L} \left(\frac{m}{M}\right)^{1/2} \frac{v_{thi}}{R_c} \quad (12.C)$$

For higher values of shear, corresponding to $\omega < 2\kappa v_{the}/L$ we need to turn our attention to non-normal mode types of solutions which are discussed elsewhere.

3) For $b = 2hr/R_c \tau$, a resonance between the gravitational and the drift wave occurs. Then, in lowest order, the relevant equation is:

$$\left\{ \left[2 \frac{v_{thi}^2}{L^2} \frac{1+\tau}{\tau} + k v_d (1 + i\rho)(\delta\omega + \Delta\omega_o) \right] \left[b k v_d (1 + 2\tau)\, \delta\omega \right. \right.$$

$$\left. + \Sigma\, b k^2 v_d^2 \tau (1+\tau) \frac{d^2}{d\kappa^2} - 2 \frac{(\delta\omega + \Delta\omega_o)}{k v_d \tau^2} \frac{v_{the}^2}{L^2} \kappa^2 \right]$$

$$\left. - \frac{1}{2} k^2 v_g^2 k^2 v_d^2 (1+\tau)^2 \right\} \bar{\phi}(o) = 0 \quad . \qquad (13.C)$$

Using the notations of Sec. B, this can be written as:

$$\left\{ \delta\bar{\omega} + \Sigma_o \frac{d^2}{d\kappa^2} - (\delta\bar{\omega} + I)\, I\, \frac{M}{m} \frac{b}{b(1+2\tau)} \kappa^2 - \frac{C}{I + (\delta\bar{\omega} + I)(1 + i\rho)} \right\} \bar{\phi}(o) = 0 ,$$

where $\Sigma_o = (L^2/4 L_s^2)(1+\tau)/(1+2\tau)$ and $\Delta\bar{\omega}_o$ has been taken equal to I .

Then, the dispersion relation for the solution $e^{-\sigma u^2/2}$ is

$$(I + \delta\bar{\omega}) \left(\frac{M}{m} \frac{I \tau \Sigma_o}{b(1+2\tau)} \right) = \left[\delta\bar{\omega} - \frac{C}{I + (\delta\omega + I)(1 + i\rho)} \right]^2 ,$$

with

$$\Sigma_o \sigma = \delta\bar{\omega} - \frac{C}{I + (\delta\bar{\omega} + I)(1 + i\rho)}$$

We know from the analysis of the dispersion relation, without shear, that we have two roots, $\delta\omega_1^o >$ and $\delta\omega_2^o < 0$, of which only $\delta\omega_1^o$ satisfied the conditions of validity of the equation. Now, the dispersion relation with shear has four roots. If we ignore the small $i\rho$ terms, two of these roots are real and close to $\delta\omega_1^o$. The root which is smaller than $\delta\omega_1^o$ is nonlocalized, the other one, when increasing the shear, goes into the root studied in case 2. For the two other roots which are in the vicinity of $\delta\omega_2^o$ and correspond to the drift wave, the equation is not valid. We can conclude that a very small shear is sufficient to make the resonance disappear.

SECTION D

The Drift Wave

1) As previously said, examining the order of magnitude of the diagonal terms in the Hill determinant, we see that the other interesting wave is the drift wave with frequency $\omega = -k v_d \tau$, for $b \lesssim 1$. Since $k v_d = \sqrt{b/2}\,(v_{thi}/r)\,\tau$, we conclude that, for values of b such that:

$$b \lesssim 8\left(\frac{r}{L\tau}\right)^2 , \qquad (1.D)$$

this wave is stabilized as $\omega \lesssim 2\,v_{thi}/L$ and good ion communication exists. This is actually too stringent a condition as we will see that for $b < 1$ the requirement

$$\frac{L}{r} \lesssim \left(\frac{2}{k_y a_i}\right)^2 \frac{1}{\tau} \qquad (1'.D)$$

is sufficient. If in addition,

$$\frac{L}{r} < \frac{2\sqrt{2}}{\tau} , \qquad (2.D)$$

the wave is stabilized for all values of b. For $b > 1$, $\omega \approx -k v_d \tau/\sqrt{b}$ and the same condition holds. Condition (2.D) eliminates also the instability associated with the gravitational wave as it occurs for $|\omega| \leq k v_d \tau$, i.e., $b\tau \gtrsim 2hr/R_c$.

However, it is clear that condition (2.D) does not apply to the case where we consider perturbations of the form $e^{ik_\parallel z} \sum_n \phi_n e^{2inz/L}$ with $k_\parallel < 1/L$, as we do at the end of this section. This is an important point for consideration, as in configurations with closed lines of force $1/k_\parallel$

cannot be chosen arbitrarily large but will have to be of the order of the circumference length. If L is of the order of this length, the criteria given above then hold for all possible low-frequency instabilities.

2) We now assume that condition (1. D) is not satisfied.

In order to derive some conclusions valid for $b < 1$ we return to consider the 3×3 determinant with the intention of justifying later its consistency. Then, for $\omega = -k v_d \tau + \delta\omega$ and $F(b) \equiv 1 - I_o(b)\, e^{-b}$ we obtain

$$\left\{ h(1+\tau)\, k^2 v_g v_d - k^2 v_d^2 \tau(1+\tau)\, F(b) \right\} \left\{ h(1+\tau)\, k^2 v_g v_d - k^2 v_d^2 \tau(1+\tau)\, F(b) + \frac{2 v_{thi}^2}{L^2 \tau}(1+\tau)\, I_o(b)\, e^{-b} + k v_d\, \delta\omega \left(1 + i\sqrt{\pi}\, \frac{k v_d \tau L}{2 v_{the}}\right) \right\} = \frac{1}{2}\left[k^2 v_g v_d (1+\tau) I_o e^{-b}\right]^2 \tag{3. D}$$

so that, for $b \ll 1$,

$$\delta\omega = (1+\tau)\left(1 - i\sqrt{\pi}\, \frac{k v_d \tau L}{2 v_{the}}\right) k v_d \left\{ \frac{1}{2}\, \frac{v_g^2/v_d^2}{h v_g/v_d - \tau b} + \left(b\tau - \frac{h v_g}{v_d}\right) - \frac{2 v_{thi}^2}{L^2 k^2 v_d^2 \tau} \right\}, \tag{4. D}$$

where $v_g/v_d = 2r/R_c$.

Consider the three terms inside brackets:

- the first one represents the effect of the bad curvature and is destabilizing for $b\tau < 2hr/R_c$.

- the second one represents the usual drift instability mechanism, connected with the ion inertia across the field which is counteracted by the average good curvature, and is destabilizing for $b\tau > 2hr/R_c$.

- the third one represents the stabilizing effect of the ion inertia along the lines of force. We can then derive the following conclusions:

(a) For $b\tau < 2hr/R_c$ the drift mechanism is ineffective and, since a fortiori $b\tau < 2hrR_c/L^2$ for $R_c^2 > L^2$, the stabilizing ion term prevails over the one due to the bad curvature.

(b) For $2h\ r/R_c < b\tau < 2\ r/L$ the ion term dominates the drift destabilizing one and the wave is again stable.

(c) For $b\tau > 2\ r/L > 2h\ r/R_c$, the wave is unstable to the usual drift mechanism.

In all cases for consistency of the derivation, we require that the ion term in the $n = 2$ diagonal element be larger than the off diagonal term. This is verified for $b\tau < r\,R_c/2L^2$, which is compatible with the limits considered above.

3) For $b\tau > r R_c/2L^2$ we cannot reduce the Hill determinant to a 3×3 determinant. We have to keep all the elements as given by the equations:

$$\tilde{\phi}_o \left\{ - \omega(\omega - k v_d) F(b) + h(1+\tau) k^2 v_g v_d \right\} - \left\{ \tilde{\phi}_1 + \tilde{\phi}_{-1} \right\} \frac{k v_g}{2} (k v_d - \omega) I_o(b) e^{-b} = 0 ,$$

$$\tilde{\phi}_n \left\{ - \omega(\omega - k v_d) \left[F(b) - I_o(b) e^{-b} \frac{2 n^2 v_{thi}^2}{\omega^2 L^2} \right] + h(1+\tau) k^2 v_g v_d - \frac{\omega}{\tau}(\omega + k v_d \tau)(1 + i\sqrt{\pi} \frac{k v_d \tau L}{2 h v_{the}}) \right\}$$

$$- \left\{ \tilde{\phi}_{n+1} + \tilde{\phi}_{n-1} \right\} \frac{k v_g}{2} (k v_d - \omega) I_o e^{-b} = 0 , \qquad (5.D)$$

for values of n_1 such that $\omega > 2 n_1 v_{thi}/L$, and $\tilde{\phi}_n \sim 0$ for values for n_2 such that $\omega \approx 2 n_2 v_{thi}/L$. Now looking for the drift wave with $b \lesssim 1$, we set $\omega = k v_d \tau (\delta \omega - 1)$ and we have for $n \neq 0$:

$$= \tilde{\phi}_n \left\{ \delta \bar{\omega} (1 + i \left| \frac{\delta}{n} \right|) - (1+\tau) b + \frac{4 n^2 (1+\tau)}{\tau^2 b} \frac{r^2}{L^2} + \frac{2 h (1+\tau) r}{R_c \tau} \right\}$$

$$- \frac{r}{R\tau}(1+\tau) \left[\tilde{\phi}_{n+1} + \tilde{\phi}_{n-1} \right] \qquad (6.D)$$

with $$|\delta| = \sqrt{\pi} \frac{k v_d \tau}{L v_{the}}$$

If we neglect the resonant particle effects, proportional to $|\delta|$, we can reduce the relevant dispersion equation to a Mathieu equation. We can then compute the growth rate of the drift wave by perturbation. In Appendix II we construct a general quadratic form which could give the same result. The corresponding Mathieu equation to Eq. (6.D) is:

$$L^2 \frac{d^2 \tilde{\phi}}{dz^2} - (a + q \cos 2z) \tilde{\phi} = 0 , \qquad (7.D)$$

with

$$a = \left\{ \delta \bar{\omega}_R - (1+\tau) \quad b \quad + \frac{2h(1+\tau)\, r}{R\tau} \right\} \times \left(\frac{L\tau}{r}\right)^2 \frac{b}{1+\tau} \ ,$$

$$q = - \left\{ \frac{2r}{R\tau}(1+\tau) \right\} \times \left(\frac{L\tau}{r}\right)^2 \frac{b}{1+\tau} \ . \tag{8.D}$$

For the values of b we consider, we have $q > 1$ and we can obtain a localized solution of the Mathieu equation, expanding the cosine function for $z < L$; then Eq. (7.D) reduces to a Weber type equation:

$$L^2 \frac{d^2\phi}{dz^2} - \left(\frac{L\tau}{r}\right)^2 \frac{b}{1+\tau} \left\{ \delta\, \delta\bar{\omega} + \frac{4r}{R\tau}(1+\tau) \frac{z^2}{L^2} \right\} \tag{9.D}$$

with

$$\delta\, \delta\bar{\omega} = \delta \bar{\omega}_R - (1+\tau) \left\{ b + \frac{2r}{R_C\tau} (1-h) \right\}$$

the lowest eigensolution, $\exp(-\sigma z^2/2L^2)$, leads to find:

$$\sigma^2 = \frac{4r\tau}{R} \left(\frac{L}{r}\right)^2 b$$

and

$$\delta\, \delta\bar{\omega} = - \frac{2r}{L} \left(\frac{1+\tau}{b\tau}\right)^{1/2} \tag{10.D}$$

then if we set $\delta\bar{\omega} = \delta\bar{\omega}_R - i\bar{\gamma}$, we obtain:

$$\bar{\gamma} = \delta\bar{\omega} \; |\delta| \; \frac{4r \sum_{n_0 < n < n_1} \frac{|\tilde{\phi}_n|^2}{n}}{\Sigma_n |\tilde{\Phi}_n|^2} \tag{11.D}$$

with

$$\tilde{\phi}_n = \frac{1}{2r} \int_{-r}^{r} \frac{dz}{L} \, e^{-\sigma z^2/2L^2 - 2inz/L} \approx \frac{1}{\sqrt{2r\sigma}} \, e^{-2n^2/\sigma}$$

$$\sum_n |\tilde{\phi}_n|^2 = \sqrt{\frac{\pi}{\sigma}} \tag{12.D}$$

then using the expression of $\delta \delta \overline{\omega}$ we get:

$$\overline{\gamma} = \frac{2|\delta|}{r\sigma}(1+\tau)\left\{b + \frac{2r}{R\tau}(1-h) - \frac{2r}{L}\left[b\tau(1+\tau)\right]^{-1/2}\right\}\sum_{n_o \leq n < n_1} \frac{e^{-4n^2/\sigma}}{n} \quad (13.D)$$

In the expression for $\overline{\gamma}$, we can recognize the stabilizing contributions (i.e., favorable curvature term, in this case always smaller than the term representing the unfavorable curvature, and longitudinal ion inertia term) and the destabilizing ones. The growth rate $\overline{\gamma}$ is of the same order as usual if we take $n = 1$, $\sigma \approx 4$. On the other hand this "ballooning" type of mode cannot be found if $\sigma^2 \lesssim 1$, implying the stability criterion:

$$\frac{L^2}{r R_c} < \frac{1}{\tau} \quad (14.D)$$

valid for any value of b.

We come to the conclusion that as long as it is possible to establish a drift mode in the region where curvature is unfavorable to stability, the criterion of stabilizing the relevant mode by a deep average well depth is not valid. Stability is instead achieved by having relatively short connection lengths. We recall that, contrary to this case, "ballooning" types of mode localized in regions of unfavorable curvature had not been found in the high temperature collisional regime. The reason is that in the two cases the equations for the relevant modes have two different parts which are set equal zero to lowest order to obtain the real part of the frequency. Therefore in one case we have to lowest order solutions which can involve all the terms of the Hill determinant and on the other solutions involving just the 3×3 center elements of it.

In the first case it is important to observe that since the drift wave can also be localized in one region of unfavorable curvature, this wave is insensitive to the fact that $L_t > L$ or $L_t >> L$, or that the lines of force do not close, provided that no shear exist in the equilibrium. The relevant stability criterion has been given (14. D) and, as reasonable to expect, is insensitive to the value of the average well depth. Let us suppose now that criterion (14. D) is satisfied. Then, as previously shown on the 3×3 determinant, the drift wave is unstable for values of b such that $b\tau > 2r/L$ and is expected to be cured only by shear as indicated in Ref. 5.

Introducing shear in the equilibrium we may expect to make possible the existence of waves with $k_{\parallel} < 1/L$. The relevant perturbation then sees only the average magnetic curvature and in low b limit we obtain out of the Hill determinant the following equation in $\bar{\phi}(\kappa)$

$$\left\{ -\omega(\omega - k v_d)\, b\left(1 - \Sigma \frac{d^2}{d\kappa^2}\right) + k^2 v_g v_d h(1+\tau) + \omega(\omega - k v_d) \frac{2\kappa^2 v_{thi}^2}{L^2 \omega^2} - \omega\left(\frac{\omega}{\tau} + k v_d\right)(1 + i\rho(\kappa)) \right\} \bar{\phi}(\kappa) = 0 , \qquad (15.\,D)$$

with $\rho(\kappa) = -\sqrt{\pi}\, \omega L / 2\kappa v_{the}$. For values of $b > 2hr/R\tau$, this equation reduces to the known equation for drift instabilities with shear, after the change of variable $2\kappa/L = k_{\parallel} = kx/L_s$, and provides the stability criterion[5] approximately of the form

$$L_s/r \lesssim (M/m)^{1/2}\, \tau^{-1/2} (k\; a_i)^{-1} \qquad (16.\,D)$$

SECTION E

Influence of Trapped Particles

The model we have so far proposed and studied has been taking into account the longitudinal modulation of the magnetic field, such as that existing in a system with negative V'' , only by considering the relevant periodic variation along the lines of force for the azimuthal (y) component of the curvature drift. In particular, the instabilities we have considered involve particles resonating with the wave and having velocity $\omega L \leq v_{the}$. Particles of this type are likely to be trapped in the varying magnetic field and it is interesting to study the modification to the obtained conclusions due to this effect. Obviously, this problem did not arise in studying the resistive regime where $\nu_{ei} > v_{the}/L$. Since we think of a configuration with a constant strong magnetic field on which a modulated one is superimposed, we consider a model ad hoc as in Sec. A, where B_o is replaced by $\overline{B}_o(1 - \alpha \cos 2z/L)$. In fact we do not consider here flute instabilities, which may be due to the trapped particles. Then, for our purpose it is sufficient to insert the orbits of the trapped electrons into Eq. (6.A) and leave the ion equation, Eq. (5.A) unchanged. So, if we imagine writing the dispersion relation, setting the relevant Hill determinant equal to zero, we can again recognize in lowest order two types of waves: the gravitational wave, obtained assuming that the perturbation is constant along the magnetic field lines, and the drift wave. Therefore, the resonant electrons do not affect, in lowest order, the real part of the frequency but just the imaginary part. Now, on the basis of the quadratic form discussed in Appendix II, where we see that the sign of the imaginary part of the frequency depends on that of the real part, we may argue that introducing the effect of trapped particles will modify the growth rate but not change its sign.

We can assume for our purpose $\alpha < 1$, i.e., small field modulation. Then we can see from a simple argument that if $\omega L/v_{the} >> \alpha^{1/2}$, the resonating particles giving rise to the relevant Landau damping are free or have their trajectories slightly modulated as a consequence of the magnetic field variation. To evaluate the influence of this on the growth rate we have in fact computed the power associated with the perburbed electron current and verified that the growth rate is not significantly changed. The relative variation is found to be

$$\sum_{\ell} \int_{o}^{\epsilon^2/\alpha(\ell+1)^2} d\xi \; e^{-\xi} \; J_{\ell}^2 \left[2\xi\alpha \frac{(\ell+1)^2}{\epsilon^2} \right]$$

where $\epsilon^2 = (\omega L/v_{the})^2$ and $\xi = v_{\perp}^2/v_{the}^2$. This expression reduces to 1 for $\epsilon^2 >> \alpha$.

The most interesting case is that in which $\omega L/v_{the} \lesssim \alpha^{1/2}$ and the particles resonating with the wave are trapped. We consider for simplicity the case where these are so slow that $\epsilon^2 << \alpha$. Then their equation of motion along ζ is

$$\frac{d\zeta}{dt} = \sqrt{\mathcal{E} - \mu B} = \sqrt{\mathcal{E} - \mu B_o + \mu B_o \alpha \cos \frac{2\zeta}{L}} \qquad (1.E)$$

where $\mathcal{E} = v_{\parallel}^2 + v_{\perp}^2$ represents the energy and $\mu = v_{\perp}^2/B$ the magnetic moment of each particle, and since $z << L$

$$\frac{d\zeta}{dt} = \sqrt{\mathcal{E} - \mu B_o(1 - \alpha) - \mu B_o \alpha \frac{2\zeta^2}{L^2}} \qquad (2.E)$$

The solution of this is

$$\zeta = \frac{L}{2} A \cos (t/T + \psi) \qquad (3.E)$$

with

$$A = \left[2 \frac{\mathcal{E} - \mu B_o(1 - \alpha)}{\alpha \mu B_o} \right]^{1/2}, \quad \text{and} \quad T = \frac{L}{(2 \alpha \mu B_o)^{1/2}} \sim \frac{L}{\alpha^{1/2} v_\perp}$$

In particular, the power associated with the perturbed electron current for a tube of flux of length, πL, is proportional to

$$P = B_o \int_o^{\pi L} \tilde{\underset{\sim}{E}} \cdot \tilde{\underset{\sim}{j}}_e \frac{d\zeta}{B} = - \int_o^{\pi L} \nabla \tilde{\phi} \cdot \tilde{\underset{\sim}{j}}_e \frac{d\zeta}{B} = e B_o \int_o^{\pi L} d\zeta \, i \omega \tilde{\phi} \int \tilde{f}_e \frac{d\mathcal{E} \, d\mu}{|v_\parallel|}$$

$$= i\omega \, e B_o \int d\mathcal{E} \, d\mu \int_o^{2\pi} d\psi \, T \, \tilde{f}_e \tilde{\phi} \quad , \quad \text{where} \quad T \, d\psi = \frac{d\zeta}{|v_\parallel|}$$

Now we recall that

$$\tilde{f}_e = \frac{f_e}{e \lambda_D^2} \sum_n \tilde{\phi}_n \left\{ e^{i(2n/L)z} - i(\omega + k v_d \tau) \int_{-\infty}^t dt' \, e^{i\omega t' + i(2n/L)z(t')} \right\}$$

where $z \approx \zeta$, and we have neglected terms of order $k v_g$ in comparison with ω. Clearly we are interested in the real part of P. Therefore if we take the first three terms in the expansion for $\tilde{\phi}$ we are reduced to consider

$$\mathrm{Re}\,(P) = \mathrm{Re} \left\{ \frac{1}{\lambda_D^2} \omega(\omega + k v_d i \tau) \left| \tilde{\phi}_1 \right|^2 B_o \int d\mathcal{E} \, d\mu \, f_e \, T \right.$$

$$\int_{-\infty}^{o} d\hat{t} \, e^{i\omega\hat{t}} \int_o^{2\pi} d\psi \, \exp \left[i \, A \cos \left(\frac{t + \hat{t}}{T} + \psi \right) - i \, A \cos \left(\frac{t}{T} + \psi \right) \right] ,$$

where we have set $\hat{t} = t' - t$. Now we carry out the integration over ψ expanding the exponentials in series of Bessel functions as indicated in Ref. 2. The real part of P is then proportional to

$$B_o \int T f_e \, d\mathcal{E} \, d\mu \sum_{\ell} J_{\ell}^2(A) \int_{-\infty}^{0} e^{i(\omega + \ell/T)t'} dt' \quad , \qquad (4.E)$$

i.e.,

$$B_o \int T \, f_e \, d\mathcal{E} \, d\mu \sum_{\ell} J_{\ell}^2(A) \; \delta(\omega + \ell/T) \qquad (5.E)$$

$$= \sum_{\ell} \int f_e \, d\mathcal{E} \, \frac{1}{\ell} \, \delta(\omega + \frac{\ell}{T}) \, \frac{L^2}{\alpha} \quad d\left(\frac{\ell}{T}\right) J_{\ell}^2(A)$$

$$= \sum_{\ell} \int f_e \, d\mathcal{E} \quad \frac{L^2}{\alpha \ell} \quad J_{\ell}^2(A) \qquad (6.E)$$

where now $A = [2\mathcal{E} - (1 - \alpha) \; \omega^2 \; L^2/(\alpha \, \ell^2)]/[\omega^2 L^2/(2\,\ell^2)]$. In order for $A < 1$, we require that $\mathcal{E} \approx \omega^2 L^2/(2\alpha\ell^2) = \left[\epsilon^2/(2\alpha\ell^2)\right] v_{the}^2$. Therefore the expression in Eq. (6.E) becomes nearly equal to

$$\frac{n_o L^2}{v_{the}} \left\{ \sum_{\ell} \frac{\epsilon^2}{\alpha} e^{-\epsilon^2/(\alpha\ell^2)} \, \frac{1}{\ell^3} \int dA \; J_{\ell}^2 \; (A) \right\} \quad . \qquad (7.E)$$

The factor within brackets then represents the relative change of the growth rate when the wave resonates with particles trapped at the "bottom" of the magnetic well so that the wave frequency is equal or in a multiple of the bouncing frequency. So we see that the relative reduction of the growth rate is of the order of

$$\frac{\omega^2 L^2}{\alpha v_{the}^2} < 1 \ .$$

CONCLUSIONS

We have considered a one-dimensional model of a toroidal equilibrium configuration. The aim is to study the influence of the periodically varying magnetic curvature along the field lines on stability in the collisionless regime. For this we have made use of the Vlasov equation including finite Larmor radius and wave-particle resonance effects. Then considering a system with negative V'', i.e., with average favorable curvature around the torus, the existence of two types of wave, with the same periodicity as that of magnetic curvature, is recognized:

a) a "gravitational" wave with frequency $\omega_o \approx 2h v_{thi}/(R_c k_y a_i)(1 + T_e/T_i)$, k_y representing the azimuthal wave number, R_c the maximum radius of magnetic curvature, v_{thi} the ion thermal velocity, a_i the ion Larmor radius, h/R_c the average favorable curvature (well depth), and $T_{e,i}$ the temperature;

b) a drift wave with frequency $\omega_1 \approx k_y a_i (v_{thi}/2r) T_e/T_i$, r being the scale length associated with the density gradient.

In addition drift waves localized in one region of unfavorable curvature can also be found, therefore giving rise to a sort of "ballooning" mode. The main difference from hydromagnetic "ballooning" modes is that in the present case the magnetic curvature is not driving the instability but just determining its topology. As a consequence this type of wave is not stabilized by increasing the well depth nor resent from the fact that the lines of force close around the toroid. The relevant stability criterion comes from inforcing that the "connection" length L be sufficiently short as to insure that the stabilizing effects of longitudinal ion inertia (e.g., sound wave along the magnetic field and ion Landau damping) are not negligible. It reads

$$\frac{L^2}{r R_c} < \frac{T_i}{T_e}$$

L representing the extension of the unfavorable curvature. We recall that localized waves of this type were not found in the high temperature collisional regime.[1]

The gravitational wave is unstable if $\omega_o < \omega_1$ (more precisely, for $T_e/T_i \cdot k_y^2 a_i^2 \geq 4 h r/R_c$) and corresponds to the instability driven by the "drift" mechanism and due to favorable curvature discussed in the appendix of Ref. 1 for the collisional regime.[4] The periodically varying curvature couples this wave with the drift wave having periodicity L so that for $k_y^2 a_i^2 = (4 h r/R_c) T_i/T_e$ a resonance occurs. There a growth rate of order $(m/M)^{1/2} h (L/R_c)^3 v_{thi}/r$ can be obtained. For instability of these waves bad ion communication is also required so that $v_{thi} < \omega_o L < v_{the}$ and ion Landau damping can be neglected. Then the stability condition corresponding to good ion communication is, for the gravitational wave,

$$\frac{h L^2}{r R_c} < 4 \frac{T_i}{T_e} \quad .$$

The drift wave with periodicity L can be unstable to the periodic magnetic curvature for long transverse wavelengths, such that $(k_y a_i)^2 < 4(hr/R_c) T_i/T_e$ and for $h < \sqrt{1/2}$. For this however the longitudinal ion inertia (sound wave) provides a relatively easy stability condition

$$L < 2 R_c$$

For $(k_y a_i)^2 > (2hr/R_c)\, T_i/T_e$ the same drift wave is little affected by magnetic curvature and the relevant instability is driven by the known "drift" mechanism associated with transverse ion inertia and radial gradient of the longitudinal electron pressure. Then the stabilizing longitudinal ion inertia provides the criterion

$$\frac{L}{r} \lesssim \frac{T_i}{T_e}\left(\frac{2}{k_y a_i}\right)^2$$

valid for $k_y^{-1} < a_i$. Now we distinguish two situations:

1. The lines of force close around the torus and have a length $L_t \gtrsim L$. Then if $L \sim L_t$, the waves discussed above are the only significant ones and the same criteria as given above apply, with the only reservation that if $L_t \sim R_c$ they can be fulfilled only in toroidal multipole devices. If $L_t >> L$, then we have to consider besides the waves with periodicity L, drift and gravitational waves with longitudinal wavelength $L < 1/k_{\parallel} \lesssim L_t$. The stability criterion against drift waves of this type is

$$\frac{L_t}{r} \lesssim \frac{T_i}{T_e}\left(\frac{2}{k_y a_i}\right)^2$$

assuming $2hr/R_c < T_e/T_i$ as is usually the case. This condition can be of practical importance if, by estimate of diffusion coefficients, we rule out short transverse wavelengths $k_y^{-1} \sim a_i$ as being irrelevant. If, instead waves with $(k_y a_i/2)^2 > (r/L_t)(T_i/T_e)$ give an important diffusion coefficient shear stabilization has to be considered. Finally gravitational waves with $(1/L_t) < k_{\parallel} < 1/L$ are to be ruled out if

$$L_t^2 < \frac{M}{m} R_c^2 \quad ,$$

an easy condition to be satisfied.

2. The lines of force do not close around the torus and magnetic shear exists in the equilibrium. Then drift waves with $k_{\parallel} < 1/L$ which "see" only the average good curvature can be considered as the most relevant ones. Assuming $2hr/R_c < T_e/T_i$, the stability condition against the resulting convective modes is, with good approximation,

$$\frac{r}{L_s} > \left(\frac{m}{M}\frac{T_e}{T_i}\right)^{1/2} \frac{k_{\perp} a_i}{n^o}$$

where L_s is the shearing distance and n^o the number of exponentation after which a convective mode can be considered as dangerous. If instead $2hr/R_c > T_e/T_i$ stability occurs for all drift waves with $k_{\parallel} < 1/L$. For waves with periodicity L this limit has not been investigated here because when it is occurs the relevant dispersion relation cannot be solved analytically. The drift wave can be unstable to the effect of magnetic curvature if $\frac{1}{2}(k_y a_i)^2 < (T_e/T_i)\, 2hr/R_c$. In this case the condition for the disappearance of the relevant normal mode is

$$\frac{r}{L_s} > \left(\frac{m}{M}\frac{rh}{R_c}\right)^{1/2}$$

On the other hand the condition against gravitational waves is

$$\frac{R_c}{L_s} > h\left(\frac{m}{M}\frac{T_i}{T_e}\right)^{1/2}$$

and is quite easy to satisfy.

in conclusion the present analysis shows that,

- a toroidal configuration possessing MHD stability can be made stable against microinstabilities more easily in the collisionless regime than in the collisional one

- short connection lengths L , defining the periodicity of magnetic curvature along the lines of force, and strong shear, if the total length L_t of the lines of force around the torus is larger than L , are indicated as the safe mean to achieve stability

- configurations with closed lines of force and L_t not much larger than L can also be made stable to low frequency modes provided they satisfy the criteria listed above. This last circumstance may serve as an explanation for the low diffusion coefficients recently observed in toroidal multipole devices.

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APPENDIX I

Now we shall demonstrate that if there is good communication for ions and electrons the gravitational wave is stable. The relevant equations are in fact:

$$\left\{- \omega(\omega - k v_d) b + k^2 v_g v_d h(1 + \tau) - \frac{i\sqrt{\pi}}{4} \frac{L\omega}{v_{thi}} k^2 v_g^2 \frac{(k v_d - \omega)}{\omega}\right\} \tilde{\phi}(o)$$

$$- \frac{1}{2} k v_g \left\{(k v_d - \omega) \frac{i\sqrt{\pi} L\omega}{2 v_{thi}}\right\} \left\{\tilde{\phi}(1) + \tilde{\phi}(-1)\right\} = 0$$

$$\left\{\omega(1 + \tau) + \tau(\omega - k v_d) \left(\frac{i\sqrt{\pi} L\omega}{2 v_{thi}}\right)\right\} \tilde{\phi}(1)$$

$$- \frac{1}{2} k v_g \left\{(k v_d - \omega) \frac{i\sqrt{\pi} L}{2 v_{thi}}\right\} \tilde{\phi}(o) = 0$$

where we have neglected the electron Landau damping in comparison with the one due to the ions.

If we proceed as in Sec. B and call $-\omega_o$ the frequency of the gravitational wave, we have, for $\tau = 1$ and $\rho = \omega_o L\sqrt{\pi}/(2v_{thi})$,

$$b(2\omega_o + k v_d)\,\delta\omega = \frac{i}{2}\rho k^2 v_g^2 \frac{(k v_d + \omega_o)}{\omega_o} - \frac{1}{2} k^2 v_g^2 \frac{(\omega_o + k v_d)^2 \rho^2}{\omega_o^2(1 + \frac{1}{\tau}) - \omega_o(\omega_o + k v_d) i\rho}$$

so that

$$b(2\omega_o + k v_d)\,\delta\omega_I = \frac{\rho}{2} k^2 v_g^2 \frac{(\omega_o + k v_d)}{\omega_o} \left\{1 - \frac{(\omega_o + k v_d)^2 \rho^2}{\omega_o^2(1 + \frac{1}{\tau})^2 + (\omega_o + k v_d)^2 \rho^2}\right\}$$

Then $\delta\omega_I > 0$ and the wave is stable.

APPENDIX II

Here we construct a quadratic form which can be useful to verify the validity of the results of the analysis made by the Hill determinant. For this purpose, let us compute the quantity $1/(L\pi)\int_o^{L\pi} dz\, \tilde{n}_i\, \tilde{\phi}^*(z)$. Using Eq. (5.A) we have:

$$\frac{1}{L\pi}\int_o^{L\pi} dz\, \tilde{n}_i\, \tilde{\phi}^*(z) = \frac{\tau}{\lambda_D^2 e}\left\{\frac{1}{L\pi}\int_o^{L\pi} |\tilde{\phi}|^2 dz - i(\omega - k v_d)\, I_o\, e^{-b}\, \frac{\alpha_i}{\pi}^{1/2} \times\right.$$

$$\frac{1}{L\pi}\int_o^{L\pi} dz\, \tilde{\phi}^*(z)\int_{-\infty}^{+\infty} d v_{\parallel}\, e^{-\alpha_i v_{\parallel}^2}\int_{-\infty}^{o} dt'\, \tilde{\phi}(z + v_{\parallel}t')\, e^{i\omega_i t - i\frac{k_y v_g L}{2 v_{\parallel}}[\sin(\bar{z}+\bar{v}_{\parallel}t) - \sin\bar{z}]}$$

If we study solutions of the type: $\tilde{\phi}(z) = \sum \tilde{\phi}_n\, e^{2inz/L}$ and adopt the expansion:

$$e^{-i\frac{k_y v_g L}{2 v_{\parallel}}[\sin(\bar{z}+\bar{v}_{\parallel}t) - \sin\bar{z}]} = \sum_{m,\ell} e^{-\frac{2im v_{\parallel} t}{L} + 2i(\ell - m)\frac{z}{L}}\, J_m\left(\frac{k_y v_g L}{v_{\parallel}}\right) J_\ell\left(\frac{k_y v_g L}{v_{\parallel}}\right)$$

we can write after integration upon time:

$$\frac{1}{L}\int_o^{L\pi} dz\, \tilde{n}_i\, \tilde{\phi}^*(z) = -\frac{\tau}{\lambda_D^2 e}\left\{\sum_n |\tilde{\phi}_n|^2 - (\omega - k v_d)\, I_o\, e^{-b}\left(\frac{\alpha_i}{\pi}\right)^{1/2}\int d v_{\parallel}\, e^{-\alpha_i v_{\parallel}^2} \times\right.$$

$$\left.\sum_{n',n,m,\ell} \tilde{\phi}^*_{n'}\, \tilde{\phi}_n\, e^{2i\frac{z}{L}(n-n'+\ell-m)}\, \frac{J_m(k_y v_g L/v_{\parallel})\, J_\ell(k_y v_g L/v_{\parallel})}{\omega_i + 2(n-m) v_{\parallel}/L}\right\} .$$

We have indicated $\bar{z} \equiv z\,2/L$, $\bar{v}_{\parallel} = \bar{v}_{\parallel} 2/L$, and $I_o \equiv I_o(b)$.

We then integrate over z and, making the change of variables $n - m = M$, we obtain:

$$\frac{1}{L\pi}\int_0^{L\pi} dz\, \tilde{n}_i\, \tilde{\phi}^*(z) = -\frac{\tau n_o}{e\lambda_o^2}\left\{\sum_n |\tilde{\phi}_n|^2 - (\omega - k v_d)\left(\frac{\alpha_i}{\pi}\right)^{1/2} I_o e^{-b}\int_{-\infty}^{+\infty} dv_{\|}\, e^{-\alpha_i v_{\|}^2} \times \right.$$

$$\left. \sum_M \frac{\left|\sum_n \tilde{\phi}_n J_{n-M}(k_y v_g L/v_{\|})\right|^2}{\omega_i + 2 M v_{\|}/L}\right\}$$

Then we can construct the quadratic form as:

$$0 = -\frac{e\lambda_D^2 n_o}{\tau L\pi}\int_0^{L\pi} dz\, (\tilde{n}_i - \tilde{n}_e)\, \phi^*(z) =$$

$$= (1+\tau)\sum_n |\tilde{\phi}_n|^2 - (\omega - k v_d)\left(\frac{\alpha_i}{\pi}\right)^{1/2} I_o e^{-b}\int_{-\infty}^{+\infty} dv_{\|}\, e^{-\alpha_i v_{\|}^2}\sum_M \frac{\left|\sum_n \tilde{\phi}_n J_{n-M}(k_y v_g L/v_{\|})\right|^2}{\omega_i + 2 M v_{\|}/L}$$

$$- (\omega + k v_d \tau)\left(\frac{\alpha_e}{\pi}\right)^{1/2}\int_{-\infty}^{+\infty} dv_{\|}\, e^{-\alpha_e v_{\|}^2}\sum_M \frac{\left|\sum_n \tilde{\phi}_n J_{n-M}(k_y v_g L/v_{\|})\right|^2}{\omega_e + 2 M v_{\|}/L}$$

The interesting property of this quadratic form is that the sign of the contribution due to resonant particles depends only on $(\omega - k v_d)$ for ions and on $(\omega + k v_d \tau)$ for electrons.

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