# Collisions for Step-Reduced SHA-256 

Ivica Nikolić * and Alex Biryukov<br>University of Luxembourg<br>\{ivica.nikolic, alex. biryukov\}@uni.lu


#### Abstract

In this article we find collisions for step-reduced SHA-256. We develop a differential that holds with high probability if the message satisfies certain conditions. We solve the equations that arise from the conditions. Due to the carefully chosen differential and word differences, the message expansion of SHA-256 has little effect on spreading the differences in the words. This helps us to find full collision for 21-step reduced SHA-256, semi-free start collision, i.e. collision for a different initial value, for 23 -step reduced SHA-256, and semi-free start near collision (with only 15 bit difference out of 256 bits) for 25 -step reduced SHA- 256 .


## 1 Introduction

The SHA-2 family of hash functions was introduced to the cryptographic community as a new, more complex, and hopefully, more secure variant of MD4-family of hash functions. The recent results on the widely used MD4-family hash functions SHA-1 and MD5 [6],[7] show flaws in the security of these functions, with respect to collision attacks. The question arises, if the most complex member of MD4-family, the SHA-2 family, is also vulnerable to collision attacks.

Known Results for the SHA-2 Family. Research has been made on finding a local collisions for the SHA-2 family. Gilbert and Handschuh [2] reported a 9step local collision with probability of the differential path of $2^{-66}$. Later, Mendel et al [4] estimated the probability of this local collision to be $2^{-39}$. Somitra and Palash obtained a local collision with probability $2^{-42}$. Using modular differences Hawkes, Paddon and Rose [3] were able to find a local collision with probability $2^{-39}$. As far as we know, the only work on finding a real collision for SHA-2 was made by Mendel et al[4]. They studied message expansion of the SHA-256 and reported a 19-step near collision.

Our contributions. We find a 9 -step differential that holds with probability of $\frac{1}{3}$ by fixing some of the intermediate values and solving the equations that arise. We show that it is not necessary to introduce differences in message words on each step of the differential. This helps us to overcome the message expansion. We use modular substraction differences. Using only one instance of this

[^0]differential we find 20 and 21-step collisions (collisions for the original initial value) with probabilities $\frac{1}{3}$ and $2^{-19}$ respectively. Also, using slightly different differential we were able to find a 23 -step semi-free start collision (collisions for a specific initial value) with probability $2^{-21}$. Our final result is a 25 -step semi-free start near collision with Hamming distance of 15 bits and probability $2^{-34}$.

Let $H\left(M, h_{0}\right)$ be a hash function, where $M$ is the input message, and $h_{0}$ is the initial chaining value. The following attacks are considered in the paper: Collision attack: Find messages $M_{1}$ and $M_{2}$ such that $M_{1} \neq M_{2}$ and $H\left(M_{1}, h_{0}\right)=$ $H\left(M_{2}, h_{0}\right)$.
Semi-free start collision attack: Find messages $M_{1}, M_{2}$ and hash value $h_{0}^{*}$ such that $M_{1} \neq M_{2}$ and $H\left(M_{1}, h_{0}^{*}\right)=H\left(M_{2}, h_{0}^{*}\right)$.
Near collision attack: Find messages $M_{1}$ and $M_{2}$ such that $M_{1} \neq M_{2}$ and Hamming distance between $H\left(M_{1}, h_{0}\right)$ and $H\left(M_{2}, h_{0}\right)$ is small compared to the output size $n$ of the hash function.

## 2 Description of SHA-2

SHA-2 family consists of iterative hash functions SHA-224, SHA-256, SHA-384, and SHA-512. For our purposes, we will describe only SHA-256. The definitions of the rest of the functions can be found in [1]. The SHA-256 takes a message of length less than $2^{64}$ and produces a 256 -bit hash value. First, the input message is padded so the length becomes a multiple of 512 , and afterwards each 512bit message block is processed as an input in the Damgard-Merkle iterative structure. Each iteration calls a compression function which takes for an input a 256 -bit chaining value and a 512 -bit message block and produces an output 256 -bit chaining value. The output chaining value of the previous iteration is an input chaining value for the following iteration. The initial chaining value, i.e. the value for the first iteration, is fixed, and the chaining value produced after the last message block is proceeded is the hash value of the whole message. Internal state of SHA-256 compression function consists of 832 -bit variables A, $\mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}$, and H , each of which is updated on every of the 64 steps. These variables are updated according to the following equations:

$$
\begin{aligned}
A_{i+1} & =\Sigma_{0}\left(A_{i}\right)+\operatorname{Maj}\left(A_{i}, B_{i}, C_{i}\right)+\Sigma_{1}\left(E_{i}\right)+C h\left(E_{i}, F_{i}, G_{i}\right)+H_{i}+K_{i}+W_{i} \\
B_{i+1} & =A_{i} \\
C_{i+1} & =B_{i} \\
D_{i+1} & =C_{i} \\
E_{i+1} & =\Sigma_{1}\left(E_{i}\right)+C h\left(E_{i}, F_{i}, G_{i}\right)+H_{i}+K_{i}+W_{i}+D_{i} \\
F_{i+1} & =E_{i} \\
G_{i+1} & =F_{i} \\
H_{i+1} & =G_{i}
\end{aligned}
$$

The $\operatorname{Maj}(X, Y, Z)$ and $C h(X, Y, Z)$ are bitwise boolean functions defined as:

$$
\begin{array}{r}
C h(X, Y, Z)=(X \wedge Y) \vee(\neg X \wedge Z) \\
M a j(X, Y, Z)=(X \wedge Y) \vee(X \wedge Z) \vee(Y \wedge Z)
\end{array}
$$

For SHA-256 $\Sigma_{0}(X)$ and $\Sigma_{1}(X)$ are defined as:

$$
\begin{aligned}
& \Sigma_{0}(X)=\operatorname{ROTR}^{2}(X) \oplus \operatorname{ROTR}^{13}(X) \oplus \operatorname{ROT}^{22}(X) \\
& \Sigma_{1}(X)=\operatorname{ROTR}^{6}(X) \oplus \operatorname{ROTR}^{11}(X) \oplus \operatorname{ROTR}^{25}(X)
\end{aligned}
$$

State update function uses constants $K_{i}$, which are different for every step. The 512 -bit message block itself is divided in 1632 -bit bit words: $m_{0}, m_{1}, \ldots, m_{16}$. Afterwards, the message block is expanded to 6432 -bit words according to the following rule:

$$
W_{i}= \begin{cases}m_{i}, & 0 \leq i \leq 15 \\ \sigma_{1}\left(W_{i-2}\right)+W_{i-7}+\sigma_{0}\left(W_{i-15}\right)+W_{i-16}, & i>15\end{cases}
$$

For SHA-256 $\sigma_{0}(X)$ and $\sigma_{1}(X)$ are defined as:

$$
\begin{aligned}
& \sigma_{0}(X)=\operatorname{ROTR}^{7}(X) \oplus \operatorname{ROTR}^{18}(X) \oplus S H R^{3}(X) \\
& \sigma_{1}(X)=\operatorname{ROTR}^{17}(X) \oplus \operatorname{ROTR}^{19}(X) \oplus S H R^{10}(X)
\end{aligned}
$$

The compression function after the 64 -th step adds the initial values to the chaining variables, i.e. the hash result of the compression function is:
$h(M)=\left(A_{64}+A_{0}, B_{64}+B_{0}, C_{64}+C_{0}, D_{64}+D_{0}, E_{64}+E_{0}, F_{64}+F_{0}, G_{64}+G_{0}, H_{64}+H_{0}\right)$.
These values become the initial chaining value for the next compression function.

## 3 Technique for Creating Collisions

Differences used in this paper are subtractions mod $2^{32}$ differences.
We use the following notation:

$$
\begin{aligned}
& \Delta X_{i}=X_{i}^{\prime}-X_{i}, X \in\{A, B, D, D, E, F, G, H, W, m\}, \\
& \Delta M a j^{i}(\Delta a, \Delta b, \Delta c)=\operatorname{Maj}\left(A_{i}+\Delta a, B_{i}+\Delta b, C_{i}+\Delta c\right)-\operatorname{Maj}\left(A_{i}, B_{i}, C_{i}\right), \\
& \Delta C h^{i}(\Delta e, \Delta f, \Delta g)=C h\left(E_{i}+\Delta e, F_{i}+\Delta f, G_{i}+\Delta g\right)-C h\left(E_{i}, F_{i}, G_{i}\right) . \\
& \Delta \Sigma_{0}\left(A_{i}\right)=\Sigma_{0}\left(A_{i}^{\prime}\right)-\Sigma_{0}\left(A_{i}\right) \\
& \Delta \Sigma_{1}\left(E_{i}\right)=\Sigma_{1}\left(E_{i}^{\prime}\right)-\Sigma_{1}\left(E_{i}\right) \\
& \Delta \sigma_{0}\left(m_{i}\right)=\sigma_{0}\left(m_{i}^{\prime}\right)-\sigma_{0}\left(m_{i}\right) \\
& \Delta \sigma_{1}\left(m_{i}\right)=\sigma_{1}\left(m_{i}^{\prime}\right)-\sigma_{1}\left(m_{i}\right)
\end{aligned}
$$

We introduce perturbation on step $i$ and in the following 8 steps we try to correct the differences in the internal variables. We use the following differential:

Table 1. A 9 step differential for SHA-2 family. Notice that only 5 differences are introduced, i.e. in steps $i, i+1, i+2, i+3$, and $i+8$.

| step | $\Delta A$ | $\Delta B$ | $\Delta C$ | $\Delta D$ | $\Delta E$ | $\Delta F$ | $\Delta G$ | $\Delta H$ | $\Delta W$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| i | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $\mathrm{i}+1$ | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | $\delta_{1}$ |
| $\mathrm{i}+2$ | 0 | 1 | 0 | 0 | -1 | 1 | 0 | 0 | $\delta_{2}$ |
| $\mathrm{i}+3$ | 0 | 0 | 1 | 0 | 0 | -1 | 1 | 0 | $\delta_{3}$ |
| $\mathrm{i}+4$ | 0 | 0 | 0 | 1 | 0 | 0 | -1 | 1 | 0 |
| $\mathrm{i}+5$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | -1 | 0 |
| $\mathrm{i}+6$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $\mathrm{i}+7$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $\mathrm{i}+8$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | $\delta_{4}$ |
| $\mathrm{i}+9$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

As you can see from the table (column $\Delta W$ ), only the perturbation has been fixed. All the other differences are to be determined.

### 3.1 Conditions for the Local Collision

From the definition of SHA-2, focusing on registers $A_{i+1}$ and $E_{i+1}$, we get:

$$
\begin{aligned}
& \Delta A_{i+1}-\Delta E_{i+1}=\Delta \Sigma_{0}\left(A_{i}\right)+\Delta M a j^{i}\left(\Delta A_{i}, \Delta B_{i}, \Delta C_{i}\right)-\Delta D_{i} \\
& \Delta E_{i+1}=\Delta \Sigma_{1}\left(E_{i}\right)+\Delta C h^{i}\left(\Delta E_{i}, \Delta F_{i}, \Delta G_{i}\right)+\Delta H_{i}+\Delta D_{i}+\Delta W_{i}
\end{aligned}
$$

We will keep in mind that if $\Delta A_{i}=\Delta B_{i}=\Delta C_{i}=0$ then $\Delta \operatorname{Maj}^{i}(0,0,0)=0$. Also if $\Delta E_{i}=\Delta F_{i}=\Delta G_{i}=0$ then $\Delta C h^{i}(0,0,0)=0$.

We fix the differences for the registers $A$ and $E$ (as shown in the table). The variables $B, C, D, F, G, H$ can only inherit the values from $A$ and $E$. So, for each step we get some equations with respect to $\delta_{i}$ and $A_{i}$ or $E_{i}$.

Step i+1. We have that $\Delta D_{i}=0, \Delta H_{i}=0, \Delta \Sigma_{0}\left(A_{i}\right)=0, \Delta \Sigma_{1}\left(E_{i}\right)=0$. We require $\Delta A_{i+1}=1, \Delta E_{i+1}=1$. So we deduce:

$$
\begin{equation*}
\Delta W_{i}=1 \tag{1}
\end{equation*}
$$

Step i+2. We have that $\Delta D_{i+1}=0, \Delta H_{i+1}=0$. We require $\Delta A_{i+2}=0$, $\Delta E_{i+2}=-1$. We want also $\Delta \Sigma_{0}\left(A_{i+1}\right)=1$ to be satisfied. So we deduce:

$$
\begin{align*}
& \Delta M a j^{i+1}(1,0,0)=0  \tag{2}\\
& \Delta W_{i+1}=-1-\Delta C h^{i+1}(1,0,0)-\Delta \Sigma_{1}\left(E_{i+1}\right)  \tag{3}\\
& \Delta \Sigma_{0}\left(A_{i+1}\right)=1 \tag{4}
\end{align*}
$$

Step $\mathbf{i}+\mathbf{3}$. We have that $\Delta D_{i+2}=0, \Delta H_{i+2}=0, \Delta \Sigma_{0}\left(A_{i+2}\right)=0$. We require $\Delta A_{i+3}=0, \Delta E_{i+3}=0$. So we deduce:

$$
\begin{align*}
& \Delta M a j^{i+2}(0,1,0)=0  \tag{5}\\
& \Delta W_{i+2}=-\Delta \Sigma_{1}\left(E_{i+2}\right)-\Delta C h^{i+2}(-1,1,0) \tag{6}
\end{align*}
$$

Step i+4. We have that $\Delta D_{i+3}=0, \Delta H_{i+3}=0, \Delta \Sigma_{0}\left(A_{i+3}\right)=0, \Delta \Sigma_{1}\left(E_{i+3}\right)=$ 0 . We require $\Delta A_{i+4}=0, \Delta E_{i+4}=0$. So we deduce:

$$
\begin{align*}
& \Delta M a j^{i+3}(0,0,1)=0  \tag{7}\\
& \Delta W_{i+3}=-\Delta C h^{i+3}(0,-1,1) \tag{8}
\end{align*}
$$

Step i+5. We have that $\Delta D_{i+4}=1, \Delta H_{i+4}=1, \Delta \Sigma_{0}\left(A_{i+4}\right)=0, \Delta \Sigma_{1}\left(E_{i+4}\right)=$ 0 . We require $\Delta A_{i+5}=0, \Delta E_{i+5}=1$. So we deduce:

$$
\begin{equation*}
\Delta C h^{i+4}(0,0,-1)=-1 \tag{9}
\end{equation*}
$$

Step i+6. We have that $\Delta D_{i+5}=0, \Delta H_{i+5}=-1, \Delta \Sigma_{0}\left(A_{i+5}\right)=0$. We require $\Delta A_{i+6}=0, \Delta E_{i+6}=0$. We want also $\Delta \Sigma_{0}\left(E_{i+5}\right)=1$ to be satisfied. So we deduce:

$$
\begin{align*}
& \Delta C h^{i+5}(1,0,0)=0  \tag{10}\\
& \Delta \Sigma_{1}\left(E_{i+5}\right)=1 \tag{11}
\end{align*}
$$

Step i+7. We have that $\Delta D_{i+6}=0, \Delta H_{i+6}=0, \Delta \Sigma_{0}\left(A_{i+6}\right)=0, \Delta \Sigma_{1}\left(E_{i+6}\right)=$ 0 . We require $\Delta A_{i+7}=0, \Delta E_{i+7}=0$. So we deduce:

$$
\begin{equation*}
\Delta C h^{i+6}(0,1,0)=0 \tag{12}
\end{equation*}
$$

Step i+8. We have that $\Delta D_{i+7}=0, \Delta H_{i+7}=0, \Delta \Sigma_{0}\left(A_{i+7}\right)=0, \Delta \Sigma_{1}\left(E_{i+7}\right)=$ 0 . We require $\Delta A_{i+8}=0, \Delta E_{i+8}=0$. So we deduce:

$$
\begin{equation*}
\Delta C h^{i+7}(0,0,1)=0 \tag{13}
\end{equation*}
$$

Step i+9. We have that $\Delta D_{i+8}=0, \Delta H_{i+8}=1, \Delta \Sigma_{0}\left(A_{i+8}\right)=0, \Delta \Sigma_{1}\left(E_{i+8}\right)=$ 0 . We require $\Delta A_{i+9}=0, \Delta E_{i+9}=0$. So we deduce:

$$
\begin{equation*}
\Delta W_{i+8}=-1 \tag{14}
\end{equation*}
$$

### 3.2 Solution of the System of Equations

Lets first observe (4) and (11). From the differential we can see that $\Delta A_{i+1}=$ $\Delta E_{i+5}=1$. It means that we want the functions $\Delta \Sigma_{0}\left(A_{i+1}\right), \Delta \Sigma_{1}\left(E_{i+5}\right)$ to preserve the difference 1 , in other words:

$$
\begin{aligned}
& \Sigma_{0}\left(A_{i+1}+1\right)-\Sigma_{0}\left(A_{i+1}\right)=1, \\
& \Sigma_{1}\left(E_{i+5}+1\right)-\Sigma_{1}\left(E_{i+5}\right)=1 .
\end{aligned}
$$

The only solution to these equations is $A_{i+1}=E_{i+5}=-1$, so we get:

$$
\begin{array}{ll}
A_{i+1}=-1, & A_{i+1}^{\prime}=0 \\
E_{i+5}=-1, & E_{i+5}^{\prime}=0 \tag{16}
\end{array}
$$

Now lets consider the function $\Delta M a j^{i}=\operatorname{Maj}\left(A_{i}^{\prime}, B_{i}^{\prime}, C_{i}^{\prime}\right)-\operatorname{Maj}\left(A_{i}, B_{i}, C_{i}\right)$. Lets suppose that $B_{i}^{\prime}=B_{i}, C_{i}^{\prime}=C_{i}$ and $A_{i}$ and $A_{i}^{\prime}$ differ in every single bit, i.e. $A_{i} \oplus A_{i}^{\prime}=0 \mathrm{xffffffff}$. Then:

$$
\Delta M a j^{i}=0 \Leftrightarrow B_{i}=C_{i}
$$

Therefore (2) gives us $B_{i+1}=C_{i+1}$, which is $A_{i}=A_{i-1}$. With the same reasoning we can deduce from (5) that $A_{i+2}=A_{i}$, and from (7) that $A_{i+3}=A_{i+2}$. So, from (2),(5) and (7) we get that

$$
\begin{equation*}
A_{i-1}=A_{i}=A_{i+2}=A_{i+3} \tag{17}
\end{equation*}
$$

Similarly to what we have done with $M a j$, now lets consider $\Delta C h^{i}$ and suppose that $F_{i}^{\prime}=F_{i}, G_{i}^{\prime}=G_{i}$ and $E_{i}$ and $E_{i}^{\prime}$ differ in every single bit. Then:

$$
\Delta C h^{i}=0 \Leftrightarrow F_{i}=G_{i}
$$

Therefore (10) and the result (16) gives us $F_{i+5}=G_{i+5}$, which is:

$$
\begin{equation*}
E_{i+4}=E_{i+3} \tag{18}
\end{equation*}
$$

Solving (12) requires slightly different reasoning; if we have $E_{i+6}=E_{i+6}^{\prime}, G_{i+6}=$ $G_{i+6}^{\prime}$ and $F_{i+6}$ and $F_{i+6}^{\prime}$ would differ in every bit (and they do, see (16)) then :

$$
\begin{equation*}
\Delta C h^{i+6}=0 \Leftrightarrow E_{i+6}=0 . \tag{19}
\end{equation*}
$$

Analogously, from (13) we get:

$$
\begin{equation*}
E_{i+7}=-1 \tag{20}
\end{equation*}
$$

The only remaining condition is (9):
$\Delta C h^{i+4}=C h\left(E_{i+4}, F_{i+4}, G_{i+4}^{\prime}\right)-C h\left(E_{i+4}, F_{i+4}, G_{i+4}\right)=-1, G_{i+4}^{\prime}-G_{i+4}=-1$.
The words $E_{i+4}, F_{i+4}, G_{i+4}$ are already determined to satisfy the previous conditions. So, we don't have any degrees of freedom left to control precisely the solution of this equation. Therefore we will try to find the probability that this condition holds. We can see that it holds if and only if register $E_{i+4}$ has 0 's in the bits where $G_{i+4}^{\prime}$ and $G_{i+4}$ are different. The $G_{i+4}^{\prime}$ and $G_{i+4}$ can differ in the last $i$ bits, where $1 \leq i \leq 32$., and these bits are uniquely determined. So, for the probability we get:
$\sum_{i=1}^{i=32} \mathrm{P}\left\{\right.$ Last i bits of $E_{i+4}$ are zero $\} \times \mathrm{P}\{$ Difference in the exactly i last bits $\}=$

$$
=\sum_{i=1}^{i=32} \frac{1}{2^{i}} \frac{1}{2^{i}} \approx \frac{1}{3} .
$$

So, the overall probability of our differential is $\frac{1}{3}=2^{-1.58}$.
The differences in message words of the differential as in Table 1 are the following:

$$
\begin{aligned}
\delta_{1} & =-1-\Delta C h^{i+1}(1,0,0)-\Delta \Sigma_{1}\left(E_{i+1}\right), \\
\delta_{2} & =-\Delta \Sigma_{1}\left(E_{i+2}\right)-\Delta C h^{i+2}(-1,1,0), \\
\delta_{3} & =-\Delta C h^{i+3}(0,-1,1) \\
\delta_{4} & =-1
\end{aligned}
$$

Notice that the condition (17) shows us that $A_{i}=B_{i}$ has to hold.

## 4 Full, Semi-free and Near Collisions for Step-reduced SHA-256

Our attack technique is the following:

1. Introduce perturbation at step $i$;
2. Correct the differences in the following 8 steps (probability of success is the probability of our differential, i.e. $\frac{1}{3}$ ). After the last step of the differential, the differences in the internal variables are zero;
3. All the the message words that follow the last step of the differential have to have zero differences;

### 4.1 20-Step Collision

From the Table 3 of Appendix A we can see that the words $m_{5}, m_{6}, m_{7}, m_{8}$, and $m_{13}$ are used only once in the first 20 steps of SHA-2, i.e. i.e. they are not used to compute the values of expanded words $W_{16}, W_{17}, W_{18}$, and $W_{19}$. This means that message expansion doesn't introduce any difference after the last step of the differential. So, we get collision for 20 step reduced SHA-2, and the collisions can be found practically by hand. The probability of collision is $2^{-1.58}$.

### 4.2 21-Step Collision

From the Table 3 of Appendix A we can easily see that we have to consider message expansion since there are no message words that are used only once in the first 21 steps and that have the proper indexes for the differential.
We will introduce differences in the words $m_{6}, m_{7}, m_{8}, m_{9}$, and $m_{14}$. The words $m_{6}, m_{7}, m_{8}$ are used only once in the first 21 steps. Therefore the message expansion in the first 21 steps is irrelevant with respect to these words, i.e. differences in these words don't introduce any other new differences, after the last step of the differential(step 14). Now, we want to find words $m_{9}, m_{9}^{\prime}, m_{14}, m_{14}^{\prime}$ such that after the 14 -th step, the message expansion will not introduce any difference in
the following steps. From the Table 3 of Appendix A we can see that the words $m_{9}$ and $m_{14}$ are used in $W_{16}, W_{18}$, and $W_{20}$. So, from the definition of $W_{i}$ we get the equations:

$$
\begin{align*}
& \Delta W_{16}=\Delta \sigma_{1}\left(m_{14}\right)+\Delta m_{9}+\Delta \sigma_{0}\left(m_{1}\right)+\Delta m_{0}=0  \tag{21}\\
& \Delta W_{17}=\Delta \sigma_{1}\left(m_{15}\right)+\Delta m_{10}+\Delta \sigma_{0}\left(m_{2}\right)+\Delta m_{1}=0  \tag{22}\\
& \Delta W_{18}=\Delta \sigma_{1}\left(W_{16}\right)+\Delta m_{11}+\Delta \sigma_{0}\left(m_{3}\right)+\Delta m_{2}=0  \tag{23}\\
& \Delta W_{19}=\Delta \sigma_{1}\left(W_{17}\right)+\Delta m_{12}+\Delta \sigma_{0}\left(m_{4}\right)+\Delta m_{3}=0  \tag{24}\\
& \Delta W_{20}=\Delta \sigma_{1}\left(W_{18}\right)+\Delta m_{13}+\Delta \sigma_{0}\left(m_{5}\right)+\Delta m_{4}=0 \tag{25}
\end{align*}
$$

Obviously if $m_{i}^{\prime}=m_{i}\left(W_{i}^{\prime}=W_{i}\right)$ then $\Delta \sigma_{0}\left(m_{i}\right)=0\left(\Delta \sigma_{0}\left(W_{i}\right)=0\right)$. This means that $\Delta W_{17}=\Delta W_{19}=0$. If we can make so that $\Delta W_{16}=0$ then $\Delta W_{18}=$ $\Delta W_{20}=0$. So, we get the equation:

$$
\begin{equation*}
\Delta \sigma_{1}\left(m_{14}\right)+\Delta m_{9}=0 \tag{26}
\end{equation*}
$$

Considering that $\Delta m_{14}=\delta_{4}=-1$, and $m_{9}$ can take any value, our experimental results (Monte Carlo method with $2^{32}$ trials) give us a probability of $2^{-17.5}$ that $\Delta m_{14}$ and $\Delta m_{9}$ satisfy this equation. Therefore, the overall probability of 21 step collision is around $2^{-19}$.

### 4.3 23-Step Semi-free Start Collision

For 23 step collision we introduce differences in the words $m_{9}, m_{10}, m_{11}$, and $m_{12}$.
If we would follow our differential, we are supposed to introduce difference in the message word $W_{17}$. We can not control $W_{17}$ directly because it is an expanded word. From the condition $W_{17}=\delta_{4}=-1$ (differential) and the message expansion, we get:

$$
\Delta W_{17}=\Delta \sigma_{1}\left(m_{15}\right)+\Delta m_{10}+\Delta \sigma_{0}\left(m_{2}\right)+\Delta m_{1}=-1 .
$$

Since $\Delta m_{15}=\Delta m_{2}=\Delta m_{1}=0$, we get:

$$
\begin{equation*}
\Delta m_{10}=-1 . \tag{27}
\end{equation*}
$$

In our original differential there are no message differences in the word $W_{16}$. But for $W_{16}$ we have:

$$
\Delta W_{16}=\Delta \sigma_{1}\left(m_{14}\right)+\Delta m_{9}+\Delta \sigma_{0}\left(m_{1}\right)+\Delta m_{0} .
$$

Obviously only $\Delta m_{9} \neq 0$ and therefore $\Delta W_{16}=\Delta m_{9}=1 \neq 0$. Therefore we shall use slightly different differential:one were there is a difference in the word $W_{16}$. To keep everything else unchanged, the equations for the step 17 become the following:

$$
\Delta E_{17}=\Delta \Sigma_{1}\left(E_{16}\right)+\Delta C h^{16}(0,0,1)+\Delta D_{16}+\Delta H_{16}+\Delta W_{16} .
$$

From the differential we can see that: $\Delta E_{17}=\Delta \Sigma_{1}\left(E_{16}\right)=\Delta D_{16}=\Delta H_{16}=0$. Therefore we get:

$$
\begin{equation*}
\Delta C h^{16}(0,0,1)+\Delta W_{16}=0 \tag{28}
\end{equation*}
$$

Now, lets observe the other words of the message expansion.
For $W_{18}$ we have:

$$
W_{18}=\Delta \sigma_{1}\left(W_{16}\right)+\Delta m_{11}+\Delta \sigma_{0}\left(m_{3}\right)+\Delta m_{2}=0
$$

Since $\Delta m_{3}=\Delta m_{2}=0, \Delta W_{16}=1$ we get the equation:

$$
\begin{equation*}
\Delta \sigma_{1}\left(W_{16}\right)+\Delta m_{11}=0 \tag{29}
\end{equation*}
$$

For $W_{19}$ we have:

$$
W_{19}=\Delta \sigma_{1}\left(W_{17}\right)+\Delta m_{12}+\Delta \sigma_{0}\left(m_{4}\right)+\Delta m_{3}=0
$$

Since $\Delta m_{4}=\Delta m_{3}=0, \Delta W_{17}=-1$ we get the equation:

$$
\begin{equation*}
\Delta \sigma_{1}\left(W_{17}\right)+\Delta m_{12}=0 \tag{30}
\end{equation*}
$$

For $W_{20}$ we have:

$$
W_{20}=\Delta \sigma_{1}\left(W_{18}\right)+\Delta m_{13}+\Delta \sigma_{0}\left(m_{5}\right)+\Delta m_{4}=0
$$

Since $\Delta W_{18}=\Delta m_{13}=\Delta m_{5}=\Delta m_{4}=0$ we get that this equation is satisfied for all values of $W_{18}, m_{13}, m_{5}, m_{4}$.
For $W_{21}$ we have:

$$
W_{21}=\Delta \sigma_{1}\left(W_{19}\right)+\Delta m_{14}+\Delta \sigma_{0}\left(m_{6}\right)+\Delta m_{5}=0
$$

Since $\Delta W_{19}=\Delta m_{14}=\Delta m_{6}=\Delta m_{5}=0$ we get that this equation is satisfied for all values of $W_{19}, m_{14}, m_{6}, m_{5}$.
For $W_{22}$ we have:

$$
W_{22}=\Delta \sigma_{1}\left(W_{20}\right)+\Delta m_{15}+\Delta \sigma_{0}\left(m_{7}\right)+\Delta m_{6}=0
$$

Since $\Delta W_{20}=\Delta m_{15}=\Delta m_{7}=\Delta m_{6}=0$ we get that this equation is satisfied for all values of $W_{20}, m_{15}, m_{7}, m_{6}$. For $W_{23}$ we have:

$$
W_{23}=\Delta \sigma_{1}\left(W_{21}\right)+\Delta W_{16}+\Delta \sigma_{0}\left(m_{8}\right)+\Delta m_{7}=0
$$

Since $\Delta W_{21}=\Delta m_{8}=\Delta m_{7}$ and $\Delta W_{16} \neq 0$ we get that this equation has no solution. That is why we can not get more than 23 step collision.

Lets try to solve (27), (28), (29) and (30).
For (27) and the value of the register $E_{11}$ from the differential's conditions we have:

$$
\left.\Delta E_{11}=\Delta \Sigma_{1}\left(E_{10}\right)\right)+\Delta C h^{10}(1,0,0)+\Delta m_{10} .
$$

Since $\Delta E_{11}=m_{10}=-1$ we get:

$$
\Delta \Sigma_{1}\left(E_{10}\right)+\Delta C h^{10}(1,0,0)=0
$$

We solve this equation by setting $\Delta \Sigma_{1}\left(E_{10}\right)=1$ and $\Delta C h^{10}(1,0,0)=-1$. The first one has solution:

$$
\begin{equation*}
E_{10}=-1, E_{10}^{\prime}=0 \tag{31}
\end{equation*}
$$

The second equation holds for the values:

$$
\begin{equation*}
F_{10}=G_{10}+1 \tag{32}
\end{equation*}
$$

Now lets turn to the solution of (28). Using the fact that $G_{16}=-1$ and $G_{16}^{\prime}=0$, we get that this equation is satisfied if:

$$
\begin{equation*}
E_{16}=0 x f f f f f f f e \tag{33}
\end{equation*}
$$

Lets observe the equation (30). From the conditions of the differential we have:

$$
\Delta E_{13}=\Delta \Sigma_{1}\left(E_{12}\right)+\Delta C h^{12}(0,-1,1)+\Delta H_{12}+\Delta D_{12}+\Delta m_{12}
$$

Since $\Delta E_{13}=\Delta E_{12}=\Delta H_{12}=\Delta D_{12}=0$ we get:

$$
\Delta C h^{12}(0,-1,1)+\Delta m_{12}=0
$$

If we substitute $m_{12}$ from (30) we can get:

$$
\Delta C h^{12}(0,-1,1)=\Delta \sigma_{1}(-1) .
$$

This equation can be satisfied if we can control $E_{12}$ and $F_{12}$.
For $E_{12}$, from the definition of $A_{12}$ and $E_{12}$ we have:

$$
A_{12}-E_{12}=\Sigma_{1}\left(A_{11}\right)+C h\left(A_{11}, B_{11}, C_{11}\right)-D_{11}
$$

Considering that $A_{12}=A_{11}=C_{11}=D_{11}$ from the differential's conditions, we get:

$$
E_{12}=A_{9}-\Sigma_{1}\left(A_{9}\right)
$$

Since $A_{9}$ can take any value (we consider semi-free start collision) we deduce that $E_{12}$ can take any value.
The $F_{12}$ value, which is $E_{11}$ can be controlled through $H_{10}$. Notice that changing $H_{10}$, which is $G_{9}$, doesn't effect $E_{10}$, because from (31) we can see that $E_{10}$ always takes the arranged value.
We proved that we can fully control $E_{12}$ and $F_{12}$. We can choose some specific value for $\Delta \sigma_{1}(-1)$ which is possible to get from $\Delta C h^{12}(0,-1,1)$, and set the $A_{9}$ and $G_{9}$ so that the equation (30) will hold.
The last equation, i.e. (29), is satisfied for some specific values of $W_{16}$ and $m_{11}$. Our experimental results show that with probability $2^{-19.5} W_{16}$ and $m_{11}$ satisfy (29). Therefore the overall probability for 23 step semi-free start near collision is around $2^{-21}$.

### 4.4 25-Steps Semi-free Start Near Collision

Lets suppose we have a semi-free start collision on the 23-rd step. Each following step introduces differences in the chaining variables $A$ and $E$. The variables $B, C, D, F, G, H$ can only inherit differences from $A$ and $E$. Therefore, for each step, we should try to minimize the differences in $A$ and $E$. When we say to minimize the differences we mean to minimize the Hamming distances between $A^{\prime}$ and $A$, and between $E^{\prime}$ and $E$.
Step 24.

$$
\min _{W_{23}^{\prime}-W_{23}=1} h_{d}\left(E_{24}^{\prime}, E_{24}\right)=\min _{W_{23}^{\prime}-W_{23}=1} h_{d}\left(C_{1}+1, C_{1}\right)=1,
$$

where $C_{1}=\Sigma_{1}\left(E_{23}\right)+C h\left(E_{23}, F_{23}, G_{23}\right)+H_{23}+D_{23}+K_{23}+W_{23}$.

$$
\min _{W_{23}^{\prime}-W_{23}=1} h_{d}\left(A_{24}^{\prime}, A_{24}\right)=\min _{W_{23}^{\prime}-W_{23}=1} h_{d}\left(C_{2}+1, C_{2}\right)=1,
$$

where $C_{2}=\Sigma_{0}\left(A_{23}\right)+\operatorname{Maj}\left(A_{23}, B_{23}, C_{23}\right)+\Sigma_{1}\left(E_{23}\right)+C h\left(E_{23}, F_{23}, G_{23}\right)+H_{23}+$ $K_{23}+W_{23}$.
We have the minimal Hamming distances when $C_{1}^{32}=C_{2}^{32}=0$, which means with probability $2^{-2}$.

Step 25.

$$
\begin{aligned}
& \min _{W_{24}^{\prime}-W_{24}=-1+\Delta \sigma_{0}(1)} h_{d}\left(E_{25}^{\prime}, E_{25}\right)= \\
& =\min h_{d}\left(\Sigma_{1}\left(E_{24}^{\prime}\right)+C h\left(E_{24}^{\prime}, F_{24}, G_{24}\right)-1+\sigma_{0}\left(m_{9}+1\right)+C_{1}, \Sigma_{1}\left(E_{24}\right)+C h\left(E_{24}, F_{24}, G_{24}\right)+\sigma_{0}\left(m_{9}\right)+C_{1}\right), \\
& \text { where } C_{1}=H_{24}+D_{24}+K_{24}+\sigma_{1}\left(W_{22}\right)+m_{8} . \text { If } F_{24}^{32}=1 \text { and } G_{24}^{32}=0(\text { probability } \\
& \left.2^{-2}\right) \text { then, considering that } E_{24}^{\prime 32}=1, E_{24}^{32}=0 \text {, we have } C h\left(E_{24}^{\prime}, F_{24}, G_{24}\right)-1= \\
& C h\left(E_{24}, F_{24}, G_{24}\right) \text {, and we can rewrite the last expression as: }
\end{aligned}
$$

$$
\min h_{d}\left(\Sigma_{1}\left(E_{24}^{\prime}\right)+\sigma_{0}\left(m_{9}+1\right)+C_{2}, \Sigma_{1}\left(E_{24}\right)+\sigma_{0}\left(m_{9}\right)+C_{2}\right)
$$

where $C_{2}=C_{1}+C h\left(E_{24}, F_{24}, G_{24}\right)$.
If no carry occurs due to the differences, then the above minimum is:

$$
\min h_{d}\left(\Sigma_{1}\left(E_{24}^{\prime}\right)+\sigma_{0}\left(m_{9}+1\right)+C_{2}, \Sigma_{1}\left(E_{24}\right)+\sigma_{0}\left(m_{9}\right)+C_{2}\right)=5
$$

For $\Sigma_{1}\left(E_{24}^{\prime}\right)$ (difference in three bits) there are no carries with probability $2^{-3}$. For $\sigma_{0}\left(m_{9}+1\right)$ (two differences if $m_{9}^{32}=0$ ) with probability $2^{-3}$. Therefore the minimum is 5 with probability $2^{-8}$.
Using the same methods we can get:

$$
\min _{W_{24}^{\prime}-W_{24}=-1+\Delta \sigma_{0}(1)} h_{d}\left(A_{25}^{\prime}, A_{25}\right)=8,
$$

with probability $2^{-11}$. Notice that if minimum holds for $A_{25}$ then it holds for $E_{25}$.

So, for the whole hash value, we have:
$\left.h_{d}\left(\left(A_{25}^{\prime}, B_{25}^{\prime}, C_{25}^{\prime}, D_{25}^{\prime}, E_{25}^{\prime}, F_{25}^{\prime}, G_{25}^{\prime}, H_{25}^{\prime}\right)\right),\left(A_{25}, B_{25}, C_{25}, D_{25}, E_{25}, F_{25}, G_{25}, H_{25}\right)\right)=$
$=h_{d}\left(\left(A_{25}^{\prime}, A_{24}^{\prime}, C_{25}, D_{25}, E_{25}^{\prime}, E_{24}^{\prime}, G_{25}, H_{25}\right),\left(A_{25}, A_{24}, C_{25}, D_{25}, E_{25}, E_{24}, G_{25}, H_{25}\right)\right)=$
$=h_{d}\left(A_{25}^{\prime}, A_{25}\right)+h_{d}\left(E_{25}^{\prime}, E_{25}\right)+h_{d}\left(A_{24}^{\prime}, A_{24}\right)+h_{d}\left(E_{24}^{\prime}, E_{24}\right)=$
$=8+5+1+1=15$

Therefore we get a 25 -step semi-free start near collision with the Hamming weight of 15 bits and probability $2^{-34}$. Notice that we haven't investigated all the possible outcomes of the carry effects. Therefore, it is possible that the real probability is higher.

Table 2. Collision search attacks for SHA-256.

| \# of steps | Type of collision | Complexity(*) | Paper |
| :---: | :---: | :---: | :---: |
| 19 | Near collision | $\left({ }^{* *}\right)$ | $[4]$ |
| 20 | Collision | $2^{1.58}$ | This paper |
| 21 | Collision | $2^{19}$ | This paper |
| 22 | Pseudo-collision | $\left({ }^{* *}\right)$ | $[4]$ |
| 23 | Semi-free start collision | $2^{21}$ | This paper |
| 25 | Semi-free start near collision | $2^{34}$ | This paper |

(*) Complexity is measured in reduced SHA-256 calls
(**) Complexity not mentioned in the paper

## 5 Conclusion

We created a 9-step differential for SHA-256 that holds with high probability. Using the characteristics of this differential, precisely, the fact that not all of the input message words have differences, we were able to overcome the beginning steps of the message expansion. We created a full collisions for 20 and 21-step reduced SHA-256. Also, we found a 23 -step reduced semi-free start collision, and 25 -step reduced near collision with Hamming distance of 17 out of 256 bits. The complexities of these collisions search attacks are showed in Table 2. Obviously, our results hold for SHA-224 too. For SHA-384 and SHA-512 different equations arise. We have not analyzed them, but our guess is that complexities of the attacks should stay the same.

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## A Message Expansion

Table 3. Message expansion of SHA-2. There is ' $x$ ' in the intersection of row with index $i$ and column with index $j$ if $W_{i}$ uses $m_{j}$.

| W | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | x |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 |  | x |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  | x |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 |  |  |  | x |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  | x |  |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  | x |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  | x |  |  |  |  |  |  |  |  |  |
| 7 |  |  |  |  |  |  |  | x |  |  |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |  |  | x |  |  |  |  |  |  |  |
| 9 |  |  |  |  |  |  |  |  |  | x |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |  |  |  |  | x |  |  |  |  |  |
| 11 |  |  |  |  |  |  |  |  |  |  |  | x |  |  |  |  |
| 12 |  |  |  |  |  |  |  |  |  |  |  |  | x |  |  |  |
| 13 |  |  |  |  |  |  |  |  |  |  |  |  |  | x |  |  |
| 14 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | x |  |
| 15 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | x |
| 16 | x | x |  |  |  |  |  |  |  | x |  |  |  |  | x |  |
| 17 |  | x | x |  |  |  |  |  |  |  | x |  |  |  |  | x |
| 18 | x | x | x | x |  |  |  |  |  | x |  | x |  |  | x |  |
| 19 |  | x | x | x | x |  |  |  |  |  | x |  | x |  |  | x |
| 20 | x | x | x | x | x | x |  |  |  | x |  | x |  | x | x |  |
| 21 |  | x | x | x | x | x | x |  |  |  | x |  | x |  | x | x |
| 22 | x | x | x | x | x | x | x | x |  | x |  | x |  | x | x | x |

## B Conditions for collision

Table 4. The differences propagation for 20, 21, and 23-step collisions for SHA256. Notice that for each collision initial difference is introduced in different steps (steps 5,6,9 respectively).

| $\begin{gathered} 20 \\ \text { step } \end{gathered}$ | $\begin{gathered} 21 \\ \text { step } \end{gathered}$ | $\left\lvert\, \begin{gathered} 23 \\ \text { step } \end{gathered}\right.$ | $\Delta A$ | $\Delta B$ | $\Delta C$ | $\Delta D$ | $\Delta E$ | $\Delta F$ | $\Delta G$ | $\Delta H$ | $\Delta W$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 6 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 6 | 7 | 10 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | $\delta_{1}$ |
| 7 | 8 | 11 | 0 | 1 | 0 | 0 | -1 | 1 | 0 | 0 | $\delta_{2}$ |
| 8 | 9 | 12 | 0 | 0 | 1 | 0 | 0 | -1 | 1 | 0 | $\delta_{3}$ |
| 9 | 10 | 13 | 0 | 0 | 0 | 1 | 0 | 0 | -1 | 1 | 0 |
| 10 | 11 | 14 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | -1 | 0 |
| 11 | 12 | 15 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 12 | 13 | 16 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | $\delta_{5}$ |
| 13 | 14 | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 |
| 14 | 15 | 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 5. The values of the word differences in 20, 21, and 23 -step collisions for SHA-256. Notice that 23 -step semi-free start collision has a word difference in $\delta_{5}$. That is why its collision path is slightly different than the one used for 20 and 21-step collision.

|  | $\delta_{1}$ | $\delta_{2}$ | $\delta_{3}$ | $\delta_{5}$ |
| :---: | :---: | :---: | :---: | :---: |
| 20-step $-1-\Delta C h^{6}(1,0,0)-\Delta \Sigma_{1}\left(E_{6}\right)$ | $-\Delta \Sigma_{1}\left(E_{7}\right)-\Delta C h^{7}(-1,1,0)$ | $-\Delta C h^{8}(0,-1,1)$ | 0 |  |
| 21-step $-1-\Delta C h^{7}(1,0,0)-\Delta \Sigma_{1}\left(E_{7}\right)$ | $-\Delta \Sigma_{1}\left(E_{8}\right)-\Delta C h^{8}(-1,1,0)$ | $-\Delta C h^{9}(0,-1,1)$ | 0 |  |
| 23-step | -1 | $-\Delta \Sigma_{1}\left(E_{11}\right)-\Delta C h^{11}(-1,1,0)$ | $-\Delta C h^{12}(0,-1,1)$ | 1 |

Table 6. The additional conditions that have to hold in order to get a 20,21 , and 23 -step collisions for SHA-256.

| 20-step | $A_{4}=A_{5}=A_{7}=A_{8}$ | $E_{9}=E_{8}, E_{10}=-1, E_{10}^{\prime}=0$ | $\Delta C h^{9}(0,0,-1)=-1$ |
| :---: | :---: | :---: | :---: |
|  | $A_{6}=-1, A_{6}^{\prime}=0$ | $E_{11}=0, E_{12}=-1$ |  |
| 21-step | $A_{5}=A_{6}=A_{8}=A_{9}$ | $E_{10}=E_{9}, E_{11}=-1, E_{11}^{\prime}=0$ | $\Delta C h^{10}(0,0,-1)=-1$ |
|  | $A_{7}=-1, A_{7}^{\prime}=0$ | $E_{12}=0, E_{13}=-1$ | $\Delta \sigma_{1}(-1)+\delta_{3}=0$ |
| 23-step | $A_{8}=A_{6}=A_{9}=A_{10}$ | $E_{13}=E_{12}, E_{14}=-1, E_{14}^{\prime}=0$ | $\Delta C h^{13}(0,0,-1)=-1$ |
|  | $A_{10}=-1, A_{10}^{\prime}=0$ | $E_{15}=0, E_{16}=0 x f f f f f f f e$ | $\Delta \sigma_{1}(-1)+\delta_{3}=0$ |
|  |  | $E_{9}=E_{8}+1, E_{10}=-1, E_{10}^{\prime}=0$ | $\Delta \sigma_{1}(1)+\delta_{2}=0$ |

## C Collision Examples

Table 7. A 21-step collision for SHA-256.

|  | $0004024 f$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ae18 | 1d1 | 21 | ab551b5f | a48 | 00 | 19 | 000 |
|  |  | 00 | 00000000 | 00 | 0000000 | 2c51 |  | 7c6 |
|  | b238a344 | 1 d 11 dac 8 | 21d0617 | ab551 | a48e9a8 | 00 | 18ffffff | 00000000 |
|  |  |  |  |  |  |  |  |  |

Table 8. A 23 -step semi-free start collision for SHA-256.

|  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{0}$ |  |  | 00 | 0000000 | 00000000 | 00000000 | 0000000 |  |
|  | 0000 | a385 | 8 | 6e523f5 | 34 e 27 | 9740 | £36566 |  |
| $M_{0}^{\prime}$ |  |  |  | 0000000 | 000 | 0000000 | 000 |  |
|  | 00000000 | a385a0ec | dbcdde47 | 22e583f0 | 34 e 27 dbb | 97406f01 | f3656653 | cdc5da0 |
|  |  |  |  |  |  |  |  |  |

Table 9. A 25 -step semi-free start near collision with Hamming distance of 17 bits for SHA-256.

| $H_{0}$ | 8e204f9e | bca27aea | 42da63d7 | 00f2f219 | fd1db715 | 6389ae13 | c6f57538 | de4e655c |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{0}$ | c63714eb | 13d5fa9c | 00000000 | 00000000 | 00000000 | 00000000 | 00000000 | 00000 |
|  | 00000000 | d51b4dba | aeb6f738 | 61dce9b7 | 0ab5c01a | 83406f01 | df65666b | cdc5da |
| $M_{0}^{\prime}$ | c63714eb | 13d5fa9c | 00000000 | 00000000 | 00000000 | 00000000 | 00000000 | 00000000 |
|  | 00000000 | d51b4dbb | aeb6f737 | 71dd499a | 0ab5bf 1b | 83406f01 | df65666b | cdc5da00 |
| H | 2e2fcb73 | 8192d3a4 | f85b5a7d | 801c4583 | 9307e51c | cf57fb61 | 11c48b0d | 7131ccd |
| H | 6c478ef3 | 8192d3a5 | f85b5a7d | 801c4583 | 9127a49c | cf57fb62 | 11c48b0d | 7131 |


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