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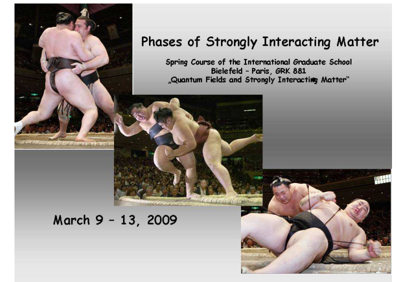



# Color superconductivity in dense quark matter

For a review, see

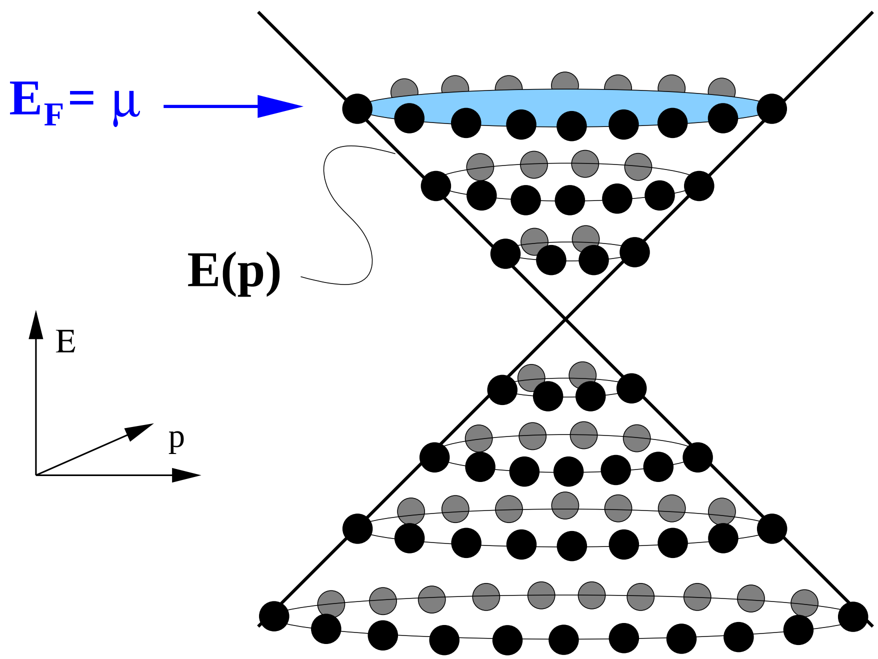
M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, Rev. Mod. Phys. 80, 1455 (2008)

- **Basics of (color) superconductivity**
- **Color-flavor locking (CFL) – highest densities**
- **Stressed pairing – below CFL densities**
- **QCD calculations**
- **Effective theory of CFL – kaon condensation**
- **Transport properties – quark matter in compact stars**

- **Part I**

- **Basics of (color) superconductivity**
- Color-flavor locking (CFL) – highest densities
- Stressed pairing – below CFL densities
- QCD calculations
- Effective theory of CFL – kaon condensation
- Transport properties – quark matter in compact stars

# • Basics of (color) superconductivity (page 1/5)




free energy $\Omega = E - \mu\, N$

- no interactions: add fermion at $E = \mu$ without cost
- attractive interaction: add pair with gain
- pairs condense $\rightarrow$ superconductivity

This Bardeen-Cooper-Schrieffer (BCS) argument holds for electrons in metal, $^3$He atoms, . . .,
and quarks in quark matter

# • Basics of (color) superconductivity (page 1/5)

## Fermi sphere

free energy $\Omega = E - \mu N$

- no interactions: add fermion at $E = \mu$ without cost
- attractive interaction: add pair with gain
- pairs condense $\longrightarrow$ superconductivity

This Bardeen-Cooper-Schrieffer (BCS) argument holds for electrons in metal, $^3$He atoms, . . .,
and quarks in quark matter

# • Basics of (color) superconductivity (page 1/5)

## Cooper pairing


free energy $\Omega = E - \mu N$

- no interactions: add fermion at $E = \mu$ without cost
- attractive interaction: add pair with gain
- pairs condense $\rightarrow$ superconductivity

This Bardeen-Cooper-Schrieffer (BCS) argument holds for electrons in metal, $^3$He atoms, . . .,
and quarks in quark matter

- **Basics of (color) superconductivity (page 2/5)**
  **Electromagnetic vs. color superconductor**

| | **Where?** | **What?** | **Attractive force** | **Cooper pairs** | **Meissner effect** |
|---|---|---|---|---|---|
| "usual" superconductor | metals, alloys | ion lattice & electrons | phonons | electrons | photon |
| **color superconductor** | **neutron stars** | **quarks & gluons** | **gluons** | **quarks** | **gluons (and photon)**$^{(*)}$ |

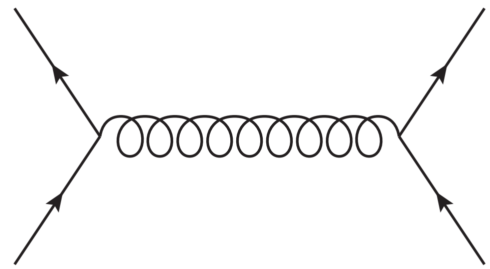
$^{(*)}$ Most color superconductors are not electromagnetic superconductors ("rotated electromagnetism")

Exception: Spin-1 color superconductors

A. Schmitt, Q. Wang, D. H. Rischke, PRL 91, 242301 (2003)

- **Basics of (color) superconductivity (page 3/5): attractive quark-quark interaction**

  - one-gluon exchange



attractive in antisymmetric antitriplet channel $[\bar{\mathbf{3}}]_c^a$

$$SU(3)_c : \qquad [\mathbf{3}]_c \otimes [\mathbf{3}]_c = [\bar{\mathbf{3}}]_c^a \oplus [\mathbf{6}]_c^s$$

  - flavor space

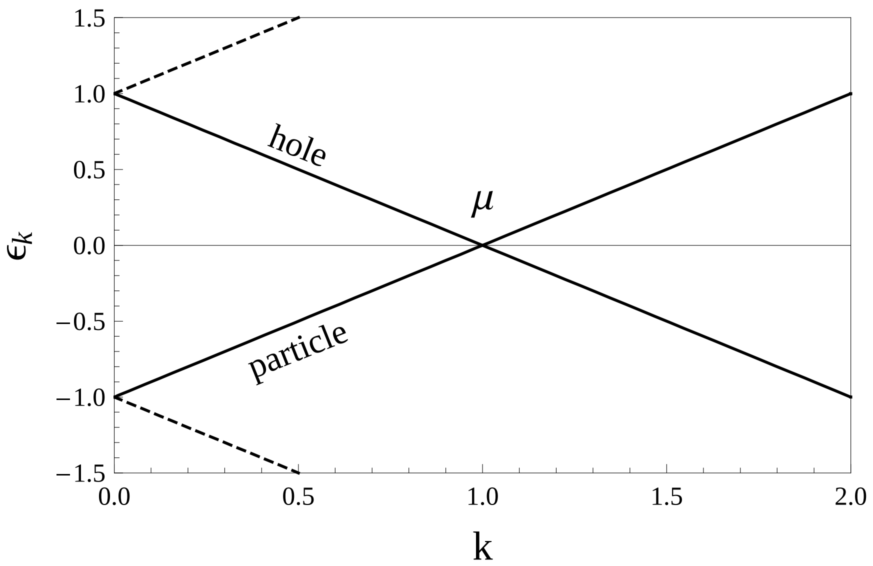
$$SU(3)_f : \qquad [\mathbf{3}]_f \otimes [\mathbf{3}]_f = [\bar{\mathbf{3}}]_f^a \oplus [\mathbf{6}]_f^s$$

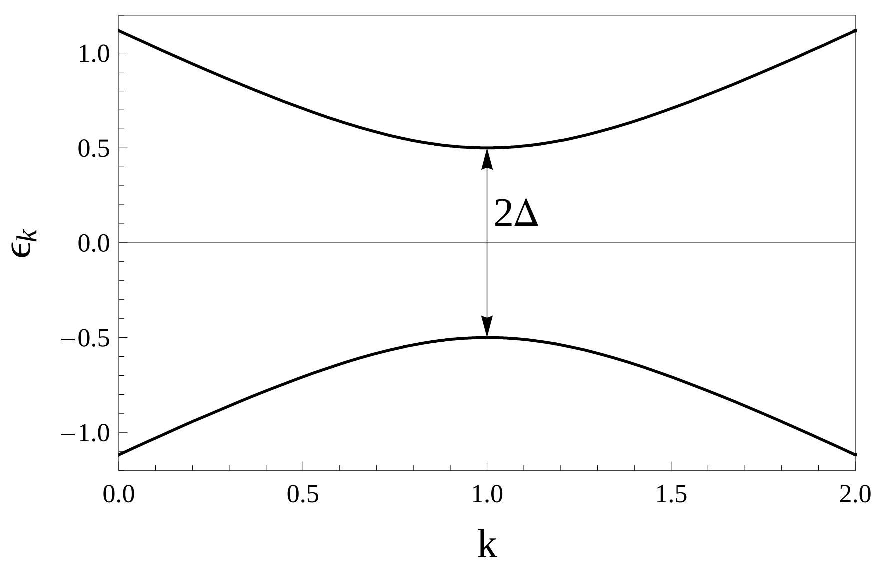
  - order parameter (for spin-0 pairing):

$$\langle \psi_i^\alpha C \gamma_5 \psi_j^\beta \rangle \; \propto \; \epsilon^{\alpha\beta A} \, \epsilon_{ijB} \, \phi_B^A \;\; \in \;\; [\bar{\mathbf{3}}]_c^a \otimes [\bar{\mathbf{3}}]_f^a$$

- **Basics of (color) superconductivity (page 4/5): energy gap**

## Fermion dispersions



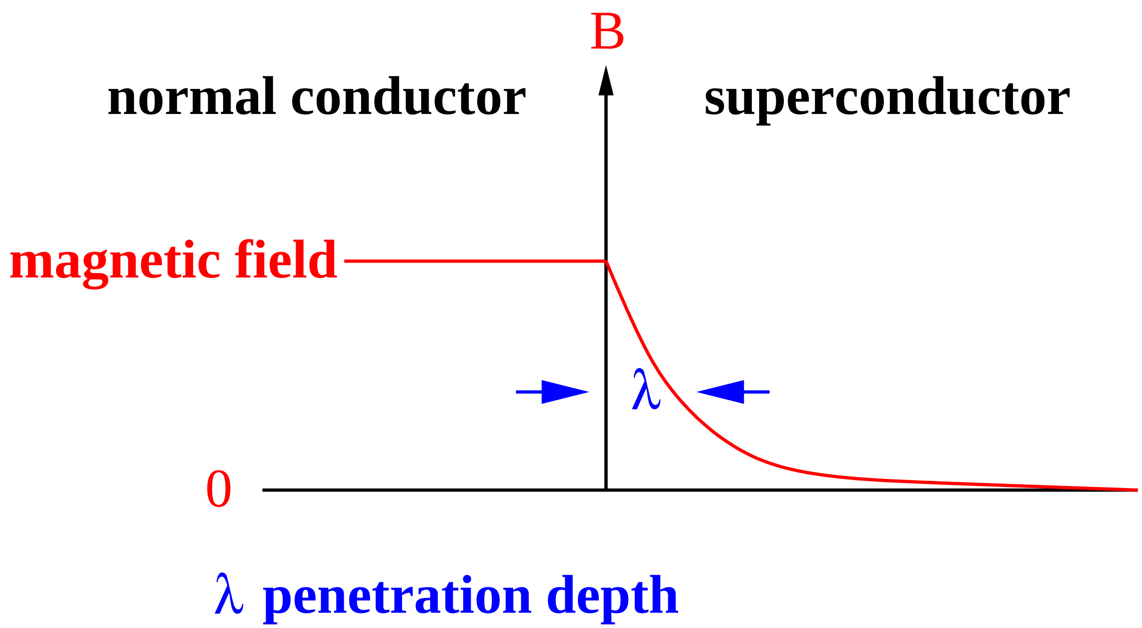

$$\epsilon_k = \pm(k - \mu)$$

$$\epsilon_k = \pm\sqrt{(k - \mu)^2 + \Delta^2}$$

$\rightarrow$ suppression of specific heat, viscosity, neutrino emissivity, etc.

**Critical temperature (in BCS theory):** $T_c \simeq 0.57\Delta$

- **Basics of (color) superconductivity (page 5/5): Meissner effect**

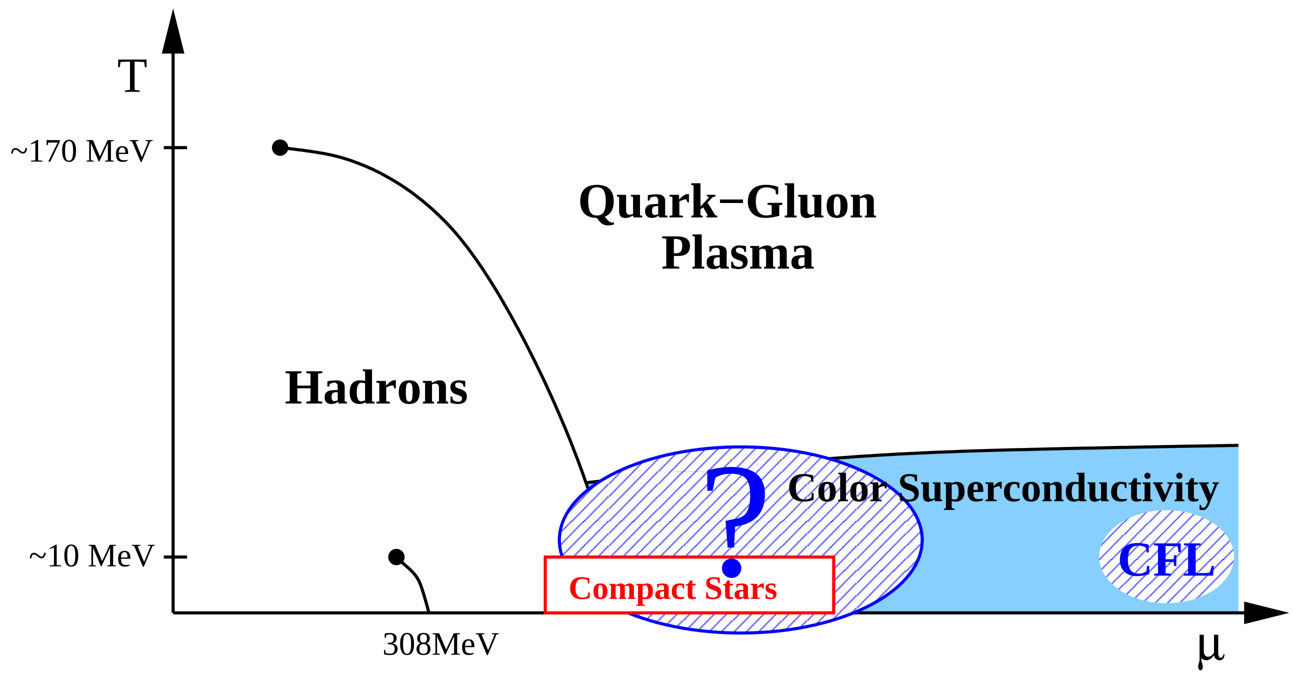



- spontaneous symmetry breaking: $U(1)_{\text{em}} \to \mathbb{Z}_2$
- Anderson-Higgs mechanism: photon Meissner mass $m_M$

$$m_M = \frac{1}{\lambda}, \qquad B \propto e^{-m_M r}$$

- gluon Meissner masses in color superconductors

- **QCD phase diagram (page 1/3): Known and unknown territories**



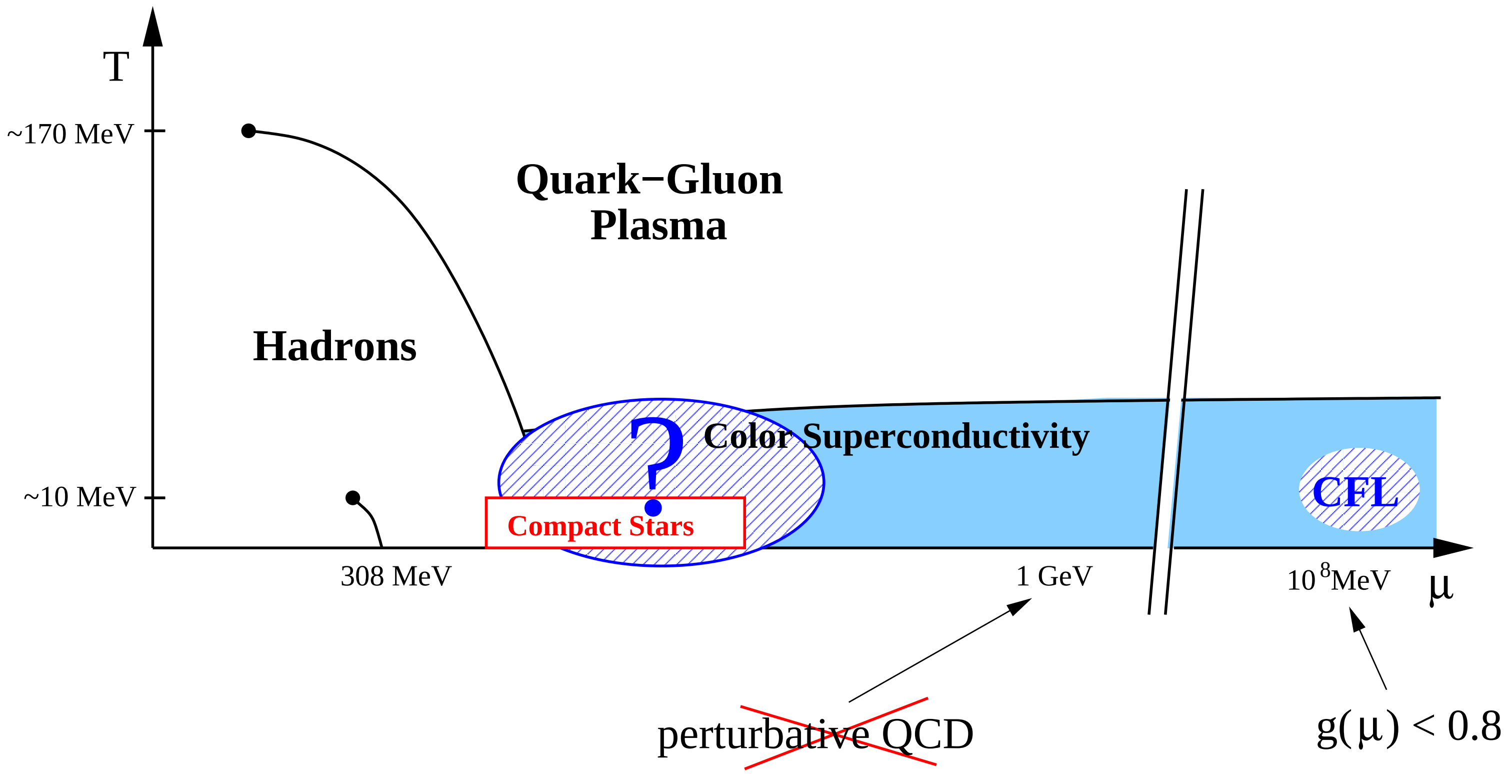

**High densities:**

- rigorous theoretical control
- no nonperturbative gaps in our understanding

**Moderate densities:**

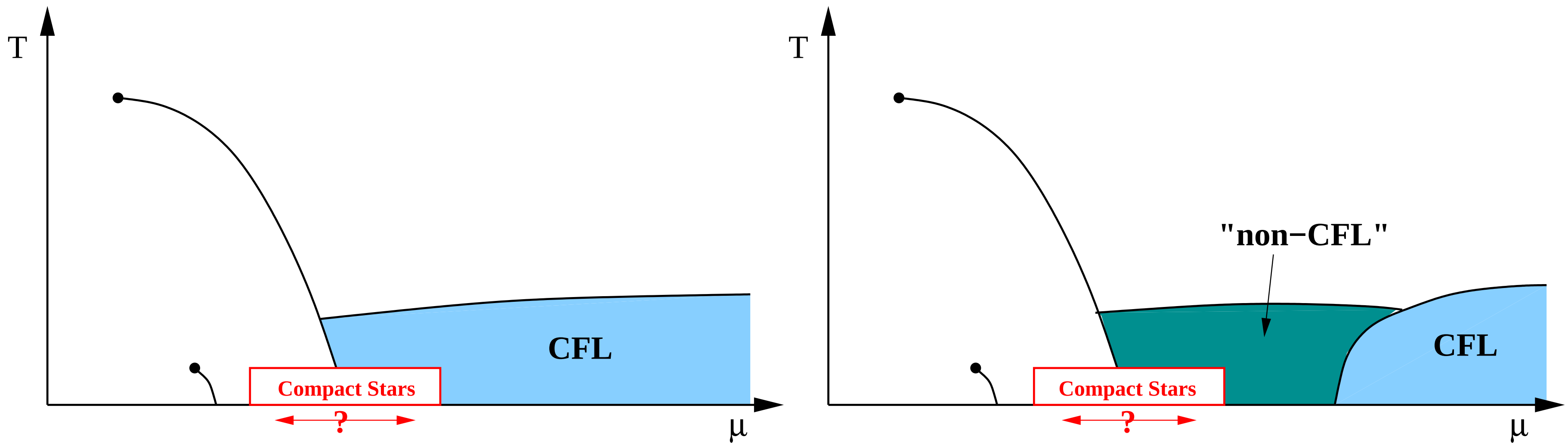
- perturbative QCD not valid
- strange mass & neutrality: stress on Cooper pairing

# • QCD phase diagram (page 2/3): Validity of perturbative QCD




- here: always $N_f = 3$ (ignore heavy quarks)
- $N_f > 3$: T. Schäfer, NPB 575, 269 (2000)

# • QCD phase diagram (page 3/3): 2 Possible scenarios




## CFL superseded by nuclear matter:

- effective theory for CFL in strongly-coupled regime
- CFL matter in the core of compact stars?

## CFL superseded by "non-CFL" matter:

- complicated phase structure?
- rely on Nambu-Jona-Lasinio-type models

## Question:

**What is the ground state of deconfined quark matter at moderate densities (in the interior of compact stars)?**

1. **Theoretical approach: start from CFL and ask "what is next phase down in density?" (if not hadronic matter)**
2. **Phenomenological approach: "guess" possible phase, compute its properties and compare with astrophysical observations**

3. (Tabletop approach: learn from parallels to cold fermionic atoms in magnetic trap)

- **Part II**

- Basics of (color) superconductivity
- **Color-flavor locking (CFL) – highest densities**
- Stressed pairing – below CFL densities
- QCD calculations
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- **On safe grounds: Asymptotically large density**

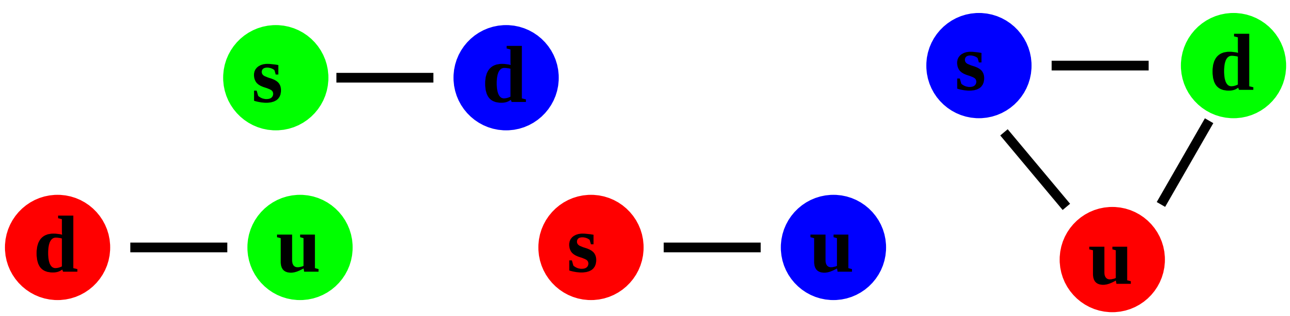
$$0 \simeq m_s \simeq m_u \simeq m_d \ll \mu \qquad \text{all quark masses negligible}$$

**“color-flavor locked phase (CFL)”**

M. Alford, K. Rajagopal, F. Wilczek, NPB 537, 443 (1999)

$$\phi^{A}_{B} = \delta^{A}_{B} \quad \Rightarrow \quad \langle \psi^{\alpha}_{i} C\gamma_5 \psi^{\beta}_{j} \rangle \ \propto \ \epsilon^{\alpha\beta A}\, \epsilon_{ijA}$$

$$\Rightarrow \quad SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \rightarrow \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$



- **Properties of CFL (page 1/3)**

## (1) chiral symmetry breaking

- usual chiral symmetry breaking: LR pairing $\langle\bar{\psi}_R\psi_L\rangle$
- here: LL, RR pairing $\langle\psi_R\psi_R\rangle$, however

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}}$$

- chiral symmetry broken through "locking" to color
- octet of pseudo-Goldstone modes $K^0$, $K^\pm$, $\pi^0$, ...
- quark-hadron continuity? T. Schäfer, F. Wilczek, PRL 82, 3956 (1999)

# • Properties of CFL (page 2/3)

## (2) superfluidity

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}}$$

- *exactly massless* Goldstone mode $\phi$
- vortices in rotating CFL M. M. Forbes, A. R. Zhitnitsky, PRD 65, 085009 (2002)

## (3) rotated electromagnetism

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}}$$

- Cooper pairs neutral under $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$
- photon-gluon mixing with (small) mixing angle

$$\cos^2\theta = \frac{3g^2}{3g^2 + 4e^2} \simeq 1$$

# • Properties of CFL (page 3/3)

## (3) rotated electromagnetism (continued)

- Analogy to standard model

| **Weinberg-Salam** | **CFL** |
|---|---|
| $SU(2)_I \times U(1)_Y$<br>isospin, hypercharge | $SU(3)_c \times U(1)_Q$<br>color, electromagnetism |
| $W_1, W_2, W_3, W_0$ | $A_1, \ldots, A_8, A$ |
| $SU(2)_I \times U(1)_Y \to U(1)_Q$ | $SU(3)_c \times U(1)_Q \to U(1)_{\tilde{Q}}$ |
| $W^+, W^-$<br>$Z = \cos\theta_W W_3 + \sin\theta_W W_0$<br>$A = -\sin\theta_W W_3 + \cos\theta_W W_0$ | $A_1, \ldots, A_7$<br>$\tilde{A}_8 = \cos\theta A_8 + \sin\theta A$<br>$\tilde{A} = -\sin\theta A_8 + \cos\theta A$ |
| $W^+$, $W^-$, $Z$ (massive) | $A_1, \ldots, A_7, \tilde{A}_8$ (massive) |
| $A$ (massless) | $\tilde{A}$ (massless) |

- **Why CFL is favored (page 1/2)**

- general order parameter

$$\langle \psi_i^\alpha C\gamma_5 \psi_j^\beta \rangle \ \propto \ \epsilon^{\alpha\beta A} \, \epsilon_{ijB} \, \phi_B^A$$

- complex $3 \times 3$ matrix $\phi_B^A \rightarrow$ *huge configuration space?!*

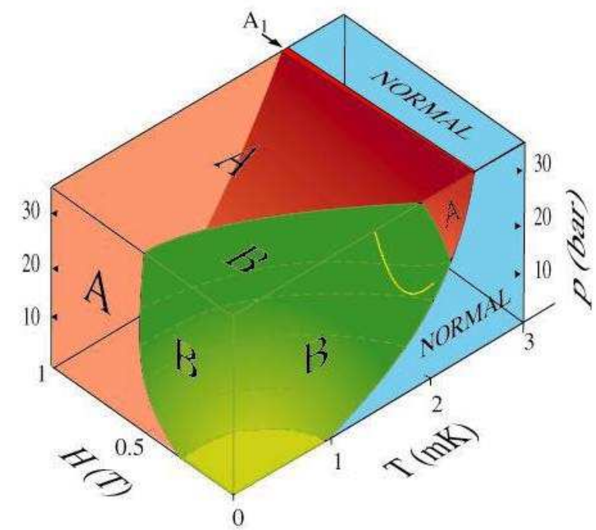
  – cf. **superfluid $^3$He**

  $$SO(3)_L \times SU(2)_S \times U(1)$$

  – $3 \times 3$ order parameter in angular momentum $L$, spin $S$

  – **A phase**: $\phi_B^A = \delta^{A3}(\delta_{B1} + i\delta_{B2})$

  – **B phase**: $\phi_B^A = \delta_B^A$

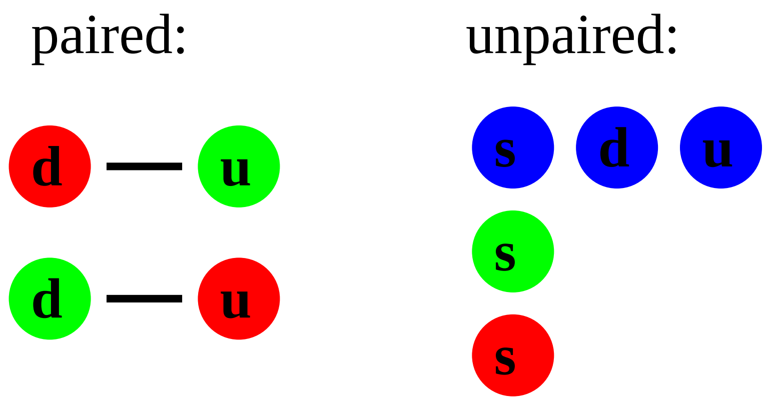


- **Why CFL is favored (page 2/2)**

  - *simple at high densities!* $\longrightarrow$ full $SU(3)_c \times SU(3)_f$ symmetry
  - can restrict to diagonal $\phi^A_B$

$$\forall\,\phi\ \ \exists\, U \in SU(3)_c\,,\ V \in SU(3)_f : \quad U^T\,\phi\,V \text{ diagonal}$$

  - then $\phi^A_B = \delta^A_B$ is only phase where **all quarks pair**
  - for instance **2SC phase** $\phi^A_B = \delta^{A3}\delta_{B3}$



- **Part III**

- Basics of (color) superconductivity
- Color-flavor locking (CFL) – highest densities
- **Stressed pairing – below CFL densities**
- QCD calculations
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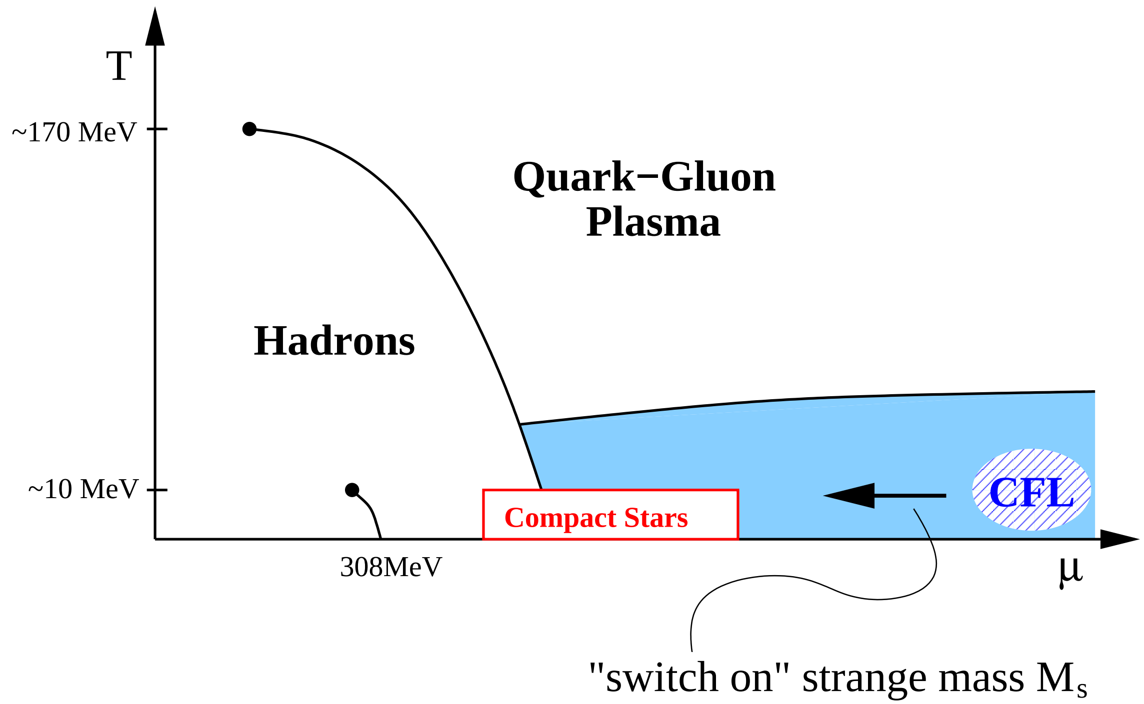
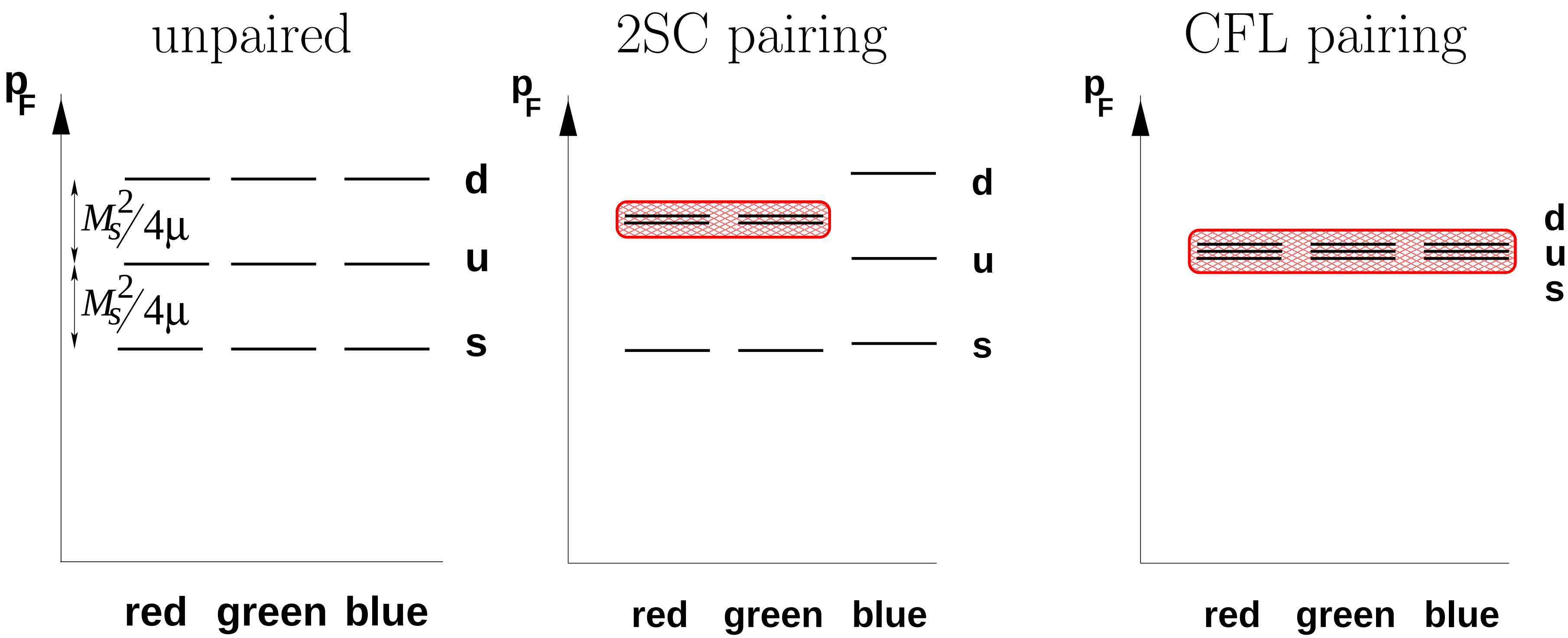
- **Going down in density:**
  **Large, but not asymptotically large, densities**
  - strange mass $M_s \simeq 120\,\mathrm{MeV}$ no longer $\ll \mu \simeq 400\,\mathrm{MeV}$




  - at given chemical potentials $\mu$, $\mu_e$: $M_s \neq 0$ reduces $p_F^s$
  - electric neutrality: increase in $p_F^d$ to compensate
  - **Fermi momenta "try" to split apart**

- **$p_F$'s splitting apart**

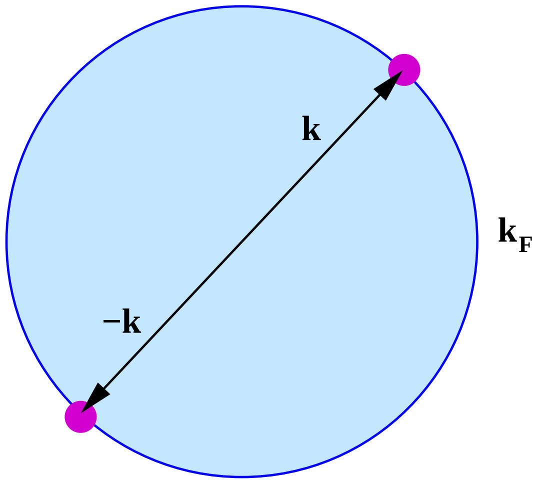



- any pairing pattern most "comfortable" with $M_s$ and neutrality?
- stressed pairing is **unavoidable**!

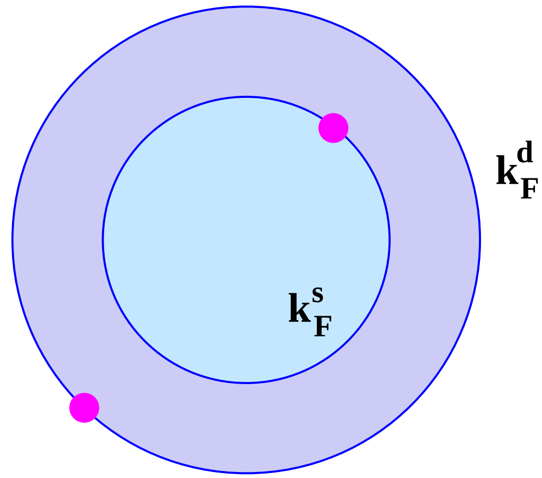
  K. Rajagopal, A. Schmitt, PRD 73, 045003 (2006)

# • Stressed Cooper pairing: a general phenomenon (page 1/2)

BCS-pairing:




split Fermi momenta:




- for instance: two species (spin states) of cold fermionic atoms
  - Experiments:

    Y. Chin, M.W. Zwierlein, C.H. Schunck, A. Schirotzek, W. Ketterle, PRL 97, 030401 (2006)

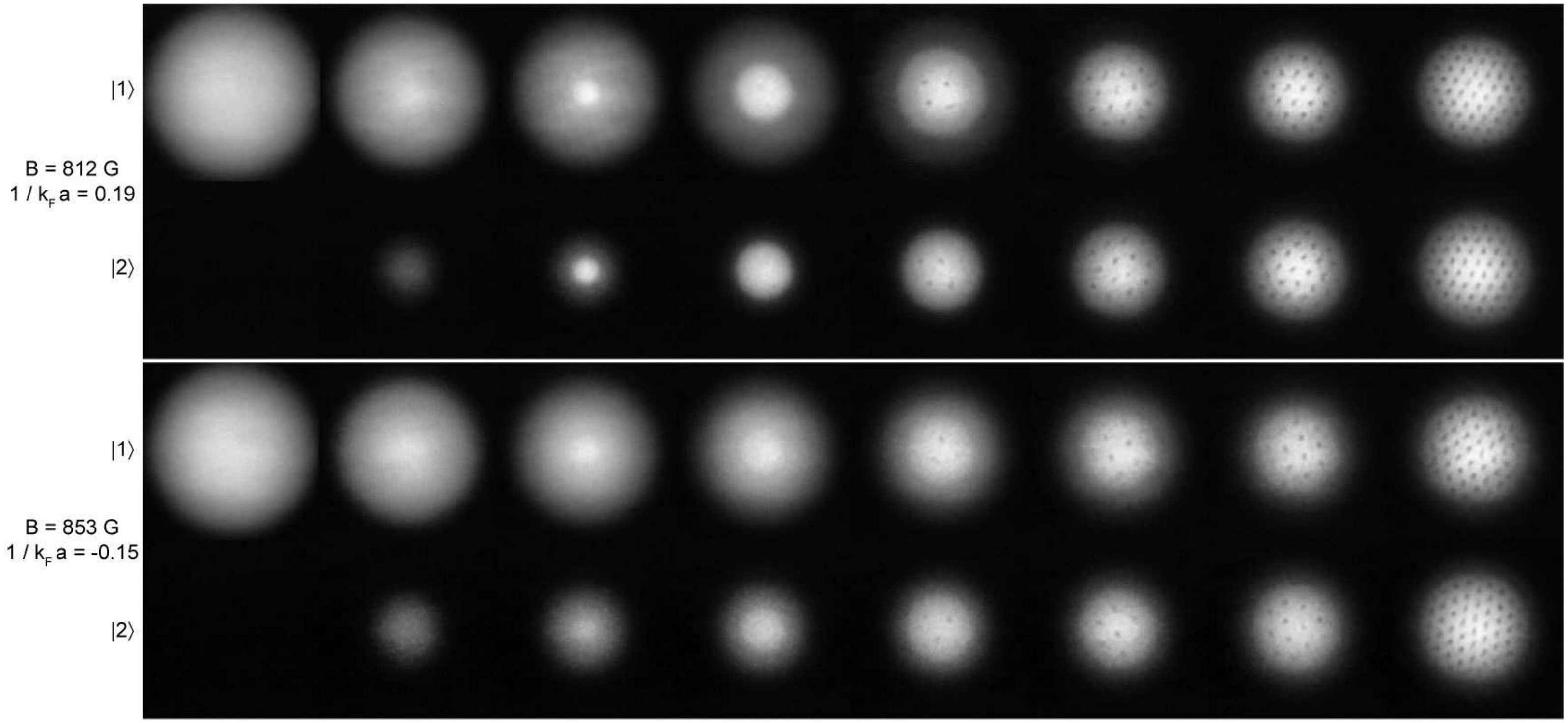
    G.B. Partridge, W. Li, R.I. Kamar, Y. Liao, R.G. Hulet, Science 311, 503 (2006)
  - Theory (review):

    D.E. Sheehy, L. Radzihovsky, Ann. Phys. 322, 1790 (2007)

- **Stressed Cooper pairing: a general phenomenon (page 1/2)**

**Pairing with mismatch**

# • Stressed Cooper pairing: a general phenomenon (page 2/2)

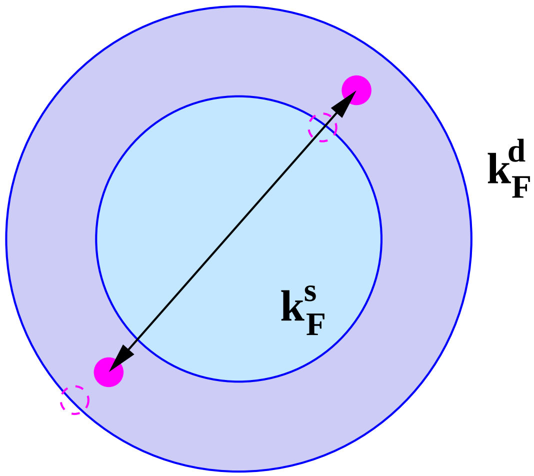



M.W. Zwierlein, A. Schirotzek, C.H. Schunck, W. Ketterle, Science 311, 492 (2006)

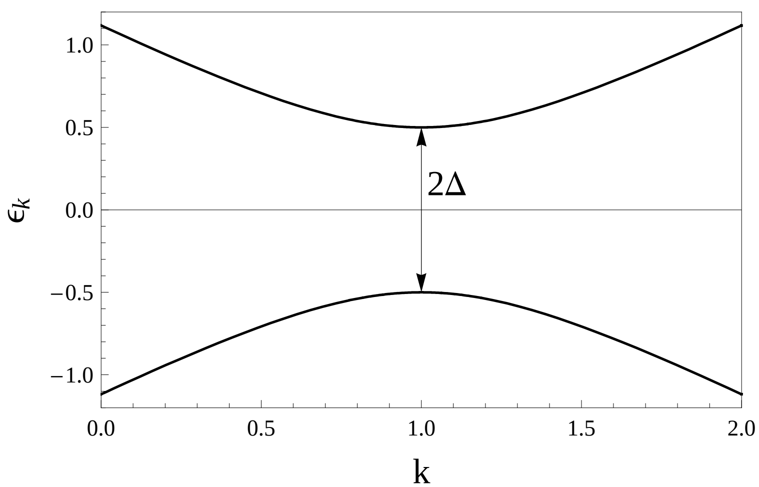
- **phase separation** of superfluid and normal components
- phase separation unlikely in quark matter (local color neutrality!)

M. Alford, C. Kouvaris, K. Rajagopal, PRL 92, 222001 (2004)

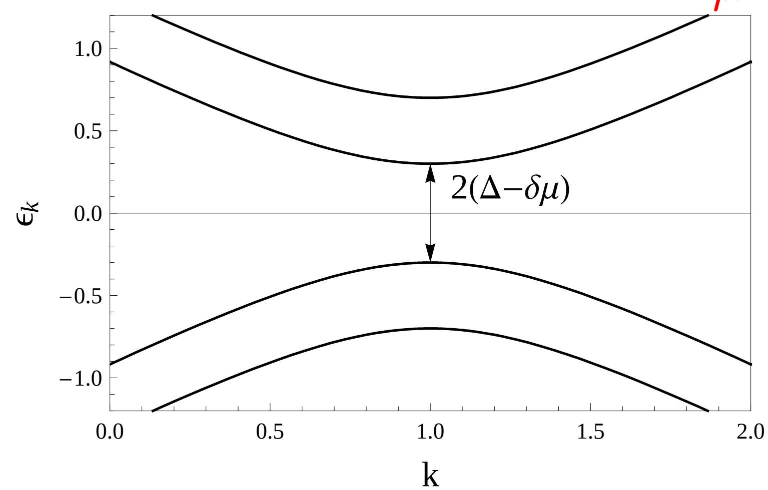
# • CFL pairing with small stress




- create common Fermi surface: cost in free energy $\sim \delta p_F^2\, \mu^2 \sim M_s^4$
- form pairs: gain in free energy $\sim \Delta^2 \mu^2$
- CFL survives for $\Delta \gtrsim \frac{M_s^2}{\mu}$




$$\epsilon_k = \pm\sqrt{(k-\mu)^2 + \Delta^2}$$

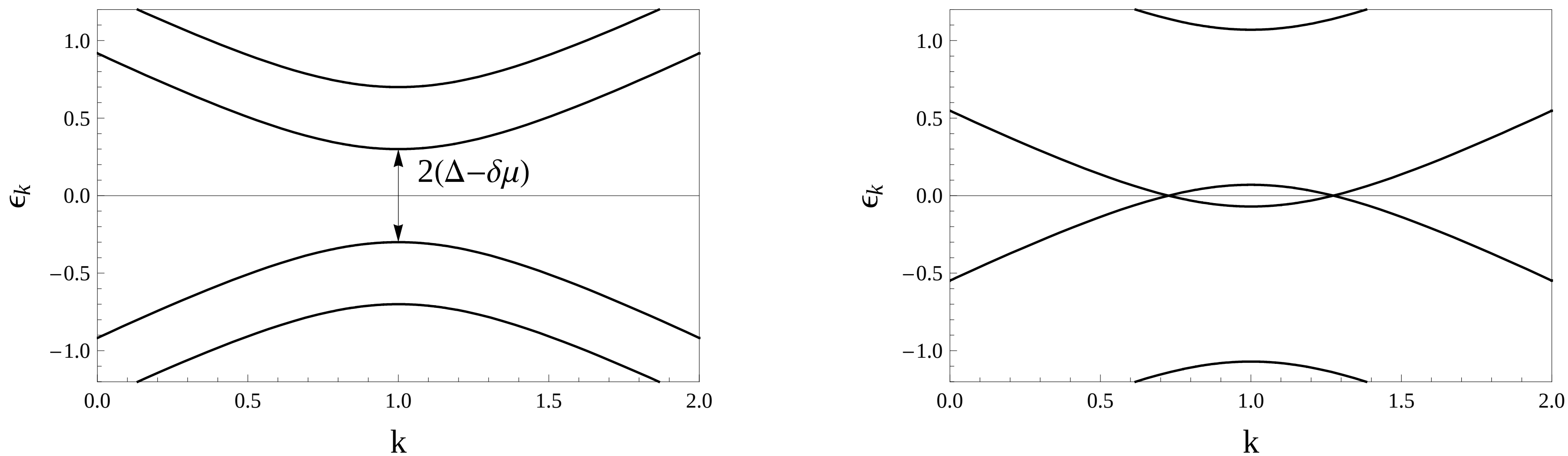



$$\epsilon_k = \pm\left(\sqrt{(k-\bar{\mu})^2 + \Delta^2} \pm \delta\mu\right)$$

- **Large stress: gapless CFL?**

→ "gapless CFL" for $\delta\mu > \Delta$

M. Alford, C. Kouvaris, K. Rajagopal, PRL 92, 222001 (2004)



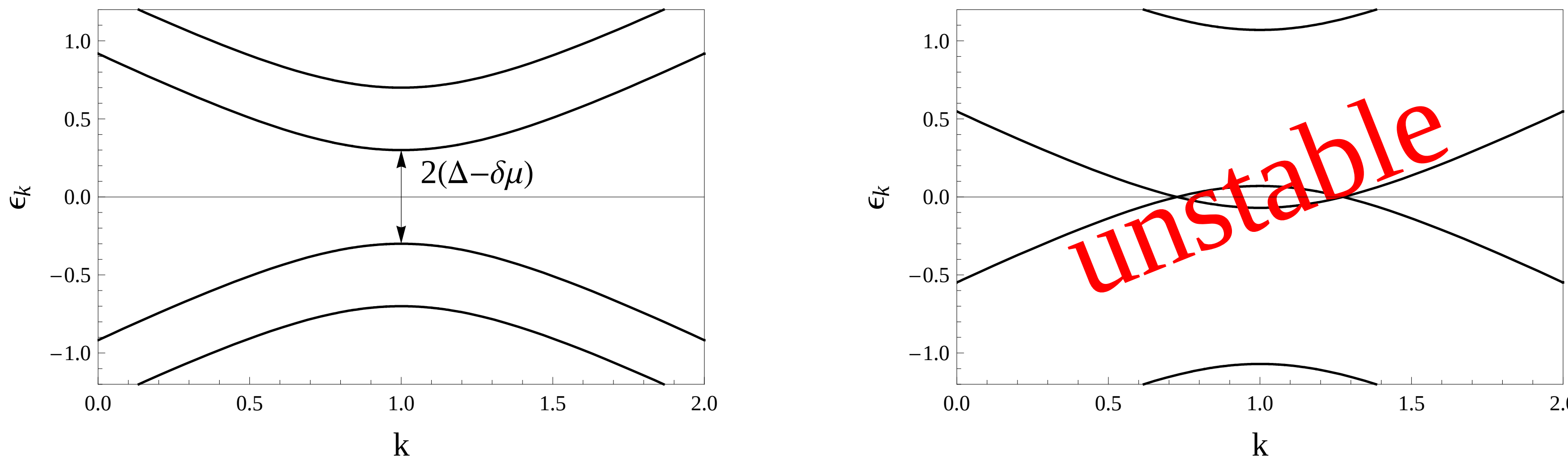

→ or: "breached" pairing

E. Gubankova, W. V. Liu and F. Wilczek, PRL 91, 032001 (2003)

- **Large stress: gapless CFL? No!**

→ "gapless CFL" for $\delta\mu > \Delta$

M. Alford, C. Kouvaris, K. Rajagopal, PRL 92, 222001 (2004)




→ or: "breached" pairing

E. Gubankova, W. V. Liu and F. Wilczek, PRL 91, 032001 (2003)

→ **chromomagnetic instability** (at $T = 0$)

M. Huang, I. A. Shovkovy, PRD 70, 051501 (2004)

R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli, M. Ruggieri, PLB 605, 362 (2005)

→ ground state must be different → **currents**

# • Less (and less symmetric) pairing (page 1/4)

## Kaon condensation "CFL-$K^0$"

P. F. Bedaque and T. Schäfer, NPA 697, 802 (2002)

- chiral field

$$\Sigma = \phi_L^\dagger \phi_R$$

- pure CFL: $\Sigma = \mathbf{1}$
- kaon condensation $\Rightarrow \Sigma = e^{i\varphi T_6}$ (relative L/R rotation)

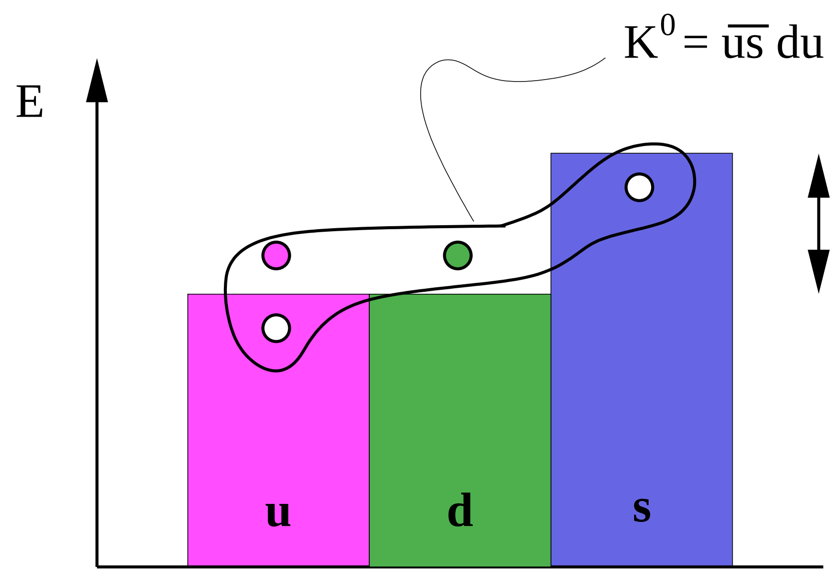
- in other words:

  create kaon with mass

  $m_{K^0}^2 = a\, m_d(m_s + m_u) \ll \Delta^2$

  from $0 \to \bar{s} + u + \bar{u} + d$

  $(a \sim \Delta^2/\mu^2)$



- **Less (and less symmetric) pairing (page 2/4)**

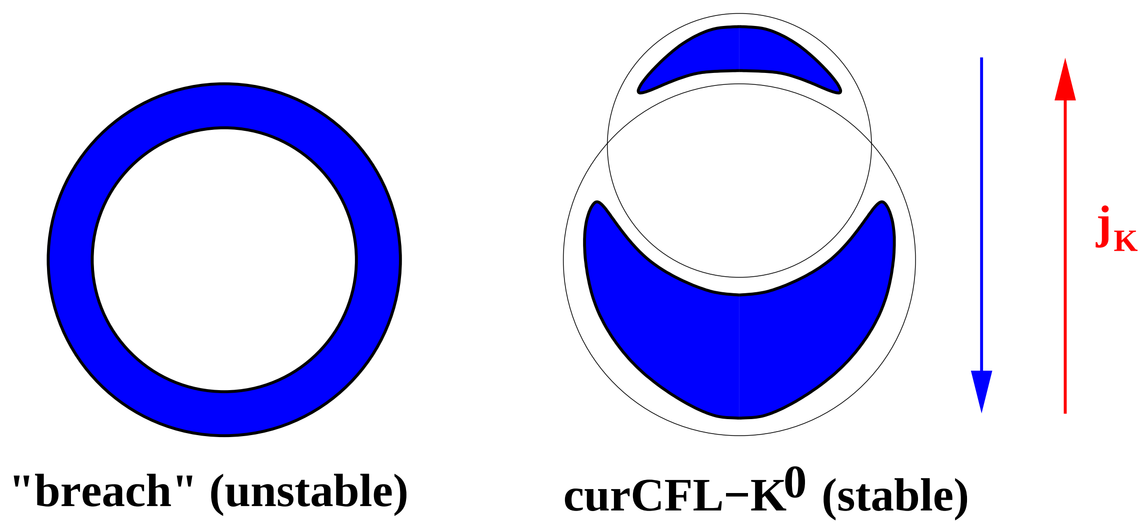
## (Super)currents in CFL: "curCFL-$K^0$"

T. Schäfer, PRL 96, 012305 (2006)

A. Kryjevski, PRD 77, 014018 (2008)

$$\phi_L(\mathbf{x}) = \Delta\, e^{i\boldsymbol{j}_K\cdot\mathbf{x}\,T_8}\; e^{i(\varphi/2)T_6}$$
$$\phi_R(\mathbf{x}) = \Delta\, e^{i\boldsymbol{j}_K\cdot\mathbf{x}\,T_8}\; e^{-i(\varphi/2)T_6}$$

- "anisotropic breach"
- stable and unstable Fermi surface topologies:



# • Less (and less symmetric) pairing (page 3/4)

## More currents in CFL: crystalline structures (LOFF)

M. Alford, J. Bowers, K. Rajagopal, PRD 63, 074016 (2001)

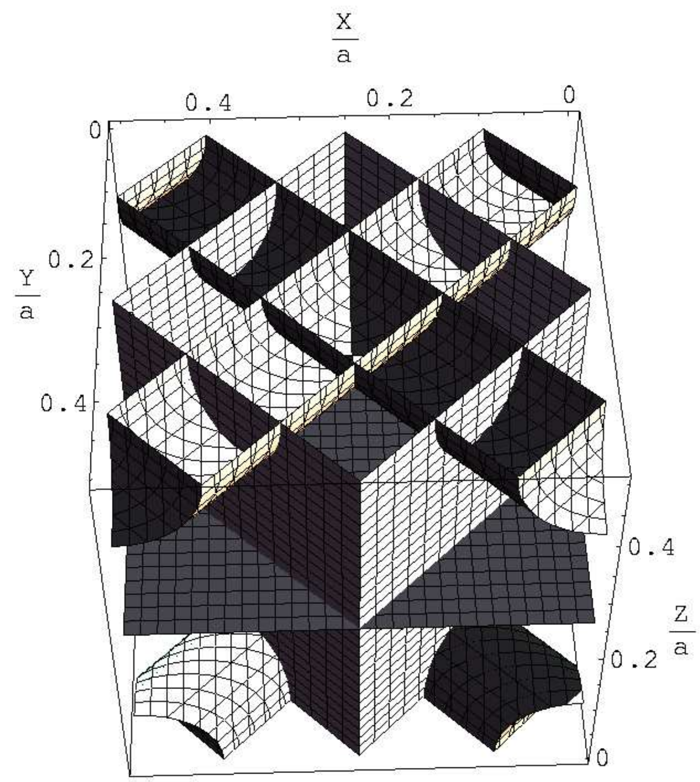
M. Mannarelli, K. Rajagopal and R. Sharma, PRD 73, 114012 (2006)

$$\langle ud \rangle \sim \Delta_3 \sum_a \exp(2i\mathbf{q}_3^a \cdot \mathbf{x})$$

$$\langle us \rangle \sim \Delta_2 \sum_a \exp(2i\mathbf{q}_2^a \cdot \mathbf{x})$$

$$\langle ds \rangle \simeq 0$$

- here: "CubeX"
- $\{\mathbf{q}_3\}$, $\{\mathbf{q}_2\}$ each contain 4 vectors, together pointing to the corners of a cube




$\Delta_3(\mathbf{x}), \Delta_2(\mathbf{x})$

# • Less (and less symmetric) pairing (page 4/4)

## Last resort: single flavor pairing

- need $J = 1$ Cooper pairs

$$\phi \in [\bar{\mathbf{3}}]^a_c \otimes [\mathbf{3}]^s_J$$

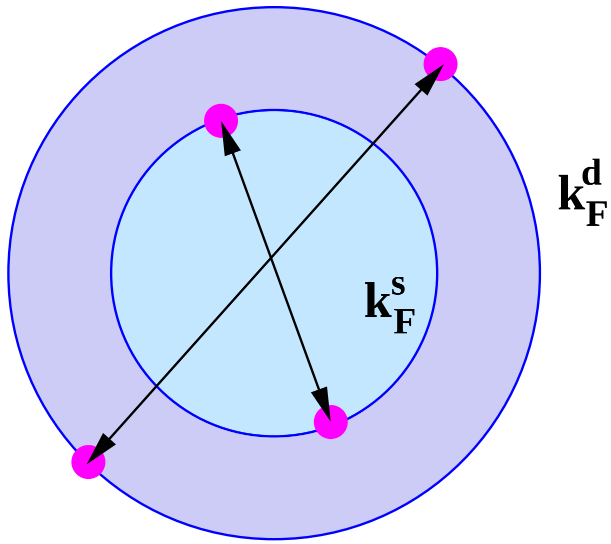
- different possible phases:

  Color-spin locking (CSL),

  $A$-phase, polar phase ...

  T. Schäfer, PRD 62, 094007 (2000)

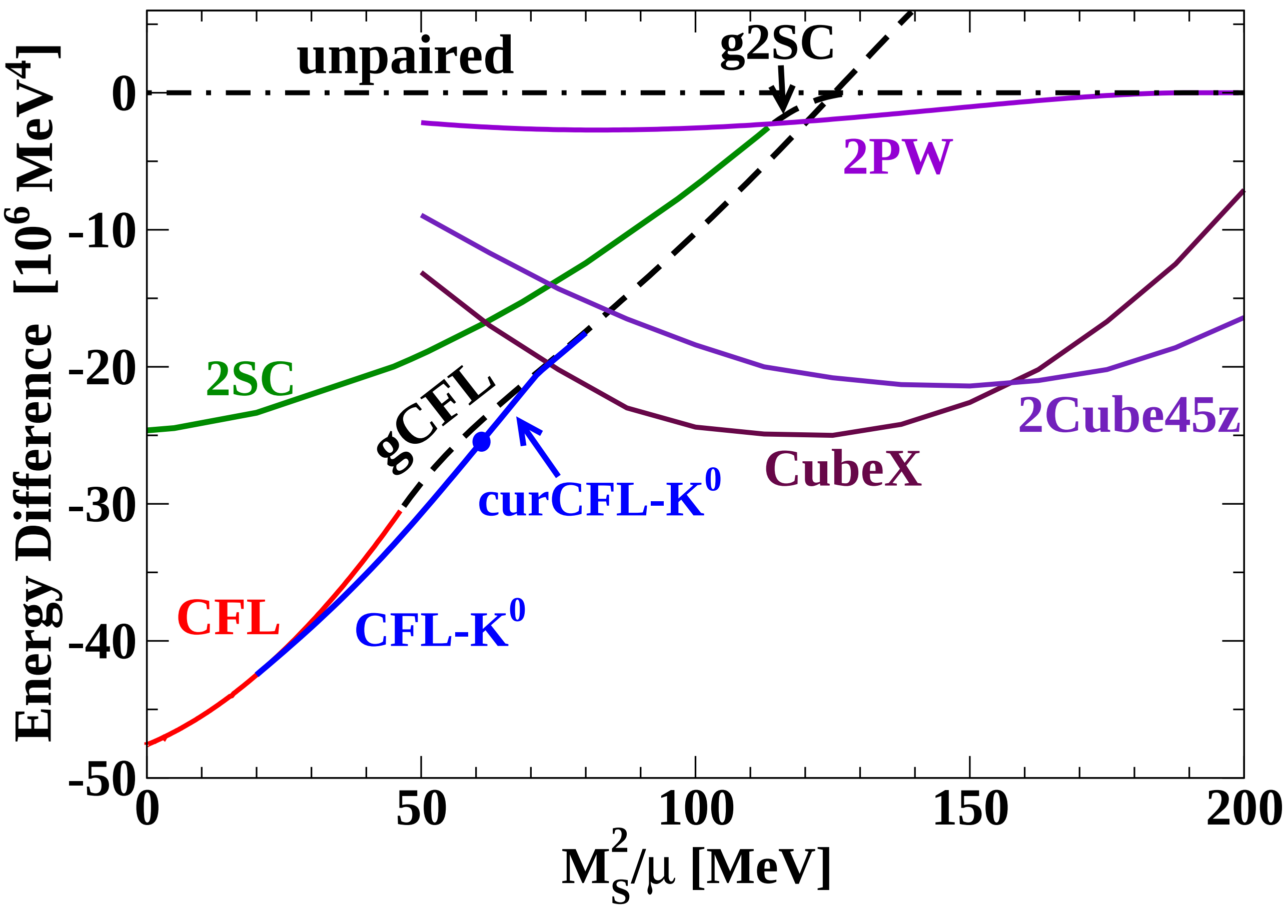
  A. Schmitt, PRD 71, 054016 (2005)




- preferred phase at high densities: CSL
- gap much smaller than in spin-0 phases:

$$\Delta_{J=1} \lesssim 10^{-2} \Delta_{J=0}$$

- **Stressed pairing: free energy comparison**




here: $\Delta_{\rm CFL} = 25$ MeV

(pert. QCD: $\Delta_{\rm CFL} \simeq 20$ MeV, NJL: $\Delta_{\rm CFL} \simeq (20 - 100)$ MeV).

# • Summary Parts I – III

- **3-flavor quark matter** at asymptotically high densities is in the **Color-Flavor locked (CFL)** state
- **CFL** quark matter
  - breaks chiral symmetry
  - is a superfluid
  - is a transparent insulator
- phase(s) between CFL and hadronic matter (if there is/are any) is/are unknown
  - **large coupling**
  - **stressed pairing** renders ground state complicated

- **Part IV**

- Basics of (color) superconductivity
- Color-flavor locking (CFL) – highest densities
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- **QCD calculations**
- Effective theory of CFL – kaon condensation
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- **QCD Lagrangian and Nambu-Gorkov basis**

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu D_\mu + \hat{\mu}\gamma_0 - \hat{m})\psi - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu}$$

- high densities: $m_u = m_d = m_s = 0$
- from weak interactions: only $\mu$, $\mu_e$ (not $\mu_u$, $\mu_d$, $\mu_s$)
- introduce charge-conjugate spinors

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}, \qquad \overline{\Psi} = (\overline{\psi}, \overline{\psi}_C)$$

- gap matrix $\mathbf{\Phi}^\pm \propto \Delta\, \epsilon^{\alpha\beta A}\, \epsilon_{ijB}\, \phi^A_B$ in NG propagator

$$S^{-1} = \begin{pmatrix} [G_0^+]^{-1} + \Sigma^+ & \mathbf{\Phi}^- \\ \mathbf{\Phi}^+ & [G_0^-]^{-1} + \Sigma^- \end{pmatrix}$$

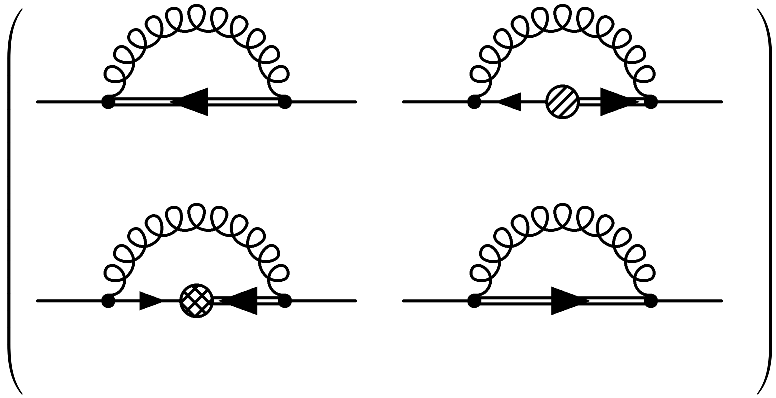
# • Gap equation

- $72 \times 72$ $(2 \cdot 4 N_f N_c)$ quark propagator
- $F^{\pm}$ "anomalous" propagators

$$S = \begin{pmatrix} G^+ & F^- \\ F^+ & G^- \end{pmatrix}$$

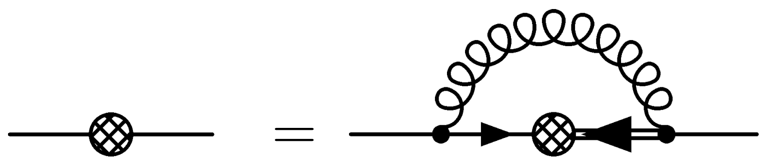
- one-loop self-energy

$$\Sigma \quad =$$



- **gap equation** (Dyson-Schwinger equation)

$$\boldsymbol{\Phi}^+(K) = g^2 \int_Q \gamma^\mu \, T_a^T \, F^+(Q) \, \gamma^\nu \, T_b \, D_{\mu\nu}^{ab}(K-Q)$$



# • Solution of the gap equation (page 1/3)

- gap parameter $\Delta$ at the Fermi surface at $T = 0$

$$\boldsymbol{\Delta} = \mathbf{512}\boldsymbol{\pi}^4 \left(\frac{\mathbf{2}}{\mathrm{N_f g^2}}\right)^{5/2} \boldsymbol{\mu}\ \mathbf{exp}\left(-\frac{\pi^2+4}{8}\right) \mathrm{e^{-d}e^{-\zeta}}\ \mathbf{exp}\left(-\frac{3\pi^2}{\sqrt{2}\mathrm{g}}\right) [\mathbf{1} + \mathcal{O}(\mathbf{g})]$$

**D.T. Son, PRD 59, 094019 (1999)**

**T. Schäfer, F. Wilczek, PRD 60, 114033 (1999); R. D. Pisarski, D. H. Rischke, PRD 61, 051501 (2000); D.K. Hong, V.A. Miransky, I.A. Shovkovy, L.C.R. Wijewardhana, PRD 61, 056001 (2000)**

**W. E. Brown, J. T. Liu and H. c. Ren, PRD 61, 114012 (2000); Q. Wang, D. H. Rischke, PRD 65, 054005 (2002)**

**T. Schäfer, PRD 62, 094007 (2000); A. Schmitt, Q. Wang, D. H. Rischke, PRD 66, 114010 (2002); A. Schmitt, PRD 71, 054016 (2005)**

- let's discuss the contributions separately ...

# • Solution of the gap equation (page 2/3)

$$\boldsymbol{\Delta = 512\pi^4 \left(\frac{2}{N_f g^2}\right)^{5/2} \mu \; \exp\left(-\frac{\pi^2+4}{8}\right) e^{-d} e^{-\zeta} \; \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) [1+\mathcal{O}(g)]}$$

• leading $1/g$ term from **long-range magnetic gluons**

$$\ln\left(\frac{\Delta}{\mu}\right) = -\frac{b_{-1}}{g} - \bar{b}_0 \ln g - b_0 - \ldots$$

• unlike BCS (point-like interaction), $\Delta \propto \exp(-1/g^2)$

$\Rightarrow$ color-superconducting gap *increases* at $\mu \to \infty$:

$$\frac{1}{g^2} \propto \ln\frac{\mu}{\Lambda} \quad \Rightarrow \quad \Delta \propto \mu \, e^{-const./g(\mu)} \text{ increases with } \mu$$

(of course $\Delta/\mu$ decreases)

# • Solution of the gap equation (page 2/3)

$$\boldsymbol{\Delta} = \mathbf{512}\boldsymbol{\pi}^4 \left(\frac{\mathbf{2}}{\mathbf{N_f g^2}}\right)^{\mathbf{5/2}} \boldsymbol{\mu}\ \mathbf{exp}\left(-\frac{\pi^2+4}{8}\right) \mathbf{e^{-d}} \mathbf{e}^{-\zeta}\ \mathbf{exp}\left(-\frac{3\pi^2}{\sqrt{2}\mathbf{g}}\right) [\mathbf{1} + \mathcal{O}(\mathbf{g})]$$

- $\exp[-(\pi^2+4)/8] \simeq 0.2$ from "normal" quark self energy $\Sigma^{\pm}$

$$S^{-1} = \begin{pmatrix} [G_0^+]^{-1} + \Sigma^+ & \boldsymbol{\Phi}^- \\ \boldsymbol{\Phi}^+ & [G_0^-]^{-1} + \Sigma^- \end{pmatrix} \qquad \Sigma^{\pm} \simeq \gamma_0 \Lambda_{\mathbf{k}}^{(\pm)} \frac{g^2}{18\pi^2} k_0 \log \frac{48 e^2 m_g^2}{\pi^2 k_0^2}$$

- smallness of spin-1 gaps

$$e^{-d} = \begin{cases} 1 & J = 0 \\ e^{-6} \simeq 2.5 \cdot 10^{-3} & J = 1;\ LL, RR \text{ pairing} \\ e^{-4.5} \simeq 0.01 & J = 1;\ LR, RL \text{ pairing} \end{cases}$$

- **Solution of the gap equation (page 3/3)**

$$\boldsymbol{\Delta = 512\pi^4 \left(\frac{2}{N_f g^2}\right)^{5/2} \mu \exp\left(-\frac{\pi^2+4}{8}\right) e^{-d} e^{-\zeta} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) [1+\mathcal{O}(g)]}$$

- $e^{-\zeta}$ depends on the **pairing pattern** (not on details of interaction)

- Define
$\mathcal{M}_{ij}^{\alpha\beta} \equiv \epsilon^{\alpha\beta A} \epsilon_{ijB} \phi_B^A$

- Denote
$\lambda_r$ ($n_r$-fold) eigenvalues of $\mathcal{M}\mathcal{M}^\dagger$

- quasiparticle dispersions

$$\epsilon_{k,r} = \sqrt{(k-\mu)^2 + \lambda_r \Delta^2}$$

- condensation energy

$$\delta P = \frac{\mu^2}{4\pi^2} \sum_r n_r \lambda_r \Delta^2$$

- ## Solution of the gap equation (page 4/4)

- then

$$\zeta = \frac{1}{2}\frac{\langle n_1\lambda_1 \log\lambda_1 + n_2\lambda_2 \log\lambda_2\rangle}{\langle n_1\lambda_1 + n_2\lambda_2\rangle}$$

- $e^{-\zeta} \neq 1$ for **(i) two-gap structure...**

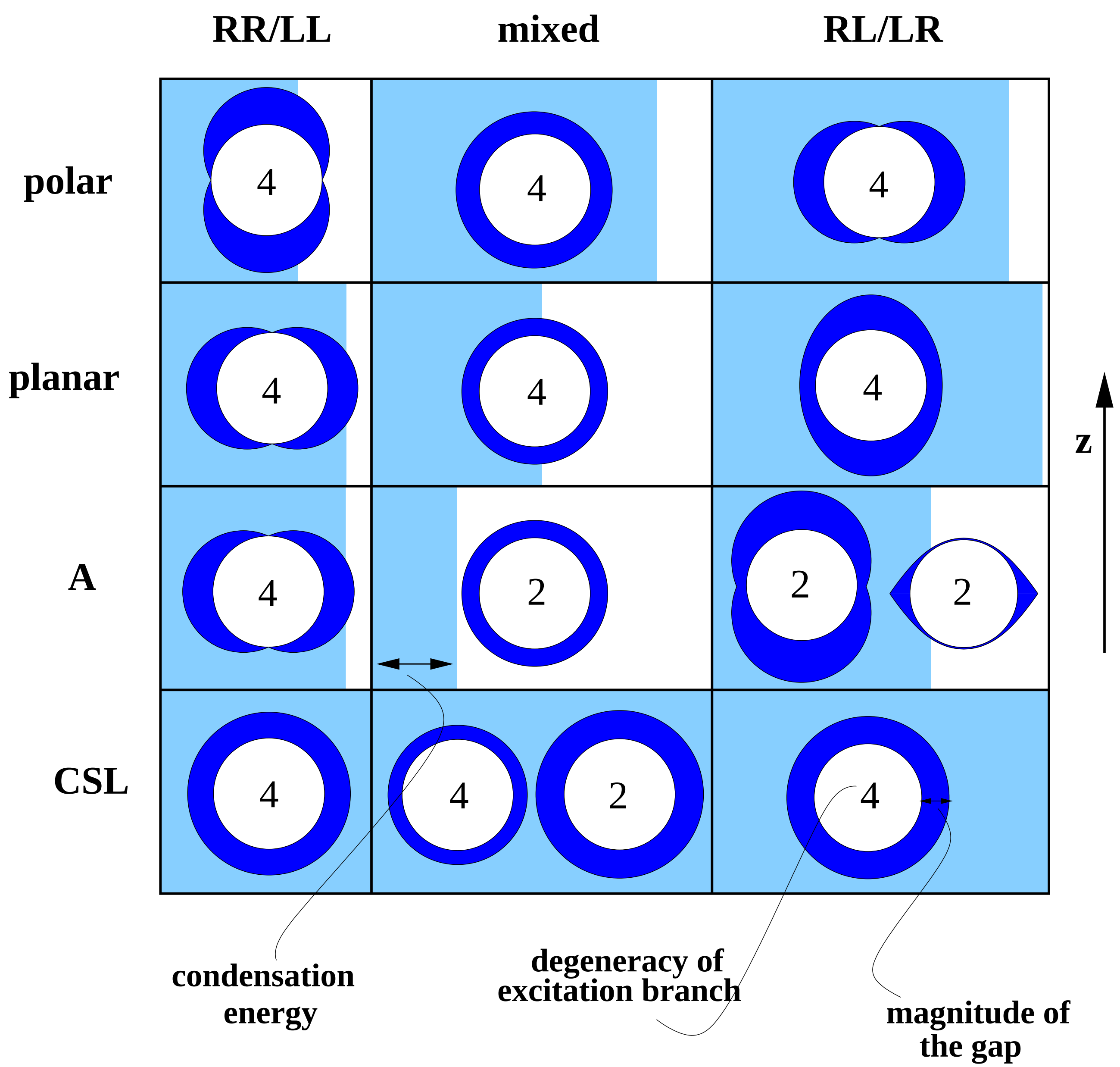
| | CFL | 2SC |
|---|---|---|
| $\lambda_1\ \ (n_1)$ | $4\ \ (1)$ | $1\ \ (4)$ |
| $\lambda_2\ \ (n_2)$ | $1\ \ (8)$ | $0\ \ (5)$ |
| $e^{-\zeta}$ | $2^{-1/3}$ | $1$ |

- consequently,

$$\Delta_{\rm CFL} = 2^{-1/3}\Delta_{\rm 2SC}$$

- **...and/or (ii) anisotropic gaps**

- **Anisotropic gaps** A. Schmitt, PRD 71, 054016 (2005)



- **Critical temperature**

- set $\Delta = 0$ in gap equation to extract $T_c$

$$T_c = e^{\zeta} \frac{e^{\gamma}}{\pi} \Delta \left[1 + \mathcal{O}(g)\right] \simeq 0.57\, e^{\zeta}\, \Delta$$

- $e^{\zeta}$ violates BCS relation $T_c \simeq 0.57\, \Delta$

A. Schmitt, Q. Wang, D. H. Rischke, PRD 66, 114010 (2002)

- include gauge field fluctuations:

second order $\longrightarrow$ first order, and

$$\frac{T_c^* - T_c}{T_c} = \frac{\pi^2}{12\sqrt{2}}\, g$$

I. Giannakis, D. f. Hou, H. c. Ren, D. H. Rischke, PRL 93, 232301 (2004)

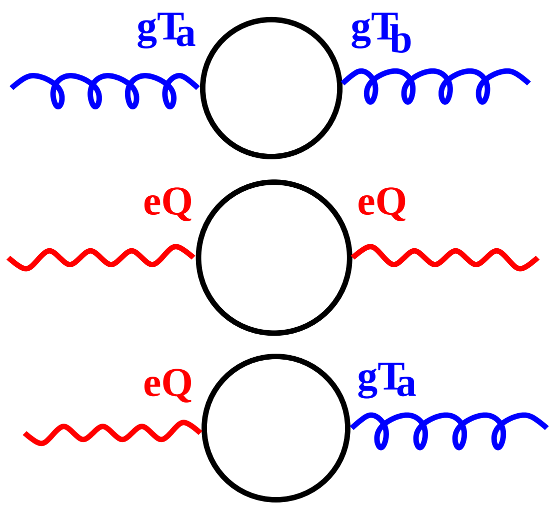
# • Meissner masses from polarization tensors

Meissner masses: $$m^2_{ab} \equiv \frac{1}{2} \lim_{p \to 0} (\delta^{ij} - \hat{p}^i \hat{p}^j)\, \Pi^{ab}_{ij}(p_0 = 0, \mathbf{p})$$

**gluon** $\Pi^{ab}_{\mu\nu}(P)$

**photon** $\Pi^{\gamma\gamma}_{\mu\nu}(P)$

**photon/gluon** $\Pi^{\gamma a}_{\mu\nu}(P)$




Elimination of unphysical, "mixed" masses . . .

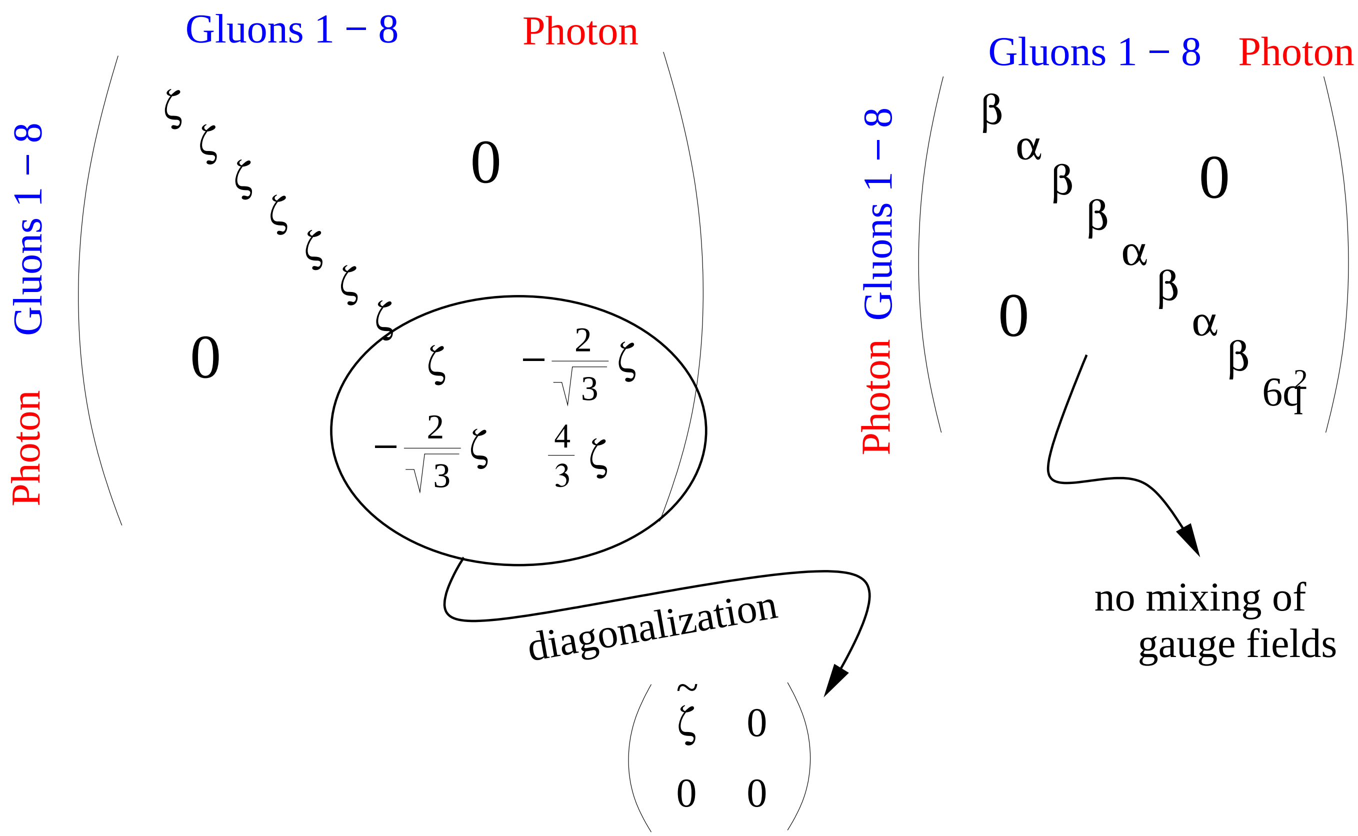
$$m^2_{ab} \;\to\; \tilde{m}^2_{ab} \equiv \mathcal{O}_{ac}\, m^2_{cd}\, \mathcal{O}^T_{db}$$

. . . goes along with rotation of gauge fields

$$A^\mu_a \;\to\; \tilde{A}^\mu_a \equiv \mathcal{O}_{ab}\, A^\mu_b$$

# • Results: CFL and CSL phases

**CFL phase (J=0)** **CSL phase (J=1)**




**Units:** $c^2 N_f \mu^2/(6\pi^2)$, $(c^2 = g^2, eg, e^2)$.

**Abbreviations:** $\zeta \equiv (21-8\ln 2)/54$, $\alpha \equiv (3+4\ln 2)/27$, $\beta \equiv (6-4\ln 2)/9$, $\tilde{\zeta} \equiv (\frac{4}{3}e^2+g^2)\zeta$.

**CFL phase:** $\tilde{m}_\gamma = 0$ $\longrightarrow$ *no* electromagnetic Meissner effect

**CSL phase:** $m_\gamma = \frac{q\mu e}{\pi}$ $\longrightarrow$ electromagnetic Meissner effect

- **Part V**

- Basics of (color) superconductivity
- Color-flavor locking (CFL) – highest densities
- Stressed pairing – below CFL densities
- QCD calculations
- **Effective theory of CFL – kaon condensation**
- Transport properties – quark matter in compact stars

- **Low-energy degrees of freedom in CFL**

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \rightarrow \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}}$$

$\rightarrow$ pseudo-Goldstone octet $(K^0, \bar{K}^0, K^\pm, \pi^0, \pi^\pm, \eta)$
$+$ "superfluid mode" $\varphi$

- **effective theory for mesons and superfluid mode**

P. F. Bedaque, T. Schäfer, NPA 697, 802 (2002)

D. T. Son, M. A. Stephanov, PRD 61, 074012 (2000)

$$\mathcal{L}_{\rm eff}^{\Sigma} = \frac{f_\pi^2}{4}{\rm Tr}\left[(\partial_0\Sigma + i[A_0,\Sigma])(\partial_0\Sigma^\dagger - i[A_0,\Sigma]^\dagger) - v_\pi^2\partial_i\Sigma\partial_i\Sigma^\dagger\right] + \frac{af_\pi^2}{2}{\rm det}\hat{M}\,{\rm Tr}[\hat{M}^{-1}(\Sigma+\Sigma^\dagger)] + \ldots$$

$$\mathcal{L}_{\rm eff}^{\phi} = \frac{1}{2}(\partial_0\phi)^2 - \frac{v_H^2}{2}(\nabla\phi)^2 - \frac{\pi}{9\mu^2}\partial_0\phi\partial_\mu\phi\partial^\mu\phi + \frac{\pi^2}{108\mu^4}(\partial_\mu\phi\partial^\mu\phi)^2 + \ldots$$

$+$ weak interactions between $\Sigma$ and $\phi$

# • Low-energy degrees of freedom in CFL

- $\mathcal{L}_{\rm eff}^{\Sigma} + \mathcal{L}_{\rm eff}^{\phi}$ valid for $T < \Delta \simeq 25\,{\rm MeV}$
- form of $\mathcal{L}_{\rm eff}^{\Sigma} + \mathcal{L}_{\rm eff}^{\phi}$ determined entirely by CFL symmetries

$$\mathcal{L}_{\rm eff}^{\Sigma} = \frac{\mathbf{f}_\pi^2}{4}{\rm Tr}\left[(\partial_0\Sigma + i[A_0,\Sigma])(\partial_0\Sigma^\dagger - i[A_0,\Sigma]^\dagger) - v_\pi^2\partial_i\Sigma\partial_i\Sigma^\dagger\right] + \frac{\mathbf{a}\mathbf{f}_\pi^2}{2}{\rm det}\hat{M}\,{\rm Tr}[\hat{M}^{-1}(\Sigma+\Sigma^\dagger)] + \ldots$$

$$\Sigma = e^{i\theta/f_\pi} = \phi_L\phi_R^\dagger\,, \qquad \theta \in SU(3)$$

- parameters (reliably given only at high densities)

$$\mathbf{a} \equiv \frac{3\Delta^2}{\pi^2\mathbf{f}_\pi^2}\,, \qquad \mathbf{f}_\pi = \frac{21-8\ln 2}{18}\frac{\mu^2}{2\pi^2}$$

$\Rightarrow$ lower densities: quantitative uncertainties, but theory applicable!

$\longrightarrow$ e.g., compute transport properties from effective theory

# • Low-energy degrees of freedom in CFL

$$\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4}{\rm Tr}\left[(\partial_0\Sigma + i[\mathbf{A_0},\Sigma])(\partial_0\Sigma^\dagger - i[\mathbf{A_0},\Sigma]^\dagger) - v_\pi^2\partial_i\Sigma\partial_i\Sigma^\dagger\right] + \frac{af_\pi^2}{2}{\rm det}\hat{M}\,{\rm Tr}[\hat{M}^{-1}(\Sigma+\Sigma^\dagger)] + \ldots$$

$$\Sigma = e^{i\theta/f_\pi} = \phi_L\phi_R^\dagger\,, \qquad \theta \in SU(3)$$

- effective chemical potential $\mathbf{A_0} = \mu_Q Q - \hat{M}^2/2\mu$ and effective masses, e.g.,

$$E_{\pi^\pm} = \mu_{\pi^\pm} + \left[v_\pi^2 p^2 + a\,(m_u+m_d)m_s\right]^{1/2} \qquad \mu_{\pi^\pm} = \mp\frac{m_d^2 - m_u^2}{2\mu}$$

$$E_{K^\pm} = \mu_{K^\pm} + \left[v_\pi^2 p^2 + a\,(m_u+m_s)m_d\right]^{1/2} \qquad \mu_{K^\pm} = \mp\frac{m_s^2 - m_u^2}{2\mu}$$

$$E_{K^0,\bar{K}^0} = \mu_{K^0,\bar{K}^0} + \left[v_\pi^2 p^2 + a\,(m_d+m_s)m_u\right]^{1/2} \qquad \mu_{K^0,\bar{K}^0} = \mp\frac{m_s^2 - m_d^2}{2\mu}$$

- **inverse mass ordering** $m_K < m_\pi < \ldots$

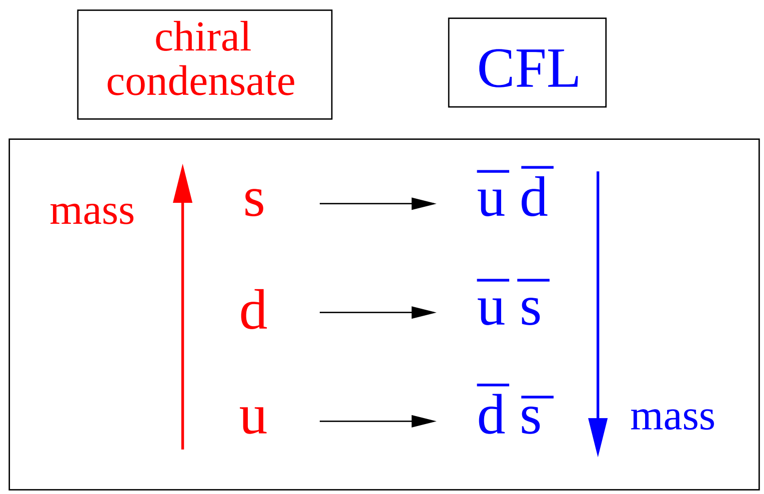
# • Inverse mass ordering

- Remember: $\phi_L$, $\phi_R$ are antitriplet diquarks

**usual pion** | **CFL pion:** $\bar{d} \to us$, $u \to \bar{d}\bar{s}$

$$\pi^+ \sim \bar{d}u \qquad \pi^+ \sim us\bar{d}\bar{s}$$

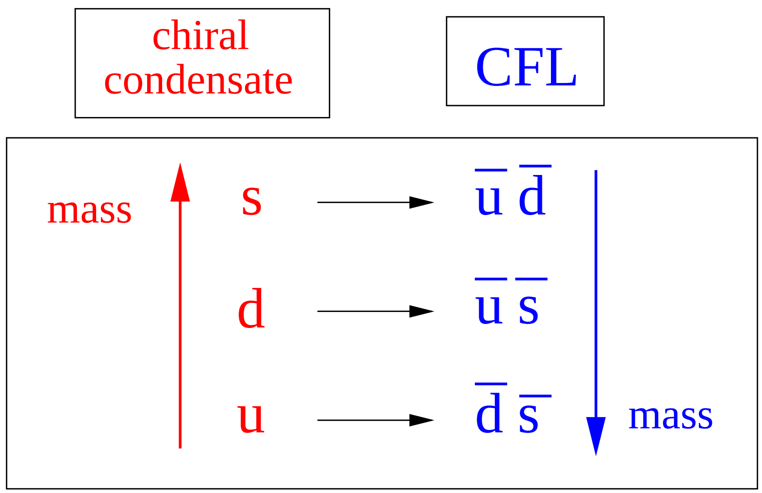
$$m_{\pi^+} \propto m_u + m_d \qquad m_{\pi^+}^2 \propto m_u m_s + m_d m_s$$



# • Inverse mass ordering

- Remember: $\phi_L$, $\phi_R$ are antitriplet diquarks

| **usual pion** | **CFL pion:** $\bar{d} \rightarrow us$, $u \rightarrow \bar{d}\bar{s}$ |
|---|---|
| $\pi^+ \sim \bar{d}u$ | $\pi^+ \sim us\bar{d}\bar{s}$ |
| $m_{\pi^+} \propto m_u + m_d$ | $m^2_{\pi^+} \propto m_u m_s + m_d m_s$ |

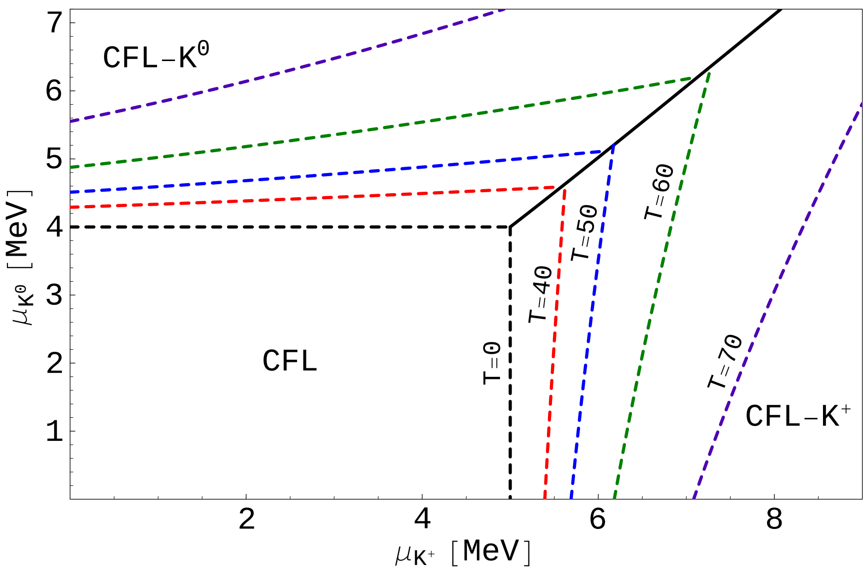



# • Kaon condensation (page 1/2)

- kaon condensation at $T = 0$ for $\mu_{K^0} > m_{K^0}$

- $T \neq 0$: 2PI formalism

  M. G. Alford, M. Braby, A. Schmitt, J. Phys. G 35, 025002 (2008)




$\Rightarrow$ condensate (if present at $T = 0$) very robust, $T_c \sim T_c^{\rm CFL}$

- Goldstone mode in CFL-$K^0$

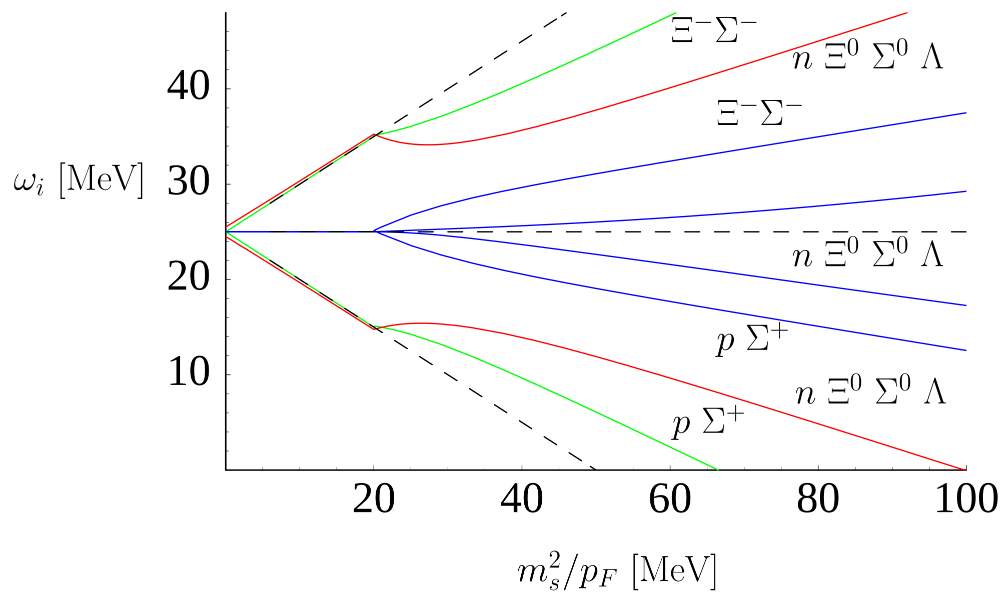
$$\epsilon_k = \begin{cases} \left\{v_\pi^2 k^2 + M^2(T) + \mu_{K^0}^2 - \sqrt{4\mu_{K^0}^2 v_\pi^2 k^2 + [M^2(T) + \mu_{K^0}^2]^2}\right\}^{1/2} & \text{for } T < T_c \\ \sqrt{v_\pi^2 k^2 + M^2(T)} - \mu_{K^0} & \text{for } T > T_c \end{cases}$$

# • Kaon condensation (page 2/2)

- **kaon condensate** shifts the onset of gapless modes (remember chromomagnetic instability)
- include fermions into effective theory as baryons $N \sim \psi\langle\psi\psi\rangle$

A. Kryjevski, T. Schäfer, PLB 606, 52 (2005)

- fermion spectrum in CFL-$K^0$
- here: $\Delta = 25\,\text{MeV}$




Onset of gapless modes: $\dfrac{M_s^2}{2p_F} = \Delta$ (CFL); $\dfrac{M_s^2}{2p_F} = \dfrac{4\Delta}{3}$ (CFL-$K^0$)

- **Part VI**

- Basics of (color) superconductivity
- Color-flavor locking (CFL) – highest densities
- Stressed pairing – below CFL densities
- QCD calculations
- Effective theory of CFL – kaon condensation
- **Transport properties – quark matter in compact stars**

# • Color superconductivity in compact stars (I)

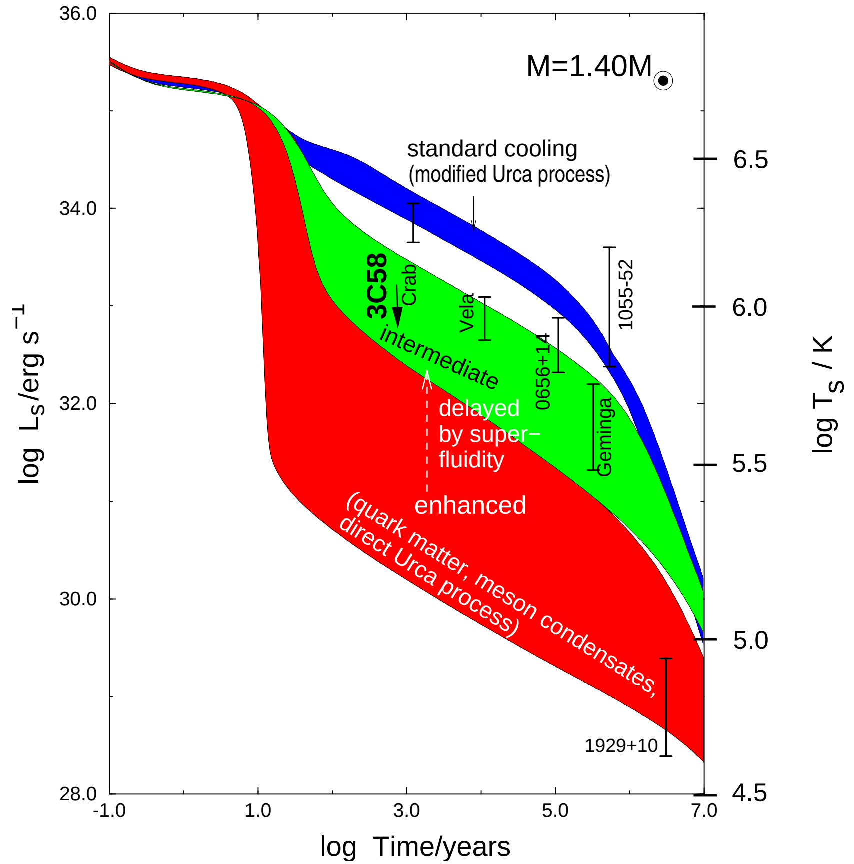
## Cooling of the star

- **neutrino emissivity**

  $T \sim 10\,\text{MeV} \to 0.1\,\text{keV}$

  $(t \sim 0 \to 10^6\,\text{yr})$

F. Weber, Prog. Part. Nucl. Phys. 54, 193 (2005)




**unpaired quark matter** Iwamoto, PRL 44, 1637 (1980) **CFL** Jaikumar, Prakash, Schäfer, PRD 66, 063003 (2002) **2SC** Jaikumar, Roberts, Sedrakian, PRC 73, 042801 (2006) **spin-1** Schmitt, Shovkovy, Wang, PRD 73, 034012 (2006) **LOFF** Anglani, Nardulli, Ruggieri, Mannarelli, PRD 74, 074005 (2006)

- **Neutrino emissivity**

**CFL**

- (pseudo-)**Goldstone processes**

$$\pi^{\pm}, K^{\pm} \rightarrow e^{\pm} + \bar{\nu}_e$$
$$\pi^0 \rightarrow \nu_e + \bar{\nu}_e$$
$$\varphi + \varphi \rightarrow \varphi + \nu_e + \bar{\nu}_e$$

$\Rightarrow$ very small emissivity

$$\epsilon_\nu \sim \frac{G_F^2 T^{15}}{f^2 \mu^4}$$

**Non-CFL**

- **direct Urca process**

$$u + e \rightarrow d + \nu_e$$
$$d \rightarrow u + e + \bar{\nu}_e$$

$\Rightarrow$

$$\epsilon_\nu \simeq \frac{457}{630} \alpha_s G_F^2 T^6 \mu_e \mu_u \mu_d$$

(unpaired quark matter)

- non-CFL (2SC, LOFF, spin-1, ...): $\rightarrow$ **ungapped quarks**
- possible exception (?): CSL

# • Color superconductivity in compact stars (II)

## magnetic fields

- spin-0 **no Meissner effect**

  M.G. Alford, J. Berges, K. Rajagopal,
  NPB 571, 269 (2000)

- spin-1 **Meissner effect**

  A. Schmitt, Q. Wang, D.H. Rischke,
  PRL 91, 242301 (2003)

- "magnetic" CFL

  E.J. Ferrer, V. de la Incera, C. Manuel,
  PRL 95, 152002 (2005)

- de Haas-van Alphen oscillations

  K. Fukushima and H. J. Warringa,
  PRL 100, 032007 (2008)
  J. L. Noronha and I. A. Shovkovy,
  PRD 76, 105030 (2007)

- ferromagnetism in the **curCFL-$K^0$** phase

  D. T. Son and M. A. Stephanov, PRD 77, 014021 (2008)

# • Color superconductivity in compact stars (III)

## glitches

- **need vortex pinning at some crystal**
- maybe: neutron superfluidity + ion lattice in crust
- however: inconsistent with precession of star on $\sim$ yr time scale

  B. Link, arXiv:astro-ph/0608319
- better (?): crystalline color superconductivity
- **large shear modulus of CubeX** (and 2Cube45z)

  M. Mannarelli, K. Rajagopal, R. Sharma, PRD 76, 074026 (2007)

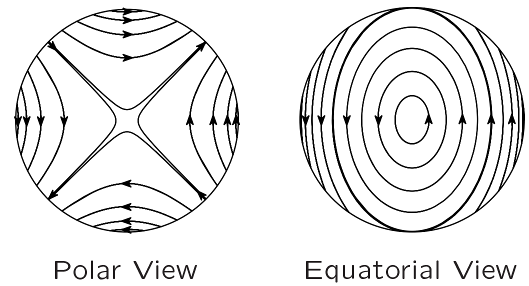
$$\nu = 3.96 \times 10^{33} \mathrm{erg/cm^3} \left(\frac{\Delta}{10\ \mathrm{MeV}}\right)^2 \left(\frac{\mu}{400\ \mathrm{MeV}}\right)^2$$

- 20 – 100 times more rigid than neutron star crust

- **Color superconductivity in compact stars (IV)**

## r-mode instability

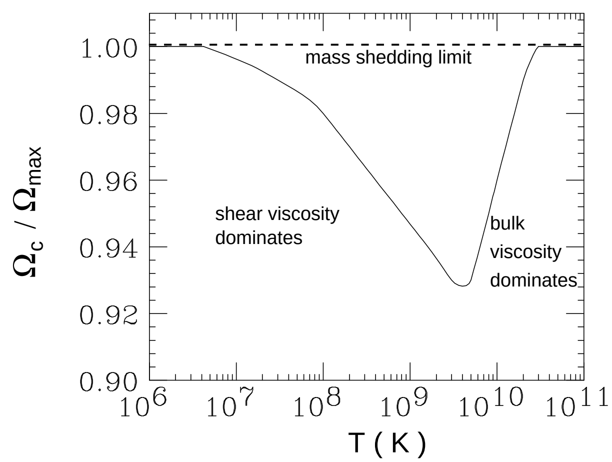
- **r-modes:**
  non-radial pulsation modes
- **grow unstable**
  in a perfect-fluid rotating star
  $\longrightarrow$ emission of gravitational waves




L. Lindblom, arXiv:astro-ph/0101136

- **spin down**
  the star drastically and quickly (within days)

- fast rotating stars are observed!
  $\omega \simeq 1\text{ms}^{-1}$
- must be some damping mechanism $\longrightarrow$ bulk/shear viscosity



# • What is bulk viscosity?

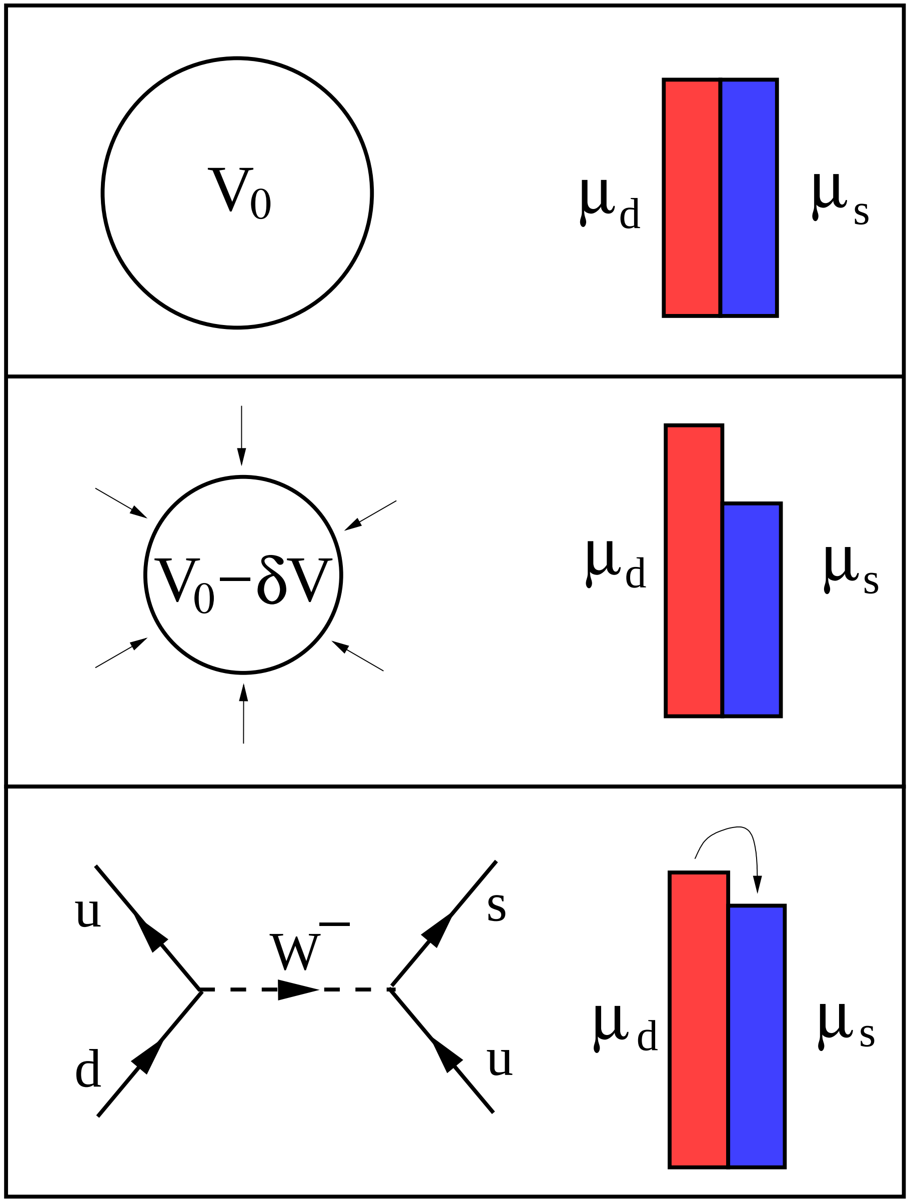
- volume oscillation
  $\longrightarrow$ chemical
  non-equilibrium

$$\mu_d - \mu_s \neq 0$$

- re-equilibration via

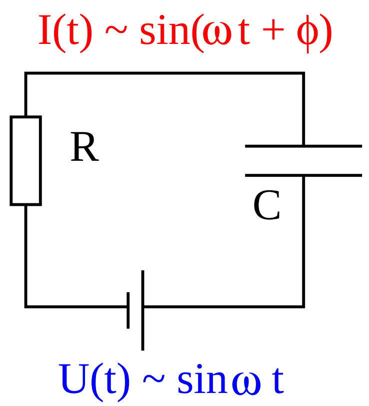
$$\boxed{u + d \leftrightarrow u + s}$$

- **resonance phenomenon**:
  external oscillation
  vs. microscopic rate

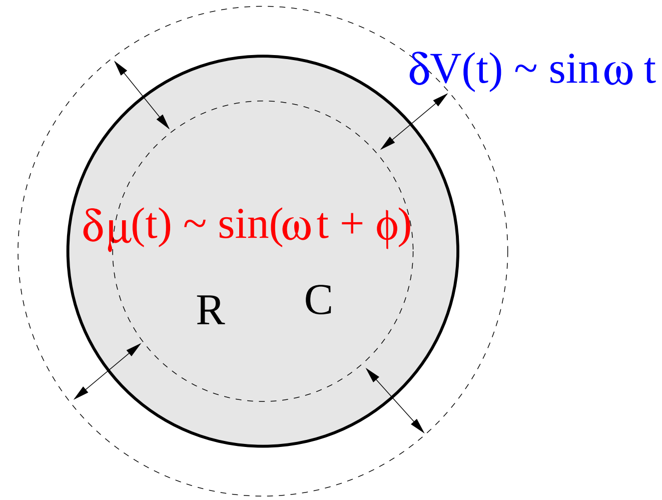


# • Bulk viscosity is a resonance phenomenon

**Just like an electric circuit!**




dissipation ~ < I(t) U(t) >




dissipation ~ < δV(t) δμ (t) >

**"capacitance"** $C \leftrightarrow$ inverse microscopic rate $\gamma^{-1}$
(slow process $\longrightarrow$ store large chemical energy)

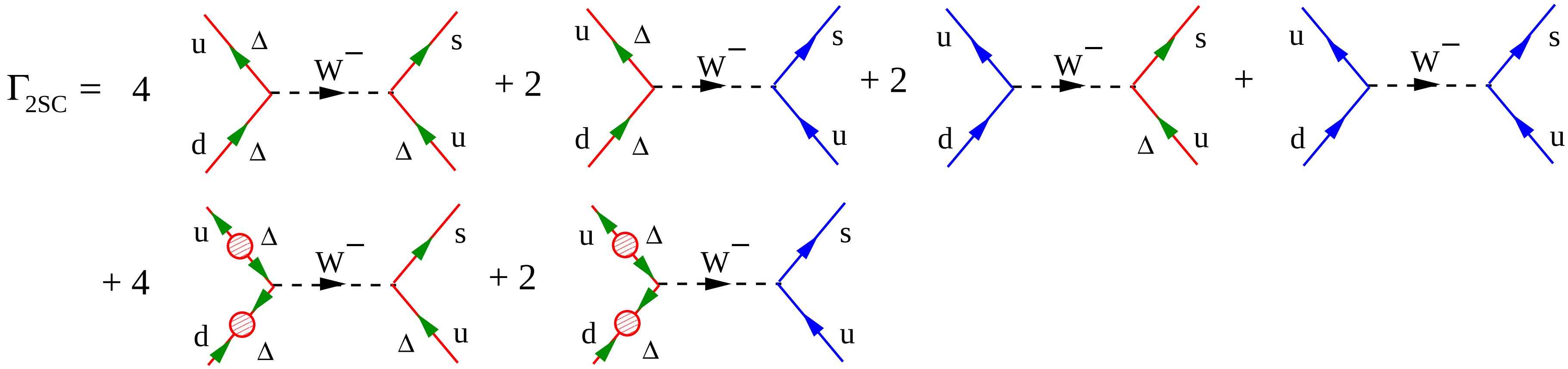
**"resistance"** $R \leftrightarrow \left(n_u \frac{\partial \mu_d}{\partial n_u} + n_d \frac{\partial \mu_d}{\partial n_d} - n_s \frac{\partial \mu_s}{\partial n_s}\right)^{-1}$
(same dispersion for $d$ and $s$ $\longrightarrow$ infinite "resistance" $\longrightarrow$ no dissipation)

**Bulk viscosity**

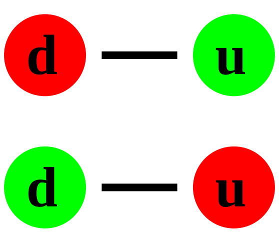
$$\zeta = \alpha \, \frac{\gamma}{\gamma^2 + \omega^2}$$

$$\alpha \equiv \frac{n_u \frac{\partial \mu_d}{\partial n_u} + n_d \frac{\partial \mu_d}{\partial n_d} - n_s \frac{\partial \mu_s}{\partial n_s}}{\frac{\partial \mu_d}{\partial n_d} + \frac{\partial \mu_s}{\partial n_s}}$$

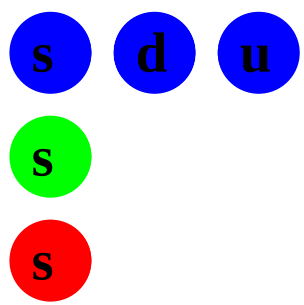
- **Compute rate for $u + d \leftrightarrow u + s$ in 2SC**




paired:




unpaired:




**small temperatures**,

$T \ll T_c \simeq 30\text{MeV}$

$$\Gamma_{2SC} = \frac{1}{9}\Gamma_{\text{unpaired}}$$

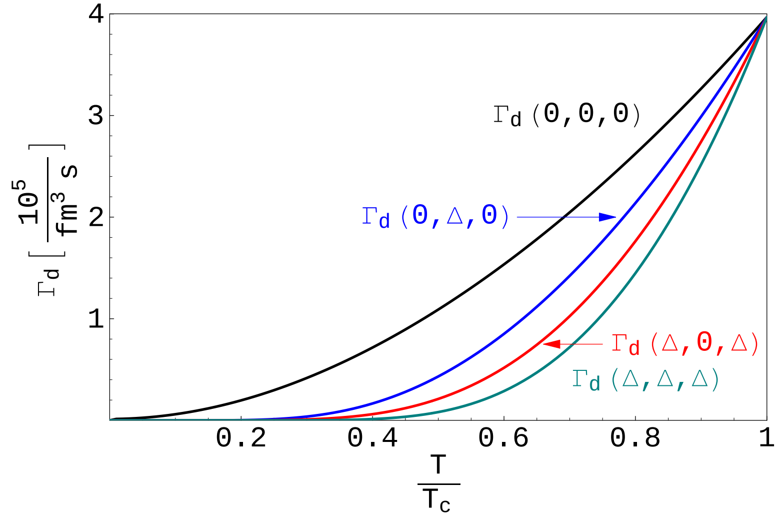
due to **exponential suppression**

$\exp(-\Delta/T)$ of gapped modes

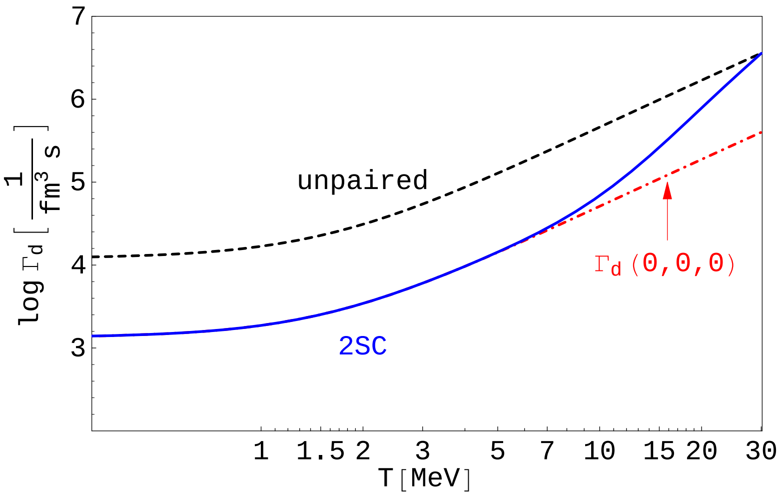
# • Results for all temperatures $T < T_c$

(i) fixed $\delta\mu = \mu_s - \mu_d > 0 \rightarrow$ net production of $d$ quarks, $\Gamma_d > 0$

- **contributions of subprocesses**



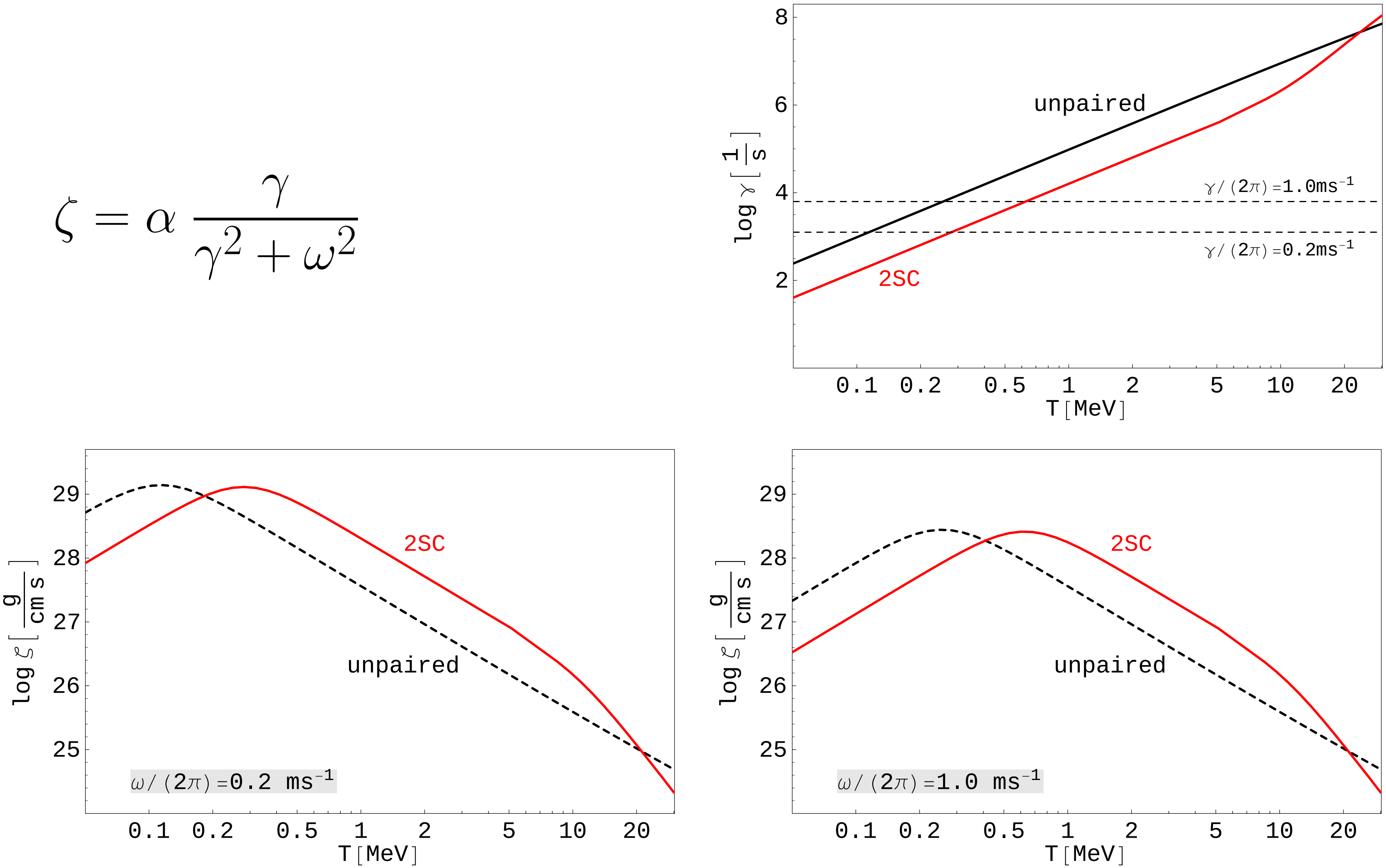

- **total rate compared to unpaired quark matter**



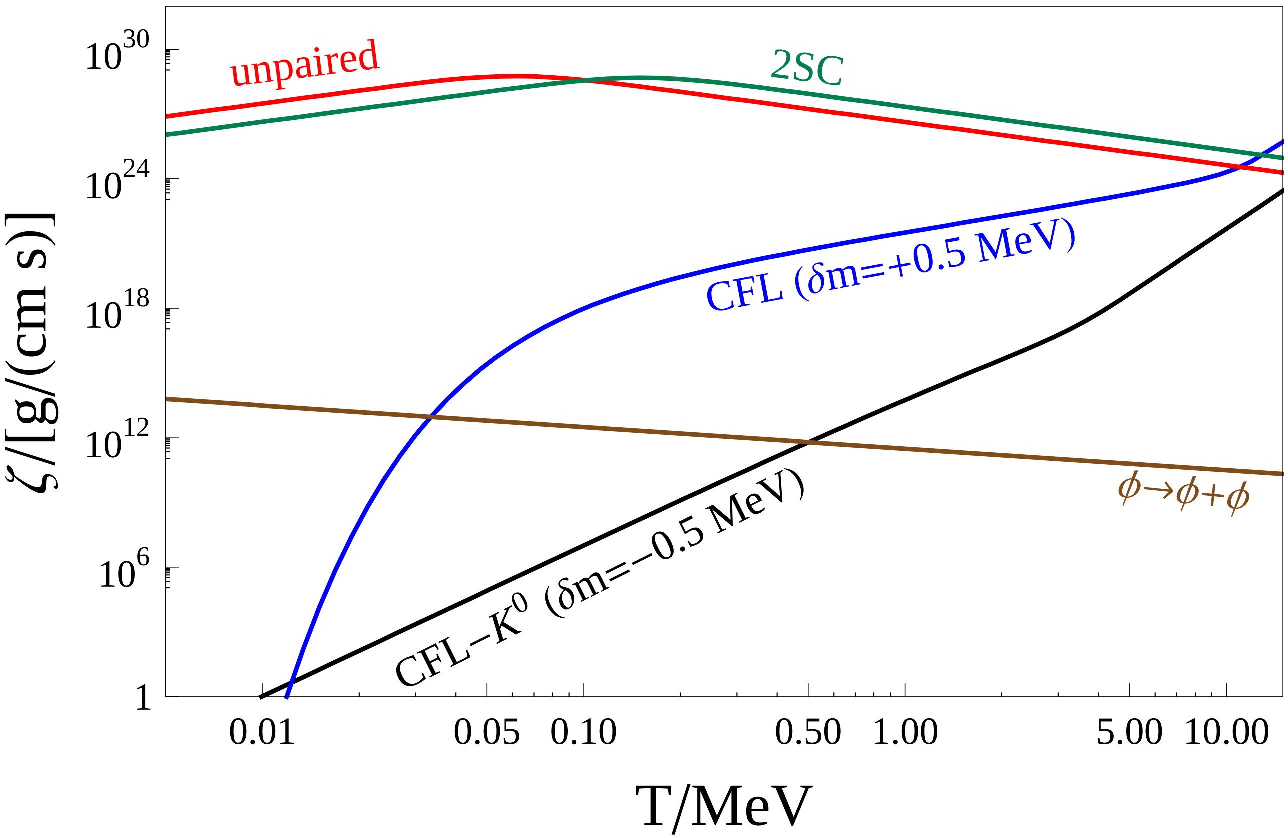

(ii) for bulk viscosity, consider $\boldsymbol{\gamma} = \dfrac{\partial \Gamma_d}{\partial \delta\mu}$

- **Results for bulk viscosity**

$$\zeta = \alpha \, \frac{\gamma}{\gamma^2 + \omega^2}$$



# • Quark matter bulk viscosity: different phases




$$\omega/(2\pi) = 1\,\mathrm{ms}^{-1}$$

$$\mu = 400\,\mathrm{MeV}$$

$$\delta m \equiv m_{K^0} - \mu_{K^0}$$

**unpaired** J. Madsen, PRD 46, 3290 (1992)

**2SC** M.G. Alford, A. Schmitt, JPG 34, 67-101 (2007)

**CFL from** $K^0 \leftrightarrow \phi + \phi$ M.G. Alford, M. Braby, S. Reddy, T. Schafer, PRC 75, 055209 (2007)

**CFL-**$K^0$ **from** $K^0 \leftrightarrow \phi + \phi$ M.G. Alford, M. Braby, A. Schmitt, JPG 35, 115007 (2008)

**CFL from** $\phi \leftrightarrow \phi + \phi$ C. Manuel, F. Llanes-Estrada, JCAP 0708, 001 (2007)

**See also: Spin-1** B.A. Sa'd, I.A. Shovkovy, D.H. Rischke, PRD 75, 065016 (2007)

**Vortices in CFL** M. Mannarelli, C. Manuel, B. A. Sa'd, PRL 101, 241101 (2008)

- **Summary Parts IV – VI**

- **QCD** gap equation gives **parametrically different** result compared to BCS, $\Delta \propto \exp(-\text{const}/g)$
- **effective theory** of CFL provides powerful tool to study **moderate densities** (all gluons & quarks are gapped)
- **transport properties** dominated by **Goldstone modes** in CFL ($\rightarrow$ small $\nu$-emissivity, specific heat, bulk viscosity ...) and **ungapped quarks** in non-CFL ($\rightarrow$ large emissivity ...)