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COLLUSION UNDER ASYMMETRIC INFORMATION

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When applied to groups, the Revelation Principle postulates a Bayesian-Nash behavior between agents. Their binding agreements are unenforceable or the principal can prevent them at no cost. We analyze instead a mechanism design problem in which the agents can communicate between themselves and *collude under asymmetric information*. We characterize the set of implementable collusion-proof contracts both when the principal offers *anonymous* and *nonanonymous* contracts. After having isolated the nexi and the stakes of collusion we proceed to a normative analysis, perform some comparative statics, discuss our concept of collusion-proofness, and provide some insights about *transaction costs* in side contracting.

KEYWORDS: Collusion, asymmetric information, mechanism design, regulation.

1. INTRODUCTION

WHEN INFORMATION IS SPREAD amongst several agents whose objectives are not aligned with that of their organization, the problem of building a *mechanism* to make them behave according to the organization's goal becomes the cornerstone of organizational design. The Revelation Principle[2] can be considered as a fundamental step towards a better understanding of this *mechanism design* problem. The reasons for the success of this Principle are twofold. First, it provides a simple way to characterize the set of implementable allocations when information is decentralized. Second, it gives the right framework for conducting normative analysis under asymmetric information, i.e., for comparing different allocation mechanisms.

However, the Revelation Principle is based on a number of relatively implausible assumptions when it applies to *groups*. First, it presumes that communication is costless between the principal and the agents. Second, it postulates a Bayesian-Nash behavior between the agents. It is indeed assumed that agents behave noncooperatively, i.e., binding agreements between them are *unenforceable* or the principal can prevent these agreements at no cost. A possible interpretation of this assumption is that communication costs between the agents of an organization are in fact very large. Another interpretation is that the principal can preclude this communication at no cost.

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[2] See Gibbard (1973), Green and Laffont (1977), Dasgupta, Hammond, and Maskin (1979), Harris and Raviv (1981), and Myerson (1979, 1982).

We will be considering a somewhat more symmetric situation in which communication between agents in an organization can be costlessly achieved through the mediation of a third party. In this case, *collusion* within subsets of agents to promote their collective goals (which again are likely to be distinct from those of the organization) becomes an important issue and certainly has an impact on the efficiency of the organization.

Since the Revelation Principle presumes the absence of collusion in the organization, it *always* underestimates the importance of informational constraints in the choice of allocation mechanisms. Clearly, allowing collusion should not be the exception but the rule in a more realistic approach to incentive theory. Therefore, it is crucial to develop a convenient conceptual framework to analyze the consequences of *group incentives* if we want to evaluate the true impact of asymmetric information on the way organizations behave.

In the present paper, we propose to develop such a framework in a particular setting. We analyze a simple regulatory model of a duopoly, each firm (the agents) being endowed with private information on its costs which is unknown to the regulator (the principal). After having derived the optimal regulation without side contracting, i.e., when the agents play a Bayesian-Nash equilibrium, we explicitly allow them to collude. Collusion imposes more constraints on the set of implementable allocations which must now be also *coalition incentive compatible*. We first characterize where are the *nexi* of collusion in the organization, i.e., which are the coalition incentive constraints binding at the optimum of the regulator's problem. Second, we conduct a normative analysis and describe the optimal *collusion-proof* mechanism.

We proceed in several steps. First, we develop a framework to model the game of coalition formation. We envision a side contract as being offered by a mediator or *third party* who maximizes the sum of expected rents for the members of the coalition subject to a set of *feasibility*, *incentive*, and *acceptance* constraints of the side contract. Second, we prove a *weak Collusion-Proofness Principle* when collusion takes place under asymmetric information. A mechanism is collusion-proof when the null side contract is *an* equilibrium of the game of coalition formation. We characterize these collusion-proof mechanisms and describe the set of coalition incentive constraints under asymmetric information. It turns out that these constraints have the same flavor as if agents were under complete information at the time of side contracting. Simply, the true costs of the coalition's members have to be replaced by their so-called *discounted virtual costs*.

We perform this analysis in two different contexts. In the first one, the agents are able to commit to a prior agreement, before the principal offers his grand contract. This agreement, sharing equally transfers from the regulator, constrains the principal to offer *anonymous* grand contracts. This contractual limitation creates a *stake for collusion* to occur also ex post, i.e., after the principal's offer of a grand contract. To prevent collusion, the principal can either propose a contract with some bunching or relax incentive constraints if he chooses to implement a separating allocation. The optimal collusion-proof and anonymous contract nevertheless always calls for some *screening*.

When the agents cannot commit to sharing transfers, the principal is not restricted to offer anonymous contracts. With *nonanonymous* contracts, he can still offer the second-best outcome and side contracting does not impose further constraints on the allocation of resources. In our context, collusion must contain an ex ante agreement to have any bite.

Finally, for both anonymous and nonanonymous grand contracts, the optimal *collusion-proof* mechanism is implemented in dominant strategy and it is *strongly collusion-proof*, i.e., robust to all continuation equilibria of the game of coalition formation.

Section 2 briefly reviews the literature. Section 3 presents our regulatory model and in particular discusses our choices for modeling collusion between agents. As a benchmark, Section 4 derives the second-best regulation without side contracting. Section 5 shows that the optimal regulation without side contracting is not robust to manipulations by the coalition. Section 6 characterizes the collusion-proof contracts and proves our Collusion-Proofness Principle. Section 7 describes the optimal collusion-proof contract under anonymity and compares this outcome with the second-best. Section 8 discusses further our concept of collusion-proofness and its robustness. Section 9 shows how asymmetric information can be used to generate endogenously some transaction costs in side contracting. Section 10 deals with the case of nonanonymous contracts. Section 11 briefly concludes and discusses possible extensions.

2. A REVIEW OF THE LITERATURE

Even since its inception, the mechanism design literature has noted that the formation of coalitions may invalidate the results obtained when only individual incentives are taken into account. Green and Laffont (1979) show for instance that the Groves mechanisms which implement the first-best decision for the level of a public good are not robust to the formation of coalitions under complete information. In the same vein, Maskin (1979) shows that the set of social choice functions which can be implemented in strong Nash equilibrium, i.e., which prevent joint deviations by coalitions, is much smaller than the set of social choice functions which are only Nash implementable. Bernheim and Whinston (1987) in their discussion of the weaker concept of *coalition-proof equilibrium* obtain also negative results along these lines. Contrary to the literature quoted above, we focus on the interaction between the coalition incentive constraints and the agents' participation constraints.

Using a differential approach, Laffont and Maskin (1980) show that there is a strong tension between individual and coalition incentives when dominant strategy implementation is required. In fact, only decision rules which entail full pooling can finally be implemented with twice differentiable mechanisms. The reason is that the number of coalition incentive constraints which must be satisfied by a collusion-proof contract increases enormously with a continuum of types. If one considers implementation in Bayesian-Nash equilibrium and if one assumes that members of a coalition who share their information are committed to stay in the coalition,[3] some decision rules which are not full pooling can also be implemented (Laffont and Maskin (1979)). To avoid the limitations arising in the analysis of adverse-selection problems with a continuum of types, we focus instead on a simple model with a discrete distribution of types. This helps to give a clear characterization of where the nexi of collusion are.

Crémer (1996) is the first to introduce asymmetric information in the formation of coalitions. He analyzes the robustness of Groves mechanisms to side-contracting. However, he still imposes that coalitions agree on side contracts which are implementable in dominant strategy. He shows that there exist Groves mechanisms which are robust to coalitions of size two. Furthermore, larger coalitions would be destroyed by manipulations from subcoalitions.

McAfee and McMillan (1992) analyze collusion in auctions.[4] They characterize a collusive ring's best strategy when side contracts do or do not involve monetary transfers and study some possible responses of the seller. Caillaud and Jehiel (1994) also deal with the robustness of simple auctions (second-price auctions with reservation prices) to the formation of rings. In their setting, ring formation is complicated by the existence of externalities between the possible buyers of the good.

As in the last three contributions, we assume that information is asymmetric between the members of a coalition and that the side contract is offered by an uninformed third party. In contrast with Crémer, and consistent with Caillaud and Jehiel, our goal is to optimize within the set of collusion-proof contracts and not to look for the implementability of the first-best efficient decision rule. However, the latter authors only optimize in a very restricted class of grand mechanisms. Since we first describe the set of collusion-proof mechanisms and then optimize in this set, we take a broader perspective on the origins of the nexi and the stakes of collusion.

Our paper is also linked to the literature on collusion in hierarchies surveyed in Tirole (1992).[5] As this author calls for, we provide a characterization of what can be implemented with collusion and relate our collusion concept to a *constrained efficiency* notion within the coalition. In particular, by appealing to a third party as a side mechanism designer, we avoid the inefficient signaling equilibria which may arise if an informed agent makes the side contract offer, an issue dealt with by Felli (1991) in a specific three-layer hierarchical model. Moreover, our approach, by stressing the role of asymmetric information at the coalition formation stage, endogenizes the transaction costs of side contracting used so far in this literature.

[3] With this assumption, one avoids the difficult issue of knowing how agents who still remain in the coalition form their conjectures about the others who leave the coalition.

[4] See also the references therein.

[5] See also Tirole (1986).

There is no definitive view of coalition formation under asymmetric information. In particular, we take in our analysis a noncooperative approach with an uninformed mediator. Earlier contributions, such as Wilson (1978), Kobayashi (1980), and Yannelis (1991) among others, have instead proposed various notions describing allocations in the core of an economy with agents having differential information.

3. THE MODEL

3.1. *The General Setting*

We consider the regulation of a duopoly, the agents A_1 and A_2, by a regulator, the principal P. Alternatively, the principal can be considered as the owner of a multidivisional firm, each agent being then considered as a separate unit.

Firm (or unit) A_1 produces a quantity q_1 of an intermediate good which is used by firm A_2 to produce a quantity q_2 of final good. The production technology of this second firm is Leontief and one-to-one. We denote by q the amount of final good produced; therefore $q = q_1 = q_2$. Each firm A_i, $i \in \{1,2\}$, faces a constant marginal cost of producing the good in quantity q_i. We denote by θ_i these marginal costs, which are drawn independently from the same common knowledge distribution with support $\Theta_i = \Theta = \{\underline{\theta}, \bar{\theta}\}$ where $\Delta\theta = \bar{\theta} - \underline{\theta} > 0$. With probability ν (resp. $1 - \nu$), a firm is efficient (resp. inefficient) and has a cost $\underline{\theta}$ (resp. $\bar{\theta}$). Each firm knows only its own cost and not that of the other firm. The regulator is uninformed on both firms' costs.

As in Baron and Myerson (1982), the regulator maximizes a linear combination of the consumer's surplus net of the transfer paid to the firms and of the firms' profits, namely:

$$W = S(q) - (t_1 + t_2) + \alpha(U_1 + U_2), \tag{1}$$

where $\alpha \in [0,1[$, $U_i = t_i - \theta_i q$ for $i \in \{1,2\}$, and where $S' > 0$, $S'' < 0$, and $S''' \geq 0$.[6]

A regulatory contract M, or *grand mechanism* between the regulator and the firms is a triplet $\{q(\hat{\theta}_1, \hat{\theta}_2); t_1(\hat{\theta}_1, \hat{\theta}_2); t_2(\hat{\theta}_1, \hat{\theta}_2)\}$ where $\hat{\theta}_i$ is firm A_i's report on his private information specifying for each pair of reports a quantity to produce and some transfers to firms A_1 and A_2.

In the absence of *collusion* between the agents the contract must satisfy two requirements:

(i) First, the contract must be *Bayesian incentive compatible* (BIC), i.e., it is a Bayesian-Nash equilibrium for the agents to truthfully reveal their types.

(ii) Second, the contract must be *Bayesian individually rational* (BIR), i.e., must guarantee a positive expected profit for each firm after it has learned its own type and before it eventually learns that of the other firm.

[6] $S''' \geq 0$ is only used in Section 10.

3.2. *Collusion*

As soon as communication between the agents is feasible and costless, they try to coordinate and to manipulate their reports into the principal's mechanism. The important theoretical question is how should we model collusive behavior. For instance, should we explicitly model the extensive form of bargaining? Should we explicitly model the repeated relationships between the colluding partners?[7] Should be allow one of the informed parties to offer the side contract?[8]

All these modeling alternatives raise difficult problems of information signaling and bargaining under asymmetric information. They all imply that the game of coalition formation which follows a given principal's offer may end up having a multiplicity of equilibria. This multiplicity would be quite appealing if our goal was to understand how history and other exogenous factors affect the amount of collusion in the organization. However, this multiplicity appears less appealing to analyze the implications of these collusions for the organization's overall efficiency. By using a third party, hereafter denoted by T, as a *side contract mechanism designer* we are able to greatly simplify the problem and to precisely characterize the set of implementable allocations with collusion.

Different timings of the collusion game are also possible. The agreement between the agents may take place after or before the offer of the grand mechanism but it can also take an intermediate form. We assume first that the agents agree ex ante to share equally any asymmetric monetary transfers they receive from the principal. This agreement is made before the principal offers his own contract and before the agents learn their information. The importance of this stage is that it constrains the principal to offer thereafter *anonymous* contracts. Second, the third party T offers also a *side contract* which takes place after the principal's grand contract offer. Following Myerson and Satterthwaite (1983), such a side contract should be considered as a reduced form for the noncooperative decentralized bargaining process between the agents which takes place after the approval of a given grand mechanism.

The benevolent third party can be viewed as a fictitious modeling device which maximizes the sum of the agents' rents,

$$U_1 + U_2,$$

subject to a set of incentive compatibility, feasibility, and acceptance constraints. If collusion was taking place under symmetric information, the Coase Theorem would tell us that decentralized bargaining would reach a Pareto efficient outcome for the coalition. Instead, under asymmetric information, decentralized

[7] See Tirole (1992) and Martimort (1996) for some first attempts. As a matter of fact, modeling collusion between the agents through a repeated relationship would also require modeling the relationship of the principal with the agents in a repeated game framework. This would lead to great technical difficulties.

[8] See Myerson (1983) and Maskin and Tirole (1992) for an analysis of the informed principal problem.

bargaining may fail to achieve such outcomes. The literature on bargaining in extensive forms (Ausubel and Deneckere (1993)) nevertheless suggests that outcomes lying on the *constrained efficient* Pareto frontier remain feasible for some particular extensive forms when the future is not discounted too much by the agents. Moreover, any particular extensive form of bargaining would implement allocations within the coalition which can be achieved by the third party. Hence, the regulator could possibly increase his profit if such an extensive form was instead used. Our approach helps therefore to focus on the *lowest* bound of what can be achieved by the principal.[9]

Alternatively, if we keep the justification of Nash behavior as deriving from the absence of feasible direct communication between agents, the third party may also be interpreted as an uninformed agent having the technology needed to communicate with the members of the coalition. The competitive supply of this technology leads to zero profits for the third party.[10]

A side-contract S is a triplet $\{\phi(\cdot), y_1(\cdot), y_2(\cdot)\}$. $\phi(\cdot)$ represents the manipulation of the report function. This is a function mapping any pair of reports by the agents to the third party into the set of possibly stochastic reports into the principal's contract. $y_1(\cdot)$ and $y_2(\cdot)$ are two monetary transfers from the third party to each agent respectively. We assume that the third party is not a *source* of money. More precisely, we require that the following *ex post budget balance* constraint be satisfied for all states of nature:

$$(2) \qquad \sum_{k=1,2} y_k(\theta_1, \theta_2) = 0, \quad \forall \theta_1, \theta_2.$$

Note that there is no loss of generality in restricting the set of feasible side contracts to direct revelation mechanisms. Indeed, the Revelation Principle applies at this stage of the game. For any given grand mechanism, any indirect mechanism proposed by the third party (with general communication spaces between the agents and this third party and with a decision space being the cross product of the set of possible reports into the mechanism M and the set of ex post balanced transfers) would give an outcome of the game of coalition formation which can be achieved with the agents sending to the third party messages on their types only.

To be accepted along an equilibrium path, the side contract must guarantee to an agent an interim utility level $U_i(\theta_i)$ greater than what he expects from playing noncooperatively the grand mechanism M and then getting a utility $V_i(\theta_i)$. The outcome of the mechanism M defines then a set of reservation utilities for the agents before accepting or not accepting the side contract. We will be more precise below on what these reservation utilities are.

[9] We note that an upper bound is obtained with the second-best outcome when the agents fail to collude at all.

[10] We do not stress this interpretation too much which would oblige us to consider also the robustness of the grand coalition to possible bilateral collusions between this third party and any of the agents, or the likely use of this third party by the principal himself as a whistleblower through an appropriate incentive scheme.

We assume also that the side contract is *enforceable* even though the secrecy of this contract implies that there is no court of justice available to enforce it. Again, the assumption of enforceability of the side contract is a short-cut to capture in a static context the reputations of the third party and the agents which guarantee that the self-enforceability of these contracts would emerge in repeated relationships.

3.3. *Timing of the Game*

The timing of the overall game of contract offers and coalition formation is the following:

0. The third party T proposes to the agents an ex ante agreement to share equally any asymmetric transfer received by the coalition. Each firm accepts or refuses this agreement. If at least one of them refuses, the ex ante agreement is not enforced and each firm gets a reservation utility normalized at 0.
1. Nature draws the value of each firm's cost θ_i, for $i \in \{1,2\}$. Each of these firms learns only its own type.
2. The regulator P proposes the grand mechanism M.
3. Each firm accepts or refuses this grand contract. If at least one firm refuses, each firm gets its reservation utility.
4. The third party offers the side contract S to the agents.
5. Each firm accepts or refuses this side contract. If at least one firm refuses, the grand contract is played noncooperatively. In this case, reports are directly made in the grand mechanism and the next two stages of the game do not occur.
6. If the side contract has been accepted, reports in the side contract take place. Each agent reports noncooperatively its type to the third party.
7. The corresponding side transfers and the reports in the grand mechanism requested by the manipulation function are made.
8. The quantity of output and the monetary transfers requested by the grand mechanism M are enforced.[11]

We are interested in characterizing the perfect Bayesian equilibria[12] of the overall game of contract offer and coalition formation and as usual we solve the game backward. Often we refer to stages 4 to 8 as the continuation game of coalition formation.

4. THE OPTIMAL CONTRACT WITHOUT SIDE CONTRACTING

First, we analyze the second-best outcome obtained when the agents are not informed on each other's types and do not collude. Since the two firms are

[11] Note that this timing gives in fact lots of flexibility to the collusion agreement which occurs both before and after the grand contract. We comment further on this assumption in Section 10. In particular, we will discuss what happens when stage [0] of our game disappears and the principal can then offer *nonanonymous* grand contracts.

[12] See Fudenberg and Tirole (1991) for a definition.

perfectly symmetric, there is no loss of generality in looking for the optimal contract without side contracting within the class of mechanisms which are *symmetric*, i.e., such that $t_1(\cdot)=t_2(\cdot)=t(\cdot)$. For the sake of simplifying our notations, we denote $t(\bar{\theta},\bar{\theta})=\bar{t}$; $t(\underline{\theta},\bar{\theta})=\hat{t}_1$; $(t(\bar{\theta},\underline{\theta})=\hat{t}_2$; $t(\underline{\theta},\underline{\theta})=\underline{t}$. Anonymous contracts obtain when $\hat{t}_1=\hat{t}_2=\hat{t}$. We use a similar notation for $q(\cdot)$.[13] We also denote by $\underline{U}=U(\underline{\theta})$ and $\bar{U}=U(\bar{\theta})$ the expected rent of a firm obtained in the grand mechanism when it is respectively efficient or inefficient.

4.1. *Bayesian Implementation*

Without side contracting, there is no constraint on the set of contracts that the principal can offer to the agents. In particular, the absence of an ex ante agreement implies that he can offer nonanonymous transfers. From the Revelation Principle, we can restrict the analysis to the set of mechanisms such that truthtelling is a Bayesian-Nash equilibrium between the agents, i.e., such that the following Bayesian incentive compatibility constraints are satisfied. For the efficient agent,

$$\underline{U}=\nu\left(\underline{t}-\underline{\theta}\underline{q}\right)+(1-\nu)\left(\hat{t}_1-\underline{\theta}\hat{q}\right)\geq\nu\left(\hat{t}_2-\underline{\theta}\hat{q}\right)+(1-\nu)\left(\bar{t}-\underline{\theta}\bar{q}\right); \tag{3}$$

for the inefficient agent,

$$\bar{U}=\nu\left(\hat{t}_2-\bar{\theta}\hat{q}\right)+(1-\nu)\left(\bar{t}-\bar{\theta}\bar{q}\right)\geq\nu\left(\underline{t}-\bar{\theta}\underline{q}\right)+(1-\nu)\left(\hat{t}_1-\bar{\theta}\hat{q}\right). \tag{4}$$

To be accepted, the regulatory contract must also satisfy the following participation constraints:

$$\underline{U}\geq 0 \tag{5}$$

and

$$\bar{U}\geq 0. \tag{6}$$

The regulator maximizes the following welfare function:

$$W=\nu^2\left(S(\underline{q})-2\underline{t}\right)+2\nu(1-\nu)\left(S(\hat{q})-\hat{t}_1-\hat{t}_2\right)$$
$$+(1-\nu)^2(S(\bar{q})-2\bar{t})+2\alpha\left(\nu\underline{U}+(1-\nu)\bar{U}\right),$$

subject to constraints (3) to (6).

The following proposition summarizes the outputs requested by the optimal contract in the absence of collusion between the agents.

PROPOSITION 1: *Optimal regulation without side contracting between the agents entails*:

(i) *A decreasing schedule of outputs*, $\underline{q}^n>\hat{q}^n>\bar{q}^n$, *given by*

$$S'\left(\underline{q}^n\right)=2\underline{\theta}, \tag{7}$$

[13] However, we simplify the notations by postulating that $q(\bar{\theta},\underline{\theta})=q(\underline{\theta},\bar{\theta})=\hat{q}$, as more generality would not have any use since the agents are perfectly symmetric.

$$S'(\hat{q}^n) = \underline{\theta} + \bar{\theta} + (1-\alpha)\frac{\nu}{1-\nu}\Delta\theta, \tag{8}$$

$$S'(\bar{q}^n) = 2\bar{\theta} + 2(1-\alpha)\frac{\nu}{1-\nu}\Delta\theta. \tag{9}$$

(ii) *Only the efficient agent's incentive constraint* (3) *and the inefficient agent's participation constraint* (6) *are binding.*

(iii) *This second-best outcome can also be implemented with anonymous contracts.*

PROOF: The proof is standard.

As usual,[14] optimal regulation entails no distortion for the output levels requested from two efficient agents. To reduce the informational rent of these efficient agents, optimal regulation calls for a decrease in the output levels requested when at least one of them is inefficient. Even though the agents' types are drawn independently, the technological complementarity between the two firms' activities requires that optimal regulation links the requested outputs to the messages of both agents. This creates some possible scope for collusion between agents.

Since under Bayesian implementation, only expected transfers are characterized by the binding incentive and participation constraints, it is also possible to implement this second-best outcome with anonymous transfers. The relevant binding incentive (3) and participation (6) constraints then become respectively:

$$\underline{U} = \nu(\underline{t} - \underline{\theta}\underline{q}) + (1-\nu)(\hat{t} - \underline{\theta}\hat{q}) \geq \nu(\hat{t} - \underline{\theta}\hat{q}) + (1-\nu)(\bar{t} - \underline{\theta}\bar{q}); \tag{10}$$

$$\bar{U} = \nu(\hat{t} - \bar{\theta}\hat{q}) + (1-\nu)(\bar{t} - \bar{\theta}\bar{q}) \geq 0. \tag{11}$$

4.2. *Dominant Strategy Implementation*

Our informational structure is one of *private values*, i.e., the agents' types are identically and independently distributed. A result due to Mookherjee and Reichelstein (1992) applies in this context to justify that any decreasing schedule of outputs can be *equivalently* implemented in Bayesian or in *dominant strategy*. The idea behind this result is the following. In the optimal anonymous Bayesian mechanism described in Proposition 1, the binding incentive (10) and participation constraints (11) are defining the values of the expected transfers received by the efficient and the inefficient agents, i.e., respectively $\nu\underline{t} + (1-\nu)\hat{t}$ and $\nu\hat{t} + (1-\nu)\bar{t}$. These two constraints are insufficient to define without ambiguity the three variables $\underline{t}, \hat{t}, \bar{t}$. This leaves some leeway to adjust these transfers to satisfy

[14] See, for instance, Baron and Myerson (1982) and Laffont and Tirole (1993) among others.

the dominant strategy constraints even when transfers are restricted to be anonymous.[15]

Hence, since the second-best schedule of outputs derived in Proposition 1 is decreasing, there is no loss of generality in looking for the optimal contract within the set of dominant strategy mechanisms. In fact, dominant strategy implementation of an anonymous contract is obtained when the following constraints are satisfied: For the efficient agent,

$$\underline{t} - \underline{\theta}\underline{q} \geq \hat{t} - \underline{\theta}\hat{q}, \tag{12}$$

$$\hat{t} - \underline{\theta}\hat{q} \geq \bar{t} - \underline{\theta}\bar{q}; \tag{13}$$

for the inefficient agent,

$$\hat{t} - \bar{\theta}\hat{q} \geq \underline{t} - \bar{\theta}\underline{q}, \tag{14}$$

$$\bar{t} - \bar{\theta}\bar{q} \geq \hat{t} - \bar{\theta}\hat{q}. \tag{15}$$

The first (respectively second) two equations express that it is a dominant strategy for an efficient (respectively inefficient) agent A_1 to reveal truthfully his information whatever the report made by the other agent A_2. We have the following proposition.

PROPOSITION 2: (i) *Optimal regulation without side contracting between the agents can be implemented in a dominant strategy.*

(ii) *The only binding constraints are the two dominant strategy incentive constraints* (12) *and* (13) *and the inefficient agent's participation constraint* (11).

For further reference, we denote this contract D^*.

5. THE OPTIMAL CONTRACT WITHOUT SIDE CONTRACTING IS NOT COLLUSION-PROOF

5.1. *The Nexi of Collusion*

Under symmetric information[16] at the collusion stage, a pair of efficient agents is indifferent between telling collectively the truth to the principal or sending him any other pair of reports since (12) and (13) are binding, and this

[15] Mookherjee and Reichelstein (1992) show in fact that the equivalence between Bayesian and dominant implementation holds when the agent's utility function also satisfies a generalized Spence-Mirrlees property. This property is trivially satisfied in our model with constant marginal costs. To use the dominant strategy implementation despite the anonymity constraint on the transfers (which is not imposed by Mookherjee and Reichelstein (1992)) we nevertheless continue to express the participation constraints of the agents at the interim stage and do not require them to be satisfied ex post.

[16] We will see later than under asymmetric information the relevant nexi of collusion are the same as under symmetric information.

indifference can be broken in favor of the principal. Collusion certainly does not affect the reports of this coalition into the grand mechanism. From (14) and (15) which are both strictly satisfied, we also conclude that a pair of inefficient agents has no incentive to collectively lie into the grand mechanism. Again collusion has no bite in this state of nature. The only important coalition incentive constraint is that of a pair made of an efficient and an inefficient agent who would prefer to claim that they are both inefficient.

Indeed, when the second-best output schedule is proposed by the regulator and is implemented in a dominant strategy, the incentive constraint (13) is binding. Since the optimal output schedule is strictly decreasing, the inefficient agent's dominant strategy incentive constraint is slack, i.e.,

$$(16) \qquad \bar{t}^n - \bar{\theta}\bar{q}^n > \hat{t}^n - \bar{\theta}\hat{q}^n.$$

But when (16) holds, truthful revelation by an efficient agent A_1 of his type exerts a *negative externality* on the agent A_2 when he is inefficient. The latter would indeed prefer that the former claims he is inefficient to obtain a share of the gain $G_{FI} = \bar{t}^n - \bar{\theta}\bar{q}^n - (\hat{t}^n - \bar{\theta}\hat{q}^n) = \Delta\theta(\hat{q}^n - \bar{q}^n)$. This quantity represents the stake of collusion under symmetric information.

Since with D^*, an efficient agent A_1 is indifferent between telling the truth or lying to the principal, any slight positive bribe he may receive from the inefficient agent A_2 to misreport will break this indifference in favor of the colluding partner. Moreover, as long as the gain from trade is strictly positive, the inefficient agent also accepts such a side contract which provides him with a strictly positive profit.

5.2. *Implementability of a Collusive Side Contract*

We show hereafter that under asymmetric information, the third party can find a strictly positive bribe $\hat{y}$ from an inefficient to an efficient agent which implements the same manipulation of reports $\phi^*(\cdot)$ as under symmetric information at the coalition formation stage. With such a transfer, the contractual externality between the agents is internalized.

Suppose that the third-party offers the following simple side contract, denoted S^*: first, a manipulation function $\phi^*(\cdot)$ such that

$$\phi^*(\underline{\theta},\underline{\theta}) = \underline{\phi}^* = (\underline{\theta},\underline{\theta}),$$
$$\phi^*(\underline{\theta},\bar{\theta}) = \phi^*(\bar{\theta},\underline{\theta}) = \hat{\phi}^* = (\bar{\theta},\bar{\theta}),$$
$$\phi^*(\bar{\theta},\bar{\theta}) = \bar{\phi}^* = (\bar{\theta},\bar{\theta});$$

second, a set of ex post budget balanced transfers between the agents such that

$$y_1^*(\underline{\theta},\underline{\theta}) = y_2^*(\underline{\theta},\underline{\theta}) = 0,$$
$$y_1^*(\underline{\theta},\bar{\theta}) = -y_2^*(\underline{\theta},\bar{\theta}) = y_2^*(\bar{\theta},\underline{\theta}) = -y_1^*(\bar{\theta},\underline{\theta}) = \hat{y},$$
$$y_1^*(\bar{\theta},\bar{\theta}) = y_2^*(\bar{\theta},\bar{\theta}) = 0.$$

As soon as such a side contract can be implemented by the third party, the optimal contract is not robust to the manipulation of reports defined by $\phi^*(\cdot)$. By implementation of this side contract, we mean that the resulting overall contract proposed to the agent (denoted $D^* \circ S^*$), which requests a quantity of output $q \circ \phi^*(\cdot)$ and a transfer to agent A_i equal to $t \circ \phi^*(\cdot) + y_i(\cdot)$, is Bayesian incentive compatible and accepted with probability one by both agents whatever their types along the equilibrium path of the continuation game.

PROPOSITION 3: *The side contract S^* can be implemented by the third party. The efficient collusion is an equilibrium of the continuation game of coalition formation. Moreover, the side contract is enforced by the threat of returning to the dominant strategy equilibrium of mechanism D^*.*

PROOF: See Appendix 1.

In the proof of this proposition, we heavily use the dominant strategy implementation of D^* in computing the reservation values of each agent when he refuses the side contract and plays noncooperatively the grand mechanism. When the initial grand contract is implemented in truthtelling dominant strategies, the expected payoff achieved by an agent if he deviates and refuses the side contract can still be computed easily. In particular, this payoff does not depend on the possible inferences made by a nondeviant agent, say A_2, after having observed A_1's deviation, since A_2 still has truthtelling as a dominant strategy.

We have not yet proved that the optimal schedule of output is not implementable when collusion may occur. After all, this schedule also could be implemented in Bayesian-Nash strategies or with a nonanonymous set of transfers and this implementation could be robust to the formation of the coalition. This example shows where the likely gains are from collusion for the agents. In particular, it highlights two factors at their origin: first, the existence of a difference between the outputs $\hat{q}$ and $\bar{q}$ when the efficient agent's incentive constraints (12) and (13) are both binding; second, the latter's indifference between telling the truth or lying in D^*. Clearly, playing on these two factors may be part of an optimal regulatory response to the threat of collusion even under asymmetric information.

6. COLLUSION-PROOF CONTRACTS

In this section, we characterize the set of collusion-proof contracts which can be offered by the principal both when he is unrestricted in his contract offer and when he is restricted to offer anonymous contracts. We proceed in two steps. First, we define our concept of *collusion-proofness* for general mechanisms and prove a *Collusion-Proofness Principle* within this class. Second, we characterize the optimal side contract offered by the third party when the agents can obtain a given set of reservation utilities by playing noncooperatively the grand mechanism. We derive from this analysis the conditions satisfied by a collusion-proof mechanism. Finally, we show that the restriction to collusion-proof mechanisms is still valid even if the principal is restricted to offer anonymous contracts.[17]

6.1. *The Collusion-Proofness Principle*

We denote by $V_i(\theta_i)$ agent A_i's reservation utility obtained by playing noncooperatively the grand mechanism M if his type is θ_i. Obviously, this number should generally depend on the beliefs that have been used by the nondeviant agent following the refusal of the side contract. For the time being, we assume that, when the grand mechanism is played noncooperatively, a nondeviant agent does not change his beliefs on the deviant one. The choice of the side contract on the equilibrium path is thus sustained with *passive beliefs* out of the equilibrium path.[18]

The third party T is benevolent and maximizes the sum of the units' expected profits, subject to (BB), (BIC), and acceptance constraints (BIR). We first need the following definition.

DEFINITION 1: A side contract $S^* = \{\phi^*(\cdot), y_1^*(\cdot), y_2^*(\cdot)\}$ is said to be *coalition-interim-efficient* with respect to a grand mechanism $M = \{q(\cdot); t_1(\cdot); t_2(\cdot)\}$ providing the reservation utilities $V_i(\theta_i)$, $i \in \{1,2\}$, when it is played noncooperatively if and only if it is a solution of the following program (thereafter denoted (T)):

$$\text{Program } (T)\begin{cases} \max\limits_{S} E_{\Theta\times\Theta}(U_1(\theta_1) + U_2(\theta_2)) \quad \text{subject to} \\ U_i(\theta_i) = E_{\Theta_{-i}}\big(y_i(\theta_i,\theta_{-i}) + t_i(\phi(\theta_i,\theta_{-i})) \\ \qquad\qquad\qquad - \theta_i q(\phi(\theta_i,\theta_{-i}))\big), \quad \forall i, \theta_i, \\ U_i(\theta_i) \geq U_i(\hat{\theta}_i) - (\theta_i - \hat{\theta}_i)E_{\Theta_{-i}}\big(q\big(\phi\big(\hat{\theta}_i,\theta_{-i}\big)\big)\big), \\ \qquad\qquad\qquad \forall i, \theta_i, \hat{\theta}_i, \quad \text{(BIC)}, \\ U_i(\theta_i) \geq V_i(\theta_i), \quad \forall i, \theta_i \qquad \text{(BIR)}, \\ \sum\limits_{k=1,2} y_k(\theta_1,\theta_2) = 0, \quad \forall \theta_1, \theta_2 \qquad \text{(BB)}. \end{cases}$$

This definition will help us to state and prove our *Collusion-Proofness Principle*. This principle says that there is no loss of generality in restricting the analysis to the study of grand mechanisms which are unchanged through the process of coalition formation, i.e., such that S^* is equal to the *null* side contract. This means that in an equilibrium of the continuation game agents do

[17] Tirole (1992) discusses why restrictions in the set of feasible grand mechanisms may sometimes lead to equilibrium collusion.

[18] This term has been coined by Rubinstein (1985) in a bargaining context and also used by Crampton and Palfrey (1995) in their analysis of ratifiable mechanisms.

not make any monetary transfers ($y^*(\cdot)=0$) and choose not to manipulate their reports into the grand mechanism ($\phi^*(\cdot)$ is the identity function).

DEFINITION 2: A mechanism $M=\{q(\cdot); t_1(\cdot); t_2(\cdot)\}$ which gives to the agents utility levels $V_i(\theta_i)$, $i \in \{1,2\}$, when it is played noncooperatively is *collusion-proof* when the null side contract is coalition-interim-efficient with respect to this mechanism.

A coalition-interim-efficient side contract defines a continuation equilibrium of our overall game. For a collusion-proof mechanism, *one* perfect Bayesian equilibrium of the continuation game of coalition formation is obtained when the third party offers the null side contract and when the agent accepts it. Moreover, this continuation equilibrium is sustained by the threat of playing the grand mechanism with passive beliefs. We can now prove the following proposition:

PROPOSITION 4: *There is no loss of generality in restricting the principal to offer collusion-proof contracts, i.e., any perfect Bayesian equilibrium of the overall game of grand contract offer and coalition formation gives an allocation for the principal and the agents which can also be achieved with a collusion-proof contract.*

PROOF: See Appendix 2.

Our concept of collusion-proofness is rather weak. The null side contract may be only *one* continuation equilibrium of the game of coalition formation sustained with passive beliefs but other equilibria in which the third party offers a non-null side contract could also be sustained. These equilibria are obtained in particular when an out-of-equilibrium refusal of the side contract triggers a change in the belief of a nondeviant agent and the corresponding payoffs obtained by playing the grand mechanism are such that the non-null side contract is accepted by both types of agents.

We would thus prefer to have grand mechanisms robust to *all* possible perfect Bayesian equilibria of the continuation game. We will see in Section 8 that we can nevertheless content ourself with a *weak collusion-proofness* concept.[19]

[19] Note also that there always exist other equilibria of the continuation game of coalition formation. In these equilibria, the third party offers any side contract and each agent refuses to enter the coalition, anticipating that the other agent does so. These equilibria are supported with passive beliefs if one agent deviates and accepts the side contract. Any grand mechanism is always robust to these equilibria based on weakly dominated strategies. For collusion-proofness to have any bite, we need to refine the set of continuation equilibria by the elimination of weakly dominated strategies.

6.2. *Characterization of the Collusion-Proof Contracts*

The next proposition fully characterizes the symmetric collusion-proof contracts with no restriction on the set of available contracts. We focus on the subset of these collusion-proof contracts in which the inefficient agent's incentive constraint is not binding.[20]

PROPOSITION 5: *A Bayesian incentive compatible contract* $M = \{\underline{t}, \hat{t}_1, \hat{t}_2, \bar{t}, \underline{q}, \hat{q}, \bar{q}\}$ *such that the inefficient agent's incentive constraint is not binding is collusion-proof if and only if the following conditions are satisfied.*

(i) *There exists* $\epsilon \geq 0$ *(with equality if and only if* (3) *is slack) such that the following coalition incentive constraints hold: First, for the coalition made of two efficient agents,*

$$(17) \qquad 2\underline{t} - 2\underline{\theta}\underline{q} \geq \hat{t}_1 + \hat{t}_2 - 2\underline{\theta}\hat{q},$$

$$(18) \qquad 2\underline{t} - 2\underline{\theta}\underline{q} \geq 2\bar{t} - 2\underline{\theta}\bar{q};$$

second, for the coalition made of one efficient agent and one inefficient agent,

$$(19) \qquad \hat{t}_1 + \hat{t}_2 - \left(\underline{\theta} + \bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)\hat{q} \geq 2\bar{t} - \left(\underline{\theta} + \bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)\bar{q},$$

$$(20) \qquad \hat{t}_1 + \hat{t}_2 - \left(\underline{\theta} + \bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)\hat{q} \geq 2\underline{t} - \left(\underline{\theta} + \bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)\underline{q};$$

third, for the coalition made of two inefficient agents,

$$(21) \qquad 2\bar{t} - 2\left(\bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)\bar{q} \geq 2\underline{t} - 2\left(\bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)\underline{q},$$

$$(22) \qquad 2\bar{t} - 2\left(\bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)\bar{q} \geq \hat{t}_1 + \hat{t}_2 - 2\left(\bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)\hat{q}.$$

(ii) *The Bayesian incentive compatibility constraint* (3) *of a high-valuation agent is strictly satisfied when* $\epsilon = 0$. *When it is binding and the grand contract is anonymous,* ϵ *is such that* $((1-\nu)/\nu) > \epsilon > 0$.

PROOF: See Appendix 3.

In the absence of any asymmetric information at the coalition formation stage, collusion would be efficient and the agents would manipulate the sum of both units' costs when making their reports to the principal. The *coalition incentive constraints* under symmetric information would be the same as (17) to (22) with the particular value $\epsilon = 0$.

When collusion occurs under asymmetric information, the same logic applies. The contract must not only be individually incentive compatible but must also prevent group deviations. In each state of nature, the transfers specified in the

[20] In fact, one can prove that collusion-proof contracts such that the inefficient agent's incentive constraint (4) is binding are never optimal for the principal.

grand contract have to induce the agents of the coalition to jointly reveal their types. However, the total cost of the coalition is not anymore the sum of the true costs of both productive units as under symmetric information but the sum of their *discounted virtual costs*.

To give an intuition for the form taken by the coalition incentive compatibility constraints, we must go back to the definition of a coalition-interim-efficient allocation. The third party maximizes a utilitarian criterion for the coalition. If we had only considered the Bayesian incentive compatibility and budget balanced constraints, we know from Aspremont and Gérard-Varet (1979) that there would exist a system of budget balanced side transfers such that the efficient complete information manipulation of reports could be implemented by the third party. Accordingly, the coalition incentive compatibility constraints would be the same as under symmetric information, i.e., we would have $\epsilon = 0$.

When the principal offers a grand mechanism which gives to the agents some reservation utility levels $V_i(\theta_i)$, $i \in \{1,2\}$, this reasoning may break down. The set of budget balanced side transfers discussed above may fail to still implement the efficient decision rule for the coalition because, with these side transfers, the third party may no longer be able to satisfy the Bayesian individual rationality constraints of the agents. The well-known tension between budget balance and participation constraints in this Bayesian context appears at the level of the coalition.

In Section 5, we showed that D^* is not collusion-proof. Indeed, D^* is such that (19) is violated for all $\epsilon \geq 0$. An infinitesimal positive transfer $\hat{y}$ from the inefficient agent A_1 to the efficient agent A_2 is then enough to internalize the contractual externality between them and to implement the efficient manipulation of reports for the coalition when collusion takes place under asymmetric information and the principal proposes D^*. Nevertheless, implementing the manipulation of report function requires a transfer $\hat{y}$ high enough to satisfy the incentive constraint of an efficient agent. To give up this high transfer may be excessively costly for an inefficient firm who may prefer to refuse the collusive agreement if it has a high enough reservation utility. To reduce this tension, the third party proposes side contracts which require less important transfers to the efficient agent. To do this he must distort the manipulation of report function. Inequalities (17) to (22) are then simply the characterization of the set of grand mechanisms such that those distorted manipulation functions are precisely still equal to the identity.

It turns out that when writing coalition incentive compatibility constraints, everything happens as if the inefficient unit had a higher cost than its true cost, namely $\bar{\theta} + (\nu/(1-\nu))\epsilon\,\Delta\theta$ instead of $\bar{\theta}$, where $(\nu/(1-\nu))\epsilon$ is a discount factor less than one. We refer to this cost as the discounted virtual cost.[21] This

[21] The same would apply also for the efficient agent if we were interested in collusion-proof contracts in which the inefficient agent's incentive constraint only may be binding. An efficient unit would then have a lower virtual cost than its true cost ($\underline{\theta} \geq \underline{\theta} - ((1-\nu)/\nu)\epsilon'\,\Delta\theta$). Then ϵ' measures the intensity with which the reservation utility offered by the principal to an efficient agent constrains the third party.

discount measures how the reservation utility offered by the principal to an inefficient agent constrains the third party in the design of the optimal side contract. When $\epsilon = 0$, this utility level does not constrain the third party who can still implement the complete information decision rule within the coalition. The Aspremont and Gérard-Varet side transfers satisfy the agent's participation constraints. The reservation utility offered by the grand mechanism constrains all the more the third party's problem that ϵ is larger.

In our description of the collusion-proof mechanisms we have not allowed the principal to ask the agents for the information that they may have learned during the course of the coalition formation. Indeed, after having received the contingent plans proposed by the third party, they are informed on each other's types. One might think that the principal could improve society's welfare by trying to extract this common information. Such a mechanism would be of the form: $\{q((\hat{\theta}_1, m_2), (\hat{\theta}_2, m_1)); t_1((\hat{\theta}_1, m_2), (\hat{\theta}_2, m_1)); t_2((\hat{\theta}_1, m_2), (\hat{\theta}_2, m_1))\}$ where $m_j \in \{\varnothing, \Theta\}$ is a report of agent A_i on what he has learned on A_j during the coalition stage. In particular, when collusion does not take place, $m_j = \varnothing$ is sent by the agent A_i. Since the third party can always coordinate the agents' reports so that $m_1 = m_2 = \varnothing$, the principal does not gain anything from using these augmented mechanisms.[22]

6.3. *Anonymity and Collusion-Proofness*

When the principal offers an anonymous contract which is collusion-proof, the following upward coalition incentive constraints obtained when $\hat{t}_1 = \hat{t}_2$ must be satisfied:

$$(23) \qquad 2\underline{t} - 2\underline{\theta}\underline{q} \geq 2\hat{t} - 2\underline{\theta}\hat{q},$$

$$(24) \qquad 2\hat{t} - \left(\underline{\theta} + \bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)\hat{q} \geq 2\bar{t} - \left(\underline{\theta} + \bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)\bar{q}.$$

The Collusion-Proofness Principle does not make any restriction on the set of available mechanisms in the first place. Even if some anonymous grand contracts are collusion-proof, it does not mean that when he is restricted to offer anonymous contracts the principal finds it optimal to offer collusion-proof ones. It could indeed be optimal to offer an anonymous grand contract and to let collusion occur in equilibrium. Equilibrium collusion could have some benefit because nonzero side transfers break the anonymity of the transfers and possibly could help to enlarge the set of implementable output allocations. Nevertheless, we are able to prove the following proposition.

PROPOSITION 6: *There is no loss of generality in restricting the principal to offer anonymous contracts which prevent collusion between an efficient agent and an inefficient one.*

[22] Itoh (1993) makes a similar point in a moral hazard context.

PROOF: See Appendix 4.

The intuition behind this result is the following. When collusion is allowed, a manipulation of reports is made by the agents. In fact, the coalition made by an efficient agent and an inefficient one prefers to claim that they are both inefficient and (24) is reversed. It implies that the final output allocation entails some pooling. Moreover, allowing side transfers to be strictly positive has no advantage. Since they are budget balanced, the expected value of their sum is always zero for the principal. Hence, when collusion is allowed, what can be achieved by the principal could also be achieved with an anonymous contract entailing some pooling.

7. THE DESIGN OF THE OPTIMAL COLLUSION-PROOF AND ANONYMOUS CONTRACT

First, we characterize the class of collusion-proof and anonymous contracts such that the efficient agent's Bayesian incentive constraint is binding, i.e., such that $\epsilon > 0$.

LEMMA 1: *A collusion-proof and anonymous contract such that $\epsilon > 0$ entails some pooling $\hat{q} = \bar{q}$.*

PROOF: See Appendix 5.

This lemma shows that there is a strong tension between the incentive compatibility constraint (10) and the coalition incentive constraints (17) and (19). To satisfy (17), it is necessary to give a transfer $\underline{t}$ high enough. For a collusion-proof contract such that $\epsilon > 0$ the incentive constraint (10) is binding and it also requires a sufficient increase of the transfer $\bar{t}$. This cannot happen without violating the coalition incentive constraint (19) of the coalition made of an efficient and an inefficient agent. To satisfy this constraint also, the regulator is obliged to propose contracts which pool the outputs given to a pair of efficient-inefficient and to a pair of inefficient-inefficient agents.

There are two ways of preventing collusion. The first one is to destroy the stake of collusion by offering a contract with some pooling as described above. No manipulation of reports can make the inefficient agent A_2 better off since, with such a contract, A_1's report does not exert any externality on A_2. To prevent collusion, the principal can nevertheless also offer a separating allocation if he increases sufficiently the transfer $\hat{t}$ so that (19) is satisfied. When this is done, the Bayesian incentive constraint (10) is relaxed and necessarily $\epsilon = 0$. Note that a partially pooling contract such that $\hat{q} = \bar{q}$ also satisfies (17) and (23) when $\epsilon = 0$ and there is no loss of generality in optimizing within the class of collusion-proof contracts such that $\epsilon = 0$.

PROPOSITION 7: *The optimal collusion-proof and anonymous contract proposed by the regulator entails*:

(i) *a strictly decreasing schedule of outputs* $\underline{q}^c > \hat{q}^c > \bar{q}^c$ *given by*

$$S'(\underline{q}^c) = 2\underline{\theta}, \tag{25}$$

$$S'(\hat{q}^c) = \underline{\theta} + \bar{\theta} + (1-\alpha)\frac{1+\nu}{2(1-\nu)}\Delta\theta, \tag{26}$$

$$S'(\bar{q}^c) = 2\bar{\theta} + (1-\alpha)\frac{\nu}{1-\nu}\Delta\theta; \tag{27}$$

(ii) *a set of transfers* $(\underline{t}^c, \hat{t}^c, \bar{t}^c)$ *such that the following constraints are binding*:

$$\underline{t}^c - \underline{\theta}\underline{q}^c \geq \hat{t}^c - \underline{\theta}\hat{q}^c; \tag{28}$$

$$\hat{t}^c - \tfrac{1}{2}(\underline{\theta} + \bar{\theta})\hat{q}^c \geq \bar{t}^c - \tfrac{1}{2}(\underline{\theta} + \bar{\theta})\bar{q}^c; \tag{29}$$

$$\bar{t}^c - \bar{\theta}\bar{q}^c \geq \tfrac{1}{2}\nu\Delta\theta(\hat{q}^c - \bar{q}^c). \tag{30}$$

PROOF: See Appendix 6.

We denote this contract by C^*. The regulator never gives up the benefit of discriminating between a pair with two inefficient agents and a pair with one efficient and one inefficient agent even if it requires some slack on the efficient agent's incentive constraint. There is always some screening at the optimum.

7.1. *Comparative Statics*

In this section we compare the optimal contracts with and without side contracting. Under asymmetric information, the binding participation constraint on an inefficient agent needs only to be satisfied in expectation. The transfer $\hat{t}$ can be reduced in a way that does not guarantee a positive ex post profit to an inefficient agent when he faces an efficient one as soon as the principal sufficiently increases the transfers given when the inefficient agent also faces another inefficient agent. By reducing sufficiently $\hat{t}$, the regulator is able to prevent the deviation of the coalition of agents $(\underline{\theta}, \bar{\theta})$ at the lowest possible cost, i.e., (30) is binding.

PROPOSITION 8: *Optimal regulation with asymmetric information at the collusion stage entails*:

(i) *no distortion for the output requested for a pair of efficient agents, i.e., the first best level of output is still implemented*:

$$\underline{q}^n = \underline{q}^c; \tag{31}$$

(ii) *a higher distortion than without side contracting for the output requested for a pair made of an efficient agent and an inefficient one*:

$$\hat{q}^c < \hat{q}^n; \tag{32}$$

(iii) *a lower distortion than that needed without side contracting for the output requested for a pair made of inefficient agents*:

$$\bar{q}^n < \bar{q}^c. \tag{33}$$

PROOF: Immediate. See also Figure 1.

With side contracting and when (19) is binding, it is easy to check that the information rent of an efficient agent is as follows:

$$\underline{U}^c = \Delta\theta(\nu\hat{q}^c + (1-\nu)\bar{q}^c) + \frac{(1-\nu)}{2}\Delta\theta(\hat{q}^c - \bar{q}^c). \tag{34}$$

Without side contracting, this informational rent is instead

$$\underline{U}^n = \Delta\theta(\nu\hat{q}^n + (1-\nu)\bar{q}^n). \tag{35}$$

With respect to the case without side contracting, there is now a new term on the right-hand side which represents the value of collusion for an efficient agent. It is the share of the stake obtained when he faces an inefficient agent. Since this stake is proportional to the difference between $\hat{q}$ and $\bar{q}$, the cost of collusion is reduced when $\hat{q}$ and $\bar{q}$ are respectively downwards and upwards

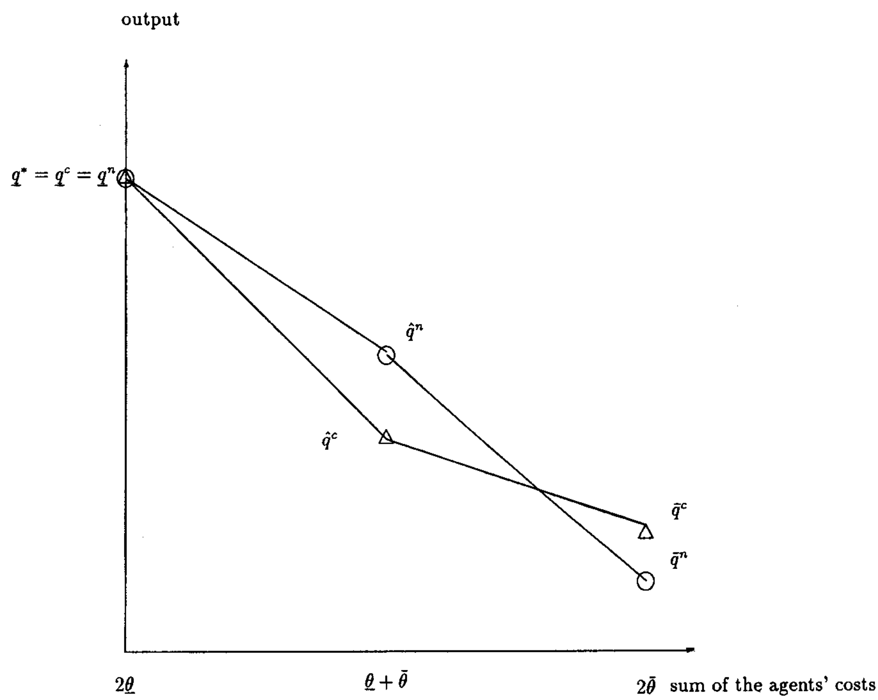

FIGURE 1.—Output distortions with and without side contracting.

distorted with respect to their values without side contracting. The allocative consequences of collusion are not clear cut. Some outputs are closer to the first-best, others are instead more distorted.

The comparison between the informational rents with and without side contracting shows that collusion does not necessarily increase the rent of the efficient agent once the reaction of the principal has been taken into account. In fact, the second term on the right-hand side of (34) is the source of an increase in the agent's informational rent. At the same time, since $\hat{q}$ is distorted downwards with side contracting, it is possible that the efficient agent loses from forming the coalition and achieves a lower payoff than if collusion does not take place. However, the following example shows that the efficient agent can nevertheless always gain from the collusion even when the reaction of the principal has been taken into account.

PROPOSITION 9: *Assume that $\Delta\theta$ is small enough or that $|S''|$ is constant; then the efficient agent's informational rent always increases with side contracting, $\underline{U}^c > \underline{U}^n$.*

PROOF: See Appendix 7.

Lastly, the regulator can strictly improve society's welfare by making sure that collusion does not take place since he then faces a less constrained problem. Hence, designing better communication channels within the regulated firm may hurt the regulator since they help the agents to collude.

8. STRONG COLLUSION-PROOFNESS

The contract derived in Section 7 characterizes the optimal way of preventing collusion with the weak collusion-proofness principle as stated in Proposition 4. We explain here why it is useless to appeal to a stronger notion of collusion-proofness to characterize the optimal contract in the presence of collusion. Let us first define such a concept.

DEFINITION 3: A mechanism M is *strongly collusion-proof* if and only if there is no perfect Bayesian equilibrium of the continuation game in which the third party offers a non-null side contract and both agents accept it.

Strong collusion-proofness differs from the concept used so far because it does not require the null side contract to be supported as a continuation equilibrium with passive beliefs out-of-the-equilibrium path. Instead, strong collusion-proofness does not put any restriction on the out-of-equilibrium beliefs which support the choice of this null side contract as a continuation equilibrium. This is therefore a more demanding implementation concept since a mechanism which is robust to one possible continuation equilibrium may generally fail to be robust to other continuation equilibria when the latter are sustained by out-of-equilibrium beliefs which are not passive. We are nevertheless able to prove the following proposition.

PROPOSITION 10: *The optimal collusion-proof and anonymous contract C^* is strongly collusion-proof. Hence, there is no loss of generality in looking for the optimal collusion-proof contract within the class considered in Definition* 2.

PROOF: See Appendix 8.

The key to the previous result is that the grand mechanism C^* is in fact implemented in dominant strategy. Since C^* must satisfy the coalition-proofness incentive constraint of a coalition of efficient agents and the Bayesian incentive constraint of an efficient agent, it is immediate that it is a dominant strategy for an efficient agent to always tell the truth. More interestingly, the transfer $\hat{t}$ determined by the binding coalition proofness constraint (29) is sufficiently high to induce an inefficient agent to reveal his type when facing an efficient one, and simultaneously sufficiently low to prevent an inefficient agent to lie if he faces another inefficient one. Hence, this is also a dominant strategy for the inefficient agent to truthfully reveal his type. Indeed, whatever grand contract is offered by the principal, the third party implements a manipulation of reports which maximizes state by state the sum of the agents' utilities where costs have been replaced by virtual discounted costs. As a result of the optimization, revealed preference arguments show that the coalition chooses a higher manipulation of outputs the lower their total cost, i.e., $q \circ \phi^*(\cdot)$ is a decreasing schedule of outputs even if $q(\cdot)$ itself is not. Hence, from Mookherjee and Reichelstein (1992), this schedule of manipulated outputs could be implemented in dominant strategy by the principal and it turns out that it can also be implemented by an anonymous grand mechanism. The play of such a collusion-proof mechanism is therefore independent of the agents' belief.

9. TOWARDS UNDERSTANDING TRANSACTION COSTS IN SIDE CONTRACTING

The recent literature on collusion in hierarchies has put forward the idea that bribes are associated with some shadow costs which undermine the efficiency of the exchanges between the colluding agents. For instance, Laffont and Tirole (1991) build a political economy theory of regulation based on the analysis of three-tier hierarchical models, regulator-supervisor-agent. In these models, one can easily link the power of a lobbying group to the importance of this shadow cost. Also Holmstrom and Milgrom (1990) analyze in a moral hazard model the consequences of transaction costs in side contracting in the context of the corporation. Even though it is very useful to understand how the organizational responses change for exogenously given transaction costs, these transaction costs need to be given more theoretical foundations. With such foundations, it would become easier to discuss how the principal designs not only the agents' incentive schemes but also the internal communication channels to prevent their collusion at the lowest possible cost.[23]

If the agents were able to sign their side contract under symmetric information but before the knowledge of their respective types, the coalition incentive constraint for a coalition made of an efficient and an inefficient agent would be as follows:

$$2\hat{t} - (\underline{\theta} + \bar{\theta})\hat{q} \geq 2\bar{t} - (\underline{\theta} + \bar{\theta})\bar{q}.$$

When this constraint is binding the gain which accrues to the inefficient agent when the efficient one lies to the principal is

$$\bar{t} - \bar{\theta}\bar{q} - (\hat{t} - \bar{\theta}\hat{q}) = \frac{\Delta\theta}{2}(\hat{q} - \bar{q}). \tag{36}$$

Under asymmetric information, the coalition incentive constraint which is binding is (25) and the gain which accrues to the inefficient agent when the efficient one lies to the principal is

$$\bar{t} - \bar{\theta}\bar{q} - (\hat{t} - \bar{\theta}\hat{q}) = \frac{\Delta\theta}{2}\left(1 - \frac{\nu\epsilon}{1-\nu}\right)(\hat{q} - \bar{q}). \tag{37}$$

In our model, we trace out the existence of such transaction costs to the asymmetric information between the colluding partners. The transaction cost of side contracting can be identified as

$$k = \frac{\nu\epsilon}{1-\nu} < 1.$$

Contrary to the previous literature which uses exogenous transaction costs, the principal can now also play on ϵ through its grand contract offer by committing to different levels of rent. In our rather stylized model, the value of ϵ is either 0 for separating contracts or indeterminate for pooling ones.

Finally, note that our approach using a third party to coordinate the agents on a coalition-interim efficient allocation underestimates transaction costs. With decentralized and possibly inefficient bargaining, the value of these transaction costs will be strictly positive and dependent on the precise extensive form used.

10. NONANONYMOUS MECHANISMS

In this section we come back to our assumption of anonymity of the grand contract. That the grand contract is anonymous is in fact implied by the extensive form of our game. With the possibility for the coalition to have an ex ante agreement in which its members commit to share equally the sum of any nonanonymous transfers they receive, there is no point for the principal in preventing collusion by offering different transfers to the two agents. If stage [0] of our game is suppressed or if one agent refuses the ex ante agreement, the principal now has the ability to offer nonanonymous transfers to the two agents. This simply means that for a pair of reports that are different, for instance A_1

[23] See Laffont and Martimort (1995) where a hierarchical organization is compared with centralization along these lines.

reports $\underline{\theta}$ and A_2 reports $\bar{\theta}$, the regulator may offer different transfers $\hat{t}_1 \neq \hat{t}_2$ to each agent depending on his report.

In particular, allowing nonanonymous transfers gives the principal the opportunity to always destroy the stake of collusion between the two agents when $S''' \geq 0$.

PROPOSITION 11: *When $S''' \geq 0$, the optimal collusion-proof and nonanonymous contract implements the second-best outcome in dominant strategy.*

PROOF: See Appendix 9.

To see this it is enough to consider the case where agents have symmetric information on each other's types at the time of collusion or equivalently to exhibit a set of transfers which satisfy incentive compatibility, (3) and (4), participation, (5) and (6), and coalition incentive contraints, (17) to (22) (with a particular value of ϵ and one can take $\epsilon = 0$), when the agents have asymmetric information on each other. By offering a sufficiently low transfer to the inefficient agent when he faces an efficient one, the principal can guarantee that the former makes the same profit (namely 0) whatever the latter's report. The inefficient agent is therefore indifferent to the report made by the efficient one. There is no contractual externality between them and there is no point in reaching a collusive agreement. Indeed, the extra degree of freedom obtained when the principal can offer nonanonymous contracts can be used to get both dominant strategy implementation and zero expected profit for the inefficient firm whatever the state of nature. The same outcome as in optimal regulation under asymmetric information without side contracting can then be achieved by the principal.

Note that a relatively high transfer $\hat{t}_2$ is required to have the inefficient agent's participation constraint be binding state by state. It could be the case that the coalition incentive constraint (20) is then violated. The assumption that $S''' \geq 0$ ensures that this is not the case. However, let us stress that allowing for nonanonymous contracts is in general not sufficient to destroy the stake of collusion. In our two-type setting this peculiar result is due to the fact that the coalition incentive constraints are the sums of individual incentive constraints. We could indeed build three-type extensions of our basic setup in which the possibility of collusion introduces coalition incentive constraints which cannot be replicated by individual incentives and which introduce also effective nexi of collusion even when nonanonymous contracts are used. The major difficulty of such an analysis will be the determination of which coalition incentive constraints are binding.

Adding together the fact that nonanonymous contracts lead to the implementation of the second-best outcome and the result of Proposition 9 tells us that there exists an equilibrium of the overall game including stage [0] in which both agents agree to share asymmetric transfers. Obviously, there may be coordination failures and there exists also another equilibrium of the overall game including stage [0] in which each agent chooses not to accept the side contract, anticipating that the other does so. However, this equilibrium relies on the use of weakly dominated strategies and can be eliminated with a standard refinement.

11. CONCLUDING REMARKS

Even if our results have been obtained in a very specific context, we have offered a methodology for studying more complex economic situations in which collusion matters. Indeed, to systematically allow collusion in our modeling of organizations and to compare different organizations with respect to their scope for collusion should give us more insights into the emergence of particular institutions. Collusion is not only important to understand the allocation of resources within the firm and its internal structure,[24] but also to understand the impact of capture on the regulatory process, the organization of the government itself,[25] and finally how markets generally behave.[26] Hence, the analysis of the present paper would certainly deserve to be extended in these other contexts.

The important theoretical question faced in any of these settings is precisely to determine where the nexi of collusion are and how the coalition incentive constraints can be written. In particular, several factors may crucially affect these contraints.

First there is the degree of risk aversion of the members of the coalition. We have chosen to analyze an example with risk neutrality.

Second, there is the degree of "competitiveness" of the environment. In our model, the agents perform complementary activities and the unique externality between them is informational and is due to the anonymity of the grand contract.

Third, the exact information structure between the agents may play an important role. We have assumed private values, but certainly this is a polar case and some correlation in the agent's information is generally present.

Fourth, the exact nature of the communication with the principal is certainly important in the determination of the nexi of collusion. For instance, in the literature on collusion in three-tier hierarchies it is generally assumed that collusion occurs on the report of a piece of verifiable information. Here, we have instead assumed that collusion occurs on the report of a piece of unverifiable information.

Fifth, the extension of our model to the case of more than two agents may raise several important questions.[27] Should we allow the grand-coalition agreements to be robust to the formation of subcoalitions? Along the same lines of ideas should we only focus on self-enforceable agreements as suggested by the work of Bernheim, Peleg, and Whinston (1987)? Should we allow for competition between several organized subgroups? Etc....

[24] See Tirole (1986).
[25] See Laffont and Tirole (1991) and Laffont and Martimort (1994).
[26] See the literature on collusion in auctions already quoted.
[27] See Laffont and Martimort (1996) for a first step in this direction.

Sixth, we could have motivated our assumption of anonymity by a lack of completeness of the grand contract offered by the principal. For instance, he may only be able to use one-dimensional mechanisms contingent on the sum of the agents' reports.[28] How incomplete contracting and collusion interact is more generally an important issue that certainly deserves more work.

Seventh, it is important to relax the anonymity assumption. This will require combining the approach proposed here with the recent progresses of mechanism design with multidimensional asymmetric information (Rochet (1995), Armstrong (1996)).

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APPENDIX 1

We first want to ensure that the composition of D^* and S^* denoted $D^* \circ S^*$ is Bayesian incentive compatible. The following constraints must therefore be satisfied: For the efficient agent

$$\nu(\underline{t} - \underline{\theta}\underline{q}) + (1-\nu)(\hat{y} + \bar{t} - \underline{\theta}\bar{q}) \geq \nu(-\hat{y} + \hat{t} - \underline{\theta}\hat{q}) + (1-\nu)(\bar{t} - \underline{\theta}\bar{q}); \tag{38}$$

and for the inefficient agent

$$\nu(-\hat{y} + \hat{t} - \bar{\theta}\bar{q}) + (1-\nu)(\bar{t} - \bar{\theta}\bar{q}) \geq \nu(\underline{t} - \bar{\theta}\underline{q}) + (1-\nu)(\hat{y} + \bar{t} - \bar{\theta}\bar{q}). \tag{39}$$

To be accepted along the equilibrium path, the side contract must give each type of agent at least the utility level he could obtain by playing the grand contract noncooperatively. Since the grand contract D^* is implemented in dominant strategy, it is an equilibrium for both types of agent to truthfully reveal their types whatever their beliefs on the other agent. Moreover, this equilibrium is unique. In particular, suppose that if an agent, say A_1, deviates and does not choose to agree on the side contract, A_2 does not change his beliefs on A_1. By playing their truthtelling dominant strategy in D^*, both the deviant agent and the nondeviant one get the following levels of rent: if they are efficient,

$$\nu(\underline{t} - \underline{\theta}\underline{q}) + (1-\nu)(\hat{t} - \underline{\theta}\hat{q}); \tag{40}$$

if they are inefficient,

$$\nu(\hat{t} - \bar{\theta}\hat{q}) + (1-\nu)(\bar{t} - \bar{\theta}\bar{q}) = 0. \tag{41}$$

Thus acceptance of the side-contract by both types of agent occurs with probability one when the following constraints are satisfied:

$$\nu(\underline{t} - \underline{\theta}\underline{q}) + (1-\nu)(\hat{y} + \bar{t} - \underline{\theta}\bar{q}) > \nu(\underline{t} - \underline{\theta}\underline{q}) + (1-\nu)(\hat{t} - \underline{\theta}\hat{q}), \tag{42}$$

[28] In Laffont and Martimort (1995), we discuss this issue. We compare different organizations of the firm when communication costs impose limits on the dimensions of the messages used in the hierarchy. The anonymity condition is then equivalent to a dimensionality restriction.

and

$$\nu(-\hat{y}+\bar{t}-\bar{\theta}\bar{q})+(1-\nu)(\bar{t}-\bar{\theta}\bar{q})>\nu(\hat{t}-\bar{\theta}\hat{q})+(1-\nu)(\bar{t}-\bar{\theta}\bar{q})=0. \tag{43}$$

Taking into account that (12) and (13) are binding for the optimal contract without side contracting, constraints (38), (39), (42), and (43) become respectively:

$$\hat{y}\geq 0,$$

$$\hat{y}\leq \nu\Delta\theta(\underline{q}-\bar{q}),$$

$$\hat{y}>0,$$

$$\hat{y}<\Delta\theta(\hat{q}-\bar{q}).$$

Since the second-best output schedule is strictly decreasing, it is possible to find strictly positive values of $\hat{y}$ which satisfy the last four equations. Therefore, both types of agents' utility levels can be improved with respect to their noncooperative values. The third party can offer the efficient side contract S^* to improve the agents' payoffs.

APPENDIX 2

Consider any perfect Bayesian equilibrium of the overall game. It consists of three elements: First, an initial grand mechanism M^*; second, a pair of beliefs ν_1 and ν_2 that are used respectively by A_2 and A_1 to assess the type of respectively A_1 and A_2 when these agents deviate and refuse the side contract S^*; and lastly a side contract S^* that is coalition interim efficient for the reservation utilities given by $V_i(\theta_i)$, the payoff of agent A_i when he plays M^* with A_j having the beliefs ν_i.

The composition of the initial grand mechanism cum the side contract gives rise to a new mechanism $M^*\circ S^*$ with output requirements $q^*\circ\phi^*(\cdot)$ and a set of transfers $t_i^*\circ\phi^*(\cdot)+y_i^*(\cdot)$, $i\in\{1,2\}$.

We denote this contract by M^{**}. From program (T), we know that it is (BIC). Since both agents accept M^* before accepting and playing the side contract S^*, we know that M^{**} must give them a positive profit whatever their types, i.e., M^{**} is individually rational.

Suppose that the regulator commits to M^{**}. This contract gives the agents the utility levels $U_i(\theta_i)=E_{\Theta_{-i}}(y_i(\theta_i,\theta_{-i})+t_i(\phi(\theta_i,\theta_{-i}))-\theta_i q(\phi(\theta_i,\theta_{-i})))$.

Consider problem (T) with reservation utilities given by $U_i(\theta_i)$. We claim that it is optimal for T to offer the null side contract, or that any other side contract which would be coalition interim efficient with respect to $U_i(\theta_i)$ gives all types of agents the same levels of rents. Suppose not; then there would exist a side contract $\tilde{S}$ different from the null side contract which could be proposed by T and be accepted by all types of agents by the threat of playing M^{**} with beliefs unchanged on a deviant type. This side contract would give the third-party a sum of expected profits for the agents *strictly* greater than that achieved with the null side contract signed on top of M^{**}. This contradicts the interim efficiency of contract S^* in the first place since a side contract $S^*\circ\tilde{S}$ could improve the expected profit of at least one of the types of agent without reducing the others' utility levels.

APPENDIX 3

To characterize the set of collusion-proof contracts, we proceed in several steps. First, we consider an optimal manipulation function which is part of a coalition-interim-efficient side contract with respect to a pair of symmetric reservation utilities $(V(\underline{\theta}),V(\bar{\theta}))$ given by the grand mechanism when it is played truthfully.[29] We prove that this manipulation function is a solution of a reduced program. Then, we check that a solution of this reduced program (T') which is different from the identity can be implemented by the third party and is accepted by the agent. This proves that a grand

[29] Note that these reservation utilities are such that $\Delta\theta\, E_{\Theta}(q(\bar{\theta},\theta_2))\leq V(\underline{\theta})-V(\bar{\theta})\leq \Delta\theta\, E_{\Theta}(q(\underline{\theta},\theta_2))$ from Bayesian incentive compatibility of the grand mechanism.

mechanism such that the identity is not a solution of the reduced program (T') is not collusion-proof. Writing that the identity is a solution of (T') yields a characterization of the collusion-proof mechanisms. Finally, we deduce from the implementability condition an upper bound on ϵ.

(i) *The reduced program* (T'): We start by showing the following lemma.

LEMMA 2: *A manipulation of report function $\phi^*(\cdot)$ which is implementable by the third party with a set of reservation utilities $(V(\underline{\theta}), V(\bar{\theta}))$ satisfies the following constraints*:

(a) $$E_{\Theta\times\Theta}\left(\sum_{k=1,2} t_k(\phi(\theta_1,\theta_2)) - \left(\theta_k + \delta_k \frac{\nu}{1-\nu}\Delta\theta\right) q(\phi(\theta_1,\theta_2))\right) \geq 2V(\bar{\theta}),$$

(b) $$E_{\Theta\times\Theta}\left(\sum_{k=1,2}\left(t_k(\phi(\theta_1,\theta_2)) - \left(\theta_k - \mu_k \frac{1-\nu}{\nu}\Delta\theta\right) q(\phi(\theta_1,\theta_2))\right)\right) \geq 2V(\underline{\theta}),$$

where $\delta_k = 0$ (resp. 1) if and only if $\theta_k = \underline{\theta}$ (resp. $\theta_k = \bar{\theta}$) and $\mu_k = 0$ (resp. 1) if and only if $\theta_k = \bar{\theta}$ (resp. $\theta_k = \underline{\theta}$).

PROOF: Using the upward incentive constraint, we have

(44) $$\begin{aligned} U_1(\underline{\theta}) &= \nu y_1(\underline{\theta},\underline{\theta}) + (1-\nu) y_1(\underline{\theta},\bar{\theta}) + E_\Theta(t_1(\phi(\underline{\theta},\theta_2)) - \underline{\theta} q(\phi(\underline{\theta},\theta_2))) \\ &\geq U_1(\bar{\theta}) + \Delta\theta\, E_\Theta(q(\phi(\bar{\theta},\theta_2))) \end{aligned}$$

or

(45) $$E_\Theta(y_1(\underline{\theta},\theta_2)) \geq U_1(\bar{\theta}) + \Delta\theta\, E_\Theta(q(\phi(\bar{\theta},\theta_2))) - E_\Theta(t_1(\phi(\underline{\theta},\theta_2)) - \underline{\theta} q(\phi(\underline{\theta},\theta_2))).$$

Moreover, we also have by definition,

(46) $$E_\Theta(y_1(\bar{\theta},\theta_2)) = U_1(\bar{\theta}) - E_\Theta(t_1(\phi(\bar{\theta},\theta_2)) - \bar{\theta} q(\phi(\bar{\theta},\theta_2))).$$

Multiplying (45) by ν and (46) by $(1-\nu)$ and summing the two yields

(47) $$E_{\Theta\times\Theta}(y_1(\theta_1,\theta_2)) \geq U_1(\bar{\theta}) - E_{\Theta\times\Theta}\left(t_1(\phi(\theta_1,\theta_2)) - \left(\theta_1 + \delta_1\frac{\nu}{1-\nu}\Delta\theta\right) q(\phi(\theta_1,\theta_2))\right).$$

By permuting indices, a similar inequality can be also obtained for agent 2, denoted (47′). Summing (47) and (47′), using (BB) and $U_i(\bar{\theta}) \geq V(\bar{\theta})$ we obtain (a). Using downward incentive constraints and proceeding as above we get (b). *Q.E.D.*

Now, we note that using (BB) it is immediate to show that for all manipulations of report functions,

(48) $$E_{\Theta\times\Theta}(U_1(\theta_1) + U_2(\theta_2)) = E_{\Theta\times\Theta}\left(\sum_{k=1,2}(t_k(\phi(\theta_1,\theta_2) - \theta_k q(\phi(\theta_1,\theta_2))))\right).$$

We can now consider the following reduced program (T') which is less constrained than (T):

$$\max_{\phi(\cdot)} E_{\Theta\times\Theta}\left(\sum_{k=1,2} t_k(\phi(\theta_1,\theta_2)) - \theta_k q(\phi(\theta_1,\theta_2))\right)$$

subject to (a) and (b).

A solution $\phi^{**}(\cdot)$ to this problem is a solution to (T) when there exist balanced side transfers which satisfy acceptance and incentive constraints for both types of agents, i.e., when it is implementable by the third party.

It is immediate to derive the necessary and sufficient condition that must be satisfied by the optimal manipulation function $\phi^{**}(\cdot)$.

LEMMA 3: *A necessary and sufficient condition for* $\phi^{**}(\cdot)$ *to be a solution to* (T') *is that there exist* ϵ *and* ϵ' *such that*:

First, $\epsilon+\epsilon'\leq 1$; $\epsilon\geq 0$, *with equality if* (a) *is slack*; $\epsilon'\geq 0$, *with equality if* (b) *is slack.*

Second,

$$\phi^{**}(\theta_1,\theta_2)\in \arg\max_{\phi(\cdot)} \sum_{k=1,2}\Bigg(t_k(\phi(\theta_1,\theta_2)) - \left(\theta_k+\delta_k\frac{\nu}{1-\nu}\epsilon\,\Delta\theta-\mu_k\frac{1-\nu}{\nu}\epsilon'\,\Delta\theta\right)q(\phi(\theta_1,\theta_2))\Bigg). \tag{49}$$

PROOF: Since the set of possible manipulations of report functions is the set of probability measures on $\Theta\times\Theta$, this is a compact and convex set. Since we allow for stochastic manipulation functions, the objective function of Lemma 2 is also convex. Therefore, Lagrangean techniques apply. We use L to denote the Lagrangean of the previous program. We have

$$\begin{aligned}L={}&E_{\Theta\times\Theta}\sum_{k=1,2}(t_k(\phi(\theta_1,\theta_2))-\theta_k q(\phi(\theta_1,\theta_2)))\\&+\gamma\left(E_{\Theta\times\Theta}\sum_{k=1,2}\left(t_k(\phi(\theta_1,\theta_2))-\left(\theta_k+\delta_k\frac{\nu}{1-\nu}\Delta\theta\right)q(\phi(\theta_1,\theta_2))\right)-2V(\bar\theta)\right)\\&+\gamma'\left(E_{\Theta\times\Theta}\sum_{k=1,2}\left(t_k(\phi(\theta_1,\theta_2))-\left(\theta_k-\mu_k\frac{1-\nu}{\nu}\Delta\theta\right)q(\phi(\theta_1,\theta_2))\right)-2V(\underline{\theta})\right),\end{aligned}$$

where γ and γ' are the two nonnegative Lagrange multipliers of constraints (a) and (b). Moreover, they are equal to zero only if these constraints are slack. Setting $\epsilon=\gamma/(1+\gamma+\gamma')$ and $\epsilon'=\gamma'/(1+\gamma+\gamma')$ the optimization of this Lagrangean is equivalent to (49). *Q.E.D.*

Importantly, we should note that (as shown in the proof of the two previous lemmata) as soon as ϵ (resp. ϵ') is not equal to zero, the incentive constraints of both efficient agents (resp. inefficient) are binding in program (T). In the sequel, we are interested only in collusion-proof contracts such that the inefficient agent's incentive constraint is slack. For such a collusion-proof contract, (b) reduces to (3).

As a first step towards the analysis of these mechanisms, we focus on the solutions of (T') such that only (a) may be binding.

(ii) *Implementability by the third party*: Reciprocally, one may wonder if a solution $\phi^{**}(\cdot)$ to program (T') which is different from the identity can be implemented by the third party.[30] To be implementable the manipulation of report function $\phi^{**}(\cdot)$ must be such that the incentive and the participation constraints of both types of agents are satisfied in program (T). Moreover, the grand mechanism is then not collusion-proof if at least one type of agent gets a strictly higher payoff by accepting the side contract than by revealing truthfully in the grand mechanism.

The first step is to prove the following lemma.

LEMMA 4: *A solution* $\phi^{**}(\cdot)$ *to* (T') *such that* (b) *is slack is such that* $q(\phi^{**}(\theta_1,\theta_2))$ *is weakly decreasing in* θ_1 *and* θ_2.

[30] The identity can always be implemented with side transfers which are identically equal to zero.

PROOF: The fact that $q(\phi^{**}(\cdot))$ is weakly decreasing in each of its arguments comes from using a bunch of revealed preference arguments. For instance, from the definition of $\phi^{**}(\cdot)$ we have

$$\sum_{k=1,2} t_k(\phi^{**}(\underline{\theta},\underline{\theta})) - 2\underline{\theta}q(\phi^{**}(\underline{\theta},\underline{\theta})) \geq \sum_{k=1,2} t_k(\phi^{**}(\underline{\theta},\bar{\theta})) - 2\underline{\theta}q(\phi^{**}(\underline{\theta},\bar{\theta})).$$

Similarly, we also have

$$\sum_k t_k(\phi^{**}(\underline{\theta},\bar{\theta})) - \left(\underline{\theta}+\bar{\theta}+\frac{\nu}{1-\nu}\epsilon\,\Delta\theta\right)q(\phi^{**}(\underline{\theta},\bar{\theta}))$$
$$\geq \sum_k t_k(\phi^{**}(\underline{\theta},\underline{\theta})) - \left(\underline{\theta}+\bar{\theta}+\frac{\nu}{1-\nu}\epsilon\,\Delta\theta\right)q(\phi^{**}(\underline{\theta},\underline{\theta})).$$

Summing the last two inequalities, we obtain

$$\Delta\theta\left(1+\frac{\nu}{1-\nu}\epsilon\right)(q(\phi^{**}(\underline{\theta},\bar{\theta})) - q(\phi^{**}(\underline{\theta},\underline{\theta}))) \leq 0.$$

One can proceed similarly for all the pairs (θ_1,θ_2) and obtain that $q(\phi^{**}(\cdot))$ is weakly decreasing in all its arguments. *Q.E.D.*

Note that for any grand contract (both anonymous and nonanonymous ones) the manipulation function $\phi^{**}(\cdot)$ solution of (T') is also symmetric and anonymous. We denote $\phi^{**}(\underline{\theta},\underline{\theta}) = \underline{\phi}$, $\phi^{**}(\underline{\theta},\bar{\theta}) = \phi^{**}(\bar{\theta},\underline{\theta}) = \hat{\phi}$, and $\phi^{**}(\bar{\theta},\bar{\theta}) = \bar{\phi}$. From the previous lemma, we have $q(\underline{\phi}) \geq q(\hat{\phi}) \geq q(\bar{\phi})$.

LEMMA 5: *Assume that the manipulation of report $\phi^{**}(\cdot)$ solution to (T') achieves a strictly higher payoff than the identity; then it can be implemented by the third party, i.e., there exist balanced side transfers such that the (BIC) and the acceptance constraints are satisfied.*

PROOF: The proof consists in finding a set of side transfers which are budget-balanced and satisfy incentive and participation constraints for both types. Since the manipulation function is anonymous and symmetric, we need only to find a transfer $\hat{y}$ from an inefficient agent to an efficient one. We denote $\underline{U}_{\phi^{**}} = \nu(t(\underline{\phi}^{**}) - \underline{\theta}q(\underline{\phi}^{**})) + (1-\nu)(t(\hat{\phi}^{**}) - \underline{\theta}q(\hat{\phi}^{**}))$ and $\bar{U}_{\phi^{**}} = \nu(t(\hat{\phi}^{**}) - \bar{\theta}q(\hat{\phi}^{**})) + (1-\nu)(t(\bar{\phi}^{**}) - \bar{\theta}q(\bar{\phi}^{**}))$. The side contract is Bayesian incentive compatible when the following constraints are satisfied:

$$\hat{y}_1 = \Delta\theta\,E(q(\phi^{**}(\underline{\theta},\theta_2))) - \underline{U}_{\phi^{**}} + \bar{U}_{\phi^{**}}$$
$$\geq \hat{y} \geq \Delta\theta\,E(q(\phi^{**}(\bar{\theta},\theta_2))) - \underline{U}_{\phi^{**}} + \bar{U}_{\phi^{**}} = \hat{y}_2$$

where the two inequalities are not incompatible from the monotonicity of $q(\phi^{**}(\cdot))$. The side contract is individually rational when the following two constraints are satisfied:

$$\bar{U}_{\phi^{**}} - \nu\hat{y} \geq V(\bar{\theta})$$

and

$$\underline{U}_{\phi^{**}} + (1-\nu)\hat{y} \geq V(\underline{\theta}),$$

or put differently when $\hat{y}$ is such that

$$\hat{y}_3 = \frac{\bar{U}_{\phi^{**}} - V(\bar{\theta})}{\nu} \geq \hat{y} \geq \frac{-\underline{U}_{\phi^{**}} + V(\underline{\theta})}{1-\nu} = \hat{y}_4.$$

We note that the latter two inequalities are not incompatible since by definition of ϕ^{**} we have $\nu\underline{U}_{\phi^{**}} + (1-\nu)\bar{U}_{\phi^{**}} \geq \nu V(\underline{\theta}) + (1-\nu)V(\bar{\theta})$.

The manipulation of reports $\phi^{**}(\cdot)$ is implementable when the subsets $[\hat{y}_2, \hat{y}_1]$ and $[\hat{y}_4, \hat{y}_3]$ have a nonempty intersection. Their intersection is empty in two cases: first, when $\hat{y}_2 > \hat{y}_3$, i.e., when

$$V(\bar{\theta}) > \nu \underline{U}_{\phi^{**}} + (1-\nu)\bar{U}_{\phi^{**}} - \nu \Delta\theta\, E_\Theta(q(\phi^{**}(\bar{\theta}, \theta_2))),$$

which is false since the opposite constraint is satisfied by $\phi^{**}(\cdot)$ solution to (T'); second, when $\hat{y}_4 > \hat{y}_1$, i.e., when

$$V(\underline{\theta}) > \nu \underline{U}_{\phi^{**}} + (1-\nu)\bar{U}_{\phi^{**}} + (1-\nu)\, \Delta\theta\, E_\Theta(q(\phi^{**}(\underline{\theta}, \theta_2))),$$

which is also false since the opposite constraint is satisfied by $\phi^{**}(\cdot)$ solution to (T'). *Q.E.D.*

This proves that a solution of (T') is also a solution of the more constrained problem (T). Writing that $\phi(\cdot) = Id(\cdot)$ solves (T') gives us the exact characterization of the collusion-proof mechanisms.

(iii) *A further condition on* ϵ: We consider a nonincreasing schedule of outputs $\underline{q} \geq \hat{q} \geq \bar{q}$ which is implemented by the principal in a collusion-proof and anonymous manner and such that the incentive constraint (10) is binding. The following coalition incentive constraints are satisfied:

$$\frac{1}{2}\left(\underline{\theta} + \bar{\theta} + \frac{\nu}{1-\nu}\epsilon\,\Delta\theta\right)(\underline{q} - \hat{q}) \geq \underline{t} - \hat{t} \geq \underline{\theta}(\underline{q} - \hat{q}), \tag{50}$$

$$\left(\bar{\theta} + \frac{\nu}{1-\nu}\epsilon\,\Delta\theta\right)(\hat{q} - \bar{\theta}) \geq \hat{t} - \bar{t} \geq \frac{1}{2}\left(\underline{\theta} + \bar{\theta} + \frac{\nu}{1-\nu}\epsilon\,\Delta\theta\right)(\hat{q} - \bar{q}), \tag{51}$$

$$\left(\bar{\theta} + \frac{\nu}{1-\nu}\epsilon\,\Delta\theta\right)(\underline{q} - \bar{q}) \geq \underline{t} - \bar{t} \geq \underline{\theta}(\underline{q} - \bar{q}). \tag{52}$$

Summing (50) and (51) and taking into account that (10) is binding implies that

$$\left(1 - \frac{\nu}{1-\nu}\epsilon\right)\Delta\theta(\hat{q} - \bar{q}) \geq 0.$$

Hence, we have necessarily $1 - (\nu/(1-\nu))\epsilon > 0$.

APPENDIX 4

We consider an anonymous grand contract which is not collusion-proof. Thus, the third party can offer a Bayesian incentive compatible contract which makes the agents better off than what they obtain by playing the grand contract noncooperatively with passive beliefs. Since the nexus of collusion is between an efficient and an inefficient agent, we focus on a grand contract which lets this coalition manipulate its report to the principal. We denote this manipulation by $\hat{\phi}$. The other coalitions do not manipulate.

(i) From acceptance of the side-contract, the final utilities are: for an efficient agent

$$\underline{\tilde{U}} = \nu(\underline{t} - \underline{\theta}\underline{q}) + (1-\nu)(t(\hat{\phi}) - \underline{\theta}q(\hat{\phi}) + \hat{y}) \geq \nu(\underline{t} - \underline{\theta}\underline{q}) + (1-\nu)(\hat{t} - \underline{\theta}\hat{q}); \tag{53}$$

for an inefficient agent, it is

$$\bar{\tilde{U}} = \nu(t(\hat{\phi}) - \bar{\theta}q(\hat{\phi}) - \hat{y}) + (1-\nu)(\bar{t} - \bar{\theta}\bar{q}) \geq \nu(\hat{t} - \bar{\theta}\hat{q}) + (1-\nu)(\bar{t} - \bar{\theta}\bar{q}) \tag{54}$$

with at least one of these inequalities being strict. Simplifying both equations and summing implies that a coalition incentive constraint is violated since

$$2t(\hat{\phi}) - (\underline{\theta} + \bar{\theta})q(\hat{\phi}) > 2\hat{t} - (\underline{\theta} + \bar{\theta})\hat{q}. \tag{55}$$

(ii) From incentive compatibility of the final allocation, we also have

(56) $\underline{\tilde{U}} \geq \overline{\tilde{U}} + \Delta\theta(\nu q(\hat{\phi}) + (1-\nu)\bar{q})$

and

(57) $\overline{\tilde{U}} \geq \underline{\tilde{U}} - \Delta\theta(\nu \underline{q} + (1-\nu)q(\hat{\phi}))$.

(iii) From individual rationality of the final allocation, we also have

(58) $\underline{\tilde{U}} \geq 0$

and

(59) $\overline{\tilde{U}} \geq 0$.

(iv) When collusion is allowed, the regulator realizes

$$\max_{\underline{q}, q(\hat{\phi}), \bar{q}} \nu^2(S(\underline{q}) - 2\underline{\theta}\underline{q}) + 2\nu(1-\nu)(S(q(\hat{\phi})) - (\underline{\theta}+\bar{\theta})q(\hat{\phi}))$$
$$+(1-\nu)^2(S(\bar{q}) - 2\bar{\theta}\bar{q}) - 2(1-\alpha)\nu\Delta\theta(\nu\underline{\tilde{U}} + (1-\nu)\overline{\tilde{U}})$$

subject to (56)-(57)-(58)-(59)-(17)-(18)-(55)-(21)-(22).

Moreover, from Lemma 4, we know that $\underline{q} \geq q(\hat{\phi}) \geq \bar{q}$.

Since the strict inequality (56) holds, the message $(\underline{\theta}, \bar{\theta})$ is never contained in the support of $\hat{\phi}$. Only $(\underline{\theta}, \underline{\theta})$ and $(\bar{\theta}, \bar{\theta})$ can be sent. The optimal payoff for the principal is obtained when either one or the other of these messages is sent with probability one in states $(\underline{\theta}, \bar{\theta})$ and $(\bar{\theta}, \underline{\theta})$. Hence, it is obtained with a contract that entails some partial pooling. Such a contract could have also been implemented by the principal with an anonymous and collusion-proof mechanism at the same cost.

APPENDIX 5

From (7), which is binding, we have

$$\nu(\underline{t} - \underline{\theta}\underline{q} - (\hat{t} - \underline{\theta}\hat{q})) = -(1-\nu)((\hat{t} - \underline{\theta}\hat{q}) - (\bar{t} - \underline{\theta}\bar{q})).$$

From (17), the left-hand side of the equation above is nonnegative; therefore,

$$\hat{t} - \underline{\theta}\hat{q} \leq \bar{t} - \underline{\theta}\bar{q}.$$

But using (19) yields

$$\left(\underline{\theta} + \bar{\theta} + \frac{\nu\epsilon}{1-\nu}\Delta\theta\right)(\hat{q} - \bar{q}) \leq 2\underline{\theta}(\hat{q} - \bar{q}).$$

Since $\hat{q} \geq \bar{q}$ is requested by Lemma 4, one needs to have in fact $\hat{q} = \bar{q} = q^p$.

APPENDIX 6

First, we assume that constraints (17), (19), and (6) are the only constraints which are binding at the optimum. We later check these assertions. Inserting the expressions of the transfers $\hat{t}$ into the inefficient agent's participation constraint, one gets

$$\bar{t} - \bar{\theta}\bar{q} \geq \tfrac{1}{2}\nu\Delta\theta(\hat{q} - \bar{q}).$$

Obviously, this constraint must also be binding at the optimum. Inserting all the expressions of the transfers into the regulator's objective function, we obtain

$$\begin{aligned} W = {} & \nu^2(S(\underline{q}) - 2\underline{\theta}\underline{q} - \Delta\theta(\hat{q} + \bar{q}) - \nu\Delta\theta(\hat{q} - \bar{q})) \\ & + 2\nu(1-\nu)(S(\hat{q}) - (\underline{\theta} + \bar{\theta})\hat{q} - \Delta\theta\,\bar{q} - \nu\Delta\theta(\hat{q} - \bar{q})) \\ & + (1-\nu)^2(S(\bar{q}) - 2\bar{\theta}\bar{q} - \nu\Delta\theta(\hat{q} - \bar{q})) + 2\alpha\nu\underline{U}, \end{aligned}$$

where after computations we find that

$$\underline{U} = \frac{\Delta\theta}{2}((1+\nu)\hat{q} + (1-\nu)\bar{q}).$$

Optimizing the previous expression with respect to the schedule of outputs yields equations (25) to (27). It is then routine to check that $\hat{q}^c > \bar{q}^c$. One also needs to check that indeed the inefficient agent's incentive constraint is slack under this contract. For this to be true, some manipulations show that one must have

$$\nu\Delta\theta(\underline{q}^c - \hat{q}^c) > \hat{t}^c - \bar{\theta}\hat{q}^c = -\frac{1-\nu}{2}\Delta\theta(\hat{q}^c - \bar{q}^c).$$

Since the right-hand side of the latter inequality is negative and the left-hand side is positive, the inefficient type's incentive constraint is slack.

APPENDIX 7

For $\Delta\theta$ small enough, we have:

$$\hat{q}^c - \hat{q}^n = -\frac{\Delta\theta(1-\alpha)}{2|S''|},$$

$$\bar{q}^c - \bar{q}^n = \frac{\nu\Delta\theta(1-\alpha)}{|S''|(1-\nu)},$$

and

$$\hat{q}^c - \bar{q}^c = \frac{\Delta\theta(1+\alpha)}{2|S''|}.$$

We have therefore

$$\underline{U}^c - \underline{U}^n = \frac{(\nu(1-\alpha) + (1-\nu)(1+\alpha))\,\Delta\theta^2}{2|S''|}.$$

APPENDIX 8

To show that C^* is implemented in dominant strategy, we have to check that the constraints (12) to (15) hold with this contract. First, (12) holds with an equality. Second, $\hat{t} - \bar{t} = \frac{1}{2}(\underline{\theta} + \bar{\theta})(\hat{q} - \bar{q}) > \underline{\theta}(\hat{q} - \bar{q})$, and the Bayesian incentive constraint of an inefficient agent also holds. Third, $\hat{t} - \bar{t} = \frac{1}{2}(\underline{\theta} + \bar{\theta})(\hat{q} - \bar{q}) < \bar{\theta}(\hat{q} - \bar{q})$, and (15) also holds. Lastly, $\hat{t} - \underline{t} = \underline{\theta}(\hat{q} - \underline{q}) > \bar{\theta}(\hat{q} - \underline{q})$, and (14) also holds.

Since the mechanism has a unique equilibrium that is independent of the expectations that the players have on each other play, an agent who considers deviating and refusing the null side contract knows that he will get the same level of utility as with acceptance anyway, whatever his partner's expectations.

APPENDIX 9

With nonanonymous mechanisms, the efficient agent's incentive constraints are now the following:[31]

$$(60)\qquad \underline{t} - \underline{\theta}\underline{q} \geq \hat{t}_2 - \underline{\theta}\hat{q},$$

$$(61)\qquad \hat{t}_1 - \underline{\theta}\hat{q} \geq \bar{t} - \underline{\theta}\bar{q}.$$

Similarly, the upward coalition incentive constraints can now be written as follows:[32]

$$(62)\qquad 2\underline{t} - 2\underline{\theta}\underline{q} \geq \hat{t}_1 + \hat{t}_2 - 2\underline{\theta}\hat{q},$$

$$(63)\qquad \hat{t}_1 + \hat{t}_2 - (\underline{\theta} + \bar{\theta})\hat{q} \geq 2\bar{t} - (\underline{\theta} + \bar{\theta})\bar{q}.$$

Participation constraints for the efficient agent are the following:[33]

$$(64)\qquad \hat{t}_1 - \underline{\theta}\hat{q} \geq 0,$$

$$(65)\qquad \underline{t} - \underline{\theta}\underline{q} \geq 0.$$

For the inefficient one, one can choose $\hat{t}_2$ and $\bar{t}$ such that the state by state participation constraints are binding:

$$(66)\qquad \hat{t}_2 - \bar{\theta}\hat{q} \geq 0,$$

$$(67)\qquad \bar{t} - \bar{\theta}\bar{q} \geq 0.$$

At the optimum of the regulator's program, the participation constraints (66) and (67) are both binding. Inserting the corresponding expressions of the transfers $\hat{t}_2$ and $\bar{t}$, we find respectively for (61) and (63),

$$(68)\qquad \hat{t}_1 - \underline{\theta}\hat{q} \geq \Delta\theta\,\bar{q},$$

for (62),

$$(69)\qquad 2\underline{t} - 2\underline{\theta}\underline{q} \geq \Delta\theta(\hat{q} + \bar{q}),$$

and for (60),

$$(70)\qquad \underline{t} - \underline{\theta}\underline{q} \geq \Delta\theta\,\hat{q}.$$

As soon as $\hat{q} \geq \bar{q}$, the incentive constraint (70) implies the global coalition incentive constraint (69). Inserting the expressions of the transfers when the constraints (68) and (70) are binding into the regulator's objective function, we find

$$W = \nu^2(S(\underline{q}) - 2\underline{\theta}\underline{q} - 2\Delta\theta\,\hat{q}) + 2\nu(1-\nu)(S(\hat{q}) - (\underline{\theta} + \bar{\theta})\hat{q} - \Delta\theta\,\bar{q})$$
$$+ (1-\nu)^2(S(\bar{q}) - 2\bar{\theta}\bar{q}) + 2\alpha\nu\Delta\theta(\nu\hat{q} + (1-\nu)\bar{q}).$$

One omitted constraint may be important however; this is the coalition incentive constraint of a coalition made of an efficient and an inefficient agent who could claim that they are both efficient. In fact, when (60) is binding, the transfer $\underline{t}$ is rather high and we have to check that the following downward coalition incentive constraint is indeed satisfied:

$$(71)\qquad \hat{t}_1 + \hat{t}_2 - (\underline{\theta} + \bar{\theta})\hat{q} \geq 2\underline{t} - (\underline{\theta} + \bar{\theta})\underline{q}.$$

[31] We do not write here the inefficient agent's incentive constraints which are not relevant.

[32] We only write for the moment the downward coalition incentive constraints which are the only relevant ones.

[33] Again, we omit the efficient agent's participation constraints that can easily be checked ex post.

Rearranging, we find that this amounts to

$$\underline{q} + \bar{q} \geq 2\hat{q}.$$

Using the expressions of the second-best outputs (7), (8), and (9) and denoting the inverse of S' by $\psi = S'^{-1}$, the inequality above is satisfied when

$$\psi(q\underline{\theta}) + \psi\left(2\bar{\theta} + 2\frac{\nu}{1-\nu}(1-\alpha)\,\Delta\theta\right) \geq 2\psi\left(\underline{\theta} + \bar{\theta} + \frac{\nu}{1-\nu}(1-\alpha)\,\Delta\theta\right).$$

But the latter condition holds strictly when $\psi(\cdot)$ is convex. Since $\psi'' = -(S'''/S''^3)$, it holds when $S''' \geq 0$.

REFERENCES

ARMSTRONG, M. (1996): "Multiproduct Nonlinear Pricing," *Econometrica*, 64, 51–75.

ASPREMONT, C. D', AND L. A. GÉRARD-VARET (1979): "Incentives and Incomplete Information," *Journal of Public Economics*, 11, 25–45.

AUSUBEL, L., AND R. DENECKERE (1993): "Efficient Sequential Bargaining," *Review of Economic Studies*, 60, 435–461.

BARON, D., AND R. MYERSON (1982): "Regulating a Monopolist with Unknown Costs," *Econometrica*, 50, 911–930.

BERNHEIM, D., D. PELEG, AND M. WHINSTON (1987): "Coalition-Proof Nash Equilibria 1: Concepts," *Journal of Economic Theory*, 42, 1–12.

BERNHEIM, D., AND M. WHINSTON (1987): "Coalition-Proof Nash Equilibria 2: Applications," *Journal of Economic Theory*, 42, 13–29.

CAILLAUD, B., AND P. JEHIEL (1994): "Collusion in Auctions with Externalities," Mimeo, CERAS.

CRAMPTON, P., AND T. PALFREY (1995): "Ratifiable Mechanisms: Learning From Disagreement," *Games and Economic Behavior*, 10, 255–283.

CRÉMER, J. (1996): "Manipulation of Groves Mechanisms," forthcoming in *Games and Economic Behavior*.

DASGUPTA, P., P. HAMMOND, AND E. MASKIN (1979): "The Implementation of Social Choice Rules," *Review of Economic Studies*, 46, 185–216.

FELLI, L. (1991): "Collusion in Incentive Contracts: Does Delegation Help?," Mimeo, MIT.

FUDENBERG, D., AND J. TIROLE (1991): *Game Theory*. Cambridge: MIT Press.

GIBBARD, A. (1973): "Manipulation of Voting Schemes," *Econometrica*, 41, 587–601.

GREEN, J., AND J.-J. LAFFONT (1977): "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods," *Econometrica*, 45, 427–438.

——— (1979): "On Coalition Incentive Compatibility," *Review of Economic Studies*, 46, 243–254.

GROVES, T. (1973): "Incentives in Teams," *Econometrica*, 41, 617–631.

HARRIS, M., AND A. RAVIV (1981): "Allocation Mechanisms and the Design of Auctions," *Econometrica*, 49, 1477–1500.

HOLMSTROM, B., AND P. MILGROM (1990): "Regulating Trade Among Agents," *Journal of Institutional and Theoretical Economics*, 146, 85–105.

ITOH, H. (1993): "Coalitions, Incentives, and Risk Sharing," *Journal of Economic Theory*, 60, 410–427.

KOBAYASHI, T. (1980): "Equilibrium Contracts For Syndicates with Differential Information," *Econometrica*, 48, 1635–1664.

LAFFONT, J. J., AND D. MARTIMORT (1994): "Separation of Regulators Against Collusive Behavior," Mimeo, IDEI.

——— (1995): "Collusion and Delegation," Mimeo, IDEI.

——— (1996): "The Collusion-Proof Samuelson Conditions for Public Goods," Mimeo, IDEI.

LAFFONT, J. J., AND E. MASKIN (1979): "A Differential Approach to Expected Utility Maximizing Mechanisms," in *Aggregation and Revelation of Preferences*, ed. by J. J. Laffont. Amsterdam: North-Holland.

——— (1980): "A Differential Approach to Dominant Strategy Mechanisms," *Econometrica*, 48, 1507–1520.

LAFFONT, J. J., AND J. TIROLE (1991): "The Politics of Government Decision Making: A Theory of Regulatory Capture," *Quarterly Journal of Economics*, 106, 1089–1127.

——— (1993): *A Theory of Incentives in Regulation and Procurement*. Cambridge: MIT Press.

MARTIMORT, D. (1996): "The Life Cycle of Regulatory Agencies: Dynamic Capture and Transaction Costs," Mimeo.

MASKIN, E. (1979): "Implementation and Strong-Nash Equilibrium," in *Aggregation and Revelation of Preferences*, ed. by J. J. Laffont. Amsterdam: North-Holland, Chapter 23.

MASKIN, E., AND J. TIROLE (1992): "The Principal-Agent Relationship with an Informed Principal 2: Common Values," *Econometrica*, 60, 1–42.

MCAFEE, P., AND J. MCMILLAN (1992): "Bidding Rings," *American Economic Review*, 82, 579–599.

MOOKHERJEE, D., AND S. REICHELSTEIN (1992): "Dominant Strategy Implementation of Bayesian Incentive Compatible Rules," *Journal of Economic Theory*, 56, 378–399.

MYERSON, R. (1979): "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 47, 61–73.

——— (1982): "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, 10, 67–81.

——— (1983): "Mechanism Design by an Informed Principal," *Econometrica*, 51, 1767–1797.

MYERSON, R., AND M. SATTERTHWAITE (1983): "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 28, 265–281.

ROCHET, J-C. (1995): "Ironing, Sweeping and Multidimensional Screening," Mimeo, IDEI.

RUBINSTEIN, A. (1985): "Choices of Conjectures in a Bargaining Game with Incomplete Information," in *Game Theoretic Models of Bargaining*, ed. by A. Roth. Cambridge: Cambridge University Press, 99–144.

TIROLE, J. (1986): "Hierarchies and Bureaucracies: On the Role of Collusion in Organizations," *Journal of Law, Economics and Organization*, 2, 181–214.

——— (1992): "Collusion and the Theory of Organizations," in *Advances in Economic Theory*, Vol. 2, ed. by J. J. Laffont. Cambridge: Cambridge University Press, 151–206.

WILSON, R. (1978): "Information, Efficiency and the Core of an Economy," *Econometrica*, 46, 807–816.

YANNELIS, N. (1991): "The Core of an Economy with Differential Information," *Economic Theory*, 1, 183–197.