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Gautam Goswami, Thomas H. Noe, Michael J. Rebello

Institutions: Fordham University, Federal Reserve Bank of Atlanta, Georgia State University

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Goswami, Gautam; Noe, Thomas; Rebello, Michael

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Gautam Goswami, Thomas Noe, and Michael Rebello

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Abstract: In uniform-price auctions of shares there exist collusive equilibria in which bidders capture the entire surplus from the auction as well as competitive equilibria in which the auctioneer captures the entire surplus from the auction. We provide *experimental* evidence that, in uniform-price auctions, non-binding pre-play communication facilitates convergence to collusive equilibrium outcomes. On the other hand, regardless of the opportunities for communication, in discriminatory-auction experiments subject strategies conform closely with the unique equilibrium in undominated strategies in which bidders' gains are equal to the smallest "tick size" in the bidding schedule. This evidence suggests that uniform-price auctions of Treasury securities may result in lower revenues than the currently employed discriminatory procedure.

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Please address questions of substance to Gautam Goswami, Thomas H. Noe, or Michael Rebello, Finance Department, College of Business Administration, Georgia State University, Atlanta, Georgia 30303, 404/651-2628 (office), 404/651-2630 (fax).

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COLLUSION IN UNIFORM-PRICE AUCTIONS: EXPERIMENTAL EVIDENCE AND IMPLICATIONS FOR TREASURY AUCTIONS

INTRODUCTION

Historically, the U.S. Treasury has marketed Treasury bills employing a sealed bid discriminatory mechanism in which multiple units of Treasury bills are auctioned. Individual bidders can submit bids at multiple prices and vary the number of units bid for at each price. The price paid for each unit received is the actual price bid by the winning bidders. Until recently, economists have advocated the use of sealed bid uniform-price auctions for Treasury bills (see, for example, Milgrom 1989). In these auctions, too, individual bidders can submit a schedule of bids at multiple prices. However, in uniform-price auctions, unlike discriminatory auctions, all units are sold at the same price, the market-clearing price.

Economists' advocacy of uniform-price auctions is based on the theory of auctions of units—auctions in which each participant can bid only for a single unit of the good being auctioned. However, as demonstrated by Back and Zender (1993) and Wilson (1979), the theory of single unit auctions is *not* readily extended to "auctions of shares," multiple-unit auctions in which bidders can submit bids at multiple prices.[1] In fact, in uniform-price auctions of shares, there exist self-enforcing strategies for bidders that allow them to "collude." In doing so, they are able to maximize their payoffs at the expense of the auctioneer. In these auctions other self-enforcing strategies also exist in which "competitive" outcomes obtain, and the auctioneer is able to extract the entire surplus from the auction. In contrast, in discriminatory auctions of shares,

[1] The term "auctions of shares" was used by Wilson (1979) to describe such auctions.

there exist no collusive equilibria. In fact, if prices at which bidders can submit bids are discrete, there exists a unique equilibrium in undominated strategies in which the auction clears at the highest price that is lower than the value of the good. Thus, the optimal mechanism from the perspective of the auctioneer depends on the equilibria most likely to obtain in uniform-price auctions of shares.

In this paper, we provide experimental evidence on the effect of the mechanism design and non-binding preplay communication on clearing prices and demand schedules in auctions of shares. Our experiments on uniform-price and discrimnatory auctions indicate that the sensitivity of outcomes to preplay communication varies significantly with the auction mechanism. Clearing prices and auctioneer's surplus are significantly lower, and aggregate demand at the lowest permitted price is significantly higher in uniform-price experiments permitting preplay communication. The highest clearing prices, auctioneer's surplus, and aggregate demand at the competitive price are observed in uniform-price experiments without preplay communication. In stark contrast to the uniform-price treatments, preplay communication has little impact in the discriminatory treatments.

The observed patterns of clearing prices and demands in uniform-price experiments permitting communication resulted from subjects' demand schedules conforming closely to those characterizing collusive equilibria. In the discriminatory treatments, regardless of communication opportunities, strategies approximate the unique equilibrium in undominated strategies from the inception of the experiment. In the uniform treatment without communication, a tendency to diverge from both collusive and competitive behavior is observed. We also examine the symmetry and stability of subject strategies. The symmetry of subjects' strategies increases over time in all treatments. The tendency toward the adoption of symmetric strategies is pronounced in the uniform-price treatment with communication as well as in the discriminatory treatments.[2] Variation in subject strategies diminished over time, with the tendency to increased stability being more pronounced in the absence of opportunities for communication.

This is not the first paper to analyze the issuance process for Treasury securities. A number of researchers have investigated the choice of auction mechanism by the Treasury.[3] Some of this analysis has investigated the primary market for Treasury bills from an industrial organization perspective: examining the degree of market concentration and participants' profits (see, for example, Meltzer and von der Linde 1960 and Reiber 1964). Friedman (1960), taking a different approach, examined the range of bids in 13 successive auctions. A second approach, adopted by Simon (1992), relied on a comparison of the markup of Treasury auction yields over when-issued yields. A third approach was adopted by the Treasury itself. In September 1992, the Treasury undertook a one-year experiment using the uniform-price auction format for its two-year and five-year note auctions.

These studies, while providing insights into the auctions of shares, do not permit the isolation of strategic bidder behavior from institutional factors. For example, if the group of bidders is small enough and they either have other linkages or they expect to participate in a number of auctions, collusive behavior may emerge even when it is not self-enforcing in any given auction (see, for example, Fudenberg and Maskin 1986). Similarly, institutional factors make studies, such as the Treasury's experiment, difficult to interpret. If dealers believe that, by eschewing profits during the experimentation period, they can ensure that the uniform-price auction mechanism is adopted and they can earn larger profits after the adoption, they may have

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[2] Even in uniform-price treatments with subject communication in which the parameters did not allow for the existence of completely symmetric collusive Nash equilibria, subjects gravitated towards symmetric strategies that were "close" to almost-symmetric equilibrium strategies. However, in these cases the convergence was slower and less stable.

[3] In addition to empirical investigations of the U.S. Treasury auctions, researchers have also studied auction mechanisms in other contexts. For example, Umlauf (1993) examines Mexican Treasury auctions and Tenorio (1993) examines the Zambian foreign exchange markets.

an incentive to utilize the self-enforcing competitive strategies during the experimentation period, switching to collusive strategies afterwards.

Our experimental data complement these empirical studies. In a controlled experimental setting, such as ours, it is possible to isolate the effects of communication and alternative allocation mechanisms. In addition to providing evidence on equilibrium selection in auctions of shares, our paper extends the extant experimental literature on auctions. This literature is extensive (see, for example, Smith 1967 and Cox, Smith, and Walker 1984). Much of this literature has also focused on comparing the uniform and discriminatory multi-unit auction mechanisms. However, this strand of research on multi-unit auctions has been limited to examining the outcomes of auctions of units. These auctions do not allow for self-enforcing bidding strategies that extract the auctioneer's surplus. Thus, the issues we attempt to address cannot be addressed in such settings.

The paper is organized as follows. In Section 1, a simple model is developed that we use to characterize the Nash equilibrium strategy vectors in the various experimental treatments. In Section 2, we describe the procedures followed in performing the experiments. In Section 3, we describe the results of the experiments in detail. Section 4 contains some concluding remarks. The Appendix presents the proofs of some of the results derived in Section 1.

1. Uniform and Discriminatory Auctions of Divisible Goods

As a first step to examining bidder strategies in an experimental setting, we characterize equilibrium behavior in auctions. This involves the specification of bidder and auctioneer payoffs and strategies. Our parameterizations of auctions have been selected to make the auction mechanisms transparent to the subjects, ease their computational burden, and conform with some salient institutional characteristics of Treasury auctions. To simplify the computations and make the auction mechanism transparent, we specify a common unit value. To conform with the institutional characteristics of Treasury auctions, we restrict the number of units that can be bid for by a subject and restrict subjects to placing bids at fixed and discrete price levels.[4]

1.1 Description of the Experimental Setting

In each of the experiments, 100 units of a good are for sale. Each unit of the good has a value of 20 to the bidders and a value of 10 to the auctioneer. Bidders in the auction attempt to maximize their monetary payoffs. There are 11 bidders in the auction.[5] Bidders simultaneously specify demand schedules for the good. Each schedule specifies the number of units the bidder is willing to purchase at each of three prices: 10, 15, and 20. Each bidder can submit only non-negative integer-valued bids that sum to no more than 100. Let d_{ip} represent the number of units of the good demanded by bidder i at price p. Then bidder i's demand schedule can be represented by a 3-tuple, $d_i \equiv (d_{i20}, d_{i15}, d_{i10})$.

Each bidder's allocation of the good is determined by her demand schedule and the aggregate demand schedule. Let d represent a vector of demand schedules, where $d \equiv (d_1, d_2, \ldots\ldots, d_{11})$. Let $A_p(d)$ represent the aggregate demand of bidders at a price p, where $A_p(d) \equiv \Sigma_i d_{ip}$. Similarly, let $C_p(d)$ represent the cumulative aggregate demand at price p, where

$$C_p(d) \equiv \Sigma_{p' \geq p} A_{p'}(d).$$

[4] These specifications differentiate our model from those of Back and Zender (1993) and Wilson (1979), who allow for bidding strategies to range over a continuum of price and quantities and allow for more general informational structures. These points of difference, however, make no qualitative difference to the nature of the Nash equilibrium outcomes.

[5] Most experiments involved exactly 11 subjects. Formal analysis of subject behavior and experimental outcomes is conducted only on this core group of experiments. In some experiments, because of unavoidable circumstances, the number of subjects differed from 11. The rationale for using 11 subjects in our experiments is elucidated in footnote 8.

Bidders' payoffs are determined by their allocations and the "clearing price" in the auction. A clearing price exists only if at least 100 units of the good are demanded. If less than 100 units are demanded, the auction is canceled and bidders receive no payoffs. On the other hand, if the auction is successful, the clearing or "stop-out" price for the auction is the highest price at which cumulative demand first equals or exceeds 100. To complete the description of the auction, let $C^+(d)$ represent the cumulative aggregate demand at the price immediately above the clearing price, p^*, where $C^+(d) \equiv C_{p^*}(d) - A_{p^*}(d)$. Given the clearing price, each bidder receives the number of units she demanded at prices above the clearing price and a pro-rated share of her demand at the clearing price. No allocations are received for demand at prices lower than the clearing price. Let the pro-ration factor at price p be represented by the function $\pi_p(d)$, where $\pi_p(d) = 1$ at prices above the clearing price and $\frac{100 - C^+(d)}{A_{p^*}(d)}$ at the clearing price. Thus, bidder i's allocation at prices greater than or equal to the clearing price is $r_{ip}(d) = d_{ip}\ \pi_p(d)$.

The bidder's payoff, however, is determined by the amount she is required to pay for her allocation. This amount varies with the rules of the auction. In a uniform-price auction, the price paid for all units is the clearing price. Thus, bidder i's payoff in a successful auction can be represented by $V_i(d)$, where

$$V_i(d) \equiv (20 - p^*)\ \Sigma_{p \geq p^*}\, r_{ip}(d).$$

In a discriminatory auction, for each unit that they receive, bidders pay the price at which the bid was submitted. Thus, bidder i's payoff in successful auction can be represented by $V_i(d)$, where

$$V_i(d) \equiv \Sigma_{p \geq p^*}\, r_{ip}(d)\ (20 - p).$$

Under both auction mechanisms, the auctioneer's surplus can be represented by $10 * 100 - \Sigma_i V_i(d)$.

1.2 Outcomes of the Auctions

In characterizing the outcomes of these auctions, we focus our attention on Nash equilibria. A Nash equilibrium of these auction games is a strategy vector such that the demand schedule submitted by each bidder is a best response to the demand schedules of other bidders. In line with much of the literature on strategic decision-making, we formally examine only Nash equilibria in which bidders adopt pure strategies. When multiple equilibria exist producing the same clearing price, following Back and Zender (1993), we focus on the symmetric equilibria supporting these outcomes. These equilibria are focal for two reasons: (i) all the bidders are identically endowed and it is more likely that coordination would implement outcomes that would not discriminate between bidders; (ii) communication between subjects seems to indicate that they expected equal treatment.[6]

Some properties of these Nash equilibria are fairly obvious. Regardless of the auction mechanism employed, submitting a demand vector of less than 100 units is a dominated strategy. The logic behind this result is simple, submitting demand at the lowest price of 10 is never worse than, and is sometimes better, than not submitting any demand at all.[7]

Lemma 1: *In both uniform-price and discriminatory auctions, demanding less than 100 units is a dominated strategy.*

[6] Almost all the literature on auctions has focused on symmetric equilibria (see, e.g., Vickrey 1961). Symmetric equilibria have also been the focus of research in related problems such as corporate takeovers (Holmstrom and Nalebuff 1992).

[7] Using the elimination of dominated strategies as a solution concept is common in the literature (see, e.g., Kohlberg and Mertens 1986). The support for this solution concept is based on both classical decision theory (Luce and Raiffa 1957) and the theory of evolutionary stable strategies (Samuelson 1991). However, Samuelson (1992) points out that this solution concept cannot be deduced from the common knowledge of rationality. Further discussion on this subject and its relationship with our research appears below.

The above result implies the following obvious corollary. In any Nash equilibrium in undominated strategies, d^*, total demand is no lower than the number of units for sale and the auction is successful.

In a uniform-price auction, a multiplicity of equilibria exist. Some of the outcomes supported by these equilibria are "competitive" in that they ensure that the good is sold at the reservation price of the buyers. These competitive outcomes are supported by strategy vectors in which total demand at a price of 20 is large enough to ensure that no individual bidder can lower the clearing price by withholding her demand at a price of 20. This implies that, for all bidders, the sum of all units demanded by all other bidders must be no lower than 100, or equivalently, each bidder must demand at least 10 units at the price of 20. Given the equilibrium strategy vector, the payoff from all possible strategies available to any agent is 0. This argument is presented in Proposition 1.

Proposition 1: *In any symmetric Nash equilibrium of a uniform-price auction, the clearing price equals the competitive price if and only if each bidder demands at least 10 units at a price of 20.*

In addition to the competitive outcomes characterized in Proposition 1, in uniform-price auctions there also exist "collusive" outcomes. In these outcomes, bidders are able to extract the maximum possible value from the auction. As Back and Zender (1993) and Wilson (1979) demonstrate, in a similar framework, the cumulative aggregate demand schedule induced by equilibrium strategies is highly inelastic. Thus, any attempt by an individual bidder to increase her allocation by placing a larger demand at a higher price results in a large jump in the clearing price. Because of this large increase in the clearing price, the bidder is subjected to a large loss on her original allocation. Further, given the inelasticity of the cumulative demand schedule, her allocation increases by only a small amount. Thus, her loss from the increase in the clearing price more than offsets the gain from the increased allocation.

As is demonstrated in the following proposition, 99 shares have to be purchased at the price of 20 in any collusive equilibrium. Symmetric equilibria exist because the number of bidders is a divisor of 99.[8]

Proposition 2: *(i) In a uniform-price auction, there exists a symmetric Nash equilibrium such that bidders extract all the surplus from the auction. (ii) In this equilibrium, bidders demand 9 units at a price of 20, 0 units at a price of 15, and 91 units at a price of 10.*

It is easy to demonstrate that the equilibrium outcomes characterized in Proposition 2 are coalition-proof (see Bernheim, Peleg, and Whinston 1987), while outcomes in which the clearing price is either 20 or 15 are not coalition-proof. Further, Bernheim, Peleg, and Whinston 1987 argue that the logical candidate equilibria that result from costless pre-play communication between agents are coalition-proof. Thus, outcomes in which the auction clears at 10 seem of particular interest.

In addition to equilibria in which the clearing price is 20 or 10, the uniform-price auction mechanism also has equilibria in which the clearing price is 15. The enforcement mechanism that sustains these equilibria is virtually identical to that which sustains equilibria with a clearing price of 10—the cumulative aggregate demand schedule induced by equilibrium strategies is highly inelastic, ensuring that penalties for deviations from equilibrium strategies through the placement of bids at prices above the clearing price are sufficient to deter deviations. Symmetry and the requisite inelasticity of the cumulative schedule are achieved by individual bidders demanding 9 units at a price of 20, 91 units at the price of 15, and 0 units at the price of 10.

[8] In a more general setting Nash equilibria of uniform-price auctions, the clearing price equals the auctioneer's value only if 99 units are demanded at the competitive price. This ensures that any shift in demand to a higher price raises the clearing price without inducing a significant increase in allocation. In order for the equilibrium to be symmetric all bidders must submit identical bids at the competitive price. Thus, for an equilibrium to be both collusive and symmetric, the number of shareholders must divide 99. Non-symmetric collusive equilibria, however, exist for many other parameterizations. With this caveat, all of our results extend to settings with at least three players.

As can be seen from the above discussion, there exists great variation in the equilibrium clearing prices of uniform-price auctions. There is less variation for discriminatory auctions (see Back and Zender 1993). Demanding any units at 20, because of the discriminatory nature of the auction, locks in a 0 profit on those units and thus is a dominated strategy. However, Nash equilibria exist in which demand at 20 is submitted. In these equilibria, each bidder is held to a zero profit regardless of her strategies, with the other bidders playing dominated strategies and forcing the auction to clear at 100. The equilibrium strategies in this case are identical to the strategies that induce a clearing price of 20 in uniform price auctions.

As we demonstrate in Proposition 3, the only equilibrium in undominated strategies for discriminatory auctions ensures that the clearing price is 15. In equilibrium, demand is concentrated at a price of 15, with every bidder maximizing her demand at this price. To see the uniqueness of this equilibrium, note that collusive outcomes with a clearing price of 10 are not sustainable. Because of the discriminatory nature of the auction, a bidder is able to switch some of her demand to a higher price without affecting her profits on the unchanged portion of her demand. The increased allocation resulting from this switch increases the bidder's payoff.

Proposition 3: *In a discriminatory-price auction, a unique Nash equilibrium in undominated strategies exists in which all bidders submit all demand at a price of 15, the price one tick below the competitive price.*

The above results provide predictions regarding equilibrium behavior in both uniform-price auctions and discriminatory auctions. In the following sections we describe the experimental methodology and examine subjects' behavior in light of the predictions of these results.

2. Experimental Methodology

Experiments were conducted on groups of graduate business students. The allocation mechanism as well as the opportunities for subject communication varied. Four treatments were performed: (i) uniform-price auction without communication (U), (ii) uniform-price auction with subject communication (UC), (iii) discriminatory auction without communication (D), and (iv) discriminatory auction with subject communication (DC). The relevant details of all the experiments are presented in Table 1. Henceforth, each experiment will be referred to by its name that denotes both the treatment and a number to distinguish it from other repetitions of the same treatment. For example, UC2 refers to repetition 2 of the uniform-price treatment with subject-communication. An asterisk is affixed to the name of each experiment involving a number of subjects unequal to 11.

Each experiment was performed in a computer laboratory using a local area network to communicate subjects' bids, their allocations, and their payoffs. Subjects were seated so as to prevent others from observing their computer screens. Most experiments lasted approximately 45 minutes, with the experiments involving communication between subjects lasting 10-15 minutes longer. First, subjects were presented an instructional handout that explained the rules of the game and the process used in determining their payoffs. They were given 5 minutes to peruse the instructions. After this, one of the experimenters verbally explained the auction mechanism and the computer interface. This took approximately 10 minutes. The logistics involved in running the experiments allowed for subject communication and discussion after the instructions were completed but before the experiment commenced. The time available for such discussion varied across experiments. However, the opportunities for communication were similar across all treatments.

An experiment commenced when subjects first entered their demand schedules into their terminals. Once all bids had been entered, the results of the auction were electronically computed. Then each subject was electronically informed of the clearing price, her allocation, and her payoff from the auction. Other than the clearing price, subjects were not presented any information regarding other subjects' demand schedules or the aggregate demand schedule. At this point, subjects were given an opportunity to record their payoffs and allocations for their own reference. Once this process, or round, was complete, the auction was repeated. Each of the first four rounds took approximately 4 minutes to complete. Subsequent rounds took approximately 2 minutes to complete. All experiments were run for at least twelve rounds, with most consisting of exactly twelve rounds. Variations in the number of rounds across experiments resulted from attempts to maximize the number of rounds subject to time constraints. Subjects were not informed of the number of rounds to be played, and a perusal of the results indicates that the deletion of results from rounds after round twelve would have no qualitative impact on our conclusions.

In treatments in which communication was not permitted, subjects were not allowed to speak to each other once the experiment commenced. In treatments allowing communication, subjects were allowed to speak to each other every two rounds. They were allowed 5 minutes for the first discussion and 3 minutes for subsequent discussions. However, no communication was allowed when subjects were entering their strategies or recording their payoffs. Communication was governed by the following rules: subjects were not allowed leave their terminals or show any of their personal records or notes to other subjects. However, verbal communications were unrestricted in that subjects were allowed to propose strategy for future rounds and comment on the outcomes of previous rounds. Three experimenters enforced these rules for communication.

Subjects' payoffs in each round were determined using the payoff functions described in the previous section. All prices and their payoffs were denominated in a notional currency that we called "francs." Subjects' payoffs were summed across all rounds to determine their payoff in each experiment. Their experiment payoffs were translated into monetary payoffs using the following formula:

$$\$\text{ payoff} = \text{sum of round by round payoffs} \times \mathcal{F},$$

where $\mathcal{F}$ represents a scaling factor. The scaling factor, which varied between 0.1 and 0.2, was made known to subjects before each experiment. We expected large payouts to be made in treatments involving subject communication. In order to most efficiently utilize our limited budget, we attempted to keep the total payoff the same across treatments and experiments. This called for using lower scaling factors in experiments involving communication. The resulting payoffs ranged between $0.50 and $10.00. The average payoff was approximately $5.00.[9] Despite the lower scaling factors, subjects' payoffs were significantly higher in treatments that allowed for subject communication. Subject payoffs were also relatively sensitive to the nature of the adopted strategy.[10] Because coordination to collusive equilibria requires significantly more effort than playing competitive strategies, any bias against effort induced by lower scaling factors for the treatments with communication would bias results against collusion. However, our results seem to indicate that variations in the scaling factors did not significantly affect our results. Subjects' payoffs were not revealed to other subjects and were dispensed in sealed envelopes. These payments were made from funds provided by university research funding. Subjects were informed regarding both the payment procedure and the source of the funding at the beginning of each experiment.

Attempts were made to ensure that the same number of subjects participated in each experiment. In some instances, however, the number of subjects could not be controlled. At least three repetitions of each treatment were conducted with 11 subjects. The uniform-price treatment

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[9] The average subject's payoff was not a large one for 45 minutes of work. Despite the magnitude of their payoffs, we observed many obvious signs of subject interest in the experiments. For example, subjects asked a number of questions of the experimenters during their explanations of the rules of the experiments, and they entered into animated discussions when they were permitted to communicate with one another. While we are fairly confident that our results are representative of the types of subject behavior that would obtain in similar experiments, we realize that higher payoffs might lead to greater frequency of collusive behavior by subjects.

[10] This contrasts with the relatively small sensitivity of payoffs to subject strategies observed in unit sealed-bid auction experiments. Thus, the criticisms of experimental methodology by Harrison (1989) have less force in our setting.

without communication was repeated four times for groups of 11, 11, 11, and 14 subjects. The uniform-price treatment with communication was repeated five times with groups of 11, 11, 11, 12, and 12 subjects. The discriminatory treatment without communication was repeated four times with groups of 11, 11, 11, and 10 subjects. The discriminatory treatment with subject communication was repeated three times. Each experiment was conducted with a group of 11 subjects. No subject was involved in more than one experiment.

3. Experimental Results

In this section we examine the outcomes of the experiments. We begin with a preliminary analysis of the clearing prices, auctioneer surplus, and demand schedules in all the experiments. Then we conduct statistical tests to evaluate the effects of communication and the choice of auction mechanism on clearing prices and subject demand. To control for the biases induced by changes in group size and learning, these tests, and all subsequent analysis, are restricted to the first 12 rounds of those experiments conducted with 11 subjects. The remaining analysis represents attempts to elucidate subject behavior through the development of simple measures of the attributes of demand vectors and clearing prices.

3. 1 Preliminary Findings

Table 2 presents the outcomes of each treatment. The evolution of clearing prices over rounds is illustrated in Figure 1. Figure 2 presents the breakdown of clearing prices in each of the four treatments. In the uniform-price treatment with communication, the clearing price displayed the greatest range and variance. The collusive clearing price of 10 was observed relatively often, while the competitive price of 20 seldom obtained. With the exception of UC2 and the last round of UC1, the clearing price was never 20 in the last three rounds. Further, in UC1, UC3, and UC5*, the market tended to clear at a price of 10 in the latter rounds. In UC2, clearing prices displayed a contrasting pattern. The clearing price was 10 for the first six rounds and 20 for the last four rounds. It appeared that subjects are able to collude at the inception of the experiment, but coordination broke down as the experiment progressed. In the uniform-price treatment without communication, the clearing price was never 10 and displayed little variation. With the exception of U1, the clearing price was 20 in almost all rounds. In the discriminatory treatments both with and without communication, the clearing price of 10 was never observed, and 15 was the most frequent clearing price. In fact, in the last two rounds of all of these experiments, 15 was the only clearing price.

Data on the acutioneer's surplus are also presented in Table 2. The auctioneer's share of the surplus was smallest in the uniform-price treatment with communication and largest in the uniform-price treatment without communication. The difference in the surplus in the two uniform-price treatments was pronounced. On the other hand, there was only a slight difference between the surplus in the discriminatory treatment without communication, the uniform-price treatment without communication, and the discriminatory treatment with communication.[11]

Table 3 reveals that aggregate demand always exceeded 100 units, indicating that the auction was always successful. However, weakly dominated strategies calling for demand of less than 100 units were observed. In the uniform-price treatment without communication subjects failed to demand 100 units 28 percent of the time. In contrast, in the uniform-price treatment with subject communication, subjects failed to demand 100 units only 12.5 percent of the time. Less variation was observed in discriminatory treatments. In the treatment with subject

[11] While the auctioneer's surplus in discriminatory treatments was somewhat smaller than the surplus in the uniform-price treatments without communication, this may have been an artifact of the large "tick size" in our experiment. Note that the increment between admissible bids, the "tick size," was 5 while the range between the maximum and minimum bids was 10. Thus, the tick size was one half of the range of admissible bids. This is much larger than the proportion between range and tick size in actual auctions. If, in fact, our results indicate that Nash strategies will be played in actual discriminatory auctions, then, with smaller tick sizes, the losses from discriminatory auctions should be not be much higher than the those in uniform-price auctions where bidders adopt competitive strategies.

communication, subjects demanded less than 100 units only 12.9 percnt of the time, while they failed to demand 100 units 16.5 percent of the time in the treatment without communication.

Figure 3 provides evidence for the importance of communication and auction mechanism in shaping the outcomes of the experiments. In the uniform-price treatment without communication, subject demand at the competitive price of 20 was higher than in any of the other treatments. In contrast, not only was the relative demand at the collusive price of 10 highest in the uniform-price treatment with communication, but the *majority* of demand was placed at this price. Consistent with theory, the majority of demand was placed one tick below the competitive price at a price of 15 in the discriminatory treatments regardless of whether communication was permitted.

Figure 3, together with Table 3, provides evidence on evolution of demand over time. With the exception of UC2, in the uniform-price treatment with communication, demand at price levels of 20 and 15 tended to decline over rounds. This, combined with the fact that cumulative demand at the price level of 20 was quite close to 99 in later rounds, seems to indicate that subjects' strategies approached the collusive strategies described in Proposition 2. In fact, for UC1, UC3, and UC5*, the cumulative demand in the last six rounds corresponded almost exactly to that characterizing collusive outcomes. In UC2, on the other hand, subject strategies corresponded exactly to naive collusive strategies of placing maximal demand at the lowest price of 10, during the first six rounds. In round seven, one of the subjects demanded 100 units at the price of 15, eliminating any gains to the other subjects. At this point, coordination between subjects broke down and their demand vectors resembled those inducing the competitive outcome.[12] In contrast, in the uniform-price treatment without communication, aggregate

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[12] From subject communication subsequent to round seven, it was apparent that they were attempting to revert to the naive collusive strategies played in earlier rounds. On realizing that the strategies were not self-enforcing, they attempted, albeit unsuccessfully, to agree on a trigger mechanism to enforce penalties for future deviations from the naive strategies.

demand at price levels of 20 and 10 displayed a tendency to decline over time. The trajectories of demand in the discriminatory treatments displayed different characteristics. Demand at a price level of 20 showed a marked decline while demand at a price level of 15 increased. Demand at a price of 10 also displayed a weak tendency to decline. The decline in the demand at the price of 20 is not surprising given that any strategy calling for demand at this price is weakly dominated.

3.2 Statistical Comparisons

Tables 4 and 5 present the outcomes of statistical tests performed to examine the effects of communication and changes in the auction mechanism on subject strategies. Table 4 contains the Pearson chi-square test statistics for the effects of changes in communication opportunities and auction mechanism on the distribution of clearing prices. The results demonstrate that communication induced a significant change in the clearing price distribution in the uniform-price treatments but had a negligible effect on price distributions in discriminatory treatments. The results also demonstrate that, after controlling for communication between subjects, changes in the auction mechanism induced significant changes in the distribution of clearing prices.

Table 5 documents the impact of changes in the experimental setting on subject strategies. It presents the statistic used in the Mann-Whitney-Wilcoxon test for differences in the level of aggregate demand at the three price levels. Once again, our results indicate that changes in the opportunities for communication significantly influenced subject strategies in the uniform-price treatments but had almost no effect in the discriminatory treatments. Subjects demanded significantly fewer units at prices of 20 and at 15 while demanding significantly more at the price of 10 in the uniform-price treatment with communication. Further, changes in the auction mechanism also exerted significant influence on subject strategies despite holding constant for opportunities for communication. Subject demand at the price of 15 was significantly higher in the discriminatory treatment relative to their uniform-price counterparts. On the other hand, subject demand at 20 was significantly higher in the uniform-price auction without communication than in its discriminatory counterpart. In the treatments with communication, subject demand at a price of 10 was significantly higher in the uniform-price treatment.

3.3 Other Characteristics of Subject Strategies

Table 6 presents evidence on convergence to equilibrium strategies characterized in Section 1. For a given clearing price, convergence to equilibrium strategies is measured by the average of the Euclidean distances of subjects' strategies from the associated equilibrium demand vector. If there exist multiple demand vectors inducing the same clearing price, distance is measured from the center convex hull of this set of equilibrium demand vectors.[13] These measures are normalized by dividing by 20.[14]

As Table 6 and Figure 4 indicate, in the uniform-price treatment without communication, subject strategies did not display a marked tendency to approach any equilibrium strategy vector. However, a weak tendency to approach the equilibrium strategy vector that induces a clearing price of 15 was apparent. The ability to communicate had a marked influence on subject behavior. In experiments UC1 and UC3, subject strategies displayed a marked tendency to approach the equilibrium strategy vector inducing a clearing price of 10 and diverged from the equilibrium strategy inducing a clearing price of 15. In experiment UC2, however, the opposite tendency was observed. Subject behavior in the discriminatory treatment without communication displayed a less dramatic but more consistent pattern. Demand vectors tended to approach the strategy vector inducing a clearing price of 15 and diverged from strategies that induce a clearing price of 20. Communication between subjects in discriminatory auctions tended to increase both

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[13] We measure distance from the barycenter of this set because all Nash equilibrium demand vectors lie within close proximity of this set. For example, in the uniform-price treatment the transfer of one unit of demand at a price of 20 to augment demand at a price of 10 is all that differentiates a point in this set from the symmetric equilibrium strategy that induces a clearing price of 10. Thus, any distance measure based on minimizing distance from this set produces little cross-sectional variation and, thus, is not very informative.

[14] Note that the absolute magnitude of each of these measures is irrelevant; only their relative magnitudes can be used to make inferences. The normalizing factors for each of the measures developed in the paper have been chosen to facilitate presentation of the results in a compact form.

distance measures and to an even greater extent increase the round-to-round volatility of both distance measures.[15]

We also considered the degree of symmetry between shareholder strategies. The structure of the auctions is symmetric, in that payoffs to bidders are invariant to permutations of the index set. However, there exist asymmetric equilibria. Thus, it is of interest to determine the degree of symmetry observed in shareholder strategies. To measure symmetry, we first computed the Euclidean distance of subjects' demand in each round from the average demand vector for the round. This measure of symmetry was standardized by dividing by 10. As Table 7 shows, subjects' strategies exhibited a tendency to become more symmetric over time in all four treatments. The changes in symmetry across rounds were most dramatic in the uniform-price treatment with communication. In UC1 and UC3, there was a significant increase in symmetry while in UC2 there was a significant decrease in symmetry. Another pattern that emerges is that subject strategies in the uniform-price treatment with communication tended to be the most symmetric while the strategies of subjects in the uniform-price treatment without communication displayed the lowest degree of symmetry.

Table 8 considers the effect of the clearing price on bids submitted in the subsequent round. Theory provides little guidance as to the dynamics of convergence to the equilibrium behavior. Nevertheless, the idea of the tatonment process in classical economic thought suggests that demand may adjust based on observed prices. To investigate this effect in our experimental setting we computed the Pearson-product moment correlation coefficient between demand at each price and lagged clearing prices. The resulting outcome was standardized by multiplying by 100. The results in Table 8 indicate significantly different dynamics across treatments. In the uniform-price treatments, demand at the price of 20 is positively related to lagged prices. This is

[15] A similar, though weaker, pattern is observed in the uniform-treatment with communication. A possible explanation for this phenomenon is the attempts of some subjects to convince other subjects to adopt naive low-price strategies and subsequently capture the auction's surplus for themselves by overbidding.

consistent with the notion that subjects felt that the high price in the current round signaled a high price in future rounds and adjusted their demand at 20 upward to ensure acceptance of their bids. In the discriminatory treatments there was a marked tendency of demand to rise at the price of 20 and fall at the price of 15 subsequent to a relatively high price in the previous round.

Table 9 presents evidence on the stability of subject's strategies in the experiments. The stability of subject strategies was measured using the Euclidean distance of each subject's demand vector from her average demand vector. This statistic was normalized by dividing by 33. From Table 9 it is apparent that, with the exception of two experiments permitting subject communication, UC2 and DC1, there was a marked tendency for subjects' strategies to exhibit greater stability over time. Not surprisingly, in the uniform-price treatment with subject communication, subject strategies displayed considerable stability once a pattern of collusive behavior emerged.

4. Concluding Comments

In this paper, we provided experimental evidence on strategy choice in auctions of shares. Our experiments indicate that, in uniform-price auctions, non-binding pre-play communication facilitates convergence to equilibrium outcomes. When opportunities for communication are available, bidders are more likely to gravitate towards self-enforcing collusive strategies. In the absence of communication opportunities, a clear pattern of convergence to Nash behavior is less evident. In discriminatory auctions, however, bidder strategies approximate the unique equilibrium outcome. This produces a larger surplus for the auctioneer than the collusive outcome in the uniform-price auction.

These results have important implications for the design of Treasury auctions. Because participants in these auctions have ample opportunities to communicate, it would appear that uniform-price auctions will net the Treasury lower revenues. Further, because our results indicate that bidders' strategies will quickly converge to collusive strategies, it would appear that there would be an almost immediate drop in the Treasury's revenues once uniform-price auctions are employed. This evidence is consistent with that of Simon (1992) but is inconsistent with the predictions of researchers such as Friedman (1960).

Our results also have interesting implications for researchers. The tendency for subjects to gravitate towards symmetric strategies, especially when they are permitted to communicate and there exist totally symmetric Nash equilibria, would seem to indicate that greater emphasis should be placed on the existence of symmetric equilibria in facilitating the attainment of Pareto-optimal self-enforcing agreements. Secondly, our results also point to the dynamic instability of competitive equilibria in which agents' payoffs are minimized and all feasible strategies are best responses to the equilibrium strategy vector. When this is the case, agents' strategy choices tend to wander.[16] Although a change in any individual agent's strategy by itself can have no effect on the outcome, because all agents exhibit a tendency to change their strategies, divergence from the equilibrium competitive price is observed fairly frequently.

Our investigation focused primarily on subjects' bidding strategies. Our experimental design did not permit us to analyze the effects of private information regarding valuations, transparency of the auction process, and secondary markets on equilibrium auction behavior. Extensions of our design to incorporate these effects seem fairly obvious. There exist numerous examples of experimental auction designs in which bidders possess private information regarding their reservation prices (see, for example, Smith 1967). In fact, it is the performance of uniform-price auctions in this setting that has led to its appeal among economists. A synthesis of existing experimental designs with ours will permit the examination of the impact of incomplete

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[16] See Young (1993) for an analysis of best-reply structures and the evolutionary adaptation needed for convergence.

information on behavior in auctions. The effect of the transparency of the auction process can also be examined by a fairly straightforward extension of our design. For example, the experiment could be performed while revealing both the clearing price as well as the aggregate demand schedule to the subjects after each round. Changing the auction design to study the effect of secondary markets on subject behavior is not as simple. One alternative would be to meld the existing auction model with an experimental implementation of a double auction market, where winning auction participants can trade their allocations after the completion of each round of the auction.

APPENDIX : PROOFS OF PROPOSITIONS

Definition 1: *Let $(d \mid^{-i} d_i')$ represent the vector obtained by replacing the i^{th} element of the d vector with d_i'. That is, $(d \mid^{-i} d_i') = (d_1, d_2, \ldots d_{i-1}, d_i', d_{i+1}, \ldots, d_{11})$. A Nash equilibrium is a feasible demand vector d^* such that $V_i(d^*) \geq V_i(d^* \mid^{-i} d_i')$ for all feasible demand schedules d_i' and all bidders i.*

Proof of Lemma 1: Any strategy in which the total demand of an individual bidder is less than 100 units is dominated by a strategy in which the bidder increases her demand at the lowest price of 10 by an amount that sets total demand equal to 100 units. If some of the additional demand submitted is accepted, the bidder's payoff is strictly higher; otherwise she is no worse off. ❑

Proof of Proposition 1: Clearly, if other bidders, in aggregate, demand more than a total of 100 units at a price of 20, any demand schedule is a best response. In the event that other bidders' aggregate demand at a price of 20 is lower than 100 units, a bidder will never submit a bid that would force the clearing price to 20 as this would result in a payoff of 0. Thus, if the demand vectors are symmetric, for the auction to clear at 20, each bidder must bid for at least 10 units at this price.❑

Proof of Proposition 2: First note that the clearing price under the equilibrium strategy is 10. Now consider deviations by any bidder from the equilibrium strategy. This deviation either induces the *same* clearing price, a clearing price of 15, or one of 20. Deviations that maintain the clearing price cannot result in a higher payoff for the bidder because they cannot increase her allocation. Deviations that raise the clearing price to 20 are clearly sub-optimal. Deviations that raise the clearing price to 15 are sub-optimal by our choice of the parameters.❑

Proof of Proposition 3: (i) First, note that any strategy in which $d_{i20} > 0$ is a dominated strategy as demand at this price can never produce a positive payoff. (ii) Now we show that there is no Nash equilibrium in undominated strategies where the clearing price is either 10 or 20. The latter result follows directly from (i). To see the first result, note that the assumption that N = 11 along with our choice of parameters ensures that given a clearing price of 10, bidders can always increase their allocation and payoff by moving some demand from a price of 10 to place bids at a price of 15. (iii) Now, to complete the proof, we establish that, in any Nash equilibrium, bidders will concentrate all demand at a price of 15. To see this, suppose that bidders adopt another strategy. Switching all demand to a price of 15 will increase bidder payoffs. This follows because payoff from bids made at a prices of 10 and 20 are 0, given that the clearing price must be 15. The proof is concluded by noting that concentrating all demand at a price of 15 is a Nash equilibrium. ❑

REFERENCES

Back, K., and J. Zender, 1993, "Auctions of Divisible Goods: On the Rationale of the Treasury Experiment," *Review of Financial Studies*, **6**, 733-765.

Bernheim, D., B. Peleg, M. Whinston, 1987, "Coalition-Proof Nash Equilibria I. Concepts," *Journal of Economic Theory*, **42**, 1-12.

Cox, J., V. Smith, and J. Walker, 1984, "Theory and Behavior of Multiple Unit Discriminative Auctions," *Journal of Finance*, **39**, 983-1009.

Friedman, M., 1960, *A Program for Monetary Stability*, New York: Fordham University Press.

Fudenberg, D., and E. Maskin, 1986, "The Folk Theorem in Repeated Games with Discounting and Incomplete Information," *Econometrica*, **54**, 533-554.

Harrison, G., 1989, "Theory and Misbehavior of First-Price Auctions," *American Economic Review*, **79**, 749-762.

Holmstrom, B., and B. Nalebuff, 1992, "To the Raider Goes the Surplus: A Reexamination of the Free-Rider Problem," *Journal of Economics and Management Strategy* , **1**, 37-62.

Kohlberg, E., and J.-F. Mertens, 1986, "On the Strategic Stability of Equilibria," *Econometrica*, **54**, 1003-1039.

Luce, R., and H. Raiffa, 1957, *Games and Decisions*, New York: Wiley.

Meltzer, A., and G. von der Linde, 1960, *A Study of the Dealer Market for Federal Government Securities*, Materials Prepared for the Joint Economic Committee, 86th Congress, 2nd session.

Milgrom, P., 1989, "Auctions and Bidding: A Primer," *Journal of Economic Perspectives*, **3**, 3-22.

Reiber, M., 1964, "Collusion in the Auction Market for Treasury Bills," *Journal of Political Economy*, **72**, 509-512.

Samuelson, L., 1991, "Limit Evolutionarily Stable Strategies in Two-Player, Normal Form Games," *Games and Economic Behavior*, **3**, 110-128.

__________, 1992, "Dominated Strategies and Common Knowledge," *Games and Economic Behavior*, **4**, 284-313.

Simon, D., 1992, "The Treasury's Experiment with Single-Price Auctions in the mid 1970s: Winner's or Taxpayer's curse?" Working Paper, Federal Reserve Board of Governors.

Smith, V., 1967, "Experimental Studies of Discrimination Versus Competition in Sealed-Bid Auction Markets," *Journal of Business*, **40**, 56-84.

Tenorio, R., 1993, "Revenue-Equivalence and Bidding Behavior in Multi-Unit Auction Markets: An Empirical Analysis," *Review of Economics and Statistics*, **75**, 302-314.

Umlauf, S., 1993, "An Empirical Study of the Mexican Treasury Bill Auction," *Journal of Financial Economics*, **33**, 313-340.

Vickrey, W., 1961, "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, **16**, 8-37.

Wilson, R., 1979, "Auctions of Shares," *Quarterly Journal of Economics*, **93**, 675-698.

Young, H., 1993, "The Evolution of Conventions," *Econometrica*, **61**, 57-84.

Table 1. Descriptions of the Experiments.

This table presents a summary of the parameters for all the experiments. There were four treatments investigated: uniform-price without communication (Treatment U), uniform-price with communication (Treatment UC), discriminatory without communication (Treatment D), and discriminatory with communication (Treatment DC). An asterisk is affixed to the name of each experiment involving a number of subjects unequal to 11.

Panel A. Uniform-Price Treatments

	Uniform-Price without Communication				Uniform-Price with Communication				
Experiment	U1	U2	U3	U4*	UC1	UC2	UC3	UC4*	UC5*
Price levels	3	3	3	3	3	3	3	3	3
Units	100	100	100	100	100	100	100	100	100
Rounds	14	14	12	15	12	12	12	16	12
Subjects	11	11	11	14	11	11	11	12	12

Panel B. Discriminatory Treatments

	Discriminatory without Communication				Discriminatory with Communication		
Experiment	D1	D2	D3	D4*	DC1	DC2	DC3
Price levels	3	3	3	3	3	3	3
Units	100	100	100	100	100	100	100
Rounds	12	12	12	12	12	12	12
Subjects	11	11	11	10	11	11	11

Table 2. Clearing Prices and Auctioneer's Surplus

This table presents clearing price (*CP*) and auctioneer's surplus (*W*) in all experiments. The numbers in the first column represent the round number. The last three rows of the table present averages of the auctioneer's surplus across all rounds, the first half of each experiment, and the second half of each experiment, respectively.

Panel A. Clearing Price and Auctioneer's Surplus for Uniform-Price Auctions without Subject Communication (Treatment U)

Round	Experiment U1		Experiment U2		Experiment U3		Experiment U4*	
	CP	*W*	*CP*	*W*	*CP*	*W*	*CP*	*W*
1	20	1000	20	1000	20	1000	20	1000
2	20	1000	20	1000	20	1000	20	1000
3	20	1000	20	1000	20	1000	20	1000
4	20	1000	20	1000	20	1000	20	1000
5	15	500	20	1000	20	1000	20	1000
6	15	500	20	1000	20	1000	20	1000
7	15	500	20	1000	20	1000	20	1000
8	15	500	20	1000	20	1000	20	1000
9	15	500	20	1000	20	1000	20	1000
10	20	1000	15	500	20	1000	20	1000
11	15	500	20	1000	20	1000	20	1000
12	20	1000	20	1000	20	1000	20	1000
13	20	1000	15	500	-	-	20	1000
14	15	500	15	500	-	-	15	500
15	-	-	-	-	-	-	20	1000
All	-	750	-	893	-	1000	-	967
First	-	786	-	1000	-	1000	-	1000
Second	-	715	-	786	-	1000	-	937

Table 2.

Panel B. Clearing Price and Auctioneer's Surplus for Uniform-Price Auctions with Subject Communication (Treatment UC)

Round	Experiment UC1		Experiment UC2		Experiment UC3		Experiment UC4*		Experiment UC5*	
	CP	*W*	*CP*	*W*	*CP*	*W*	*CP*	*W*	*CP*	*W*
1	10	0	10	0	20	1000	20	1000	20	1000
2	20	1000	10	0	20	1000	20	1000	20	1000
3	20	1000	10	0	15	500	15	500	15	500
4	20	1000	10	0	15	500	20	1000	20	1000
5	20	1000	10	0	10	0	20	1000	10	0
6	15	500	10	0	10	0	15	500	10	0
7	10	0	15	500	15	500	15	500	10	0
8	10	0	15	500	10	0	20	1000	15	500
9	20	1000	20	1000	10	0	20	1000	15	500
10	10	0	20	1000	10	0	15	500	15	500
11	10	0	20	1000	10	0	15	500	10	0
12	20	1000	20	1000	10	0	15	500	10	0
13	-	-	-	-	-	-	15	500	-	-
14	-	-	-	-	-	-	15	500	-	-
15	-	-	-	-	-	-	15	500	-	-
16	-	-	-	-	-	-	20	1000	-	-
All	-	541	-	416	-	292	-	719	-	417
First	-	750	-	0	-	500	-	812	-	583
Second	-	333	-	833	-	83	-	625	-	250

Table 2.

Panel C. Clearing Price and Auctioneer's Surplus for Discriminatory Auctions without Subject Communication (Treatment D)

Round	Experiment D1		Experiment D2		Experiment D3		Experiment D4*	
	CP	*W*	*CP*	*W*	*CP*	*W*	*CP*	*W*
1	20	1000	15	750	20	1000	20	1000
2	20	1000	15	875	20	1000	15	950
3	20	1000	15	875	20	1000	15	850
4	20	1000	15	750	20	1000	20	1000
5	20	1000	15	525	15	780	20	1000
6	20	1000	15	675	15	680	20	1000
7	20	1000	15	800	15	630	20	1000
8	15	865	15	525	15	705	15	755
9	20	1000	15	950	15	650	15	700
10	15	850	15	650	15	650	15	650
11	15	845	15	800	15	600	15	685
12	15	835	15	625	15	550	15	700
All	-	950	-	733	-	770	-	857
First	-	1000	-	742	-	910	-	967
Second	-	899	-	725	-	631	-	748

Table 2.

Panel D. Clearing Price and Auctioneer's Surplus for Discriminatory Auctions with Subject Communication (Treatment DC)

Round	Experiment DC1		Experiment DC2		Experiment DC2	
	CP	*W*	*CP*	*W*	*CP*	*W*
1	20	1000	15	835	20	1000
2	20	1000	20	1000	20	1000
3	15	550	15	500	15	700
4	15	650	20	1000	20	1000
5	15	525	20	1000	15	650
6	15	550	15	550	15	655
7	15	500	15	500	15	600
8	15	800	15	500	15	680
9	20	1000	15	500	15	650
10	20	1000	20	1000	15	660
11	15	500	15	500	15	655
12	15	550	15	500	15	660
All	-	712	-	699	-	743
First	-	710	-	814	-	834
Second	-	725	-	583	-	651

Table 3. Subjects' Aggregate Demand

This table presents subjects' aggregate demand at the three price levels. A_p represents the aggregate demand at price level p. The last three rows of the table present averages of aggregate demand across all rounds, the first half of each experiment, and the second half of each experiment, respectively.

Panel A. Aggregate Demand for Uniform-Price Auctions with No Subject Communication (Treatment U)

Round	Experiment U1			Experiment U2			Experiment U3			Experiment U4*		
	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}
1	263	364	285	223	414	443	501	434	165	573	567	162
2	187	238	216	190	525	235	528	415	113	564	611	150
3	129	304	199	198	495	182	289	656	103	269	631	215
4	136	354	126	244	490	219	189	706	105	188	547	300
5	14	328	269	204	450	199	198	672	128	189	639	177
6	27	506	89	231	415	188	314	532	135	202	667	203
7	47	496	110	124	509	194	324	566	90	152	774	180
8	58	520	86	132	460	189	233	564	155	132	824	160
9	86	480	91	130	474	178	290	619	145	127	765	219
10	167	303	147	84	554	188	274	636	100	123	878	164
11	90	424	121	148	496	214	165	592	190	133	822	177
12	100	456	139	107	535	229	163	607	145	130	839	144
13	103	473	116	82	560	179	-	-	-	116	823	59
14	81	524	96	82	659	100	-	-	-	99	914	85
15	-	-	-	-	-	-	-	-	-	105	888	79
All	106	412	149	156	503	210	289	583	131	214	736	171
First	115	370	185	202	471	237	336	569	125	305	634	198
Second	98	454	113	109	534	182	242	597	138	121	844	135

Table 3.

Panel B. Aggregate Demand for Uniform-Price Auctions with Subject Communication (Treatment UC)

Round	Experiment UC1			Experiment UC2			Experiment UC3			Experiment UC4*			Experiment UC5*		
	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}
1	20	20	1060	0	0	1100	123	124	852	515	480	165	113	511	285
2	270	140	690	0	50	1050	153	301	646	470	450	155	114	362	428
3	258	145	695	0	0	1100	99	91	910	27	93	100	35	90	985
4	288	220	572	0	0	1100	99	455	546	164	318	335	184	455	224
5	190	728	182	0	0	1100	99	0	1001	146	320	143	0	5	1195
6	88	901	111	0	0	1100	99	0	1001	70	417	241	10	46	1144
7	99	0	1001	0	100	1000	99	91	910	89	403	228	5	19	1176
8	98	0	1002	86	310	704	99	0	1001	113	573	131	31	138	1031
9	190	0	910	120	70	910	99	0	1001	106	485	136	97	10	1072
10	99	0	1001	620	230	250	99	0	1001	78	554	108	83	394	633
11	99	0	1001	419	141	540	99	0	1001	76	561	84	96	0	1104
12	100	0	1000	584	171	345	99	0	1001	78	663	84	96	0	1104
13	-	-	-	-	-	-	-	-	-	89	184	927	-	-	-
14	-	-	-	-	-	-	-	-	-	89	209	780	-	-	-
15	-	-	-	-	-	-	-	-	-	98	4	1098	-	-	-
16	-	-	-	-	-	-	-	-	-	100	91	1009	-	-	-
All	150	180	769	152	89	858	105	89	906	154	387	382	72	169	865
First	186	359	552	0	8	1092	112	162	826	199	382	245	76	245	710
Second	114	0	986	305	170	625	99	15	986	89	344	528	68	94	1020

Table 3.

Panel C. Aggregate Demand for Discriminatory Auctions without Subject Communication (Treatment D)

Round	Experiment D1			Experiment D2			Experiment D3			Experiment D4*		
	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}
1	205	500	335	50	510	500	220	470	355	151	415	315
2	260	510	250	75	805	170	137	563	305	90	440	320
3	225	540	228	75	700	280	157	533	365	70	535	255
4	169	518	239	50	860	165	231	514	285	105	530	215
5	171	569	340	5	940	115	56	694	290	115	575	205
6	120	555	340	35	895	100	36	819	190	167	570	164
7	145	540	360	60	870	120	26	809	225	140	560	235
8	73	590	248	5	945	105	41	879	135	51	665	235
9	102	553	239	90	850	105	30	840	195	40	580	320
10	70	597	225	30	960	100	30	705	340	30	755	205
11	69	621	228	60	915	125	20	842	235	37	805	140
12	67	659	193	25	925	125	10	865	135	40	780	180
All	140	563	269	47	848	168	83	711	255	86	601	232
First	192	532	289	48	785	222	140	599	298	116	511	246
Second	88	593	249	45	911	113	26	823	211	56	691	219

Table 3.

Panel D. Aggregate Demand for Discriminatory Auctions with Subject Communication (Treatment DC)

Round	Experiment DC1			Experiment DC2			Experiment DC3		
	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}	A_{20}	A_{15}	A_{10}
1	160	700	240	67	679	130	183	500	272
2	170	735	190	245	620	215	145	585	125
3	10	940	150	0	100	1000	40	190	640
4	30	945	115	200	565	245	147	430	290
5	5	190	905	100	850	150	30	720	130
6	10	1045	45	10	915	175	31	651	221
7	0	330	770	0	1100	0	20	210	640
8	60	1005	35	0	1100	0	36	315	525
9	110	835	155	0	300	800	30	800	90
10	240	740	120	100	800	200	32	775	110
11	0	300	800	0	970	130	31	850	30
12	20	1040	40	0	990	110	32	855	35
All	68	734	297	60	749	262	63	573	259
First	64	759	274	104	622	319	96	513	280
Second	72	708	320	17	877	207	30	634	238

Table 4. Tests for the Effect of Communication and Auction Mechanism on the Distribution of Clearing Prices

This table presents the Pearson chi-square goodness-of-fit tests comparing the clearing price distributions across auction mechanisms and communication regimes. The null hypothesis is that, for any given cell, the price distribution of the row and column treatments are the same.

	Uniform-Price with Communication	Discriminatory with Communication	Uniform-Price without Communication	Discriminatory without Communication
Uniform-Price with Communication	0	35.61[b]	94.53[b]	NC
Discriminatory with Communication	-	0	NC	0.06
Uniform-Price without Communication	-	-	0	16.36[b]
Discriminatory without Communication	-	-	-	0

[a] Significant at the 5% level.

[b] Significant at the 1% level.

Table 5. The Effects of Communication and the Auction Mechanism on Subject Demand
This table presents the Mann-Whitney-Wilcoxon test statistic comparing aggregate subject demand at each of the three price levels across auction mechanisms and communication regimes. The null hypothesis is that, for any given cell, the sum of the ranks of aggregate demand for the row and column treatments is the same.

Panel A. The Effects of Communication on Subject Demand

	Round	Uniform-Price without Communication			Discriminatory without Communication		
		T_{20}	T_{15}	T_{10}	T_{20}	T_{15}	T_{10}
Uniform-Price with Communication	All	114[a]	78[b]	221[b]			
	First	22[b]	21[b]	56[b]			
	Second	41	21[b]	57[b]			
Discriminatory with Communication	All				120[a]	142	155
	First				31	38	37
	Second				30	35	42

Panel B. The Effects of Changing the Auction Mechanism on Subject Demand

	Round	Discriminatory with Communication			Discriminatory without Communication		
		T_{20}	T_{15}	T_{10}	T_{20}	T_{15}	T_{10}
Uniform-Price with Communication	All	193[b]	78[b]	221[b]			
	First	42	21[b]	56[b]			
	Second	55[b]	21[b]	57[b]			
Uniform-Price without Communication	All				206[b]	85[b]	104[b]
	First				53[a]	22[b]	25[a]
	Second				57[b]	21[b]	25[a]

[a] Significant at the 5% level.
[b] Significant at the 1% level.

Table 6. Deviation of Subject Strategies from Equilibrium Strategies

This table presents the average Euclidean distance of subject strategies from symmetric Nash equilibrium strategy vectors, normalized by dividing by 20. Each panel presents the average distance across the first twelve rounds, the first six rounds, and rounds seven through twelve of each experiment. EDU_{15} and EDU_{10} represent distance from the unique symmetric equilibrium demand vectors that induce clearing prices of 15 and 10 in the uniform-price treatments, respectively. ED_{20} measures distance from the barycenter of the convex hull of the set of symmetric equilibrium demand vectors that induce a clearing price of 20. EDD_{15} represents distance from the unique symmetric equilibrium strategy that induces the discriminatory treatments to clear at 15.

Panel A. Uniform-Price Treatment without Subject Communication

Round	U1			U2			U3			Average		
	ED_{20}	EDU_{15}	EDU_{10}	ED_{20}	EDU_{15}	EDU_{10}	ED_{20}	EDU_{15}	EDU_{10}	ED_{20}	EDU_{15}	EDU_{10}
1-12	131	253	453	148	242	459	175	215	589	151	237	501
1-6	112	276	402	129	251	433	182	239	607	141	255	481
7-12	151	231	504	166	233	486	169	191	572	162	218	521

Panel B. Uniform-Price Treatment with Subject Communication

Round	UC1			UC2			UC3			Average		
	ED_{20}	EDU_{15}	EDU_{10}	ED_{20}	EDU_{15}	EDU_{10}	ED_{20}	EDU_{15}	EDU_{10}	ED_{20}	EDU_{15}	EDU_{10}
1-12	278	671	183	279	798	162	331	746	70	296	738	138
1-6	270	513	354	270	907	11	375	676	127	305	699	164
7-12	287	828	13	288	688	312	287	816	13	287	777	112

Panel C. Discriminatory Treatment without Subject Communication

Round	D1		D2		D3		Average	
	ED_{20}	EDD_{15}	ED_{20}	EDD_{15}	ED_{20}	EDD_{15}	ED_{20}	EDD_{15}
1-12	163	259	273	120	206	178	214	186
1-6	149	287	252	155	156	232	186	225
7-12	177	232	294	85	256	124	242	147

Panel D. Discriminatory Treatment with Subject Communication

Round	DC1		DC2		DC3		Average	
	ED_{20}	EDD_{15}	ED_{20}	EDD_{15}	ED_{20}	EDD_{15}	ED_{20}	EDD_{15}
1-12	297	257	336	276	232	296	288	276
1-6	283	219	300	354	201	322	261	299
7-12	311	295	372	198	262	270	315	254

Table 7. The Symmetry of Subject Strategies
This table presents measures of symmetry of subjects' strategies across all rounds of each of the experiments. Symmetry is measured by the average Euclidean distance of subject strategies from the average strategy. This is normalized by dividing by 10. The columns labeled Avg present the average of the metric for the experiments in the treatment. The last three rows of the table present averages of the symmetry measures across all rounds, the first half, and the second half of each experiment, respectively.

Round	U1	U2	U3	Avg	UC1	UC2	UC3	Avg	D1	D2	D3	Avg	DC1	DC2	DC3	Avg
1	94	141	551	262	20	0	165	62	213	234	194	214	200	185	142	176
2	165	143	344	217	352	41	243	212	195	151	157	168	136	345	142	208
3	153	237	291	227	385	0	137	174	235	200	120	185	121	165	269	185
4	219	381	159	253	212	0	410	208	149	134	109	151	79	513	268	287
5	251	195	269	240	356	0	0	119	168	76	149	131	276	331	265	291
6	146	418	427	330	129	0	0	43	260	119	103	161	16	167	321	168
7	190	263	393	282	0	165	137	101	232	163	120	172	345	0	342	229
8	182	228	243	218	0	306	0	102	235	109	116	154	49	0	291	113
9	199	304	350	284	137	258	0	132	165	138	136	147	97	397	162	219
10	241	256	268	255	0	438	0	146	152	136	206	165	327	430	151	303
11	203	276	268	249	0	432	0	144	187	147	155	161	397	163	145	235
12	217	232	178	209	0	382	0	127	168	72	107	116	17	164	126	102
All	189	256	312	252	133	169	91	131	197	140	144	160	172	238	219	210
First	172	252	340	255	242	7	159	136	204	153	148	168	138	284	234	219
Second	205	260	283	250	23	330	23	125	190	127	140	152	205	192	203	200

Table 8. The Price Dependence of Subjects' Strategies
This table presents measures of the impact of the lagged clearing price on aggregate demand at each of the three price levels. The metric for measuring price dependence of demand is the Pearson product moment correlation coefficient statistic (in percent) of aggregate demand and the lagged clearing price.

Price level	U1	U2	U3	Avg	UC1	UC2	UC3	Avg	D1	D2	D3	Avg	DC1	DC2	DC3	Avg
20	28	9	NA*	44	45	94	56	62	52	NA*	83	76	45	- 28	- 27	10
15	- 69	- 4	NA*	15	30	72	52	42	- 61	NA*	- 89	- 81	- 14	- 11	- 21	- 8
10	67	- 21	NA*	10	- 40	- 55	- 96	- 60	49	NA*	67	69	3	22	11	12

* Correlation not defined because of zero sample variance for one of the variables.

Table 9. Stability of Subjects' Strategies
This table presents measures of the stability of subjects' strategies across all rounds of each of the experiments. Stability is measured using the average distance of subject strategies from their average demand vector across the rounds designated in the first column of the table. This statistic is normalized by dividing by 33. The metric is presented for the experiments indicated at the head of each column. The columns labeled Avg present the average of the metric for the experiments in the treatment.

Round	U1	U2	U3	Avg	UC1	UC2	UC3	Avg	D1	D2	D3	Avg	DC1	DC2	DC3	Avg
1-12	301	259	334	298	895	917	369	727	368	273	312	318	966	1035	757	919
1-6	284	307	401	331	1141	21	505	556	414	381	250	348	736	1009	716	820
7-12	133	93	176	134	69	1189	70	443	131	93	110	111	1116	839	719	891

Figure 1

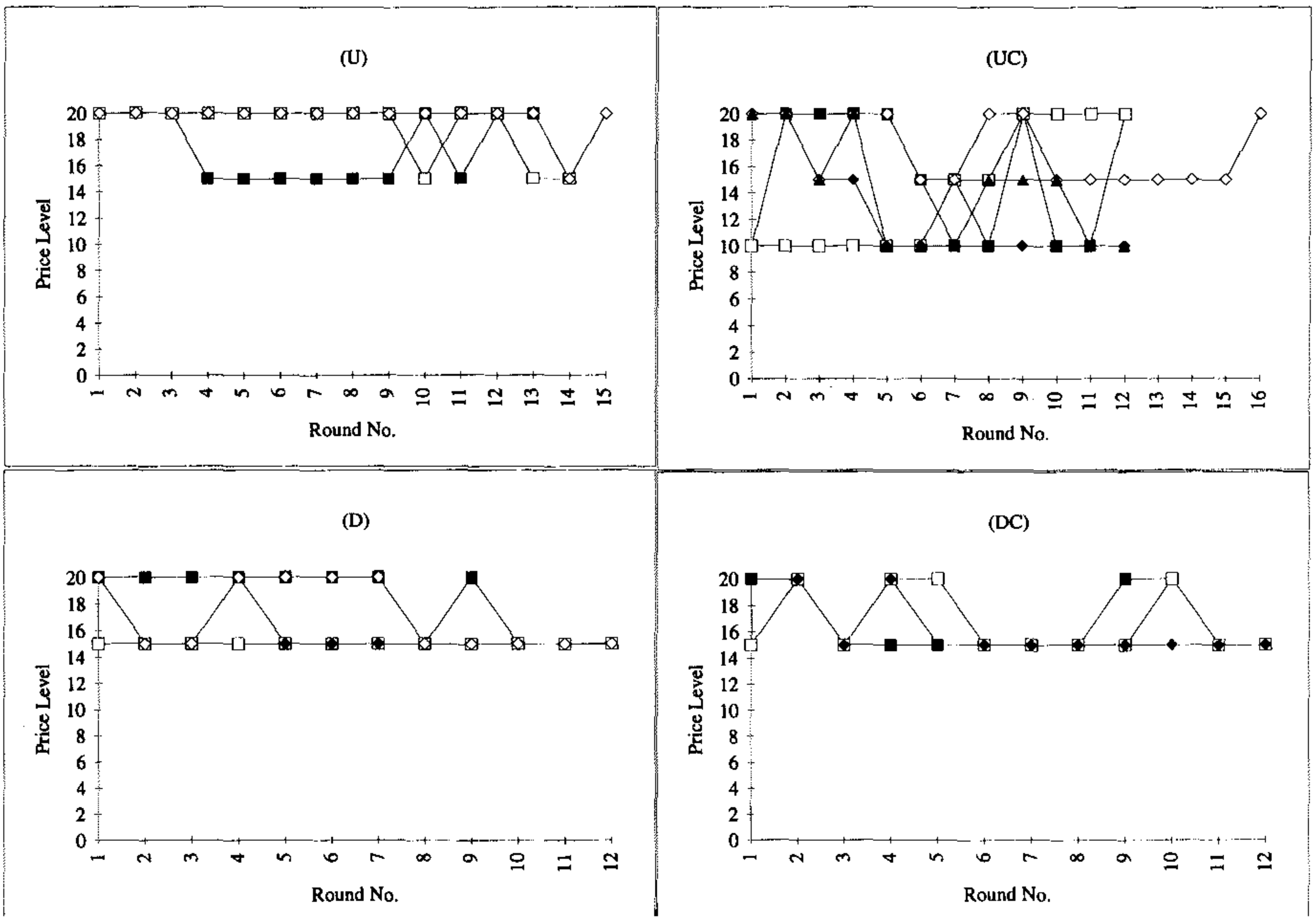

Figure 1. The Evolution of Clearing Prices.
Panel U presents the clearing price in each round of each repetition of the uniform-price treatment without communication. Panel UC presents the clearing price in each round of each repetition of the uniform-price treatment with communication. Panel D presents the clearing price in each round of each repetition of the discriminatory treatment without communication. Panel DC presents the clearing price in each round of each repetition of the discriminatory treatment without communication.

Figure 2

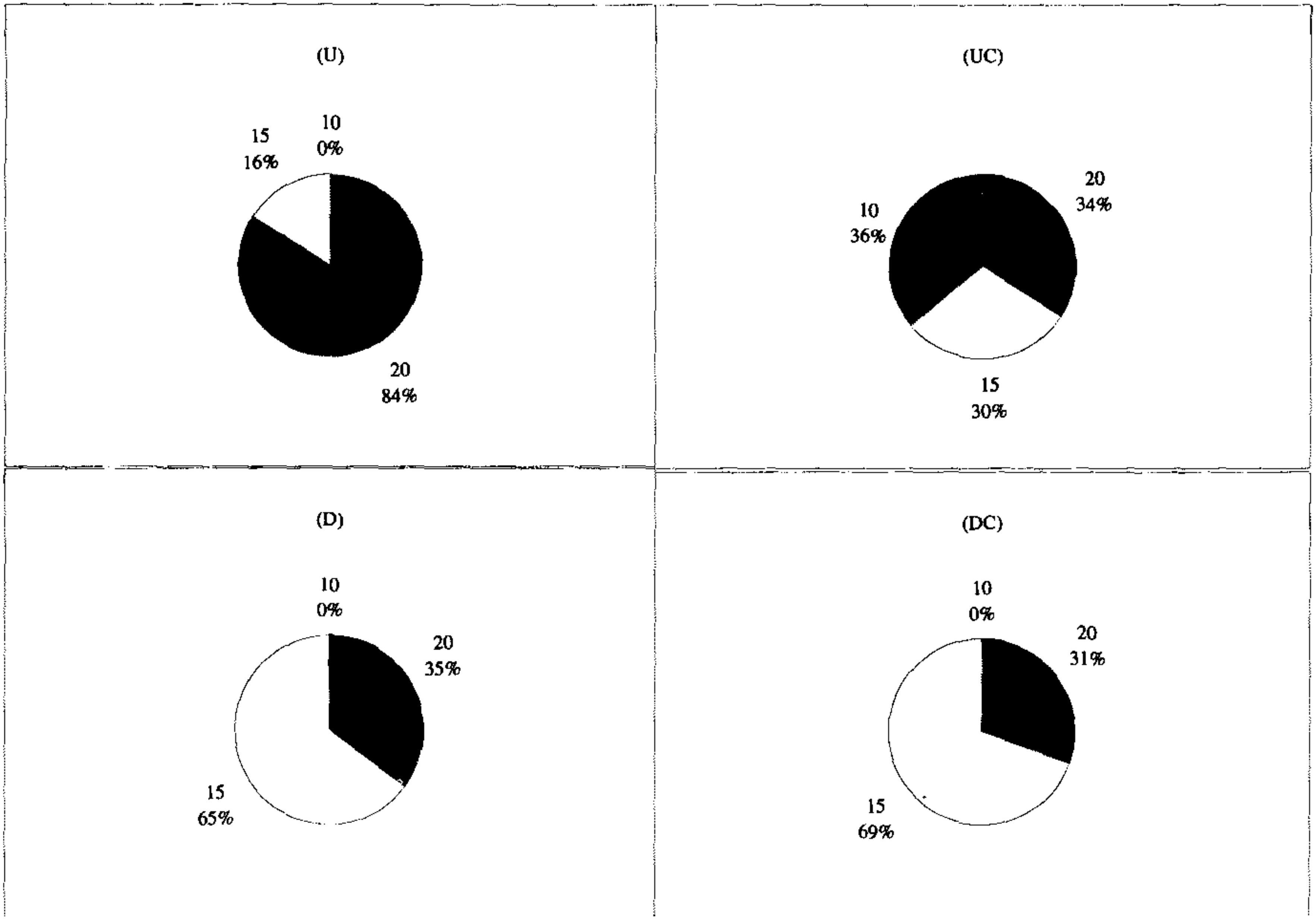

Figure 2. The Frequency of Clearing Prices.
This figure presents the frequency with which the experiments cleared at each of the three clearing prices. Panel U presents this information for the uniform-price treatment without subject communication. Panel UC deals with the uniform-price treatment with subject communication. Panels D and DC deal with the discriminatory counterparts of these two treatments.

Figure 3

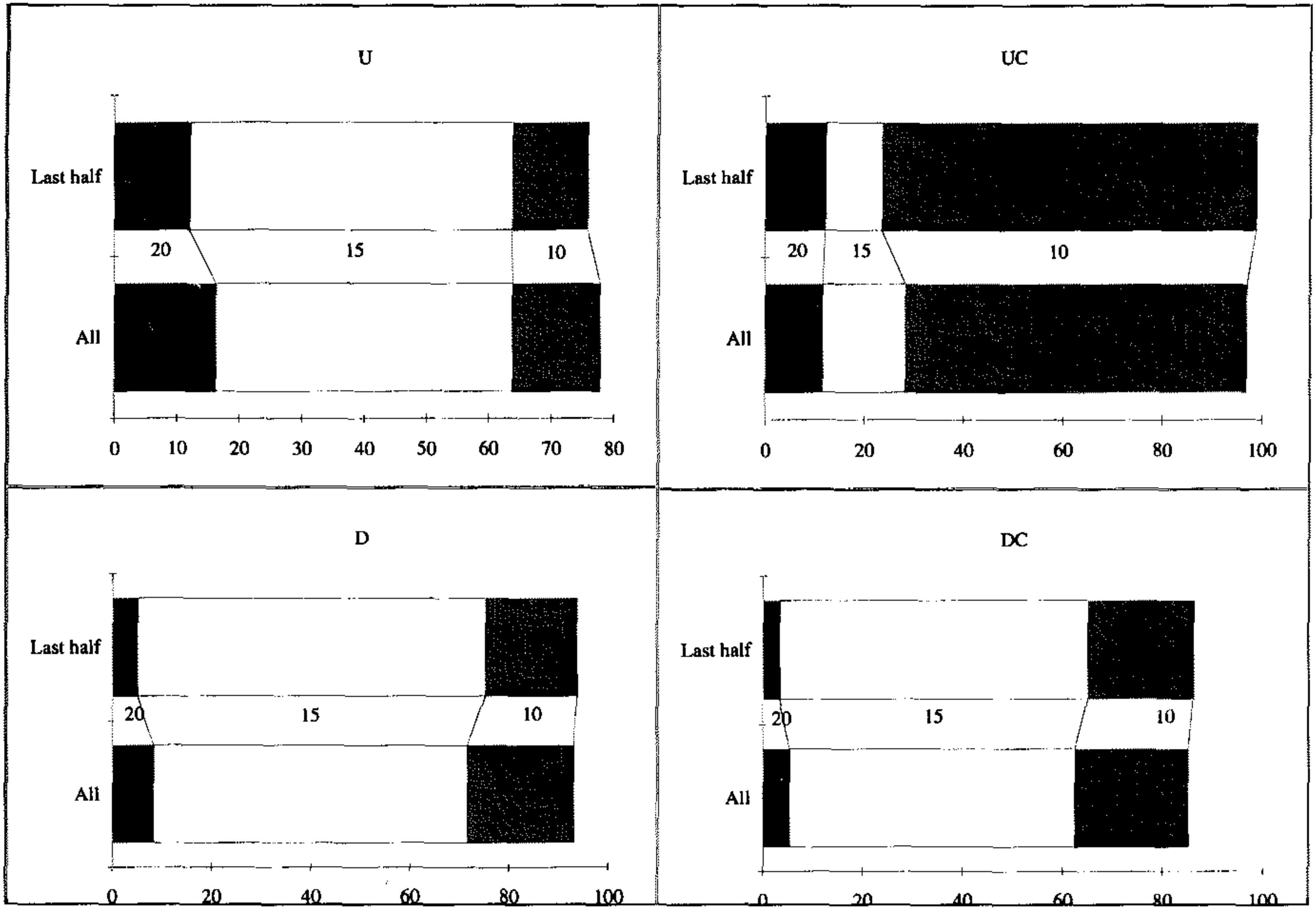

Figure 3. Aggregate Demand.
This figure presents subjects' aggregate demand at each of the three price levels across all rounds of the experiments and the second half of all repetitions in each of the four treatments. Each of the four panels presents this information for one of the four treatments. Panel U presents this information for the uniform-price treatment without subject communication. Panel UC deals with the uniform-price treatment with subject communication. Panels D and DC deal with the discriminatory counterparts of these two treatments.

Figure 4

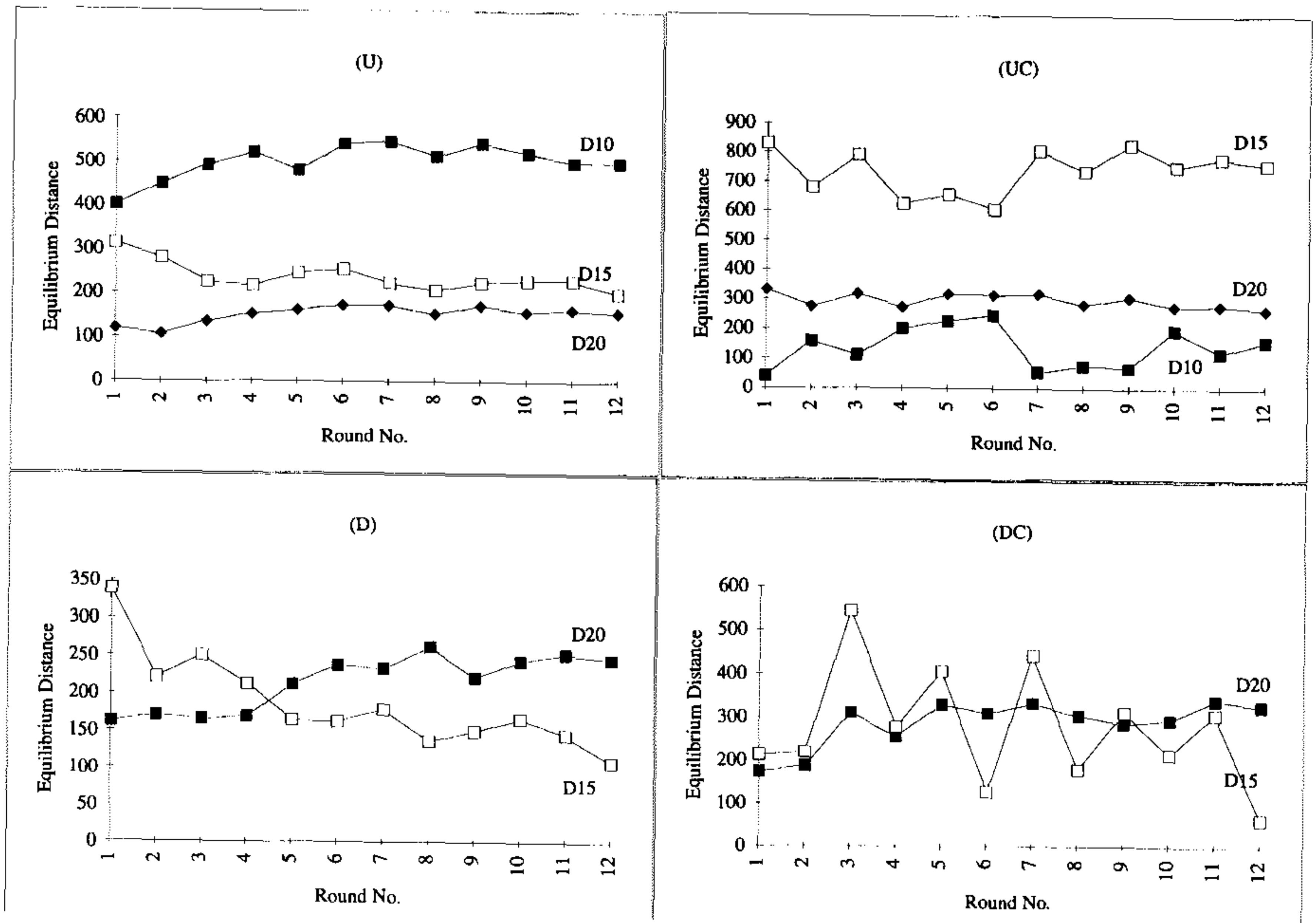

Figure 4. Distance from Symmetric Nash Equilibria.
This figure presents measures of deviations of subjects' strategies from equilibrium strategies for each of four treatments. Distances are represented by the average Euclidean distance of subject strategies from symmetric Nash equilibrium strategy vectors, normalized by dividing by 20. In panels U and UC, which present information on the uniform-price treatments without and with subject communication, respectively, D15 and D10 represent distance from the unique symmetric equilibrium demand vectors that induce clearing prices of 15 and 10 in these treatments, respectively. In all four panels, D20 measures distance from the barycenter of the convex hull of the set of symmetric equilibrium demand vectors that induce a clearing price of 20. In panels D and DC, which present information on the discriminatory treatments without and with subject communication, respectively, D15 represents distance from the unique symmetric equilibrium strategy that induces these treatments to clear at 15.