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Abstract

We consider an infinitely repeated Bertrand game, in which prices are publicly observed and each firm receives a privately observed, i.i.d. cost shock in each period. We focus on symmetric perfect public equilibria (SPPE), wherein any "punishments" are borne equally by all firms. We identify a tradeoff that is associated with collusive pricing schemes in which the price to be charged by each firm is strictly increasing in its cost level: such "fully sorting" schemes offer efficiency benefits, as they ensure that the lowest-cost firm makes the current sale, but they also imply an informational cost (distorted pricing and/or equilibrium-path price wars), since a higher-cost firm must be deterred from mimicking a lower-cost firm by charging a lower price. A rigid-pricing scheme, where a firm's collusive price is independent of its current cost position, sacrifices efficiency benefits but also diminishes the informational cost. For a wide range of settings, the optimal symmetric collusive scheme requires (i). the absence of equilibrium-path price wars and (ii). a rigid price. If firms are sufficiently impatient, however, the rigid-pricing scheme cannot be enforced, and the collusive price of lower-cost firms may be distorted downward, in order to diminish the incentive to cheat. When the model is modified to include i.i.d. public demand shocks, the downward pricing distortion that accompanies a firm's lower-cost realization may occur only when current demand is high.

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1. Introduction

In the standard model of collusion, symmetric firms interact in an infinitely repeated Bertrand game in which past prices are publicly observed. The standard model offers a number of insights, but it presumes an unchanging market environment. This is an important limitation, since the scope for testing a theory of collusion is greater when the theory offers predictions concerning the manner in which collusive prices vary with underlying market conditions.

This limitation is partially addressed in two celebrated extensions. Rotemberg and Saloner (1986) introduce publicly observed demand shocks that are i.i.d. over time.[1] When the demand shock is large, the incentive to cheat (undercut the collusive price) is acute, and collusion becomes more difficult to enforce. Rather than forego collusive activity altogether, firms then reduce the collusive price and thereby diminish the incentive to cheat. Thus, markups are countercyclical. Like the standard model, their model does not predict that actual "price wars" occur on the equilibrium path; rather, the success of collusion varies along the equilibrium path with the demand shocks that are encountered.

Following the seminal work of Stigler (1964), a second literature stresses that a firm may be unable to perfectly monitor the behavior of its rivals. Green and Porter (1984) explore this possibility in an infinitely repeated Cournot model. They assume that a firm cannot observe the output choices of rivals but that all firms observe a public signal (the market price) that is influenced both by output choices and an unobserved demand shock.[2] A colluding firm that witnesses a low market price then faces an inference problem, as it is unclear whether the low-price outcome arose as a consequence of a bad demand shock or a secret output expansion by a rival. The Green-Porter (1984) model thus represents collusion in the context of a repeated moral-hazard (hidden-action) model, and a central feature of their analysis is that wars occur along the equilibrium path following bad demand shocks.

In this paper, we propose a third extension of the standard collusion model. We consider an infinitely repeated Bertrand game, in which each firm is privately informed of its unit cost level in each period, where there is a continuum of possible costs and the cost realization is i.i.d. across firms and time. Current price selections (but not cost realizations) are publicly observed before the beginning of the next period. We thus represent collusion in the context of a repeated adverse-selection (hidden-information) model with publicly observed actions (prices).[3]

Our model is well-designed to contribute to a long-standing issue in Industrial Organization

[1]Bagwell and Staiger (1997) and Haltiwanger and Harrington (1991) consider further extensions.

[2]While the Green-Porter (1984) model is developed in the context of Cournot competition, the main insights can be captured in a repeated Bertrand setting, as Tirole (1988) shows. The Green-Porter (1984) model is further extended by Abreu, Pearce and Stacchetti (1986, 1990), Fudenberg, Levine and Maskin (1994) and Porter (1983).

[3]Our model is related to recent work that extends the Green-Porter (1984) model to allow for privately observed demand signals. Compte (1998) and Kandori and Matsushima (1998) suppose that firms publicly choose "messages" after privately observing their respective demand signals. In our setting, firms are privately informed as to their respective costs and the public action is a (payoff-relevant) price choice.

concerning the relationship between collusion and price rigidity in the presence of cost shocks. Empirical studies by Mills (1927), Means (1935) and Carlton (1986, 1989) conclude that prices are more rigid in concentrated industries, suggesting that collusion is associated with a greater tendency toward price rigidity. Similarly, Scherer (1980, pp. 184-93) summarizes a number of studies that find that collusion is often implemented with "rules of thumb," whereby the collusive price is set at some focal level, such as a fixed percentage markup above a public wholesale price, that is otherwise independent of the respective cost positions of member firms.[4] Finally, over the past several decades, antitrust enforcement has uncovered numerous price-fixing agreements in which firms coordinate on a particular price and enforce stable market shares over time. In these examples, colluding firms adjust price occasionally in response to changes in overall market conditions, but they sacrifice the efficiency advantages that could be gained by allowing a firm with a temporary cost advantage to serve a larger market share.[5]

At the same time, the Industrial Organization literature has not provided a satisfactory theory that links price rigidity with collusion. The best known theory is the "kinked demand curve" theory offered by Sweezy (1939) and Hall and Hitch (1939). As Scherer (1980) and Tirole (1988) discuss, however, this theory has important shortcomings.[6] Industrial Organization economists have thus gravitated toward the more informal view that price rigidity is appealing to collusive firms, because a rigid-price collusive scheme prevents mistrust and reduces the risk of a price war. Carlton (1989) explains:

> *"The property of the kinked demand curve theory that price is unresponsive to some cost fluctuations is preserved in most discussions of oligopoly theory whether or not based on the kinked demand curve. The reasoning is that in oligopolies prices fluctuate less in response to cost changes (especially small ones) than they would otherwise in order not to disturb existing oligopolistic discipline. Anytime a price change occurs in an oligopoly, there is a risk that a price war could break out. Hence, firms are reluctant to change price."* (Carlton, 1989, pp. 914-15).[7]

[4]Fixed markup rules are especially prevalent in oligopolies (Hall and Hitch (1939)) and are often used by "loose-knit" cartels (Helflebower (1955)). They are also commonly observed among colluding firms in retail industries or in sectors where the price of a critical input is publicly observed. For example, Safeway and Kroeger were accused of using fixed markups for dairy products (FTC Docket 7596, 1964), while European manufacturers of stainless steel used a standard markup formula called the "alloy surcharge" (European Commission, 1998).

[5]For example, a European cartel of cartonboard producers set stable market shares, but adjusted the fixed price every six months in response to changing demand conditions (European Commission, 1994a). Additional examples are discussed in Scherer (1980) and *Business Week* (1975, 1995).

[6]The kinked demand curve theory employs an *ad hoc* behavioral postulate, compresses a dynamic story into a static framework, and does not determine the collusive price. Maskin and Tirole (1988) offer an equilibrium interpretation of the kinked demand curve theory, in a model with alternating price choices, but their model does not allow for cost shocks. An alternative theory is that firms face "menu costs" when adjusting their prices. But in many examples, a firm's collusive price responds continuously to some variables (e.g., a public wholesale price) and yet does not vary with other variables (e.g., firm-specific cost positions), and so the menu-cost theory is also not fully satisfactory.

[7]For a similar view, see Scherer (1980, p. 180), who writes "*Most oligopolies...appear willing to forego the modest gains associated with micro-meter like adjustment of prices to fleeting changes in demand and costs in order to avoid the risk of more serious losses from poorly coordinated pricing policies and price warfare.*"

We develop here a rigorous evaluation of this informal reasoning. Focusing on the *private* cost fluctuations that firms experience, we explore the extent to which "mistrust" limits colluding firms' ability to respond to their respective cost positions. The costs of mistrust are formalized in terms of the price wars and pricing distortions that are required to dissuade firms from misrepresenting their private information.

We begin with a formal analysis of the static Bertrand game with inelastic demand and private cost information.[8] This game constitutes the stage game of our repeated-game model. In the unique Nash equilibrium, the symmetric pricing strategy is strictly increasing in the firm's cost level. An advantage of Nash pricing is that sales in the current period are allocated to the firm with the lowest cost. This is the *efficiency benefit* of a "fully sorting" (i.e., strictly increasing) pricing scheme. Of course, from the firms' perspective, Nash pricing also has an important limitation: sales are allocated at low prices.

We turn next to the repeated-game model and explore whether firms can then support better-than-Nash profits. We focus on the class of *symmetric perfect public equilibria (SPPE)*. An SPPE collusive scheme at a given point in time can be described by (i) a price for each cost type and (ii) an associated equilibrium continuation value for each vector of current prices, where the continuation value is symmetric across firms. In an SPPE, therefore, colluding firms move symmetrically through any cooperative or price-war phases.

We observe that a collusive scheme must satisfy two kinds of incentive constraints. First, for every firm and cost level, the short-term gain from cheating with an *off-schedule deviation* (i.e., with a price that is not assigned to any cost type and that thus represents a clear deviation) must be unattractive, in view of the (off-the-equilibrium-path) price war that such a deviation would imply. As is usual in repeated-game treatments of collusion, this constraint is sure to be met if firms are sufficiently patient. Second, the proposed conduct must also be such that no firm is ever attracted to an *on-schedule deviation*, whereby a firm of a given cost type misrepresents its private information and selects a price intended for a different cost type.

To characterize the optimal SPPE, we build on the dynamic-programming techniques put forth by Abreu, Pearce and Stacchetti (1986, 1990) and Fudenberg, Levine and Maskin (1994). We draw an analogy between our repeated hidden-information game and the static mechanism design literature, in which the on-schedule incentive constraint is analogous to the standard incentive-compatibility constraint, the off-schedule incentive constraint serves as a counterpart to the traditional participation constraint, and the continuation values play the role of "transfers." However, unlike a standard mechanism design problem in which transfers are unrestricted, the set of feasible continuation values is limited and endogenously determined. In particular, we may associate a price war with a transfer that is borne symmetrically by all firms.

[8] The Bertrand model assumes homogeneous goods, which is a common characteristic of collusive markets (Hay and Kelley (1974), Scherer (1980, p. 203)). Collusion is also often associated with inelastic demand (Eckbo (1976)).

We break our analysis of optimal SPPE into two parts. We suppose first that firms are patient, so that the off-schedule constraint is met. The on-schedule constraint then captures the *informational costs* of collusion that confront privately informed firms. The central problem is that the scheme must be constructed so that a higher-cost firm does not have an incentive to misrepresent its costs as lower, thereby securing for itself a lower price and a higher expected market share. In an SPPE, the informational costs of collusion may be manifested in two ways. First, the prices of lower-cost firms may be distorted to sub-monopoly levels. This is a potentially effective means of eliciting truthful cost information, since higher-cost firms find lower prices less appealing. Second, following the selection of lower prices, the collusive scheme may sometimes call for a future equilibrium-path price war. The current-period benefit of a lower price then may be of sufficient magnitude to compensate for the future cost of a price war, only if the firm truly has lower costs in the current period.

A rich array of collusive schemes fit within the SPPE category. One possibility is that firms incur the informational costs of collusion purely in terms of distorted pricing. An example is the *Nash-pricing scheme*, in which firms repeatedly play the Nash equilibrium of the static game. Another possibility is that firms initially achieve full sorting and adopt higher–than-Nash prices. In this case, some of the informational costs of collusion must be reflected in the future cost of a price war: such a scheme satisfies the on-schedule constraint only if equilibrium-path wars sometimes follow the selection of lower-cost prices. A further possibility is that firms may neutralize the informational costs of collusion altogether, by adopting a *rigid-pricing scheme*, in which each firm selects the same price in each period, whatever its current cost position. The downside of the rigid-pricing scheme is that it sacrifices efficiency benefits: one firm may have lower costs than its rivals, and yet the firms share the market. These schemes highlight the central tradeoff between efficiency benefits and informational costs that colluding firms must reconcile. More generally, collusive pricing schemes may be strictly increasing over some intervals of costs and rigid over other regions, with wars following some pricing realizations.

Our first main finding is that firms fare poorly under any SPPE collusive scheme that insists upon full sorting. In fact, considering the entire set of fully sorting SPPE, we find that firms can do no better than the Nash-pricing scheme. We next consider the full class of SPPE collusion schemes and report a second main finding: if firms are sufficiently patient, then an optimal SPPE collusive scheme can be achieved without recourse to equilibrium-path price wars (i.e., with stationary strategies). This finding contrasts interestingly with the predictions of the Green-Porter (1984) model.

Armed with these findings, we next add some additional structure and provide a characterization of the optimal SPPE collusive scheme. When firms are patient and the distribution of cost types is log concave, we establish a third main finding: optimal SPPE collusion is characterized by a rigid-pricing scheme, in which firms select the same price (namely, the reservation price of consumers) in each period, whatever their cost levels. We thus offer an equilibrium interpretation of the association between price rigidity and collusion described above.

We then turn to the second part of our analysis and consider impatient firms. We show that impatience creates an additional disadvantage to price wars: a scheme with high prices today sustained by wars in the future makes a deviation especially profitable today, while simultaneously reducing the value of cooperation in the future. Our second (no-wars) finding therefore continues to hold when firms are impatient. Next, we observe that the off-schedule constraint is particularly demanding for lower-cost types. Intuitively, when a firm draws a lower-cost type, the temptation to cheat and undercut the assigned price is severe, since the resulting market-share gain is then especially appealing. For impatient firms, a collusive scheme thus must ensure that lower-cost types receive sufficient market share and select sufficiently low prices in equilibrium, so that the gains from cheating are not too great.

This logic is reminiscent of the argument made by Rotemberg and Saloner (1986), although here it is private cost shocks (as opposed to public demand shocks) that necessitate modification of the collusive scheme. We confirm this logic with our fourth main finding: if firms are not sufficiently patient to enforce the rigid-pricing scheme, they may still support a partially rigid collusive scheme, in which the price of lower-cost types is reduced in order to mitigate the incentive to cheat. This finding suggests that symmetric collusion between impatient firms may be marked by occasional (and perhaps substantial) price reductions by individual firms. These departures occur when a firm receives a favorable cost shock, and they represent a permitted "escape clause" (i.e., an opportunity to cut prices and increase market share without triggering retaliation) within the collusive scheme. More generally, we establish conditions for impatient firms under which, if better-than-Nash profits can be achieved, then optimal SPPE collusion is characterized by a stationary pricing scheme in which prices are rigid over intervals of costs (i.e., the optimal pricing scheme is a weakly increasing step function).

To further develop the relationship between our theory and that of Rotemberg and Saloner (1986), we next extend our model to include public i.i.d. demand shocks. The off-schedule constraint is then most difficult to satisfy when market demand is high and a firm's cost shock is low. We thus offer a fifth main finding: in an extended model with public i.i.d. demand shocks, if firms are not sufficiently patient to enforce the rigid-pricing scheme, optimal SPPE collusion may be characterized by a stationary pricing scheme, in which an individual firm charges a lower price in high-public-demand and low-private-cost states. Rotemberg and Saloner's (1986) prediction of countercylical pricing is thus robust to private cost information. Our model has the further prediction that prices are more variable when today's demand is high.

Throughout, we restrict attention to symmetric schemes. This restriction is important. Asymmetric schemes allow one firm to enjoy a more profitable continuation value than another. Such schemes thus facilitate transfers *from* one firm *to* another. This can be accomplished if the scheme assigns history-dependent pricing behavior, so that a firm with large market share today relinquishes some market share to other firms in the future. Firms are then able to enjoy efficiency benefits with lower informational costs, and the rigid-pricing scheme is no longer optimal. In our working paper (Athey, Bagwell and Sanchirico (1998)), we present an asymmetric scheme that improves upon the rigid-pricing scheme. For a two-type model, Athey and Bagwell (forthcoming) construct an asymmetric perfect public equilibrium (APPE) that delivers first-best profits when the patience of firms exceeds a critical (finite) level.[9]

We focus on SPPE for four reasons. First, under many conditions the optimal SPPE is stationary. While we do not propose a theory of how firms coordinate upon an equilibrium, the rigid-price SPPE (and more generally, stationary symmetric equilibria) are appealingly simple. In fact, rigid-pricing schemes are widely used.[10] By contrast, APPE improve upon SPPE only for asymmetric equilibria that are nonstationary. Such schemes are more sophisticated, and may be most plausible when a small number of firms interact frequently and communicate explicitly. Second, the informal literature highlights firms' fear of industry-wide breakdowns in collusion, and it is thus interesting to analyze SPPE, which isolate this consideration. Third, SPPE are the only alternative if firms can observe the prices offered in the market but cannot observe (or infer) the identities of the firms who offer these prices. This occurs, e.g., in procurement auctions with more than two bidders, if the winning bid - but not the name of the winner - is announced. Finally, as we discuss further in the Conclusion, the methods developed here for SPPE provide a foundation as well for the analysis of repeated interactions in which a single player is privately informed.

Our findings are related to those developed by McAfee and McMillan (1992), in their analysis of bidding rings. They describe evidence that fixed-price schemes (i.e., "identical bidding") are widely used. In a static model, they show that a fixed price is the optimal strategy for bidding cartels in first-price auctions for a single object, when the cartels are "weak" (i.e., firms are unable to make transfers). Our analysis may be understood in terms of a procurement auction. In fact, in our analysis of the optimal SPPE, we generalize the weak-cartel model, since the static mechanism we analyze is directly derived from a repeated game and allows for a restricted class of transfers (corresponding to symmetric price wars). Our rigid-pricing finding thus provides additional theoretical support for the practice of identical bidding. We also extend the analysis to incorporate impatient firms.

We describe the static and repeated games in Sections 2 and 3, respectively. The latter is related to the mechanism-design approach in Section 4. We present our findings for SPPE

[9] For a family of repeated private-information games, Fudenberg, Levine and Maskin (1994) show that first-best payoffs can be reached in the limit as players become infinitely patient.

[10] There are many examples where a large number of firms successfully collude using a rigid-pricing scheme. In some cases, professional organizations coordinate prices for hundreds of individual service providers; examples include interpreters and physician groups (FTC Dockets C-3430 and 9270); industry associations may play the same role. In Europe, fixed-price rules were used by the cartonboard and steel beam cartels, each with about twenty firms (European Commission, 1994a, 1994b). In contrast, the cases we uncovered where colluding firms used nonstationary market-share allocation schemes typically involved a small number of well-organized conspirators; see Athey and Bagwell (forthcoming) for examples.

among patient firms in Section 5, and Section 6 considers impatient firms. Both of these sections close with a brief discussion of an extended model with downward-sloping demand. This extension is formally analyzed in our discussion paper (Athey, Bagwell and Sanchirico (1998)). Section 7 concludes.

2. The Static Game

We begin with a static game of Bertrand competition in which firms possess private information. This game illustrates the immediate tradeoffs that confront firms in determining their pricing policies and serves as a foundation on which our subsequent dynamic analysis builds.

We posit n *ex ante* identical firms that engage in Bertrand competition for sales in a homogenous-good market. Following Spulber (1995), we modify the standard Bertrand model with the assumption that each firm is privately informed as to its unit cost level. Firm i's "type" θ_i is drawn in an i.i.d. fashion from the support $[\underline{\theta}, \overline{\theta}]$ according to the commonly known distribution function $F(\theta)$. We assume that the corresponding density $f(\theta) \equiv F'(\theta)$ is strictly positive on $[\underline{\theta}, \overline{\theta}]$. After the firms learn their respective cost types, they simultaneously choose prices. Let $\rho_i \in \mathbb{R}_+$ denote the price chosen by firm i, with $\boldsymbol{\rho} \equiv (\rho_1, ..., \rho_n)$ then representing the associated price profile. We assume a unit mass of identical consumers, each of whom has an inelastic demand for one unit up to some reservation price r, where $r \geq \overline{\theta}$.

A price strategy for firm i is a function $p_i\left(\theta_i\right)$ mapping from the set of cost types, $[\underline{\theta}, \overline{\theta}]$, to the set of possible prices, $\mathbb{R}_+$. The function p_i is assumed continuously differentiable, except perhaps at a finite number of points (so as to allow for schedules with jumps). A price strategy profile is thus a vector $\mathbf{p}(\boldsymbol{\theta}) \equiv \left(p_i\left(\theta_i\right), \mathbf{p}_{-i}\left(\boldsymbol{\theta}_{-i}\right)\right)$, where $\boldsymbol{\theta} \equiv (\theta_i,\ \boldsymbol{\theta}_{-i})$ is the vector of cost types and $\mathbf{p}_{-i}\left(\boldsymbol{\theta}_{-i}\right)$ is the profile of rival price strategies. Each firm chooses its price strategy with the goal of maximizing its expected profit, given its cost type. To represent a firm's expected profit, we require two further definitions. First, we define $\pi\left(\rho, \theta\right) \equiv \rho - \theta$ as the profit that a firm receives when it sets the price ρ and has cost type θ and "wins" the entire unit mass of consumers. Second, we specify a Bertrand market-share-allocation function, $m_i\left(\boldsymbol{\rho}\right)$, that indicates firm i*'s* market share when the vector of realized prices is $\boldsymbol{\rho}$. This function allocates consumers evenly among firms that tie for the lowest price in the market.[11]

We may now represent firm i's *interim* profit, which is the expected profit for firm i when it has cost type θ_i, selects the price ρ_i and anticipates that rival prices will be determined by the rival pricing strategy profile, $\mathbf{p}_{-i}\left(\boldsymbol{\theta}_{-i}\right)$. With $\bar{m}\left(\rho_i; \mathbf{p}_{-i}\right) \equiv E_{\boldsymbol{\theta}_{-i}}[m_i\left(\rho_i, \mathbf{p}_{-i}\left(\boldsymbol{\theta}_{-i}\right)\right)]$, firm i's interim profit function may be written as

$$\bar{\pi}(\rho_i, \theta_i; \mathbf{p}_{-i}) \equiv \pi\left(\rho_i, \theta_i\right) \bar{m}\left(\rho_i; \mathbf{p}_{-i}\right). \tag{2.1}$$

When firms adopt a symmetric pricing strategy, $p(\cdot)$, we use the notation $\bar{m}\left(\rho_i; p\right)$ and $\bar{\pi}(\rho_i, \theta_i; p)$.

[11]More precisely, $m_i\left(\boldsymbol{\rho}\right) = 1$ if $\rho_i < \min_{j \neq i} \rho_j$, $m_i\left(\boldsymbol{\rho}\right) = 0$ if $\rho_i > \min_{j \neq i} \rho_j$, and $m_i(\boldsymbol{\rho}) = 1/L$ if there are $L - 1$ other firms that tie firm i for the lowest price.

We now describe the essential tradeoff that confronts a firm when it sets its price. As illustrated in (2.1), when a firm of a given type considers whether to lower its price, it must weigh the effect of an increase in the chance of winning (through $\bar{m}_i(\rho_i; p)$) against the direct effect of the price reduction on profit-if-win (through $\pi(\rho_i, \theta_i)$). An important feature of the model is that different types feel differently about this tradeoff. In particular, the interim profit function satisfies a *single-crossing property*: lower types find the expected-market-share increase that accompanies a price reduction relatively more appealing than do higher types, since lower types have lower total costs and thus higher profit-if-win. The single crossing property implies that higher-cost firms always select higher prices (i.e, $p_i(\theta_i)$ is non-decreasing).

The stage game may be analyzed using standard techniques from the auction literature.[12]

Proposition 1. *The static game has a unique Nash equilibrium, which 1) is symmetric: $p_i \equiv p^e$, $\forall i$; 2) is continuously differentiable and strictly increasing over $\theta \in (\underline{\theta}, \overline{\theta})$; 3) is below the monopoly price: $p^e(\theta) < r$, $\forall \theta < \overline{\theta}$ and; 4) yields positive interim profit for all types but the highest θ, who never wins and whose price $p^e(\overline{\theta}) = \overline{\theta}$ would yield zero profit even if it did.*

Notice that the symmetric equilibrium pricing strategy p^e is continuous and strictly increasing. Further, the price always falls at or below $\bar{\theta}$, no matter how high is r.

3. The Repeated Game

In this section, we define the repeated game. We also present a "Factored Program" and establish a relationship between solutions to this program and optimal SPPE.

3.1. The Model

Imagine that firms meet period after period to play the stage game described in the previous section, each with the objective of maximizing its expected discounted stream of profit. Assume further that, upon entering a period of play, a firm observes only the history of: (i) its own cost draws, (ii) its own pricing schedules, and (iii) the realized prices of its rivals. Thus, we assume that a firm does not observe rival types or rival price schedules.

Formally, we describe the repeated game in the following terms. A full path of play is an infinite sequence $\{\boldsymbol{\theta}^t, \mathbf{p}^t\}$, with a given pair in the sequence representing a vector of types and price schedules at date t. The infinite sequence implies a public history of realized price vectors $\{\boldsymbol{\rho}^t\}$, and pathwise payoffs for firm i may be thus defined as

$$u_i(\{\boldsymbol{\theta}^t, \mathbf{p}^t\}) = \Sigma_{t=1}^{\infty} \delta^{t-1} \pi\left(\rho_i^t, \theta_i^t\right) m_i\left(\boldsymbol{\rho}^t\right).$$

At the close of period τ, firm i possesses an information set, which may be written as $h_i = \{\theta_i^t, p_i^t, \boldsymbol{\rho}_{-i}^t\}_{t=1}^{\tau}$. (The null history is the firm's information set at the beginning of the first

[12] See also Spulber (1995), who establishes Proposition 1 for general demand functions.

period.) A (pure) strategy for firm i, $s_i(h_i)(\theta_i)$, associates a price schedule with each information set h_i. Each strategy profile $\mathbf{s} = (s_1, ..., s_n)$ induces a probability distribution over play paths $\{\boldsymbol{\theta}^t, \mathbf{p}^t\}$ in the usual manner. The expected discounted payoff from $\mathbf{s}$ is thus the expectation $\bar{u}_i(\mathbf{s}) = E[u_i(\{\boldsymbol{\theta}^t, \mathbf{p}^t\})]$ taken with respect to this measure on play paths.

3.2. A Dynamic Programming Approach

Under our assumptions, firm types are i.i.d. across time (and firms), and so the repeated game has a recursive structure. It is therefore natural to follow Fudenberg, Levine and Maskin (1994) [FLM] and employ a recursive solution concept; namely, we focus upon sequential equilibria in which each firm's strategy conditions only upon the publicly observed history of realized prices. Such strategies are called *public strategies* and such sequential equilibria are called *perfect public equilibria* (PPE). A public strategy may thus be abbreviated as a map from finite public histories $\{\boldsymbol{\rho}^t\}_{t=1}^{\tau}$ to price schedules. Using standard arguments (see FLM), it is then straightforward to show that the continuation of any PPE after any history is itself a PPE in the full game and yields payoffs in the continuation equal to what would have been obtained had the strategy profile been used from the start.

We further restrict attention to *symmetric perfect public equilibrium (SPPE)*, whereby following every public history, firms adopt symmetric price schedules: $s_i(\boldsymbol{\rho}^1, ..., \boldsymbol{\rho}^\tau) = s_j(\boldsymbol{\rho}^1, ..., \boldsymbol{\rho}^\tau)$, $\forall i, j, \tau,\ \boldsymbol{\rho}^1, ..., \boldsymbol{\rho}^\tau$. Symmetry means that all firms suffer future punishments and rewards together on an industry-wide basis. Symmetric equilibria thus capture the "fear of breakdown" that collusive firms may experience, as discussed in the Introduction.[13]

Drawing on the work of Abreu, Pearce and Stacchetti (1986, 1990) [APS], we apply the tools of dynamic programming to this settting. Let $\mathcal{V}_s \in \mathbb{R}$ denote the set of SPPE continuation values and write $\underline{\mathcal{V}}_s \equiv \inf \mathcal{V}_s$ and $\overline{\mathcal{V}}_s \equiv \sup \mathcal{V}_s$. Note, initially, that with a continuum of possible pricing strategies there is no *a priori* basis from which to argue that either $\overline{\mathcal{V}}_s \in \mathcal{V}_s$ or $\underline{\mathcal{V}}_s \in \mathcal{V}_s$;[14] if $\overline{\mathcal{V}}_s \in \mathcal{V}_s$, then we say that $\overline{\mathcal{V}}_s$ is an *optimal* SPPE value. Following APS, any symmetric public strategy profile $\mathbf{s} = (s, ..., s)$ can be *factored* into a first-period price schedule p and a *continuation payoff function* $v : \mathbb{R}^n_+ \to \mathbb{R}$. The continuation payoff function describes the repeated-game payoff $v(\boldsymbol{\rho})$ enjoyed by all firms from the perspective of period 2 onward after each first-period price realization $\boldsymbol{\rho} = (\rho_1, ...\rho_n) \in \mathbb{R}^n_+$. We define $\bar{v}(\rho_i; \mathbf{p}_{-i}) \equiv E_{\boldsymbol{\theta}_{-i}}[v(\rho_i, \mathbf{p}_{-i}(\boldsymbol{\theta}_{-i}))]$ as the expected continuation payoff when a firm selects ρ_i and expects other firms to price according to $\mathbf{p}_{-i}$. In view of our symmetry restriction, we may simply write $\bar{v}(\rho_i; p)$, and similarly each firm's expected payoff from $\mathbf{s}$ can be written as $E_{\theta_i}[\bar{\pi}(p(\theta_i), \theta_i; p) + \delta \bar{v}(p(\theta_i); p)]$.

[13]Despite its clear restrictiveness, a rich array of strategy profiles fit under the symmetric heading. Indeed, any amount of history dependence is allowed, and so price wars can be triggered by any number of periods of "bad behavior." For example, the law of large numbers may be utilized, in order to support "review strategies" of the type examined by Radner (1981). Likewise, price wars can vary in severity and length and thus may be tailored to the "fit the crime."

[14]We could attempt to prove compactness of the set in general terms. Instead, we establish compactness in the process of characterizing best and worst SPPE values.

We now consider the Factored Program, in which we choose factorizations directly in order to maximize a firm's expected payoff, subject to (i). the feasibility constraint that the continuation payoff rests always in the SPPE value set and (ii). the incentive constraint that a firm cannot gain by deviating to an alternative pricing schedule (given the continuation payoff function and under the assumption that other firms follow the pricing schedule):

The Factored Program: Choose price schedule p and continuation payoff function v to maximize

$$E_{\theta_i}[\bar{\pi}(p(\theta_i), \theta_i; p) + \delta\bar{v}(p(\theta_i); p)]$$

$$\text{subject to: } \forall \boldsymbol{\rho} \in \mathbb{R}^n_+,\ v(\boldsymbol{\rho}) \in \mathcal{V}_s, \text{ and}$$

$$\forall \tilde{p},\ E_{\theta_i}[\bar{\pi}(p(\theta_i), \theta_i; p) + \delta\bar{v}(p(\theta_i); p)] \geq E_{\theta_i}[\bar{\pi}(\tilde{p}(\theta_i), \theta_i; p) + \delta\bar{v}(\tilde{p}(\theta_i); p)].$$

Lemma 1. *Consider any symmetric public strategy profile* $\mathbf{s}^* = (s^*, ..., s^*)$ *with the corresponding factorization* (p^*, v^*)*. Then,* $\mathbf{s}^*$ *is an optimal SPPE if and only if* (p^*, v^*) *solves the Factored Program.*

This lemma, standard in the literature, establishes that we may characterize the set of optimal SPPE by solving the Factored Program.[15]

It is also possible to analyze the set of equilibria. If firms can randomize over continuation equilibria (e.g., using a public randomization device), then the set of SPPE values is convex and is thus fully characterized when the best and worst SPPE are found, where the worst equilibrium value is attained by minimizing rather than maximizing the objective in the Factored Program. In Section 6, we analyze the worst SPPE; until then, we focus on optimal SPPE.

3.3. The Interim Program for Games with Private Information

We next reformulate the Factored Program so that it can be analyzed using existing tools from the (static) mechanism design literature. We begin by observing that a SPPE in a repeated game with private information must be immune to two kinds of current-period deviations.

A firm deviates "off-schedule" when it chooses a price not specified for *any* cost realization (i.e., a price not in the range of p). When a firm prices in this manner, it has unambiguously deviated; consequently, if the collusive scheme prescribes a punishment (i.e., a continuation-value reduction) following such a deviation, and if the prospect of such a punishment deters a firm from undertaking the off-schedule deviation (i.e., if firms are sufficiently patient), then that punishment will never actually occur (i.e., it is off the equilibrium path). Thus, the firms can relax this constraint, without directly affecting equilibrium-path profits, by using the worst available punishment as a threat.

[15] The result follows from the fact that a strategy profile is an SPPE if and only if its factorization satisfies the Factored Problem's two constraints. We refer the reader to our working paper (Athey, Bagwell, and Sanchirico (1998)) or APS for more details.

By contrast, a firm deviates "on-schedule" when it chooses a price that is assigned under p to *some* cost level, but not its own. For example, a firm may be tempted to choose the lower price assigned to a lower cost realization in order to increase its chances of winning the market. Importantly, an on-schedule deviation is not observable, as a deviation, to rival players: a rival can not be sure that the deviating firm was not truly of the cost type that it is imitating. Thus, firms may consider a collusive scheme that imposes a punishment when low prices are chosen, as this would serve to prevent on-schedule deviations by higher-cost types. But such a punishment would be costly, since it would occur along the equilibrium path of play, whenever firms actually realized low costs.

With this distinction at hand, we take the constraints of the Factored Program, put them in interim form and parse them into two groups, and rewrite this program as:

The Interim Program: Choose price schedule p and continuation payoff function v to maximize

$$E_{\theta_i}[\bar{\pi}(p(\theta_i), \theta_i; p) + \delta \bar{v}(p(\theta_i); p)]$$

subject to:

Off-Schedule Constraints: $\forall \rho' \notin p([\underline{\theta}, \overline{\theta}])$,

(IC-off1) $\forall \boldsymbol{\theta}_{-i}$, $\qquad v(\rho', \mathbf{p}_{-i}(\boldsymbol{\theta}_{-i})) \in \mathcal{V}_s$

(IC-off2) $\forall \theta_i$, $\qquad \bar{\pi}(p(\theta_i), \theta_i; p) + \delta \bar{v}(p(\theta_i); p) \geq \bar{\pi}(\rho', \theta_i; p) + \delta \bar{v}(\rho'; p)$

On-Schedule Constraints: $\forall \hat{\theta}_i$,

(IC-on1) $\forall \boldsymbol{\theta}_{-i}$, $\qquad v(p(\hat{\theta}_i), \mathbf{p}_{-i}(\boldsymbol{\theta}_{-i})) \in \mathcal{V}_s$

(IC-on2) $\forall \theta_i$, $\qquad \bar{\pi}(p(\theta_i), \theta_i; p) + \delta \bar{v}(p(\theta_i); p) \geq \bar{\pi}(p(\hat{\theta}_i), \theta_i; p) + \delta \bar{v}(p(\hat{\theta}_i); p)$.

Notice how the on-schedule constraints are written in "direct" form: for given p and v, (IC-on2) requires that a firm with type θ_i does better by "announcing" that its type is θ_i than by announcing some other type, $\hat{\theta}_i$, when other firms are presumed to announce truthfully. This suggests that (IC-on2) may correspond to a "truth-telling" constraint in an appropriate mechanism design formulation.

4. Collusion Among Patient Firms and Mechanism Design

In this section, we build on this suggestion, showing that when firms are patient, the Interim Program can be relaxed to yield a new program that we call the Mechanism Design Program. We also use existing tools from the mechanism design literature to begin our characterization of the optimal SPPE.

4.1. The Mechanism Design Program

Our general approach has two steps. First, we relax the Interim Program, by dropping off-schedule constraints and allowing continuation payoff functions beyond those that are actually feasible in the repeated game. With some notational adjustment, we then arrive at the Mechanism Design Program. Second, we provide conditions under which the solution in the relaxed setting corresponds to a factorization that continues to satisfy all of the constraints of the Interim Program. In this way, we identify conditions under which optimal SPPE may be characterized by solving the Mechanism Design Program. The Mechanism Design Program is useful because we can apply existing tools to it directly; for example, below we make repeated use of the revenue equivalence theorem.

Our first step is to relax the constraints. In particular, we consider the Relaxed Interim Program, which is the same as the Interim Program, except that (i). the off-schedule constraints are ignored and (ii). (IC-on1) is replaced with a relaxed constraint:

$$\bar{v}(p(\hat{\theta}_i);p) \leq \overline{\mathcal{V}}_s. \tag{IC-on1$'$}$$

The first relaxation is without loss of generality if firms are sufficiently patient, since then off-schedule deviations are anyway not tempting. To appreciate the second relaxation, we recall that under (IC-on1), for every on-schedule vector of prices, the continuation value is drawn from the SPPE set, $\mathcal{V}_s$; by constrast, (IC-on1$'$) requires only that firm i's *expected* continuation value does not exceed the *supremum* of the SPPE set, $\overline{\mathcal{V}}_s$.[16] Clearly, any (p,v) that satisfies the constraints of the Interim Program satisfies the constraints of the Relaxed Interim Program.

We next use direct-form notation and define the Mechanism Design Program. Let $\Pi(\hat{\theta}, \theta; p) \equiv \bar{\pi}(p(\hat{\theta}), \theta; p)$ denote the current-period profit that a firm of type θ would expect were it to announce that its type is $\hat{\theta}$. We define as well a general "transfer" or "punishment" function, $T(\hat{\theta})$, which a firm expects to incur when it announces $\hat{\theta}$.

The Mechanism Design Program: Choose price schedule p and a punishment function T to maximize

$$E[\Pi(\theta, \theta; p) - T(\theta)]$$

$$\text{subject to : For all } \theta,\ T(\theta) \geq 0;$$

$$\text{(IC-onM) } \forall \hat{\theta}, \theta,\ \Pi(\theta, \theta; p) - T(\theta) \geq \Pi(\hat{\theta}, \theta; p) - T(\hat{\theta}).$$

Suppose (p, v) satisfies the constraints of the Relaxed Interim Program. We may then translate (p, v) into (p, T), according to $T(\hat{\theta}) \equiv \delta[\overline{\mathcal{V}}_s - \bar{v}(p(\hat{\theta}); p)]$. It is direct that (p, T) satisfies the constraints of the Mechanism Design Program; further, using this translation, the objectives of the Mechanism Design and Relaxed Interim Programs rank (p, v) pairings in the same order. The Mechanism Design Program is thus a relaxed version of the Relaxed Interim Program.

The meaning of T warrants emphasis. For a given SPPE, if a firm that announces $\hat{\theta}$ expects a continuation value below the supremum of the SPPE set (i.e., if $\overline{\mathcal{V}}_s > \bar{v}(p(\hat{\theta}); p)$), then we

[16] Note particularly that (IC-on1$'$) allows $v \equiv \overline{\mathcal{V}}_s$, even though we as yet have no assurance that $\overline{\mathcal{V}}_s \in \mathcal{V}_s$.

may interpret the SPPE as specifying (in expectation) a "war." There is then "no war," if the expected continuation value equals the supremum of the SPPE set (i.e., if $\overline{\mathcal{V}}_s = \bar{v}(p(\hat{\theta}); p)$). Using the translation $T(\hat{\theta}) \equiv \delta[\overline{\mathcal{V}}_s - \bar{v}(p(\hat{\theta}); p)]$, we thus may associate $T(\hat{\theta}) > 0$ ($T(\hat{\theta}) = 0$) with a future that follows a firm's announcement of $\hat{\theta}$ and in which there is a war (no war).

We come now to the second step in our approach, where we provide the conditions under which a solution (p^*, T^*) to the Mechanism Design Program can be translated back into a factorization (p^*, v^*) that satisfies all of the constraints of the Interim Program. The following proposition identifies an important set of conditions of this kind:[17]

Proposition 2. *(***Stationarity***) Suppose (p^*, T^*) solves the Mechanism Design Program and $T^* \equiv 0$. Then $\exists \hat{\delta} \in (0,1)$ such that, for all $\delta \geq \hat{\delta}$, there exists an optimal SPPE which is stationary, wherein firms adopt p^* after all equilibrium-path histories, and p^* solves the following program: maximize $E\Pi(\theta, \theta; p)$ subject to $\forall \hat{\theta}, \theta$, $\Pi(\theta, \theta; p) \geq \Pi(\hat{\theta}, \theta; p)$.*

To establish this result, we first explore the implications of the condition that $(p^*, T^* \equiv 0)$ is optimal in the relaxed environment. Using our translation above, if $(p^*, T^* \equiv 0)$ solves the Mechanism Design Program, then $(p^*, v^* \equiv \overline{\mathcal{V}}_s)$ is a solution to the Relaxed Interim Program. In turn, this implies that $(p^*, v^* \equiv \overline{\mathcal{V}}_s)$ is (weakly) superior to any SPPE factorization. We may therefore draw the following conclusion: if $(p^*, T^* \equiv 0)$ solves the Mechanism Design Program, then $E[\Pi(\theta, \theta; p^*) + \delta\overline{\mathcal{V}}_s] \geq \overline{\mathcal{V}}_s$.

Second, we explore the implications of the following observation: when firms are sufficiently patient and $(p^*, T^* \equiv 0)$ solves the Mechanism Design Program, the repeated play of p^* (coupled with appropriate off-schedule punishments) *is* an SPPE. It is straightforward that this pattern of play satisfies the on-schedule incentive constraint: since p^* satisfies (IC-onM) when $T^* \equiv 0$, each firm will follow p^* when future play does not vary with the on-schedule price. Likewise, we note that the repeated play of the static Nash equilibrium p^e is always an equilibrium of the repeated game; therefore, when firms are sufficiently patient, the threat of Nash reversion is sufficient to deter firms from abandoning p^* with an off-schedule deviation.[18] The observation is thus established. Recalling the definition of $\overline{\mathcal{V}}_s$, we may therefore draw the following conclusion: if firms are sufficiently patient and if $(p^*, T^* \equiv 0)$ solves the Mechanism Design Program, then $E[\Pi(\theta, \theta; p^*)]/(1-\delta) \leq \overline{\mathcal{V}}_s$.

Combining our two inequalities, we obtain the desired result: $\overline{\mathcal{V}}_s = E[\Pi(\theta, \theta; p^*)]/(1-\delta)$. In words, if the Mechanism Design Program is solved with $(p^*, T^* \equiv 0)$, and if firms are sufficiently patient, then an optimal SPPE is easily characterized: firms adopt the pricing schedule p^* in each period, where p^* is the solution to the static program stated in Proposition 2.

[17] In our working paper (Athey, Bagwell and Sanchirico (1998)), we show that the general approach also extends to cases where T^* can be strictly positive. $T^* > 0$ may be the unique solution when demand is downward-sloping, but not, as we will show, for inelastic demand.

[18] We establish below in Proposition 5(ii) that for sufficiently patient firms there always exists a stationary SPPE that gives higher per-period profits than does the static Nash equilibrium.

The off-schedule constraints play a subtle but critical role. As noted, if $(p^*, T^* \equiv 0)$ solves the Mechanism Design Program, then it must satisfy (IC-onM). This means that p^* must satisfy the on-schedule incentive constraints, period-by-period. One such pricing scheme is the Nash pricing scheme, p^e. But there are many others. For example, the on-schedule incentive constraints are trivially satisfied, period-by-period, when firms use a rigid-pricing scheme. To map the solution of the Mechanism Design Program over to a stationary SPPE of the repeated game, however, something more is required: it must also be true that no firm would ever gain from an off-schedule deviation. This is assured if firms are sufficiently patient.

It is now possible to preview the analysis that follows. In Section 4.2, we characterize the set of (p, T) that satisfy (IC-onM). This puts us in position to solve the Mechanism Design Program, using tools standard in the mechanism design literature. In Section 5, we establish that there is always a solution in which $T^* \equiv 0$. For patient firms, Proposition 2 then implies that an optimal SPPE is characterized by the stationary adoption of the the accompanying pricing schedule, p^*. Under the assumption that the distribution function $F(\theta)$ is log-concave, we find that the optimal pricing schedule takes a simple form: $p^*(\theta) \equiv r$. We thus report conditions under which for sufficiently patient firms the optimal SPPE is stationary and requires all firms to charge the same price r, regardless of their costs. This rigid-pricing scheme is supported by the threat that if any other price is observed, the firms will revert to the worst SPPE, which delivers continuation value $\underline{\mathcal{V}}_s$. When $F(\theta)$ is log-concave, we show in Section 6 that the worst SPPE is attained through Nash reversion (i.e., the Nash-pricing scheme is used in each peroid). We also establish there several additional predictions that arise when firms are less patient.

4.2. Consequences of On-Schedule Incentive Compatibility

We begin our analysis of the Mechanism Design Program by characterizing the implications of the on-schedule constraint (IC-onM). We do this in the following lemma (where we use the notation $\Pi_\theta(\theta, \theta; p) = \frac{\partial}{\partial\theta}\Pi(\hat{\theta}, \theta; p)|_{\hat{\theta}=\theta}$):

Lemma 2. (Constraint Reduction) *(p, T) satisfies (IC-onM) if and only if (p, T) also satisfies:*
(i). $p(\theta)$ is weakly increasing, and
(ii). $\Pi(\theta, \theta; p) - T(\theta) = \Pi(\overline{\theta}, \overline{\theta}; p) - T(\overline{\theta}) - \int_{\theta}^{\overline{\theta}} \Pi_\theta(\tilde{\theta}, \tilde{\theta}; p) d\tilde{\theta}.$

This result is standard in the mechanism design literature, and it follows from the single-crossing property.[19] We develop next an interpretation of Lemma 2, in order to provide the intuitive

[19] See, for example, Fudenberg and Tirole (1991). Necessity of (i) follows from the single-crossing property of $\Pi(\theta, \theta; p)$. Necessity of (ii) follows, since, by the envelope theorem, $d\Pi(\theta, \theta; p)/d\theta = \Pi_\theta(\theta, \theta; p) + T'(\theta)$ almost everywhere if $\hat{\theta}=\theta$ is optimal (see Milgrom (1999) for an appropriately general statement of the envelope theorem). Given (i), and under the single-crossing property, it is standard to show that (ii) is then sufficient.

foundation for many of the findings that follow, in particular our result that often the optimal SPPE uses the rigid-pricing scheme.

The situation analyzed here contrasts with the usual Principal-Agent formulation, since now the "agents" (i.e., firms) design their *own* schedule, with the goal of generating as much profit as possible, in light of their own incentive-compatibility constraints. The on-schedule incentive constraint puts an upper bound on the profits that a type θ can earn: if θ were to earn too much, other types would pretend to be θ. The expression in (ii) thus can be interpreted broadly as reflecting the profit that can be distributed to θ, without inducing mimicry by other types. As (ii) reveals, the interim profit (inclusive of wars), $\Pi(\theta, \theta; p) - T(\theta)$, that is "left" for type θ, after incentive-compatibility constraints are considered, consists of two terms: the "profit-at-the-top" and the "information" or "efficiency" rents earned by higher types.[20]

To interpret these terms, let us consider a type θ_k that is just below $\overline{\theta}$. How much can this type earn, without inducing mimicry by type $\overline{\theta}$? Type θ_k can earn the same profit as does type $\overline{\theta}$ plus a bit extra, where the extra portion is attributable to the greater efficiency (i.e., lower costs) that type θ_k actually enjoys. Similarly, let θ_{k-1} be a type that is slightly lower yet. Then type θ_{k-1} can earn the same profit as does type θ_k plus a bit extra. Pulling these points together, it follows that type θ_{k-1} can earn the same profit as does type $\overline{\theta}$ plus a "couple of bits" extra. But this leads to a direct interpretation of (ii): the profit for any type θ equals the profit-at-the-top plus the accumulated efficiency rents of higher types (note that $\Pi_\theta < 0$). An important implication is that an increase in the profit-at-the-top permits a corresponding increase in the profit for all lower types. Intuitively, when the profit-at-the-top is increased, the highest type has less incentive to misrepresent itself as a lower type, and this relaxation in the incentive constraints in turn permits lower types to earn higher profits.

What determines the magnitude of the efficiency rents? To answer this, let us define the market share expected by a firm when it announces $\hat{\theta}$ as $M(\hat{\theta}; p) \equiv \bar{m}(p(\hat{\theta}); p)$, where the expectation is over the announcements of other firms (assumed truthful). We observe that

$$-\int_{\theta}^{\overline{\theta}} \Pi_\theta(\tilde{\theta}, \tilde{\theta}; p) d\tilde{\theta} = \int_{\theta}^{\overline{\theta}} M(\tilde{\theta}; p) d\tilde{\theta} \tag{4.1}$$

The magnitude of the efficiency rents is thus determined by the allocation of market shares across types.

We note that the firms have two instruments, prices and wars, with which to sort between types. It is useful to consider whether the availability of the war instrument expands the set

[20] In the standard Principal-Agent problem, the principal designs a schedule that extracts as much rent from the agent as possible. Incentive compatibility then puts a *lower* bound on the rent that the principal can extract from an agent of type θ: if the principal attempts to extract too much rent from type θ, then that type would misreport. In the optimal schedule, the principal typically extracts all of the rent from the highest-cost agent (corresponding to the profit-at-the-top term in (ii)) and necessarily leaves some efficiency rents for other types. As we explain below, the optimal schedule differs markedly, when agents design their own schedule.

of incentive-compatible market-share allocations. To explore this issue, we introduce a simple restriction. Consider a scheme (p, T) and let θ_K denote the lowest θ for which $p(\theta) = p(\overline{\theta})$. We restrict attention to schemes (p, T) for which $\Pi(\theta_K, \theta_K; p) - T(\theta_K) \geq 0$. This restriction entails no loss of generality and simplifies the exposition of our findings.[21] We can now establish that the use of wars does *not* expand the range of sorting alternatives available to the firm. Formally:

Lemma 3. *Given an incentive-compatible scheme (p, T) and associated market-share allocation $M(\theta; p)$, there exists an alternative scheme $(\tilde{p}, \tilde{T} \equiv 0)$ which is also incentive compatible, and such that $M(\theta; p) = M(\theta; \tilde{p})$ and $\Pi(\bar{\theta}, \bar{\theta}; p) - T(\bar{\theta}) = \Pi(\bar{\theta}, \bar{\theta}; \tilde{p})$.*

In short, given an original incentive-compatible scheme and market-share allocation, we may construct an alternative incentive-compatible scheme that delivers the same market-share allocation without using wars, while also providing the same profit-at-the-top. This construction requires that the prices are adjusted away from their original levels, and so the lemma does not determine which firm types (if any) are better off under the alternative scheme. We explore this issue next.

Using Lemma 2 and (4.1), we observe that type θ's interim profit is determined as:

$$\Pi(\theta, \theta; p) - T(\theta) = \Pi(\overline{\theta}, \overline{\theta}; p) - T(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} M(\tilde{\theta}; p) d\tilde{\theta}. \tag{4.2}$$

The next result, which is well-known from auction theory, follows directly:

Lemma 4. (Revenue Equivalence Theorem) *Consider any (p, T) which satisfies (IC-onM). Then any other $(\tilde{p}, \tilde{T})$ which satisfies (IC-onM), $M(\theta; p) = M(\theta; \tilde{p})$ and $\Pi(\bar{\theta}, \bar{\theta}; p) - T(\bar{\theta}) = \Pi(\bar{\theta}, \bar{\theta}; \tilde{p}) - \tilde{T}(\bar{\theta})$ must also satisfy $\Pi(\theta, \theta; p) - T(\theta) = \Pi(\theta, \theta; \tilde{p}) - \tilde{T}(\theta)$ for all θ.*

Intuitively, suppose that firms start with the scheme (p, T) and that they then consider an alternative scheme $(\tilde{p}, \tilde{T})$ which is on-schedule incentive compatible and delivers the same profit-at-the-top. If in addition the alternative scheme maintains the original market-share allocation, then the efficiency rents are also preserved for every type. As the alternative scheme alters neither the profit-at-the-top nor the efficicency rents, it follows from (4.2) that this scheme maintains as well the original interim profit for all types.

At this point, we have extracted three lessons. First, after accounting for incentive compatibility, firms may be attracted to pricing schemes that raise the profit-at-the-top. Second, for a given amount of sorting, firms are free to choose whether to implement the corresponding market-share allocation with wars. Third, once the profit-at-the-top and the market-share allocation are determined, interim profit is fixed for all types. With these lessons in place, we are prepared now to characterize optimal SPPE for patient firms.

[21] If a scheme imposed negative profit (inclusive of wars) for θ_K, then all types higher than θ_K would do even worse, and an alternative scheme (in which θ_K and higher types set "non-serious" higher prices and receive zero market share while also avoiding wars) would yield a higher value for $E[\Pi(\theta, \theta; p) - T(\theta)]$. In the repeated-game context, the superior scheme would make such types inactive in the current period and follow such a realization with a no-war continuation value, $\overline{\mathcal{V}}_s$.

5. Optimal Collusion Among Patient Firms

In this section, we analyze optimal symmetric collusion among firms that are patient. We present our central points in five steps. First, we consider symmetric collusive schemes that are *fully sorting* (i.e., $p(\theta)$ is strictly increasing). Second, we explore whether an optimal SPPE for patient firms requires equilibrium-path wars (i.e., $T(\theta) > 0$ for some θ). Third, we add further structure and characterize the optimal symmetric pricing scheme. Fourth, we identify the relationships between our findings and those in the broader literature. Finally, we discuss the manner in which our results extend when demand is downward-sloping.

5.1. Fully Sorting Pricing Schemes

We now consider fully sorting collusive pricing schemes. The set of such schemes includes a variety of candidates. One possibility is that the firms employ the static Nash equilibrium in each period. Alternatively, the firms may attempt to sort in the first period with higher prices, perhaps near the monopoly level. Such schemes satisfy on-schedule incentive constraints only if they include equilibrium-path wars.

Under full sorting, the highest type makes no sales, and the profit-at-the-top is simply $-T(\overline{\theta})$.[22] The full-sorting requirement further implies that a firm wins the market if and only if all other firms announce higher types, and so efficiency rents are uniquely determined for the class of fully sorting schemes, with $M(\theta; p) = [1 - F(\theta)]^{n-1}$. Thus, as Lemma 4 confirms, any two fully sorting schemes which satisfy (IC-onM) differ only if the profit-at-the-top differs. Within the fully sorting class, the best profit-at-the-top is achieved when $T(\overline{\theta}) = 0$. But, notice that the static Nash equilibrium pricing scheme, p^e, is fully sorting and satisfies (IC-onM) with $T \equiv 0$. We conclude that under a fully sorting and on-schedule incentive-compatible pricing scheme, the interim profit (inclusive of wars) available to a type θ firm is at best equal to its Nash profit. It follows that an optimal SPPE under full sorting is simply the repeated play of the static Nash equilibrium; further, this holds for any discount factor, since the Nash-pricing scheme satisfies all off-schedule constraints as well.[23] Summarizing:

Proposition 3. *Among the class of fully sorting pricing schemes, and for any distribution function F and discount factor δ, an optimal SPPE is the repeated play of the static Nash equilibrium after all histories.*

[22] In this case, if $T(\bar{\theta}) > 0$, then $\Pi(\bar{\theta}, \bar{\theta}; p) - T(\bar{\theta}) < 0$, so this scheme does not satisfy our earlier restriction about profit-at-the-top which was required for Lemma 3. Nevertheless, a no-war scheme with the same market-share allocation can still be constructed, and it sets $\tilde{T}(\bar{\theta}) = 0$ with $\tilde{p}$ strictly increasing at the top. Moreover, in this event the no-war scheme yields a strict improvement. We allow for this possibility in our discussion, arguing that wars at the top are not optimal.

[23] Suppose that a solution to the Mechanism Design Program among the class of fully sorting pricing schemes entails $T \equiv 0$ and the Nash-pricing scheme, p^e. In this case, we do not require a high value of δ to establish that this scheme is optimal within the fully sorting SPPE class. Instead, we observe that repeating the static Nash pricing scheme in each period delivers a fully sorting SPPE. From here, the logic of Proposition 2 can be applied to show that p^e is the optimal fully sorting SPPE pricing scheme.

5.2. No Wars on the Equilibrium Path

The analysis in the previous subsection shows in the class of fully sorting pricing functions, after on-schedule incentive constraints are considered, there is no benefit in supporting higher prices with on-schedule wars. We now extend this argument for any initial market-share allocation.

To illustrate the main ideas, we refer to the three-step pricing schedule depicted in Figure 1. In this scheme, price ρ_k is used on interval $(\theta_k, \theta_{k+1}]$ for $k = 1, 2, 3$, where $\theta_1 = \underline{\theta}$ and $\theta_4 = \bar{\theta}$. Suppose that the original schedule has a war on, say, the second step. We may then construct an alternative schedule in which the second-step war is removed, and the second-step price (ρ_2) is reduced to keep θ_3, the highest type on the second step, indifferent. Given that $\pi_{\rho\theta} = 0$, it follows that all types on the second step are indifferent between the original and the alternative schedules: all types on the same step have the same market share, and thus trade off prices and wars at the same rate. The original market-share allocation therefore remains incentive compatible under the alternative schedule. In this way, we may eliminate wars on a step-by-step basis, and thereby "re-engineer" a payoff-equivalent no-war schedule.

Formally, consider a particular pricing function p, with the associated market-share allocation function $M(\theta; p)$, and suppose that the scheme (p, T) entails positive wars somewhere. We now refer to Lemma 3, which guarantees the existence of an alternative scheme $(\tilde{p}, \widetilde{T} \equiv 0)$ that is incentive compatible, induces the same market-share allocation, and generates the same profit-at-the-top. We next appeal to Lemma 4, which implies that the alternative no-war schedule $(\tilde{p}, \widetilde{T} \equiv 0)$ gives the same interim profit (inclusive of wars) as did the original schedule (p, T). As suggested by the three-step illustration above, the alternative schedule achieves this profit by exchanging any war in the original schedule for a lower price. Applying this argument, together with our stationarity result (Proposition 2), we conclude that:[24]

Proposition 4. *Allow for any distribution function F. If (p^*, T^*) is a solution to the Mechanism Design Program, then there exists as well a solution $(\widetilde{p}, \widetilde{T})$ with $\widetilde{p}(\theta) \leq p(\theta)$ and $\widetilde{T}(\theta) \equiv 0$. Thus, if firms are sufficiently patient, there then exists an optimal SPPE that is stationary: firms use the pricing scheme $\widetilde{p}(\theta)$ following every history along the equilbrium path, and $E\Pi(\theta, \theta; \widetilde{p})/(1 - \delta) = \overline{\mathcal{V}}_s$.*

We see from Proposition 4 that wars have no value: for any distribution F, if there exists an optimal SPPE that uses wars, then there exists as well an optimal SPPE that does not.

5.3. Optimal Pricing

We are now prepared to determine the optimal SPPE pricing scheme when firms are patient. Given the "no-wars" finding from the previous subsection, we seek the price strategy $p^*(\theta)$ that

[24] We include here a restriction on the discount factor, so that the stationarity proposition may be used. Below, in Proposition 7, we develop the further argument that the off-schedule constraints are relaxed in moving from a scheme with wars to a no-war scheme, and we are thus able to remove this restriction.

solves the program presented in our stationarity proposition. With some additional structure, this pricing scheme is easily characterized:[25]

Proposition 5. *For δ sufficiently large:*
(i) If either (a) F is log-concave, or (b) $r - \bar{\theta}$ is sufficiently large, then the equilibrium path of the optimal SPPE is characterized by price rigidity ($p^*(\theta) \equiv r$) and no wars ($T^*(\theta) \equiv 0$).
(ii) In any optimal SPPE that is stationary, there exists an open interval of cost types where pricing is rigid, and per-period profits above the static Nash equilibrium are attained: $\overline{\mathcal{V}}_s > \pi^{NE}/(1-\delta)$.

For patient firms, if the distribution function is log concave or r is large enough, the optimal SPPE is described as follows: firms select the price r in each period, whatever their private cost realizations, so long as all firms have selected the price r in all previous periods.[26] Further, firms can always exceed Nash payoffs, if they are sufficiently patient.

The rigid-pricing scheme, $p^*(\theta) \equiv r$, has benefits and costs. An important benefit of this scheme is that it satisfies the on-schedule incentive constraint without recourse to equilibrium-path wars. Furthermore, the price is as high as possible. However, an evident cost of the rigid-pricing scheme is that it sacrifices efficiency benefits: it may be that one firm has a low cost while another firm has a high cost, but under the rigid-pricing scheme each of these firms sells to $1/n^{th}$ of the market. The content of the proposition (part (i)) is that the benefits of the rigid-pricing scheme exceed the costs, provided that the distribution function is log concave, or the reservation price is high enough.

While a complete proof is offered in the Appendix, we may develop the intuition by posing and answering two simple questions. First, what (incentive-compatible) pricing schedule gives the greatest profit-at-the-top? Since the single-crossing property implies that $p(\theta)$ cannot decrease in θ, profit-at-the-top is highest when all firms set the same price, so that the highest type is never underpriced. The most profitable rigid-pricing scheme is the one in which firms fix the price at r. Thus, $p(\theta) \equiv r$ is the pricing scheme that offers the highest profit-at-the-top, given that downward-sloping pricing schemes are not on-schedule incentive compatible.

Second, when is it true that expected profit is maximized over on-schedule incentive-compatible schemes when profit-at-the-top is maximized? This is where log concavity comes in. Intuitively, the "contribution" of an increase in a given type's profit to the firm's expected profit is governed by the fraction of types below it (which enjoy a relaxed incentive constraint and thus earn higher profits), conditional on the "probability" that the given type will actually arise. We may thus think of $F(\theta)/f(\theta)$ as a measure of the contribution of an increase in type θ*'s* profit to the firm's expected profit. The log-concavity condition simply ensures that this

[25] This result also holds in a model with discrete types, except that an additional parameter restriction is required, one that depends on the gap between r and the highest type. The restriction is satisfied when the distance between types becomes small enough. Details are available from the authors.

[26] We have assumed $r \geq \bar{\theta}$. If this assumption were relaxed, then the optimal scheme would entail that firms with cost types greater than r sit out rather than endure negative profits.

measure is increasing, so that the type that contributes most to expected profit is the highest type. In total, under log concavity the optimal collusive scheme maximizes profit-at-the-top, and under the on-schedule incentive constraints this is achieved with a rigid price at r.[27]

A formal expression of this intuition can be derived as follows. Using our indirect-utility formulation (4.2) and setting $T(\theta) \equiv 0$, we may write a firm's expected profit as

$$E[\Pi(\theta, \theta; p)] = E[\pi(p(\overline{\theta}), \overline{\theta}) \cdot M(\overline{\theta}; p) + \int_{\theta}^{\overline{\theta}} M(\tilde{\theta}; p) d\tilde{\theta}].$$

Next, employing a standard trick from the literature on optimal auctions (Myerson (1981), Bulow and Roberts (1988)), we may integrate by parts and rewrite our objective function as

$$E[\Pi(\theta, \theta; p)] = E[\pi(p(\overline{\theta}), \overline{\theta}) \cdot M(\overline{\theta}; p) + \frac{F}{f}(\theta) \cdot M(\theta; p)]. \tag{5.1}$$

Consistent with the intuitive discussion, the first term rewards pricing schemes that raise the profit-at-the-top. Under log-concavity, the second term also rewards such schemes, since the market shares of higher types then contribute more to expected profit.

The assumption that the distribution is log-concave is common in the contracting literature, and many distributions satisfy the assumption.[28] However, we can also consider the optimal allocation of market share even if F/f is decreasing on some intervals. It is straightforward to show that, if r is close enough to $\bar{\theta}$, the optimal pricing rule entails sorting on intervals where F/f is decreasing, and rigidity elsewhere. Under our assumption that $f(\underline{\theta}) > 0$, log-concavity always holds in a neighborhood of $\underline{\theta}$. Thus, it is always optimal for sufficiently patient firms to use rigid pricing at the bottom of the pricing function, and from this part (ii) of the proposition follows: sufficiently patient firms can sustain SPPE payoffs strictly greater than the static Nash equilibrium (which entails sorting at the bottom). Finally, observe that if $r - \bar{\theta}$ is sufficiently large, so that profit-at-the-top is great enough, the benefits from pooling market share with high-cost types outweigh those from allocating market share amongst types with the highest F/f, and the rigid-pricing scheme is again optimal.

We close this subsection with an instructive example. Suppose that there are two firms and consider the family of two-step pricing schedules. Such schedules are characterized by a low price ρ_1, a high price ρ_2, and a breakpoint θ_2, such that all types below (above) θ_2 select the

[27] The rigid-pricing scheme can be implemented with or without wars, but expected profits are strictly higher when there are no wars. The no-wars rigid-pricing arrangement is uniquely optimal, since the associated profits cannot be implemented with any other feasible prices and wars, given $p \leq r$ and $T \geq 0$.

[28] For example, uniform, the class x^α where $x \in [0, 1]$, normal, log-normal, logistic, chi-squared, exponential, and Laplace satisfy the restriction, as well as any distribution whose density is log-concave. (Some of these distributions have unbounded support; but, truncating the distribution does not change its log-concavity, nor does taking the distribution of the absolute value.) In the typical Principal-Agent procurement problem, log-concavity ensures that the agent's "virtual cost," $\theta + F(\theta)/f(\theta)$, is increasing, which implies in turn that the principal's optimal contract is fully sorting (see, e.g., Myerson (1981)). In our model, since the agents' utilities are maximized, the assumption has the opposite effect: it implies that the "contract" entails pooling.

low (high) price. A rigid-pricing scheme is then an extreme case, in which $\theta_2 = \underline{\theta}$ or $\theta_2 = \overline{\theta}$. For $\theta_2 \in (\underline{\theta}, \overline{\theta})$, the single-crossing property ensures that the on-schedule incentive-compatibility constraints are satisfied if and only if a firm of type θ_2 is indifferent between the bottom and top steps: $\pi(\rho_1, \theta_2)M(\underline{\theta}; p) = \pi(\rho_2, \theta_2)M(\overline{\theta}; p)$, where $M(\underline{\theta}; p) = F(\theta_2)/2 + [1 - F(\theta_2)]$ and $M(\overline{\theta}; p) = [1 - F(\theta_2)]/2$. Given ρ_2 and θ_2, incentive compatibility thus determines ρ_1 and thereby expected profit. Suppose further that the distribution of costs is uniform over the unit interval. Direct calculations then yield: $E[\Pi(\theta, \theta; p)] = [2\rho_2(1-\theta_2)+\theta_2+(\theta_2)^2-1]/4$. Expected profit is thus maximized when $\rho_2 = r$ and θ_2 is set at a corner. With $r \geq \overline{\theta}$, the maximum is achieved when $\theta_2 = \underline{\theta}$. Thus, a rigid-pricing scheme with $p(\theta) \equiv \rho_2 = r$ is optimal within the two-step family. We may also compare the rigid-pricing and Nash-pricing schemes. Expected profit under the Nash-pricing scheme is 1/6, which is less that the value $(2r - 1)/4$ that is generated by the rigid-pricing scheme.

5.4. Related Literature

We now compare our findings to those established in related papers. We recall first McAfee and McMillan's (1992) study of bidding rings. Working with a static model of procurement auctions, McAfee and McMillan show that bidders in a "weak cartel" (where transfers are not allowed) collude best if they agree to bid the same price, r. Our Proposition 5 is closely related; indeed, their model can be mapped into the Mechanism Design Program by setting $T \equiv 0$. The analysis here extends their results, however, by formally connecting the static results to the repeated-game context and demonstrating that a rigid-pricing scheme is optimal even when schemes that sustain sorting using "wasteful" transfers ($T>0$) are allowed. In the next section, we further generalize the analysis to consider the possibility of impatient firms.

Our findings are also related to work on cartel design under private information. As Cramton and Palfrey (1990) and Kihlstrom and Vives (1992) show, if firms can design a mechanism where they communicate their cost types and make side-payments to one another, then the firms can achieve full efficiency benefits without any pricing distortion (i.e., production is allocated to the lowest-cost firm, who sets its monopoly price), although the participation constraints may fail for some cost types.[29] In our model, the SPPE restriction prevents firms from using asymmetric continuation values to mimic side-payments from one firm to another. But it is still possible for firms to achieve full efficiency benefits, if they use fully sorting pricing schemes. In this case, however, informational costs, manifested as pricing distortions and/or future price wars, must be experienced. We have seen that the costs exceed the benefits when F is log-concave.

Interestingly, communication would not have any value for firms in our model. The realization of full efficiency benefits does not require communication, while the informational

[29]Their analysis builds on earlier work by Roberts (1985), who shows that a scheme with full efficiency benefits may not be incentive compatible, when firms can communicate but are unable to make side-payments. See also McAfee and McMillan's (1992) analysis of "strong cartels," in which bidders can make transfers to one another.

costs remain, with or without communication, so long as firms cannot make side-payments. In other words, the optimal direct revelation mechanism without side-payments is characterized by Proposition 5, and no communication is required to implement it.[30]

Our findings are also related to those presented in the hidden-action collusion literature. A central feature of the Green-Porter (1984) and APS (1986, 1990) papers is that collusive conduct involves periodic reversions to price wars. Our model can be placed within their hidden-action modeling framework, if we think of a firm's strategy, $p(\theta)$, as its hidden action and the resulting price, $p = p(\theta)$, as the public signal, where the distribution of this public signal is then determined by the pricing function itself and the distributional properties of θ.

The main difference between the two modeling approaches is that we allow for an endogenous support of the public signal. Put differently, our model may be understood as a hidden-action model with *endogenous imperfect monitoring.* To see this, recall that in the Green-Porter (1984) and APS (1986, 1990) modeling framework, the support of the publicly-observed market price is independent of the private output selections made by firms. In our model, by contrast, the support of the signal is itself determined in equilibrium. In particular, if firms employ a rigid-pricing schedule in which they choose r under all cost realizations, then in equilibrium the support of the public signal is degenerate, as rival firms expect to observe the price r, no matter what cost realization the firm experiences. This in turn enables firms to limit wars to off-equilibrium-path events.

5.5. Downward-Sloping Demand

The results presented above are derived for the case of inelastic demand. In our working paper (Athey, Bagwell and Sanchirico (1998)), we consider as well the case of downward-sloping demand. This case may be treated with the notation developed above, if we re-define the profit-if-win function as $\pi(\rho, \theta) = (\rho - \theta)D(\rho)$, where the demand function D satisfies $D > 0 > D'$ over the relevant range. The monopoly price is then strictly increasing in θ.

With these adjustments, Propositions 1 and 2 hold as stated, and Proposition 3 takes an even stronger form: the repeated play of the static Nash equilibrium is *the* optimal SPPE within the fully sorting class. We show, too, that our main finding (Proposition 5) extends in the natural way: if firms are sufficiently patient, F is log-concave and demand is sufficiently inelastic, then an optimal SPPE must be characterized by a rigid price for all cost types and no wars.

For general demand functions, however, the revenue equivalence theorem (Lemma 4) no longer applies, and we cannot rule out that an optimal SPPE allows for a war when intermediate

[30] Communication can be valuable without side-payments in other oligopoly models. In the Bertrand model, prices allocate all market share to the low-price firm; by contrast, in a Cournot model, *if* efficiency benefits are sought, the firms would do better to reveal the cost types in advance and avoid unnecessary (and price-reducing) production. Even in the Bertrand model, as Athey and Bagwell (forthcoming) show, communication can be valuable if firms are able to implement "side-payments," by moving between asymmetric continuation values.

cost realizations occur. But for any demand function D and distribution function F, we establish that an optimal SPPE among sufficiently patient firms must generate greater-than-Nash profits and include an interval of lowest-cost types, $[\underline{\theta}, x)$ where $x > \underline{\theta}$, on which no wars are used and the pricing function is rigid.

6. Optimal Collusion Among Impatient Firms

We now consider collusion among impatient firms. We proceed in two general steps. First, we determine the critical patience level above which the rigid-pricing scheme can be enforced. Second, we consider less patient firms, who are unable to enforce the rigid-pricing scheme, and explore how they best collude. Impatience creates an additional motivation for the avoidance of price wars. In addition, impatient firms may use pricing schemes that entail an "escape clause," whereby a firm is allowed to depart from the rigid price and set a lower price when it experiences a favorable cost shock. In an extended model, we find that such a departure is especially likely when demand is temporarily high. We also offer further characterizations of the optimal collusive pricing schemes for impatient firms.

6.1. Enforcing Rigidity Off Schedule

We begin with the determination of the critical discount factor. The rigid-pricing scheme satisfies off-schedule constraints, if a firm always regards the current-period benefit from undercutting the rigid price as small in comparison to the discounted value of future cooperation. In turn, future cooperation is more valuable when firms are more patient and the punishment that would follow a deviation is more severe. The critical discount factor is therefore determined as the lowest discount factor at which firms can enforce the rigid-pricing scheme, when a deviation leads to the most severe punishment, $\underline{\mathcal{V}}_s$.

Formally, let us suppose that the firms attempt to maintain a rigid price $\rho \geq \underline{\theta}$ in all periods for all cost realizations. If a firm of type θ were to cheat and undercut (by ϵ) this price, then the firm would win the entire market, as opposed to just $1/n^{th}$ of the market, and so the firm's incentive to cheat is $\frac{n-1}{n}\pi(\rho, \theta)$. Importantly, this incentive is greatest for a firm with the lowest cost level, $\underline{\theta}$, since the profit-if-win is then highest and the gain in market share is thus most valuable. If a firm were to cheat, however, it would forfeit the discounted value of future cooperation. This value is measured in relation to the cost that a firm expects in the future, $E\theta$. For example, if a deviation is punished by an infinite reversion to the static Nash equilibrium, then the proposed rigid-pricing scheme is off-schedule incentive compatible if and only if

$$\frac{n-1}{n}\pi(\rho, \underline{\theta}) \leq (\delta/(1-\delta))[\frac{1}{n}\pi(\rho, E\theta) - \pi^{NE}]. \tag{6.1}$$

If the proposed scheme yields greater-than-Nash profit, (6.1) holds when δ is sufficiently large.

We note as well that both the incentive to cheat and the future value of cooperation increase with the rigid price, ρ. As long as $\delta > \frac{n-1}{n}$, however, the latter effect dominates, so that the rigid price that is easiest to support has $p(\theta) \equiv r$ in each period. Using (6.1), we find that the critical discount factor δ^* above which firms can use the Nash punishment threat to enforce the rigid-pricing scheme is

$$\delta^* \equiv \frac{(n-1)\pi(r,\underline{\theta})}{(n-1)\pi(r,\underline{\theta}) + \pi(r, E\theta) - n\pi^{NE}}. \tag{6.2}$$

It is straightforward to verify that $\delta^* \in (\frac{n-1}{n}, 1)$, if $\frac{1}{n}\pi(r, E\theta) > \pi^{NE}$. We may now state:

Proposition 6. *(i). If F is log-concave, then for all discount factors δ, $\underline{\mathcal{V}}_s = \pi^{NE}/(1-\delta)$.*
(ii). If F is log-concave and $\delta < \delta^*$, then there does not exist an SPPE with rigid pricing.
(iii). If F is log-concave or $r - \bar{\theta}$ is large enough, then $\delta^* \in (\frac{n-1}{n}, 1)$ and for all $\delta > \delta^*$ any optimal SPPE is characterized by rigid pricing at r in every period.

Thus, when F is log-concave, Nash reversion is in fact the worst punishment, and the rigid-pricing scheme can be enforced if and only if $\delta > \delta^*$, where $\delta^* \in (\frac{n-1}{n}, 1)$. To see an example, suppose there are two firms, costs are uniformly distributed over $[0, 1]$ and $r = 1$. Then, π^{NE}=1/6, and an SPPE with rigid pricing exists and is optimal if and only if $\delta \geq \delta^*$=6/7.

It is striking that the lowest SPPE continuation value, $\underline{\mathcal{V}}_s$, corresponds to Nash play when F is log-concave. This is true despite the fact that SPPE may exist in which some firm types price below cost. For example, there may exist non-stationary SPPE, in which higher-cost types price below cost in the first period, sustained by the promise of a better future equilibrium. Of course, SPPE continuation values cannot be driven too low: the scheme must offer the highest-cost type overall expected payoffs greater than $\delta\underline{\mathcal{V}}_s$ (or else the firm will deviate off-schedule) and lower-cost types cannot be deprived of the available efficiency rents. In searching for the lowest SPPE continuation value, we thus consider pricing schemes that minimize efficiency rents. Following the logic of Section 5.3, the minimum efficiency rent is attained using a strictly increasing pricing scheme when F is log-concave, and with this we can establish that it is not possible to sustain punishments worse than Nash. As we confirm in Lemma 5 in the Appendix, however, when the log-concavity assumption is relaxed, on-schedule incentive constraints are compatible with below-Nash efficiency rents. If firms are sufficiently patient, non-stationary SPPE with below-cost pricing can then be constructed that yield below-Nash payoffs.[31]

Recall that when firms have access to a public randomization device, the set of equilibrium values is convex. In that case, Proposition 6 provides a complete characterization of the SPPE set when F is log-concave and $\delta > \delta^*$.

[31]The scheme used to generate below-Nash payoffs requires some firms to price below cost, and such a firm must be dissuaded from deviating to a higher price. Indeed, a firm would undertake just such a deviation, if the market-clearing price were public but individual prices were otherwise not. In this case, the worst SPPE involves the repeated play of the static Nash equilibrium (for any F and δ). When individual prices are public, however, a firm can be induced to price below cost, and when F is not log-concave this may describe the worst SPPE.

An interesting implication of Proposition 6 is that intertemporal fluctuations in costs diminish the ability of firms to collude. Intuitively, when costs fluctuate through time, collusion requires greater patience, as the scheme must withstand the incentive imbalance that occurs when a firm draws a low current cost level ($\underline{\theta}$), and thus faces a great incentive to cheat, while assessing the long-term value of cooperation with reference to an average cost level ($E\theta$). Formally, $\delta^* > \frac{n-1}{n}$, where $\frac{n-1}{n}$ is the critical discount factor for the standard Bertrand supergame, in which firms' costs are time-invariant. This implication is broadly consistent with the common assessment (see, e.g., Scherer (1980, p. 205)) that collusion is more difficult when costs are variable across firms.

6.2. No Wars on the Equilibrium Path

How do firms best collude when they are unable to enforce the rigid-pricing scheme? In this subsection, we take a first step toward answering this question. Allowing for impatient firms, we establish that the scope for symmetric collusion cannot be improved (and may be strictly harmed) by the inclusion of equilibrium-path wars

The central idea is simple. Let us start with an original SPPE collusive scheme. Relying on Proposition 4, if there is a positive probability of an equilibrium-path war associated with some cost type, then we can re-engineer an alternative collusive scheme – by eliminating the war and reducing the price for that type a corresponding amount – that yields for this type the same expected payoff. The alternative scheme satisfies the on-schedule constraint (given that the original did) and thus constitutes a payoff-equivalent SPPE for patient firms. When firms are impatient, however, the off-schedule constraint is also a concern, and it is here that the alternative schedule offers an actual advantage: by shifting profit from the current period (price is reduced) to the future (wars are eliminated), the incentive to cheat is reduced while the future value of cooperation is enhanced. The off-schedule constraint is therefore now easier to satisfy than under the original scheme.

As the following proposition confirms, this argument is quite general:

Proposition 7. *Allow for any distribution function F and any discount factor δ. If an SPPE exists with the optimal payoff $\overline{\mathcal{V}}_s$, then there exists a stationary SPPE, where the same pricing strategy is used following every equilibrium-path history, with the optimal payoff $\overline{\mathcal{V}}_s$.*

More generally, any scheme that uses price wars is (weakly) dominated by a scheme without price wars. Thus, "revenue equivalence" does not extend to impatient firms.

6.3. Partial Rigidity and Collusion Among Impatient Firms

The propositions developed above suggest that our search for collusive schemes among impatient firms should emphasize two ingredients: the absence of rigid pricing and no equilibrium-path wars. But exactly how do impatient firms price in an optimal SPPE? In this subsection, we first present sufficient conditions under which a *two-step pricing scheme* can be enforced and is optimal for impatient firms. Second, we consider an extended model that includes publicly observed fluctuations in industry demand. Finally, we argue that optimal SPPE for impatient firms is characterized by a pricing schedule that is a step function (partial rigidity), if the collusive scheme is to offer better-than-Nash profits.

6.3.1. Introducing A Second Step: An "Escape Clause"

Recall that the rigid-price scheme fails to be enforceable when $\delta < \delta^*$, because a firm that draws the lowest-cost type is too tempted to undercut the rigid price r and increase its market share. A natural conjecture is that this problem may be overcome when a two-step pricing scheme is employed, with prices ρ_1 and ρ_2, where $\rho_1 < \rho_2$, and a break-point θ_2. In this case, the lowest-cost firm has less incentive to cheat. Firstly, this firm now expects greater than a $\frac{1}{n}^{th}$ share of the market, and so the gain in market share that accompanies a price cut is diminished. Secondly, any given gain in market share is now less profitable, since the lower-cost firm has a lower price, and thus the profit-if-win it experiences on the market share it enjoys is now lower.

This, however, is not the whole story. Balanced against this diminished incentive to cheat is a reduction in expected long-term profit: if the distribution function is log concave, a two-step scheme yields lower expected profit than does a rigid scheme, and so the firm also now has less to lose in the future if it cheats today. Complicating matters further, the net resolution of these conflicting effects for the off-schedule incentive constraint may hinge upon the nature of the distribution function. A two-step scheme will satisfy the off-schedule incentive constraint if it lowers the incentive that the lowest-cost firm has to cheat without substantially altering the expected profit that firms anticipate in the future. Intuitively, this will be the case if the density is small for lower-cost types, so that these types occur infrequently in the future.

Proposition 8. *If F is log-concave and*

$$f(\underline{\theta}) < \frac{\frac{1}{n}\pi(r, E\theta) - \pi^{NE}}{(n-1)\pi(r, \underline{\theta})\pi^{NE}} \tag{6.3}$$

then there exists $\delta^o < \delta^*$, such that, for every $\delta \in (\delta^o, \delta^*)$, there exists an optimal SPPE that is stationary and uses a two-step pricing scheme, with $p_2 = r > p_1$ and $\theta_2 \in (\underline{\theta}, \overline{\theta})$.

When F is log-concave, the two-price scheme is optimal for δ just below δ^*, since then the two-price scheme departs from the desired rigid-pricing scheme only at the lowest-cost types.

Proposition 8 describes a situation in which the realization of an unlikely and low cost type results in a marked reduction in the firm's price, suggesting that rare but pronounced price cuts may occur under symmetric collusion schemes when firms are impatient.[32] In other words,

[32]The proof constructs a two-step pricing equilibrium for θ_2 close to $\underline{\theta}$. In this equilibrium, the low-step price is approximately $[r + \underline{\theta}(n-1)]/n$ indicating a discrete reduction of amount $(r - \underline{\theta})(n-1)/n$ from the high-step price of r. Notice that the two-step pricing scheme calls for a greater price reduction when markets are less concentrated.

symmetric collusion among impatient firms may call for an "escape clause" provision, under which a firm is allowed to select a lower price in the event that a very favorable cost type is realized. The price reduction must be substantial, in order to ensure that the low price is attractive only when a firm's cost type is low.

This behavior is reminiscent of the findings of Rotemberg and Saloner (1986), but there are important differences: in their case, collusive prices adjust across *all* firms in response to a *public* demand shock. This result also may be useful when interpreting an apparent episode of "cheating" in a collusive industry. Imagine, for example, a situation in which a single firm charges a low price and yet faces no retaliation. It is difficult to reconcile such an observation with standard collusion models. In our private-information setting, however, optimal collusion among impatient firms may allow for "rare exceptions" to rigidity, in which a firm substantially cuts its price and faces no retaliation.

The "small-density condition" (6.3) plays an intuitive role, but the assumption is restrictive. Obviously, it is satisfied if $f(\underline{\theta})$ is close enough to zero. To see a more subtle example, consider the (log-concave) distribution function family, $F(\theta) = \theta^{\alpha}$, with $\underline{\theta} = 0 < 1 = \overline{\theta}$. The small-density condition is satisfied for any $\alpha > 1$, but it fails when $\alpha < 1$. The condition also fails when $\alpha = 1$ (corresponding to the uniform distribution). When (6.3) is violated, it may be that, for all $\delta < \delta^*$, no two-step pricing schedule satisfies on- and off-schedule constraints; this is the case for the uniform distribution (see Athey, Bagwell and Sanchirico (1998)).

A second example highlights an interesting prediction: if the support of the distribution increases, the optimal collusive scheme may switch from a rigid-pricing to a two-step pricing scheme. Consider a distribution $F(\theta; \mu, z)$, where the mean is constant at μ, but the support is parameterized by z, so that $\underline{\theta} = \mu - z$ and $\overline{\theta} = \mu + z$. Suppose $F(\theta; \mu, z)$ is log-concave and satisfies the small-density condition. An example is the "triangle" distribution, where the density $f(\theta; z)$ is symmetric about μ, and $f(\theta; z) = \frac{1}{z^2}(\theta - (\mu - z))$ on $[\mu - z, z]$.[33] We make two observations. First, while increasing z leaves the per-period profits from rigid pricing unchanged, it introduces lower-cost types that are especially tempted to cheat and thereby increases the critical discount factor for rigid pricing, δ^*. Second, consider increasing z while holding the discount factor fixed at $1 > \delta > \frac{n-1}{n}$. When z is small, information is appoximately complete and rigid pricing can be supported. At a critical z, however, the rigid-pricing scheme breaks down, and Proposition 8 implies that a two-price scheme is then optimal. Thus, increased "spread" in the cost distribution leads to increased price variability. In an application of this framework, Simon (1999) argues that inflation can lead to an increase in the spread of costs, and establishes that prices are more variable when inflation is high.

[33] Note that $f(\underline{\theta})$ and $f(\bar{\theta})$ are equal to 0, which strictly speaking violates our maintained assumption; but it is straightforward to show that all of our results extend as long as $f'(\underline{\theta}) > 0$ and $f'(\bar{\theta}) < 0$.

6.3.2. Observable Fluctuations in Demand

The intuition underlying Proposition 8 suggests that any exogenous variation in the economic environment that heightens the short-term incentive to cheat and/or reduces the long-term value of cooperation may result in lower and more variable prices. A variation of particular empirical relevance occurs when industry demand fluctuates over time. Following Rotemberg and Saloner (1986), we now extend our model to an environment in which industry demand fluctuates in an i.i.d. fashion between low and high states, $\phi \in \{\phi_L, \phi_H\}$ where $\phi_H > \phi_L$. Profit is proportional to the demand state, which is publicly observed at the beginning of each period, before cost shocks are realized.

In this model, the long-term value of cooperation is proportional to the demand that is expected in future periods, $E\phi$, which is independent of the current demand state. By contrast, the incentive to cheat is greatest when current demand is high. The off-schedule constraint therefore binds first for the high-demand state. Formally, we may modify (6.1) to calculate for the rigid-pricing scheme a critical discount factor,

$$\delta_H^* \equiv \frac{(n-1)\pi(r,\underline{\theta})\phi_H}{(n-1)\pi(r,\underline{\theta})\phi_H + [\pi(r,E\theta) - n\pi^{NE}]E\phi},$$

where $\delta_H^* > \delta^*$, at which the off-schedule constraint binds when current demand is high. Similarly, we may define $\delta_L^* < \delta^*$ as the critical discount factor for the rigid-pricing scheme in the low-demand state. When the discount factor falls slightly below δ_H^*, it is no longer possible to enforce a rigid price for all cost levels in the high-demand state; however, it remains possible to do so when the demand state is low. Assuming that $\frac{\phi_H}{E\phi}$ is not too large, so that $1 > \delta_H^*$, a modification of Proposition 8 implies the following:

Proposition 9. *For two firms, when the market size is i.i.d. with $\phi \in \{\phi_L, \phi_H\}$, if F is log-concave and (6.3) holds, then there exists $\delta^o \in [\delta_L^*, \delta_H^*)$, such that, for every $\delta \in (\delta^o, \delta_H^*)$, there exists an optimal SPPE that is stationary and satisfies:*
(i) in the low demand state ($\phi = \phi_L$), firms use the rigid-pricing scheme of Proposition 6;
(ii) in the high demand state ($\phi = \phi_H$), firms use the two-price scheme of Proposition 8.

This proposition extends a theme of the previous subsection: symmetric collusion between impatient firms may be marked by occasional (and perhaps substantial) price reductions by individual firms. We learn here that these departures are most likely to occur when one firm receives a favorable cost shock *and* current demand is high. Using the functional forms of the last subsection, it is straightforward to find parameters where the result applies.[34]

One implication of the model is that the countercyclical-pricing finding of Rotemberg and Saloner (1986) is robust to the presence of private cost fluctuations. This model can be generalized in a number of directions. For example, following Bagwell and Staiger (1997), we may

[34] For example, with the triangle distribution when $n = 2$, $r = 3$, $\mu = 1$ and $z = .5$, we compute $\pi^{NE} = 7/60$, $\delta^* = .74$. Thus, if $\phi_L = 1$, $\phi_H = 1.25$, and $\delta = .92$, the optimal equilibrium specifies a rigid price in the low-demand state and a two-step scheme in a high-demand state.

consider an alternative stochastic process, in which the demand growth rate follows a Markov process, so that recessions are characterized by slow growth and booms are characterized by fast growth. In such a model, recessions are the time when collusion is most difficult. Thus, given the tradeoffs we outlined above, we would expect rigid prices in booms and variable prices in recessions. This is a striking and testable prediction. Indeed, Reynolds and Wilson (1998) use this stochastic process in their empirical work, and they find that in 14 out of 15 industries, prices are more variable in recessions than in booms.

6.3.3. Optimal Pricing for Impatient Firms

Next, we consider the general features of optimal SPPE pricing schemes for impatient firms. This problem is subtle, because, as we have seen, a firm's incentive to deviate from the collusive agreement depends on its own cost type, its own price and the expected payoffs of the entire collusive agreement. The off-schedule constraint is thus analogous to a "participation constraint" in a static mechanism design model, except that the constraint depends on the type and the outside option is endogenous.

Our analysis is simplified by two observations. First, as established in Proposition 7, we may restrict attention to stationary pricing schemes. Second, we observe that all off-schedule incentive constraints are satisfied if they hold for the lowest-cost type on any step (i.e., for type θ_k on any step k defined by endpoints (θ_k,θ_{k+1}) over which the pricing schedule is flat). Clearly, this constraint is more difficult to satisfy for a given step as the step gets larger, since then the market-share gain from an off-schedule price cut is larger.

To understand the main tradeoffs, suppose that the distribution function is log-concave (at least in the relevant region) and consider whether a decrease in θ_k, and thus an increase in the length of step k, might be optimal. There are three effects. First, the off-schedule constraint for step k is exacerbated, since a deviation results in a larger increase in market share. Second, expected collusive profits increase, and this relaxes the off-schedule constraints. As in our two-step analysis, which of these two effects dominates depends on the shape of the distribution function. Finally, a multi-step scheme introduces a third effect as well: when an intermediate step is lengthened, the on-schedule constraints may require adjustments in prices on other steps, and these adjustments may in turn tighten the off-schedule constraints at these steps.

The resolution of these tradeoffs depends on the shape of F. In our discussion paper (Athey, Bagwell and Sanchirico (1998)), we specify an optimization program which can be solved, either numerically or analytically, given specific functional forms and parameters. Here, we consider a more qualitative question: Are the off-schedule incentive constraints ever so severe that the firms are induced to use an interval of strictly increasing prices, even when the distribution is log-concave? The following proposition summarizes our findings:

Proposition 10. *Suppose* $\delta > \frac{n-1}{n}$.
(i) If $r > \bar{\theta}$, *then* $\overline{\mathcal{V}}_s > \pi^{NE}/(1-\delta)$ *and* $p(\bar{\theta}) = r$.

(ii) If $\overline{\mathcal{V}}_s > \pi^{NE}/(1-\delta)$, and both F and $1-F$ are strictly log-concave, then in an optimal stationary SPPE there exists no open interval of types, (θ', θ''), where the pricing function is strictly increasing.

Proposition 10(i) establishes that, when $\delta > \frac{n-1}{n}$ and $r > \bar{\theta}$, firms can achieve above-Nash payoffs and therefore enjoy at least partial collusion.[35] Intuitively, when $r > \bar{\theta}$, the introduction of a small interval of pooling for the highest cost types at the price $p(\bar{\theta}) = r$ improves expected future profits; furthermore, when $\delta > \frac{n-1}{n}$, this improvement overwhelms the higher incentive to cheat that higher types then face. In part (ii), we assume directly that some collusion is attainable. We then show that, if both F and $1-F$ are strictly log-concave, then firms would do better to introduce tiny regions of pooling rather than strictly separate types throughout an open interval. The introduction of a small region of pooling has a first-order benefit for expected future profits when F is log-concave, and for a small step the gain in market share from undercutting the collusive price is small. There remains, however, the third effect mentioned above, associated with cross-step externalities and off-schedule constraints. For any particular type θ, an off-schedule constraint might bind above or below θ, and our assumption that both F and $1-F$ are strictly log-concave ensures that pooling is optimal, whether the cross-step externality extends to the fraction F of lower types or the fraction $1-F$ of higher types.[36]

Under the conditions of Proposition 10, then, the optimal collusive scheme is stationary, and there is no open interval of cost types where efficiency benefits are attained. Therefore, an optimal SPPE pricing scheme exhibits rigidity over regions of costs, and the observed distribution of prices will have mass points. Such an observation may offer some guidance in interpreting allegations of collusion. For example, in the NASDAQ stock exchange (see, for example, Christie and Schultz (1999)), dealers systematically restricted their price quotes to multiples of \$.25, and such behavior was associated with higher average bid-ask spreads (and thus, presumably, higher profits).

6.4. Downward-Sloping Demand

In our discussion paper (Athey, Bagwell and Sanchirico (1998)), we discuss the generalization of our findings when demand is downward-sloping. Here, we simply make two points. First, the results derived in this section (Propositions 6-10) all generalize to case in which demand is sufficiently inelastic that rigid pricing is optimal for patient firms.[37] Second, while our no-wars finding (Proposition 7) holds even for impatient firms when demand is sufficiently inelastic, we

[35] Notice that this result complements Proposition 5(ii) by offering a specific lower bound for δ, under the further assumption that $r > \bar{\theta}$.

[36] More precisely, for $\theta > \theta_j$, the on-schedule constraint implies $\Pi(\theta,\theta;p) = \Pi(\theta_j,\theta_j;p) - \int_{\theta_j}^{\theta} M(\tilde{\theta};p)d\tilde{\theta}$, from which we may derive $E[\Pi(\theta,\theta;p)|\theta > \theta_j] = \Pi(\theta_j,\theta_j;p) - E[\frac{1-F(\theta)}{f(\theta)}M(\theta;p)|\theta > \theta_j]$. If $1-F$ is strictly log-concave, so that $-\frac{1-F(\theta)}{f(\theta)}$ is strictly increasing, the expected profit for $\theta > \theta_j$ is maximized when these types are pooled.

[37] Showing that repeated play of the static Nash equilibrium yields the worst punishment when F is log-concave and demand is sufficiently inelastic entails additional work. Details are available from the authors.

can not rule out the possibility of a war in an optimal SPPE for impatient firms and general demand functions.

7. Conclusion

We propose a model of collusion in which firms are privately informed as to their current cost positions. Our five main findings are:

1. Firms fare poorly under fully sorting symmetric collusive schemes, since the efficiency benefits that such schemes afford are small relative to the informational costs.
2. Optimal symmetric collusion can be achieved without equilibrium-path price wars.
3. If firms are sufficiently patient and the distribution of costs is log-concave, optimal symmetric collusion is characterized by price rigidity and the absence of price wars on the equilibrium path.
4. If firms are less patient, optimal symmetric collusion may be characterized by price rigidity over intervals of costs (a step function), where the price of a lower-cost firms is distorted downward to diminish the incentive that such a firm has to cheat.
5. If firms are less patient and the model is modified to include i.i.d. public demand shocks, under optimal symmetric collusion, the downward pricing distortion that accompanies a firm's lower-cost realization may occur only when current demand is high.

We note that the first finding underscores the basic tradeoff present in our repeated-adverse-selection model of collusion; the second finding contrasts with the Green-Porter (1984) collusion literature on repeated moral hazard; the third finding offers an equilibrium interpretation of the empirical association between rigid pricing and industry concentration, as well as the commonly observed collusive practices of identical bidding, price fixing with stable market shares, and fixed markup rules (above a publicly observed wholesale price); and the fourth and fifth findings are reminiscent of the logic developed by Rotemberg and Saloner (1986) for collusion in markets with publicly observed demand shocks, but associate low collusive prices with individual firm behavior in high-public-demand and low-private-cost states.

Our analysis also contributes at a methodological level. In particular, we develop the precise connections between static and dynamic analyses, making clear the similarities and differences, and laying the groundwork for treating other repeated-game problems within the mechanism design modeling framework. Our work also motivates some new questions for static mechanism design, and takes some initial steps towards addressing them. For example, we examine how restrictions on transfers affect optimal mechanisms. Further, when firms are impatient, we observe that the participation constraints vary according to the firm's cost type and price as well as the collusive pricing agreement itself. Thus, the participation constraints are type-dependent and endogenous.

Important extensions remain. We mention two. First, we assume throughout that each firm receives an i.i.d. cost shock in each period. We anticipate that our price-rigidity result is robust when cost shocks are positively correlated across firms, since a rigid-pricing schedule then sacrifices a smaller efficiency benefit (the firms often have similar costs, anyway). The possibility of private cost shocks that exhibit intertemporal persistence is much more complex, since a firm's past prices may then provide information about its current cost position. Second, in some applications, players interact repeatedly but only a single player observes private information in each period (for example, a government in a "policy game"). The question then arises as to whether the informed player should follow a "rule" (i.e., adopt behavior that is never responsive to private information) or be granted "discretion" (i.e., adopt behavior that is sometimes responsive to private information). In the latter case, incentive compatibility may require that the informed player is sometimes punished. Our analysis of SPPE above provides a foundation for such applications, since in each case all informed players bear a symmetric punishment.

8. Appendix

Proof of Lemma 3: We prove the result for step functions; strictly increasing functions are analogous. Let p_k and M_k denote the price and market share for step k, and let the step cover $(\theta_k, \theta_{k+1}]$. First, notice that for each θ, π is strictly increasing in ρ for $\rho < r$. Thus, we can define $\phi(\pi; \theta) \equiv \pi^{-1}(x; \theta)$. Now observe that $\Pi(\bar{\theta}, \bar{\theta}; p) - T(\bar{\theta}) \leq \pi(r, \bar{\theta}) M(\bar{\theta}; p)$. Then, recall our restriction that the lowest type on the highest step makes non-negative profit: $\Pi(\theta_K, \theta_K; p)\ - T(\theta_K) \geq 0$. This implies $p(\theta_K) \equiv p(\bar{\theta}) \geq \theta_K$. It follows that $\Pi(\bar{\theta}, \bar{\theta}; p) - T(\bar{\theta}) \equiv \pi(p(\bar{\theta}), \bar{\theta}) M(\bar{\theta}; p) - T(\bar{\theta})$

$$= [(\theta_K - \bar{\theta}) + (p(\bar{\theta}) - \theta_K)] M(\bar{\theta}; p) - T(\bar{\theta})$$

$$= \pi(\theta_K, \bar{\theta}) M(\bar{\theta}; p) + \Pi(\theta_K, \theta_K; p)\ - T(\theta_K)$$

$\geq \pi(\theta_K, \bar{\theta}) M(\bar{\theta}; p)$. Thus, we may find $\tilde{p}(\bar{\theta}) \in [\theta_K, r]$ such that $\Pi(\bar{\theta}, \bar{\theta}; p) - T(\bar{\theta}) = \pi(\tilde{p}(\bar{\theta}), \bar{\theta}) M(\bar{\theta}; p)$. From there, no-war prices are determined according to (IC-onM), which specifies that

$$\pi(\rho_k, \theta_k) M_k = \pi(\rho_{k-1}, \theta_k) M_{k-1}.$$

Each price is then determined as

$$\rho_{k-1} = \phi(\pi(\rho_k, \theta_k) \cdot \frac{M_k}{M_{k-1}}; \theta_k).$$

Since $\frac{M_k}{M_{k-1}} < 1$ and $\pi(\rho_k, \theta_k) \leq \pi(r, \theta_k)$, and since $\pi(\tilde{p}(\bar{\theta}), \theta_K) \geq 0$, this algorithm generates prices $\tilde{p}(\theta)$ that lie between cost and r at each step.

Finally, notice that if the pricing function is strictly increasing at the top, the no-war schedule will have to give $\tilde{p}(\bar{\theta}) = \bar{\theta}$, whence $\Pi(\overline{\theta}, \overline{\theta}; \tilde{p}) = 0$. To show that the lemma holds for this case, we confirm that $\Pi(\overline{\theta}, \overline{\theta}; p) - T(\overline{\theta}) = 0$. This follows since when p is strictly increasing at the top, we have that $M(\overline{\theta}; p) = 0$ and thus $\Pi(\overline{\theta}, \overline{\theta}; p) = 0$. Our assumption that $\Pi(\overline{\theta}, \overline{\theta}; p) - T(\overline{\theta}) \geq 0$ along with the requirement that $T(\overline{\theta}) \geq 0$ then confirms the desired equality. ■

Proof of Proposition 5: Equation (5.1) gives the pointwise objective of the firms, incorporating (IC-onM). It can be expressed as the sum of two terms, the profit-at-the-top and the efficiency rent, and maximized pointwise subject to the constraint that $p(\theta)$ is nondecreasing. Consider any collusive scheme that satisfies (IC-onM). If the scheme is a candidate for the optimal SPPE, then the allocation that it achieves can be realized without recourse to an equilibrium-path war. Since the market-share allocation function $M(\theta;p)$ is non-increasing, the profit-at-the-top is maximized when $M(\overline{\theta};p) = 1/n$. This allocation is delivered by a rigid-pricing scheme. Now consider the efficiency rent term. From (5.1), it follows that when $F(\theta)/f(\theta)$ is increasing, it always pays to take market share away from lower types and give it to higher types. Thus, the rigid-price scheme maximizes both of the two terms. However, even if F/f is not increasing everywhere, if $r - \bar{\theta}$ is large enough, the profit-at-the-top term dominates, and the pointwise objective is maximized by giving as much market-share as possible to high types.

Finally, our assumption that $f(\underline{\theta}) > 0$ (or, more generally, if $f'(\underline{\theta}) > 0$) implies that F/f is increasing at $\underline{\theta}$. Thus, there exists a pricing function that is rigid at the bottom and yields per-period profits strictly greater than the static Nash equilibrium (which requires p strictly increasing). Then, when firms are sufficiently patient, this pricing scheme can be sustained in a stationary equilibrium, using the threat of reversion to the static Nash equilibrium to satisfy the off-schedule constraints.

Lemma 5. *Suppose that there exists an $x \leq \bar{\theta}$ such that F is log-concave on $[\underline{\theta}, x)$ and log-convex on $[x, \bar{\theta}]$. Define the minimum efficiency rent the firms may receive when the on-schedule constraints are satisfied by: $\hat{R} = \min_p E[\frac{F}{f}(\theta)M(\theta;p)]$. Then:*
(i) If $r > \bar{\theta}$, $\delta > \frac{n-1}{n}$ and $x < \bar{\theta}$, $\hat{R}/(1-\delta) \leq \underline{\mathcal{V}}_s < \pi^{NE}/(1-\delta)$.
(ii) If firms are sufficiently patient, $\underline{\mathcal{V}}_s = \hat{R}/(1-\delta) \in \mathcal{V}_s$.
(iii) If F is log-concave ($x = \bar{\theta}$), then for all discount factors δ, $\underline{\mathcal{V}}_s = \pi^{NE}/(1-\delta) = \hat{R}/(1-\delta)$.

Proof. We begin with part (ii). Suppose that (p, v) implements $\underline{\mathcal{V}}_s$. The off-schedule constraint for $\bar{\theta}$ requires that $-(\bar{\theta} - p(\bar{\theta}))M(\bar{\theta};p) + \delta\bar{v}(p(\bar{\theta});p) \geq \delta\underline{\mathcal{V}}_s$. Using (5.1), the on-schedule incentive constraints imply that

$$-(\bar{\theta} - p(\bar{\theta}))M(\bar{\theta};p) + \delta\bar{v}(p(\bar{\theta});p) + E[\frac{F}{f}(\theta)M(\theta)] = \underline{\mathcal{V}}_s. \tag{8.1}$$

Substituting yields

$$E[\frac{F}{f}(\theta)M(\theta;p)] \leq (1-\delta)\underline{\mathcal{V}}_s. \tag{8.2}$$

Thus, no scheme can yield a lower per-period continuation value than $\hat{R}$.

Consider the following scheme. In the first period, firms use the pricing scheme $p^L(\theta)$, where $p^L(\bar{\theta}) = \frac{1}{n}\bar{\theta} + \frac{n-1}{n}x$ and the pricing scheme is rigid on $[x, \bar{\theta}]$ and strictly increasing elsewhere. Under our assumption about the distribution function, the pricing scheme p^L minimizes expected informational rents. The continuation value function v^L specifies $v^L(\mathbf{p}(\boldsymbol{\theta})) = \underline{\mathcal{V}}_s$ if $\theta_j < x$ for any firm j (i.e. if the market price is below $p^L(\bar{\theta})$), while $v^L(\mathbf{p}(\boldsymbol{\theta})) = v_r$ otherwise (i.e. if the market price is $p^L(\bar{\theta})$). Off-schedule deviations are punished by returning to $\underline{\mathcal{V}}_s$. We choose v_r to satisfy with equality the off-schedule constraint for $\bar{\theta}$, so that profit-at-the-top is $\delta\underline{\mathcal{V}}_s$. The on-schedule constraints then imply that the payoff from this collusive scheme is $\delta\underline{\mathcal{V}}_s + \hat{R}$. Now, we know from (8.2) that $\delta\underline{\mathcal{V}}_s + \hat{R} \leq \underline{\mathcal{V}}_s$. Further, if this collusive scheme is an SPPE scheme, then the definition of $\underline{\mathcal{V}}_s$ implies that $\delta\underline{\mathcal{V}}_s + \hat{R} \geq \underline{\mathcal{V}}_s$. Hence, if this collusive scheme satisfies all other off-schedule incentive constraints, and if firms are sufficiently patient that $v_r \in V_s$, then $\underline{\mathcal{V}}_s = \hat{R}/(1-\delta) \in \mathcal{V}_s$.

To see that the proposed scheme indeed satisfies all other constraints, notice that since $p^L(\bar{\theta}) > x$, on-schedule incentive compatibility determines p^L for $\theta < x$, and in particular it requires that $p^L(\theta) > \theta$ for all θ. Since the pricing function is strictly increasing on this interval, there are no additional off-schedule constraints. Further, as may be confirmed, at the price $p^L(\bar{\theta}) = \frac{1}{n}\bar{\theta} + \frac{n-1}{n}x$, the off-schedule constraint for x (who is tempted to under-cut) is satisfied exactly when it is for $\bar{\theta}$ (who is tempted to raise price).[38]

Part (iii) follows from part (ii), together with the fact that repeated play of the static Nash equilibrium is always an SPPE. For part (i), observe that firms can use a flat scheme on an interval $[y, \bar{\theta}]$. If y is sufficiently close to $\bar{\theta}$, the incentive to deviate off-schedule can be made arbitrarily small; and Proposition 10 establishes that some future reward greater than $\pi^{NE}/(1-\delta)$ is available when $\delta > \frac{n-1}{n}$ and $r > \bar{\theta}$. ■

Proof of Proposition 6: Part (i) follows directly from Lemma 5. Parts (ii) and (iii) follow from (i) and the analysis provided in the text.

Proof of Proposition 7: Let (p, v) be the factorization of an original scheme that constitutes an optimal SPPE. As before, let $T(\theta) = \delta[\overline{\mathcal{V}}_s - \bar{v}(p(\theta); p)]$ and note that (p, T) must satisfy the constraints of the Mechanism Design Program. Using Lemma 3, and exploiting the inelastic-demand assumption, we can find an alternative scheme, $(\tilde{p}, \tilde{T} \equiv 0)$, such that $\Pi(\theta, \theta; p) - T(\theta) = \pi(\tilde{p}(\theta), \theta) M(\theta; p)$ for all θ. Clearly, this implies that $\widetilde{p}(\theta) = p(\theta)$ for any θ such that $T(\theta) = 0$ while $T(\theta) > 0$ implies that $\widetilde{p}(\theta) < p(\theta)$. The factorization associated with the alternative scheme is $(\tilde{p}, \widetilde{v} \equiv \overline{\mathcal{V}}_s)$.

Now, we compare the off-schedule incentive constraints across the two schemes. To do so, let $w \in \mathcal{V}_s$ be the punishment used following an off-schedule deviation in the original scheme. When the on-schedule incentive constraint is met, the off-schedule incentive constraint is satisfied if the lowest type on any interval of costs over which prices are rigid does not prefer to charge a slightly lower price.[39] As the original schedule satisfies the off-schedule incentive constraints, it follows that the lowest-cost type θ_k on any step k will not choose to slightly undercut the step-k price, $p(\theta_k)$:

$$\pi(p(\theta_k), \theta_k)[(1 - F(\theta_k))^{n-1} - M(\theta_k; p)] \leq \delta\{\overline{\mathcal{V}}_s - T(\theta)/\delta - w\} \tag{8.3}$$

Now consider the analogous off-schedule incentive constraint for the alternative schedule:

$$\pi(\widetilde{p}(\theta_k), \theta_k)[(1 - F(\theta_k))^{n-1} - M(\theta_k; p)] \leq \delta\{\overline{\mathcal{V}}_s - w\}. \tag{8.4}$$

In (8.3) and (8.4), the LHS's represent the current-period incentive to cheat. This incentive is either the same under the two schedules (when $T(\theta_k) = 0$) or strictly lower under the alternative schedule (when $T(\theta_k) > 0$, since then $\widetilde{p}(\theta_k) < p(\theta_k)$). The RHS's represent the expected discounted values of cooperation in the next and all future periods. The RHS is also either the same (when $T(\theta_k) = 0$) or strictly higher

[38] We note that the price described requires the least patience for implementation. If we were to raise $p^L(\bar{\theta})$, this would increase the incentive of type x to undercut the collusive price, which would tighten the off-schedule constraint for type x and thereby require a greater future reward v_r. Thus, overall firm profits go up. Lowering $p^L(\bar{\theta})$ relaxes the off-schedule constraint for type x; but it tightens the off-schedule constraint for type $\bar{\theta}$, again requiring a greater future reward v_r. However, the required increase in the reward is exactly equal to the reduction in firm profits, so that the overall scheme implements the same value. Yet, a reward of greater magnitude is required, and for firms of moderate patience, a greater reward may not always be available.

[39] This statement follows from four observations. First, over a segment for which price is strictly increasing, the lowest type clearly has nothing to gain from a small price cut. Second, over a segment for which price is flat, the on-schedule incentive constraint requires that either the lowest type on this segment is $\underline{\theta}$ or that the price schedule jumps discontinuously down for lower types. Third, over a segment for which price is flat, the incentive to undercut is greatest for the lowest-cost type. Together, these observations imply that the collusive scheme is robust against off-schedule price-cutting deviations, so long as the lowest type on a flat segment does not choose to cut price slightly. Fourth, off-schedule price-increasing deviations are unattractive under the alternative schedule (given that this is true for the original schedule and given that on-schedule constaints are satisfied). Details associated with this final observation are in our working paper.

under the alternative schedule (when $T(\theta_k) > 0$). Thus, eliminating the war in this way simultaneously raises the expected discounted value of cooperation and lowers the current incentive to cheat; as a consequence, if there is no incentive to undercut in the original collusive arrangement, then there will certainly be no such incentive under the alternative arrangement. We then conclude that $(\tilde{p}, \tilde{v} \equiv \overline{\mathcal{V}}_s)$ satisfies all of the constraints of the Interim Program. This in turn implies that $\overline{\mathcal{V}}_s = \Pi(\theta, \theta; \tilde{p})/(1-\delta)$, corresponding to an optimal, stationary SPPE where $\tilde{p}$ is used in every period. ■

Proof of Proposition 8: Consider a candidate two-price scheme, denoted $\breve{p}(\theta; \theta_2)$, with a top-step price $\rho_2 \in (\underline{\theta}, r]$ and a breakpoint $\theta_2 \in (\underline{\theta}, \overline{\theta})$. Given ρ_2 and θ_2, the on-schedule incentive constraint determines the low-step price, ρ_1, as the ρ_1 that solves

$$\pi(\rho_1, \theta_2)\mu(\underline{\theta}, \theta_2) = \pi(\rho_2, \theta_2)\mu(\theta_2, \bar{\theta}), \tag{8.5}$$

where $\mu(\theta_k, \theta_{k+1})$ is the market share for cost types on a step on $[\theta_k, \theta_{k+1}]$. If the lowest step is small enough, the gain in market share from undercutting ρ_1 is small, and the binding off-schedule constraint is the constraint for θ_2, the lowest type on the top step. The off-schedule constraint is written:

$$(\rho_2 - \theta_2)[\mu(\theta_2, \bar{\theta}) - (1 - F(\theta_2))^{n-1}] + \frac{\delta}{1-\delta}[E\Pi(\theta, \theta; \breve{p}(\cdot; \theta_2)) - \pi^{NE}] \geq 0. \tag{8.6}$$

At $\theta_2 = \underline{\theta}$, $\rho_2 = r$ and $\delta = \delta^*$, (8.6) holds with equality. Notice that increasing θ_2 affects both the incentive to cheat as well as the future value of cooperation. Since introducing a small lower step results in a low price for only a correspondingly small region of types, $E\Pi(\theta, \theta; \breve{p}(\cdot; \theta_2))$ decreases smoothly in θ_2, and thus (8.6) is differentiable in θ_2 at $\theta_2 = \underline{\theta}$. Taking the derivative of the left-hand side of (8.6) with respect to θ_2, evaluated at $\theta_2 = \underline{\theta}$ and $\rho_2 = r$, and using (8.5) yields

$$\frac{n-1}{n}\left[f(\underline{\theta})(n-1)(r - \underline{\theta}) + 1 - \frac{\delta}{1-\delta}(r - E\theta)f(\underline{\theta})\right].$$

This expression is non-negative at $\delta = \delta^*$ if and only (6.3) holds. Thus, we conclude that increasing θ_2 at $\theta_2 = \underline{\theta}$ relaxes the off-schedule incentive constraint.

When the distribution function is log-concave, rigid pricing at r is optimal so long as $\delta = \delta^*$. This implies that $E\Pi(\theta, \theta; \breve{p}(\cdot; \theta_2))$ is maximized at $\theta_2 = \underline{\theta}$ and/or $\theta_2 = \bar{\theta}$. Consider first $\theta_2 = \bar{\theta} - \varepsilon$ for ε small. When introducing a small step at or near $\bar{\theta}$, the low-step price must be set to satisfy (8.5). As $\mu(\theta_2, \bar{\theta}) \to 0$ when $\varepsilon \to 0$, (8.5) implies that $\rho_1 \to \bar{\theta}$ when $\varepsilon \to 0$. Since this low price is used by all types on $[\underline{\theta}, \theta_2)$, expected per-period profits are approximately $\pi(\bar{\theta}, E\theta)/n$ for a small step. Consider second $\theta_2 = \underline{\theta} - \varepsilon$ for ε small. We may then set $\rho_2 = r$, with ρ_1 then determined by (8.5). Since the high price is used by all types on $(\theta_2, \bar{\theta}]$, expected per-period profits are approximately $\pi(r, E\theta)/n$, as in the rigid-pricing scheme, for a small step. Recalling that $r \geq \bar{\theta}$, we conclude that $E\Pi(\theta, \theta; \breve{p}(\cdot; \theta_2))$ is maximized at $\theta_2 = \underline{\theta}$.

Finally, we must show that when δ is close to δ^*, the optimal two-step scheme dominates any other scheme with other market-share allocation schemes. Consider an alternative scheme that has more than one point of strict increase; for simplicity, suppose that this pricing scheme is a step function. Consider the bottom two steps of such a scheme. If we condition on types on the bottom two steps of this scheme, Proposition 5 can be applied without further modifications. When the distribution is log-concave, it is always better two combine the two bottom steps into one (thus reallocating market share to types with a higher $F(\theta)/f(\theta)$); or to turn a region of strict increase into one of pooling. Proceeding in this manner, a multi-step scheme is successively dominated by schemes with fewer steps. Thus, the best two-step scheme, if it clears the off-schedule constraints, is better than any other scheme with a larger number of steps. Since a two-step scheme with a small lower step is better than any other two-step scheme, and (6.3) and δ close to δ^* imply that this scheme can be supported, it must also dominate any schemes with a larger number of steps. ■

Proof of Proposition 10: We treat part (ii) first. Recall the notation developed above for pricing functions that can be represented by a finite number of subintervals. Let $M_k \equiv \mu(\theta_k, \theta_{k+1})$ be the market share for cost types on a step on $[\theta_k, \theta_{k+1}]$. Consider a candidate solution which specifies strictly increasing pricing on interval $(\theta_{j-1}, \theta_j]$. Consider introducing a tiny step on the interval $(\theta_j - \varepsilon, \theta_j]$. If there is some gain to future cooperation, then for ε small enough, introducing a tiny step does not introduce a new off-schedule incentive constraint, at least not one which is binding. Suppose it were indeed optimal for $(\theta_{j-1}, \theta_j]$ to be a region of separation of types. Then, if we chose ε to maximize the objective when $(\theta_{j-1}, \theta_j - \varepsilon]$ is a region of separation and pooling takes place on $(\theta_j - \varepsilon, \theta_j]$, the solution $\varepsilon = 0$ should be a local maximum.

Note that $\rho_k - \theta_k = \Pi(\theta_k, \theta_k; p)/M_k$. Rearranging, we thus observe that the off-schedule constraint for type θ_k, at the left endpoint of step k, can be written as:

$$\frac{\delta}{1-\delta}(E[\Pi(\theta, \theta; p)] - \underline{\mathcal{V}}_s) - \Pi(\theta_k, \theta_k; p)\frac{(1 - F(\theta_k))^{n-1} - M_k}{M_k} \geq 0. \tag{8.7}$$

We can then express the off-schedule constraint for step $k < j$ as a function of ε. Tedious calculations (using in particular the fact that $\mu_1(\theta_j, \theta_j) = \frac{1}{2}\frac{\partial}{\partial\theta_j}(1-F(\theta_j))^{n-1}$, and $\mu_{11}(\theta_j, \theta_j) = \frac{1}{3}\frac{\partial^2}{\partial\theta_j^2}(1-F(\theta_j))^{n-1} - \frac{1}{6}f'(\theta_j)(n-1)(1-F(\theta_j))^{n-2}$) establish that the first and second derivatives of (8.7) with respect to ε are equal to 0 when $\varepsilon = 0$, while the third derivative is given by:

$$\frac{\delta}{1-\delta}\left[f(\theta_j)^2 - F(\theta_j)f'(\theta_j)\right] + f'(\theta_j)\frac{(1 - F(\theta_k))^{n-1} - M_k}{M_k}. \tag{8.8}$$

If (8.8) is positive, $\varepsilon = 0$ is a local minimum; in that case as ε increases, the off-schedule constraint is relaxed, so that the strictly increasing scheme is dominated. If the density is non-decreasing, (8.8) is positive by log-concavity of the distribution. Suppose now that $f'(\theta_j) < 0$. When $\delta > \frac{n-1}{n}$, since $\frac{(1-F(\theta_k))^{n-1}-M_k}{M_k} < n-1$, (8.8) is positive if $f(\theta_j)^2 - F(\theta_j)f'(\theta_j) \geq -f'(\theta_j)$. But this is true whenever $1-F$ is log-concave.

Now return to part (i). Using a similar logic, we consider starting from a strictly increasing pricing schedule (i.e. the static Nash equilibrium) and introducing a small step on $[\bar{\theta}-\varepsilon, \bar{\theta}]$ at $p(\bar{\theta}) = r$. This will improve per-period profits if $r > \bar{\theta}$. We must then verify that it improves per-period profits faster than it tightens the off-schedule constraint for this upper step. Tedious calculations show that the first $(n-2)$ derivatives of the off-schedule constraint with respect to ε are zero, while the $(n-1)^{th}$ derivative is equal to $\frac{n-1}{n}(\frac{\delta}{1-\delta} - (n-1))(r-\bar{\theta})f(\bar{\theta})^2 > 0$. Thus, $\varepsilon = 0$ is a local minimum. Since the off-schedule constraint is satisfied at $\varepsilon = 0$, introducing a small step relaxes it. Finally, for a given market-share allocation that has a step at the top (of arbitrary size), consider the optimal $p(\bar{\theta})$. Recalling (4.2), increasing $p(\bar{\theta})$ increases $E[\Pi(\theta, \theta; p)]$ at the same rate that it increases $\Pi(\theta_k, \theta_k; p)$. Since $\frac{(1-F(\theta_k))^{n-1}-M_k}{M_k} < n-1$, inspection of (8.7) implies that increasing $p(\bar{\theta})$ increases the future value of cooperation, $\frac{\delta}{1-\delta}E\Pi(\theta, \theta; p)$, faster than the incentive to deviate off-schedule whenever $\delta > \frac{n-1}{n}$. ■

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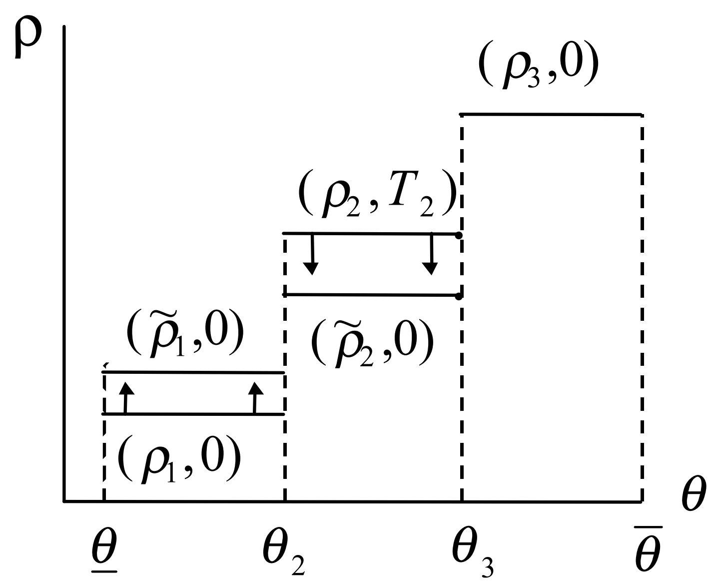

ρ
$(\rho_3, 0)$
(ρ_2, T_2)
$(\tilde{\rho}_1, 0)$
$(\tilde{\rho}_2, 0)$
$(\rho_1, 0)$
θ
$\underline{\theta}$
θ_2
θ_3
$\overline{\theta}$

Figure 1