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Color constancy: surface color from changing illumination

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Viewing the lights reflected by a set of three or more surfaces, a trichromatic visual system can recover three color-constant descriptors of reflectance per surface if the color of the surfaces' illuminant changes. This holds true for a broad range of models that relate photoreceptor, surface, and illuminant spectral properties. Changing illumination, which creates the problem of color constancy, affords its solution.

INTRODUCTION

Land's best-known demonstrations of color constancy involve changing the illumination of a Mondrian display comprising many distinctly colored surfaces.[1,2] Although changing the chromatic properties of the Mondrian illuminant alters the chromatic properties of the lights reflected toward the viewer, the perceived colors of Mondrian surfaces hardly change at all. This constancy of surface color exhibits the ability of the visual system to derive, from reflected lights, accurate descriptors of surface-color properties that do not depend on circumstances of illumination.

Theoretical work has focused on the problem of estimating surface and illuminant chromatic properties in the case in which a set of surfaces is viewed under a single illuminant.[1-10] Maloney and Wandell established a general result for this situation[6]: A trichromatic visual system viewing surfaces under a single unknown illuminant that is represented by three color descriptors[11-13] can recover, at best, two color-constant descriptors per surface. The theory of color constancy rests incomplete, however, for neither surface-color percepts[14] nor surface-color properties[15-18] are described adequately by two-dimensional models. Three descriptors per surface are required.[1,2]

The premise of the present study is that color constancy is not an issue for color perception unless the chromatic properties of illumination change. It is natural to ask whether the lights from a set of surfaces viewed under two or more distinct illuminants, as in Land's demonstrations, permit the visual system to estimate accurately both the unknown surface-color properties and the unknown illuminants. This is so; a trichromatic visual system can recover three accurate color descriptors for each unknown surface and for each illuminant when three or more surfaces are viewed under two or more lights in turn. This holds true for a variety of combinations of photoreceptor spectral sensitivities, three-dimensional models for describing the variation among surfaces' color-constant reflectance properties, and three-dimensional models for describing the variation among illuminants' spectral power distributions. Changing illumination both introduces the problem of color constancy and permits its solution.

BILINEAR MODELS

The following analysis depends on knowledge of accurate finite-dimensional linear models for surface reflectances[15-18] and illuminants[11-13] and their relationship to photoreceptoral spectral sensitivity. Such models exist and find much application in studies of color constancy.[3-7,10,19-23] A particular combination of photoreceptors and models for reflectances and illuminants provides a bilinear model,[22,23] fundamental to the analysis, that is derived as follows.

When an n-dimensional model for reflectances is used, a reflectance function $R(\lambda)$ that falls within the span of the model is a linear combination of n linearly independent basis functions $R_j(\lambda), j = 1,\ldots,n$:

$$R(\lambda) = \sum_{j=1}^{n} r_j R_j(\lambda). \tag{1}$$

The n coefficients r_j are the color-constant descriptors of surface-color properties provided by the model. An illuminant $A(\lambda)$ that falls within the span of an m-dimensional model for illuminants with linearly independent basis functions $A_i(\lambda)$, $i = 1,\ldots,m$, is expressed similarly:

$$A(\lambda) = \sum_{i=1}^{m} a_i A_i(\lambda). \tag{2}$$

Light reflected from a surface with reflectance $R(\lambda)$ viewed under illuminant $A(\lambda)$ is the product of the two functions and has the following model decomposition:

$$L(\lambda) = A(\lambda)R(\lambda) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_i r_j A_i(\lambda) R_j(\lambda). \tag{3}$$

The quantum catch q_k of the kth photoreceptoral type in a p-chromatic system, $k = 1,\ldots,p$, is given by the integral (over the visible spectrum) of the product of the kth mecha-

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nism's spectral sensitivity $Q_k(\lambda)$ and the light $L(\lambda)$:

$$q_k = \int_{400\,\text{nm}}^{700\,\text{nm}} Q_k(\lambda)L(\lambda)\mathrm{d}\lambda$$
$$= \int_{400\,\text{nm}}^{700\,\text{nm}} Q_k(\lambda)\left[\sum_{i=1}^{m}\sum_{j=1}^{n} a_i r_j A_i(\lambda)R_j(\lambda)\right]\mathrm{d}\lambda\,. \tag{4}$$

To bring descriptors outside the integrals in Eq. (4), use the bilinear model matrices $\mathbf{B}_j, j = 1,\ldots,n,$ with entries b_{jki} given by the following integral:

$$b_{jki} = \int_{400\,\text{nm}}^{700\,\text{nm}} Q_k(\lambda)A_i(\lambda)R_j(\lambda)\mathrm{d}\lambda\,. \tag{5}$$

Equation (4) in this form reads

$$q_k = \sum_{j=1}^{n}\sum_{i=1}^{m} r_j b_{jki} a_i\,. \tag{6}$$

This equation expresses the bilinear dependence of quantum catches on reflectance and illuminant descriptors.[19–23]

RECOVERY FROM MULTIPLE VIEWS

To include the effect of view, suppose that the surface with descriptors r_j is lit, in turn, by v illuminants $\mathbf{a}_w$, $w = 1,\ldots,v$, where $\mathbf{a}_w = [a_{w1}\ldots a_{wm}]^T$ (T for transpose). One may order the vp quantum catches to form a data vector $\mathbf{d} = [q_{11}\ldots q_{1p}\ldots q_{v1}\ldots q_{vp}]^T$ and the vm illuminant descriptors to form an illumination vector $\mathbf{a} = [a_{11}\ldots a_{1m}\ldots a_{v1}\ldots a_{vm}]^T$. The data are related to the illuminants by a reflectance-weighted linear combination of block–diagonal matrices $\mathbf{C}_j$, in which each of v blocks along the diagonal is $\mathbf{B}_j$: $\mathbf{C}_j = \text{diag}[\mathbf{B}_j,\ldots,\mathbf{B}_j]$. The relations among data, illuminants, and surface reflectance are then described by

$$\mathbf{d} = \sum_{j=1}^{n} r_j \mathbf{C}_j \mathbf{a}\,. \tag{7}$$

Introducing s surfaces with reflectances $\mathbf{r}_t$, $t = 1,\ldots,s$, provides s data vectors $\mathbf{d}_t$ that hold the catches of a p-chromatic system that views s surfaces under v illuminants:

$$\mathbf{d}_t = \sum_{j=1}^{n} r_{tj}\mathbf{C}_j\mathbf{a}\,. \tag{8}$$

The intuition behind this system of equations is like that underlying the results of Maloney and Wandell.[6] In the case of a trichromatic system, that is, provided with two views of three surfaces ($p = m = n = 3$, $v = 2$, $s = 3$), the three data vectors $\mathbf{d}_1$, $\mathbf{d}_2$, and $\mathbf{d}_3$ span a three-dimensional subspace of $\mathbb{R}^6$, the orientation of this subspace in $\mathbb{R}^6$ is determined uniquely by the two illuminants, and the position within this subspace of a data vector from a particular surface corresponds uniquely to a particular reflectance.

The aim is to invert this system [Eq. (8)] by using the data vectors and the known, model-dependent matrices $\mathbf{C}_j$ to provide both the s reflectances (represented by a matrix $\mathbf{R}$ with entries r_{tj}) and the v illuminants (represented by the vm-dimensional vector $\mathbf{a}$) up to an arbitrary positive scalar. With $p = m = n = s$, the matrix $\mathbf{R}$ holding the unknown reflectance descriptors and the matrices $\mathbf{C}_j$ are square; suppose further that these are nonsingular. The reflectance matrix $\mathbf{R}$ then has an inverse with entries ρ_{jt}, where $j = 1,\ldots,n$, and $t = 1,\ldots,s$, that satisfy

$$\sum_{t=1}^{s} \rho_{jt}\mathbf{d}_t = \mathbf{C}_j\mathbf{a}\,. \tag{9}$$

Setting $\mathbf{D} = [\mathbf{d}_1\ldots\mathbf{d}_s]$ and $\boldsymbol{\rho}_j = [\rho_{j1}\ldots\rho_{js}]^T$, the system [Eq. (9)] entails $\mathbf{C}_j^{-1}\mathbf{D}\boldsymbol{\rho}_j = \mathbf{a}$. In terms of the $pv \times s$ matrix $\mathbf{E}_j = \mathbf{C}_j^{-1}\mathbf{D}$, one has the n-matrix equations

$$\mathbf{E}_j\boldsymbol{\rho}_j = \mathbf{a}, \qquad j = 1,\ldots,n\,. \tag{10}$$

Taking differences for successive values of the index j, one finds the $n - 1$ equations $\mathbf{E}_h\boldsymbol{\rho}_h - \mathbf{E}_{h+1}\boldsymbol{\rho}_{h+1} = 0$. To combine these, define $\boldsymbol{\rho} = [\boldsymbol{\rho}_1^T\ldots\boldsymbol{\rho}_n^T]^T$ and construct the matrix $\mathbf{F}$, which has $n - 1 \times n$ blocks, each of which matches the size of the $\mathbf{E}_h$, namely, $pv \times s$, by placing in the diagonal blocks $\mathbf{F}_{uu}$ the matrices $\mathbf{E}_u$, placing in the adjacent blocks $\mathbf{F}_{u,u+1}$ the matrices $-\mathbf{E}_{u+1}$, and placing zeros in

Table 1. Tested Components

Photoreceptor	Illuminant Basis	Reflectance Basis
Smith and Pokorny[25]: L, M, S (human RGB; also CIE Judd-modified 1931 standard observer[13])	CIE daylight basis[11,13]: S_0, S_1 S_2 (daylight, Northern Hemisphere)	Cohen's basis[16]: I, II, III (natural surfaces,[15,17] Munsell chips[16])
Hurvich and Jameson[26]: R–G, Y–B, V_λ (human color-opponent model; also CIE 1931 standard observer[13])	Dixon's daylight basis[12]: E_λ, V_1, V_2 (daylight, Southern Hemisphere)	Parkkinen *et al.*'s basis[18]: ϕ_1, ϕ_2, ϕ_3 (Munsell chips)
CIE 1964 supplemental observer[13]: $\bar{x}_{10}$, $\bar{y}_{10}$, $\bar{z}_{10}$ (human large-field standard observer)	CIE illuminants[13]: A, D_{65}, F_2 (indoor lighting: tungsten, daylight, and fluorescent sources)	Fourier-series expansion on interval of visible wavelengths: the first three functions[19]
Sony XC-711: R, G, B (CCD camera)	Fourier-series expansion on interval of visible wavelengths: the first three functions[19]	
Sony XC-007: R, G, B (CCD camera)		

all the other entries. The single system that results is homogeneous:

$$\mathbf{F}\boldsymbol{\rho} = \mathbf{0}. \tag{11}$$

Thus the vector $\boldsymbol{\rho}$, which is determined solely by the reflectance descriptors, lies in the kernel of $\mathbf{F}$. If the dimension of ker($\mathbf{F}$) is one, then the reflectances are determined uniquely [by Eq. (11)], as are the illuminants $\mathbf{a}$ [by Eq. (10)], up to an arbitrary positive scalar [by Eq. (8)]. The kernel of $\mathbf{F}$ is readily computed by using a singular-value decomposition.[24]

SIMULATION

Whether a particular combination of photoreceptors and basis functions for illumination and reflectance actually works is a purely numerical matter. In Table 1 are listed practical choices of photoreceptoral spectral sensitivities and basis functions for illuminants and reflectances. The 60 distinct combinations formed by choosing all possible triples of receptors and illuminant and reflectance bases in Table 1 all function robustly, as was revealed by simulation of surface-color and illuminant-descriptor recovery in 128 randomly generated test cases per combination, through numerical determination[24] of ker($\mathbf{F}$) [Eq. (11)]. As expected, the technique fails if the two illuminants are linearly dependent or if the three surface reflectances are linearly dependent.

A variation suggested by Yellott[27] handles situations in which scene illumination does not change: One effectively obtains a second view of a scene by imaging it through a color filter. Two pictures, taken with and without a filter of known transmission properties, provide the input to the altered recovery procedure {the two blocks in a $\mathbf{C}_j$ matrix [Eq. (8)] differ}. The technique was simulated successfully by using the 24 camera-based combinations of Table 1 in conjunction with transmission spectra[28] for broadband (Corning 1-57 glass), orange (Schott FG9 glass), and blue (Kodak Wratten 80C) filters.

DISCUSSION

A trichromatic visual system can implement color constancy by comparing quantum catches from three (or more) surfaces viewed under two (or more) lights. The change in illumination can occur temporally or spatially or by some combination of the two. The modest requirement is that correspondence between surfaces across two views be maintained. Although a dichromatic visual system cannot take advantage of changing illumination in this way, visual systems with more than three photoreceptoral types are like trichromatic ones: Successful simulations with four- and five-dimensional systems suggest that a p-chromatic system, for $p \geq 3$, can use two views to recover p color-constant descriptors per surface.

At the heart of an extended analogy between color constancy and structure from motion lies the following. While a single view of points on a three-dimensional body (e.g., the corners of a cube) often leads to a flat, two-dimensional precept, further views of these points provided by the rigid motion of the body lead to the three-dimensional perception of a body undergoing motion.[29] Likewise, while a single view of a Mondrian permits, without further assumptions,[7] a two-dimensional representation of surface color,[6] further views can lead, in principle, to the perception of stable, three-dimensional surface colors under changing lights.

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