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Color Image Indexing Using Mathematical Morphology

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Abstract

A mathematical morphology based approach for color image indexing is explored in this paper. Morphological signatures are powerful descriptions of the image content in the framework of mathematical morphology. A morphological signature (either a pattern spectrum or a differential morphological profile) is defined as a series of morphological operations (namely openings and closings) considering a predefined pattern called structuring element. For image indexing it is considered a morphological feature extraction algorithm which includes more complex morphological operators: i.e. color gradient, homotopic skeleton, Hit-or-Miss transform. In the end, illustrative application examples of the presented approach on real acquired images are also provided.

1 Introduction

Morphological image analysis based on Mathematical Morphology (MM) theory uses the lattice theory, the set theory and Euclidian geometry to investigate the image spatial structures, the shape features of the image objects and the relationships between them ([6], [7], [8]). Based upon the MM theory, the precise detection of the object's pixels along with pertinent indexing features, proved to be more computationally efficient than other approaches (i.e. statistical methods concentrate on individual pixel values). But the extension of mathematical morphology for multivariate functions or multichannel images (e.g. color images) is a challenging approach. The definitions of the whole pyramid of the morphological operators starting from the basic ones (*erosion*

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and *dilation*) up to the next upper levels of derived operators (*opening, closing, skeleton, top-hat*, etc.) use a totally ordered complete lattice structure. This algebraic structure cannot be defined in a naturally or perceptually correct way onto the vector space of color images. The vector mathematical morphology theory have been analyzed in some fundamental papers ([1], [2], [3]). Several different orderings have been proposed for the vector space of color images:

- *Marginal ordering* uses the artificial point wise ordering (i.e. it orders color vectors component by component independently). In this case the disadvantage is that "false colors" (new color vector values that are not present in the input image) can be introduced in the processed image.
- *Conditional ordering* performs the ordering of color vectors by means of some marginal components selected sequentially according to different conditions (i.e. *lexicographic ordering*). The *conditional ordering* preserves the input color vectors and has been studied especially for HSV (Hue/Saturation/Value) representation of color images.
- *Reduced ordering* realizes the ordering of color vectors according to some scalars, computed from the components of each vector with respect to different measure criteria, typically distances or projections. The *reduced ordering* has been used to define morphological operators, in the framework of color morphology, by means of distances. It can be successfully used in filtering applications as well as the *conditional ordering.* ([9],[10]).

For the morphological operators we will use the classical functional definition from [5] and [6], where the *morphological erosion* ($\varepsilon_g(f)$) and *morphological dilation* ($\delta_g(f)$) are respectively defined as follows:

$$\varepsilon_g(f)(x) = (f \ominus g)(x) = \inf\{f(x-y) - \breve{g}(y) \mid y \in Supp(g)\}, \forall x \in Supp(f) \subseteq \mathbb{R}^n \tag{1}$$

$$\delta_g(f)(x) = (f \oplus g)(x) = \sup\{f(x-y) + \breve{g}(y) \mid y \in Supp(g)\}, \forall x \in Supp(f) \subseteq \mathbb{R}^n \tag{2}$$

where the *grayscale image* $f : E_f \subseteq \mathbb{R}^n \to \mathbb{R}$ and the *structuring element* or the *structuring function* $g : E_g \subseteq \mathbb{R}^n \to \mathbb{R}$ are semi-continuous functions and $Supp(f) = E_f$ and $Supp(g) = E_g$ are the definition domains for each function (in the following experiments $Supp(f)$ and $Supp(g)$ are Euclidian spaces, $E_f, E_g \subseteq \mathbb{R}^2$ or $E_f, E_g \subseteq \mathbb{Z}^2$, in case of an integer grid). Also $\breve{g}$ is the *symmetric structuring function* defined as follows: $\forall x \in \mathbb{R}^n, \breve{g}(x) = g(-x)$.

In case of 2-D binary images these basic morphological operators *binary erosion* ($\ominus$) and *binary dilation* ($\oplus$) are respectively rewritten more simplified:

$$A \ominus B = \{(x,y) | B_{(x,y)} \subseteq A\} = \bigcap_{(u,v)\in B} A_{(-u,-v)} = \bigcap_{(u,v)\in B^S} A_{(u,v)} \tag{3}$$

$$A \oplus B = \{(x,y)|B_{x,y} \bigcap A \neq \{\phi\}\} = \bigcup_{(u,v)\in B} A_{(-u,-v)} = \bigcup_{(u,v)\in B^S} A_{u,v} \quad (4)$$

where A is a $2-D$ *binary image* and B is a $2-D$ *structuring element* defined on an Euclidian space, $E \subseteq \mathbb{Z}^2$. $A_{(u,v)}$ and $B_{(x,y)}$ are the translations of A and B by vectors (u,v) and (x,y). B^S denotes the symmetric structuring element B:

$$B^S = \{(x,y) \in E|(-x,-y) \in B\}. \quad (5)$$

Subsequently, a multitude of MM operators are derived from *dilation* and *erosion* such as *morphological opening*:

$$\gamma_g(f)(x) = \delta_{\breve{g}}(\varepsilon_g(f))(x) = ((f \ominus g) \oplus g)(x), \forall x \in E_f \subseteq \mathbb{R}^n \quad (6)$$

and *morphological closing*:

$$\phi_g(f)(x) = \varepsilon_{\breve{g}}(\delta_g(f))(x) = ((f \oplus g) \ominus g)(x), \forall x \in E_f \subseteq \mathbb{R}^n \quad (7)$$

and *morphological gradient*:

$$\bigtriangledown_g(f)(x) = \delta_g(f)(x) - \varepsilon_g(f)(x) = ((f \oplus g)(x) - (f \ominus g)(x), \forall x \in E_f \subseteq \mathbb{R}^n \quad (8)$$

where the *structuring function* $g : E_g \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$ and the *symmetric structuring function* $\breve{g}(x) = g(-x), \forall x \in \mathbb{R}^n$, are the same semi-continuous functions defined above. In case of $2-D$ binary images *morphological binary opening* ($\circ$) and *binary closing* ($\bullet$) are respectively defined as:

$$A \circ B = (A \ominus B) \oplus B^S = \{(x,y) \in B_{(u,v)} \wedge B_{(u,v)} \subseteq A\} =$$
$$= \bigcup_{(u,v)\in A} \{B_{(u,v)}|B_{(u,v)} \subseteq A\} \quad (9)$$

$$A \bullet B = (A \oplus B) \ominus B^S = \bigcap_{(u,v)\in A^C} \{B^C_{(u,v)}|B_{(u,v)} \subseteq A^C\} = (A^C \circ B^S)^C \quad (10)$$

where A is a *2-D binary image* object and B is a *2-D structuring element* defined on an Euclidian space, $E \in \mathbb{Z}^2$. B^S denotes the symmetric structuring element B. $B_{(x,y)}$ is the translation of B by vectors (u,v). A^C is the complement of A relative to E, $A^C = \{(x,y)|(x,y) \in E \wedge (x,y) \notin A\}$. Besides morphological gradient (∇_B) we can also define *external contour* (β_B^{ext}) and *internal contour* (β_B^{int}) for the 2-D binary images:

$$\nabla_B(A) = (A \oplus B) - (A \ominus B) \tag{11}$$

$$\beta_B^{ext}(A) = (A \oplus B) - A; \quad \beta_B^{int}(A) = A - (A \ominus B) \tag{12}$$

Furthermore we can define a couple of more complex MM operators as the *Top-Hat* transforms. The *White Top-Hat* transform can detect and extract bright structures on non-uniform backgrounds and is defined as the difference between *the input* image and its *opening* by some structuring element:

$$\tau_g^W(f)(x) = f(x) - \gamma_g(f)(x) = f(x) - \delta_{\breve{g}}(\varepsilon_g(f))(x) = \\ = f(x) - ((f \ominus g) \oplus g)(x), \forall x \in E_f \subseteq \mathbb{R}^n \tag{13}$$

The *Black Top-Hat* is the dual transform and it can detect and extract dark structures on non-uniform backgrounds and is defined as the difference between the *closing* of an image by some structuring element and the same input image:

$$\tau_g^B(f)(x) = \phi_g(f)(x) - f(x) = \varepsilon_{\breve{g}}(\delta_g(f))(x) - f(x) = \\ = ((f \oplus g) \ominus g)(x) - f(x), \forall x \in E_f \subseteq \mathbb{R}^n \tag{14}$$

The skeleton of the input image $f(x)$ represents the union of all intersections of the differences between the erosion and its opening through variable sized structuring elements $g(x)$ (i.e. *White Top-Hat* transforms) [7]:

$$S(f) = \bigcup_{\lambda \geq 0} \bigcap_{\mu \geq 0} [\varepsilon_{\lambda g}(f)(x) - \gamma_{\mu g}(\varepsilon_{\lambda g}(f)(x))], \forall x \in E_f \subseteq \mathbb{R}^n. \tag{15}$$

The *Hit-or-Miss* transform, locates either pixels from erosion $X - E_1$ (i.e. "hit") and from erosion $X^c - E_2^c$, (i.e. "miss") [7]:

$$HMT_E(X) = \varepsilon_{E_1}(X) \bigcap \varepsilon_{E_2}(X^C) \tag{16}$$

2 Morphological signatures

Morphological signatures can be obtained from a series of successive openings or closings. They can be extended from a pixel-related scale to an image-related scale (i.e. from a local definition using morphological profile operators to a global definition using pattern spectrum/granulometry operators) ([4], [5]).

The *opening morphological signature series* is defined as:

$$\Gamma(f)(x) = \{\gamma_{kg}(f)(x) | kg = \delta_S^k(g)(x), \forall k \in \{0, ..., m\}, \forall x \in E_f \subseteq \mathbb{R}^n\} \tag{17}$$

where $\gamma_{kg}(f)(x)$ is the opening by a variable sized structuring element kg and $\delta_S^k(g)(x)$ is the *k-times successive dilation* of the structuring element g by another very simple or elementary structuring element S. By definition $\gamma_0(f)(x) = f(x)$.

The dual *closing morphological signature series* is defined as:

$$\Phi(f)(x) = \{\phi_{kg}(f)(x) | kg = \delta_S^k(g)(x), \forall k \in \{0, ..., m\}, \forall x \in E_f \subseteq \mathbb{R}^n\} \quad (18)$$

Where $\phi_{kg}(f)(x)$ is the closing by a similar variable sized structuring element kg, as defined above, and $\delta_S^k(g)(x)$ is the same *k-times successive dilation* of the structuring element g by another very simple or elementary structuring element S.

Subsequently, we can define the *differential series* computed from *opening morphological signature series*, which can provide more meaningful morphological information:

$$\Delta\gamma(f)(x) = \{\gamma_{kg}(f)(x) - \gamma_{(k-1)g}(f)(x) | kg = \delta_S^k(g)(x), \\ \forall k \in \{0, ..., m\}, \forall x \in E_f \subseteq \mathbb{R}^n\} \quad (19)$$

The dual *differential series* computed from *closing morphological signature series* can be defined as:

$$\Delta\phi(f)(x) = \{\phi_{(k-1)g}(f)(x) - \phi_{kg}(f)(x) | kg = \delta_S^k(g)(x), \\ \forall k \in \{0, ..., m\}, \forall x \in E_f \subseteq \mathbb{R}^n\} \quad (20)$$

Finally, morphological signatures are defined from these 4 series:

$$\begin{cases} \Gamma(f)(x) \\ \Phi(f)(x)) \\ \Delta\gamma(f)(x) \\ \Delta\phi(f)(x)) \end{cases} \quad (21)$$

where $x \in E_f \subseteq \mathbb{R}^n$.

On a local pixel scale this set of 4 series is used to define *differential morphological profile* for a given pixel $x \in E_f \subseteq \mathbb{R}^n$ in image f. On a global image scale the *pattern spectrum* is built from the image series above gathering the image pixel values through the sum operation, i.e.

$$\begin{cases} \sum_{x \in E_f} (\Gamma(f)(x) + \Phi(f)(x)) \\ \sum_{x \in E_f} (\Delta\gamma(f)(x) + \Delta\phi(f)(x)) \end{cases} \quad (22)$$

where $x \in E_f \subseteq \mathbb{R}^n$.

The *shape probability distribution function*, involves a normalization by the initial image volume:

$$\begin{cases} \dfrac{\sum\limits_{x\in E_f}(\Gamma(f)(x)+\Phi(f)(x))}{\sum\limits_{x\in E_f}(f)(x)} \\ \dfrac{\sum\limits_{x\in E_f}(\Delta\gamma(f)(x)+\Delta\phi(f)(x))}{\sum\limits_{x\in E_f}(f)(x)} \end{cases} \tag{23}$$

where $x \in E_f \subseteq \mathbb{R}^n$.

3 Morphological image indexing based on edge extraction and homotopic skeleton

One of the simplest morphological contour extraction methods is the *morphological gradient*: basically it consists of constructing an edge intensity map of the image as the difference between the *local dilation* and the *local erosion* at each image pixel (the word "local" being induced by the use of a structuring element with finite spatial support).

$$\nabla_B(f)(x) = (f \oplus g)(x) - (f \ominus g)(x), \forall x \in E_f \subseteq \mathbb{R}^2 \tag{24}$$

Starting from the above definition of *morphological gradient* for the 2-D images we can compute the *length of an object boundary or contour* which is a meaningful local morphological signature. This can also be extended to a global image scale:

$$L(\nabla_B(f)) = \sum_{x\in E_f} \nabla_B(f)(x) \tag{25}$$

where $x \in E_f \subseteq \mathbb{R}^2$.

The first step is to generate an *edge intensity map* using the *morphological gradient* operator. The *edge intensity map* consists of values proportional to the local variation within each pixel neighborhood, as defined by the support of the structuring element. The *binary edge map* is obtained by thresholding the edge intensity map and selecting the pixels with a strong color (value) variations, measured by important values of the *morphological gradient.*

A typical edge extraction result by the described method is presented in **figure 1**.

Also, we can compute another important morphological signature as the *length of the skeleton*:

$$L(S(f)) = \sum_{x \in E_f} \bigcup_{\lambda \geq 0} [\varepsilon_{\lambda g}(f)(x) - \gamma_{\lambda g}(\varepsilon_{\lambda g}(f)(x))], \forall x \in E_f \subseteq \mathbb{R}^2$$

The *length of the skeleton* was used as a very meaningful morphological signature in the experiment concerning labelling and classification of color image objects as shown in *Figure 2*.

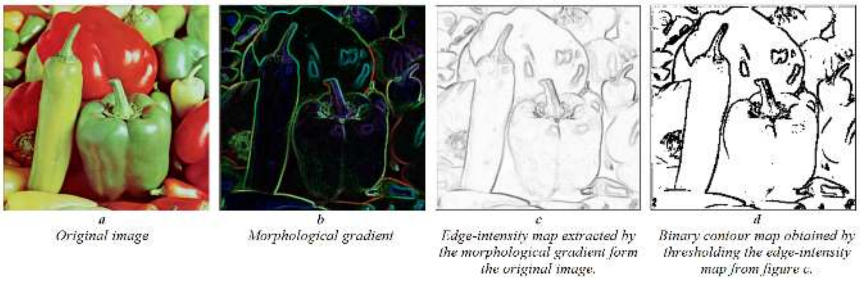

Figure 1. Morphological edge extraction for color image

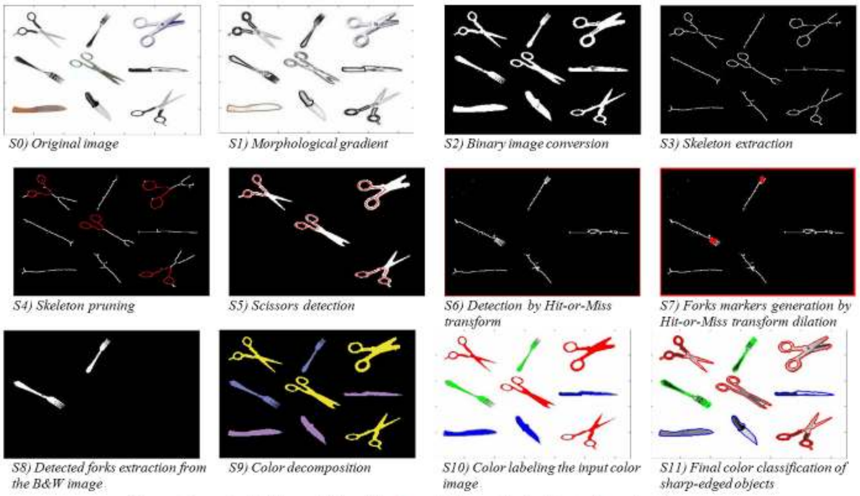

Figure 2. Labelling and classification of sharp-edged objects in a color image

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