

COLOR-SYMMETRY BREAKING AND THE BARYON SPECTRUM*

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ABSTRACT

In the colored-quark model, if the three vector gluons that correspond to an $SU(2)$ subgroup of $SU(3)$ are heavier than the other gluons, a quark-diquark structure for baryons results. Furthermore, the predicted baryon $SU(6)$ representations are the 56 for even parity and the 70 for odd parity, in agreement with recent experimental indications.

Recent analyses of the baryon spectrum suggest that the even- and odd-parity baryons correspond exclusively to the $SU(6)$ representations 56 and 70, respectively.¹ This contradicts the harmonic-oscillator quark model for all but the lowest two levels; for example, the model predicts even-parity resonances corresponding to the 56, 70 and 20 at the second excited level.²

Several years ago Lichtenberg, and later Ono, proposed that a baryon is a composite of a quark and a diquark.^{3,4} The diquarks are assumed to correspond to the symmetric $SU(6)$ representation 21, so that the unobserved 20-fold baryon representation is forbidden. There are three serious difficulties with

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this model. First, if there are just three fundamental quarks that do not satisfy Fermi statistics, it is hard to imagine a simple force that will bind two closely, and leave the third at larger distances. Second, quark-quark statistics are neglected, a proper procedure only if the diquark is pointlike. Third, 56 and 70 representations are predicted at every energy level. Lichtenberg showed that this last difficulty may be overcome by the introduction of a quark-exchange force, but this force is clearly of a different nature from that which binds the diquark.³

In this paper it is assumed that the quarks have color SU(3) indices as well as regular SU(3) and spin indices, and that the quark binding forces are transmitted by the exchange of an octet of vector gluons coupled to the color indices.⁵ It is shown that a natural form of color-symmetry breaking leads simultaneously to the quark-diquark baryonic structure and to the correspondence of the 56 and 70 representations to even and odd parities. Harmonic oscillator wave functions are used in the calculations.

The colors are labeled A, B, and C, and the interactions are assumed invariant to the SU(2) subgroup of the A and B colors. The AB potential is taken to be very strong, but to have shorter range than the AC and BC potentials. A possible cause of this range difference is mass-splitting of the vector gluon octet, if the three gluons coupled as the generators of the SU(2) of the A and B colors are heavier than the other gluons. This mechanism is similar to that used in some recent attempts to unify strong, electromagnetic and weak interactions.

This mechanism mixes color singlets and octets. The effects of the lighter five gluons are not considered negligible, however, so the baryon states must contain some color-singlet component. This requires that each baryon contain one A quark, one B quark, and one C quark. If the difference in the

ranges of the potentials is appreciable, the AB diquark will be relatively small.

A convenient set of internal variables is

$$\vec{\lambda}_{AB} = (1/\sqrt{6})(\vec{r}_A + \vec{r}_B - 2\vec{r}_C) \quad (1a)$$

$$\vec{\rho}_{AB} = (1/\sqrt{2})(\vec{r}_A - \vec{r}_B) \quad (1b)$$

The gluon-exchange potential is a sum of two-body potentials $V = V_{\alpha\beta} + V_{\beta\gamma} + V_{\gamma\alpha}$, where α, β , and γ are the three quarks. We assume that $V_{\mu\nu}$ is the sum of a short-range part (V_s) and a long-range part (V_ℓ), i. e. ,

$$V_{\mu\nu} = \sum_{i=1}^3 J_i^\mu J_i^\nu V_s(|\vec{r}_\mu - \vec{r}_\nu|) + \sum_{i=4}^8 J_i^\mu J_i^\nu V_\ell(|\vec{r}_\mu - \vec{r}_\nu|), \quad (2)$$

where J_i^μ is the i 'th Hermitean generator of SU(3), operating in the color space of the quark μ , and J_1, J_2 and J_3 are the generators of the AB SU(2) subgroup of color SU(3). The configuration-space potentials V_s and V_ℓ are positive. Exact color symmetry corresponds to $V_s = V_\ell$.

If the $\mu\nu$ state is in the representation r of AB SU(2), then $\sum_{i=1}^3 J_i^\mu J_i^\nu = \frac{1}{2}[C_2(r) - C_2(\mu) - C_2(\nu)]$, where $C_2(x)$ is the eigenvalue of the quadratic Casimir operator of AB SU(2) for the representation x . It follows from this expression that the short-range potential is attractive only between the A and B quarks, and only if they are in the singlet of AB SU(2), i. e. , the color wave function is anti-symmetric in AB exchange. We assume that the wave function is in the ground state of the strong short-range V_s potential, so that there are no excitations of the ρ variable. The quarks obey Fermi statistics, so the wave function may be written

$$\begin{aligned} \sqrt{6} \psi_{jk} &= \sum_{\mathbf{P}} \tau_{\mathbf{P}} \rho_{\mathbf{P}}^\alpha \beta \gamma U_j(\alpha\beta\gamma) R_g(\vec{\rho}_{\alpha\beta}) L_k(\vec{\lambda}_{\alpha\beta}) \\ &= \sum_{\mathbf{P}} \tau_{\mathbf{P}} \rho_{\mathbf{P}}^\alpha \beta \gamma U_j(ABC) R_g(\vec{\rho}_{AB}) L_k(\vec{\lambda}_{AB}) \end{aligned} \quad (3)$$

where U_j is the SU(6) wave function, R and L are orbital functions, and the subscript g denotes the ground state. The sum is over the six permutations of $\alpha, \beta,$ and γ , and τ is 1 and (-1) for even and odd permutations, respectively. Since the color wave function is antisymmetric in the transposition (AB), and since $(AB)\vec{\lambda}_{AB} = \vec{\lambda}_{AB}$, U_j must be symmetric in AB exchange. Therefore U_j is either the symmetric (56) representation U_6 or the λ component U_λ of the mixed (70) representation.

The potential does not depend on the SU(6) representation. Since the diquark internal wave function is the same for all states, both in color and configuration space, the relative energies of states depend on the potential between the quark and diquark. Since the diquark is in the singlet of AB SU(2), the $\sum_{i=1}^3 J_i^\mu J_i^\nu$ term does not contribute to the quark-diquark potential. Thus, only the long-range part of V contributes to quark-diquark binding, but the 1, 2, and 3 gluons may be included in the sum. The color operator of a two-quark V_ℓ term then becomes

$$\sum_{i=1}^3 J_i^\mu J_i^\nu = \frac{1}{2} [C_3 - 2C_3(q)] \quad , \quad (4)$$

where C_3 is the quadratic Casimir operator of SU(3) and $C(q)$ is its eigenvalue for the quark representation.

The states are mixtures of color octets and singlets. If two states are of the same oscillator level, that with the larger singlet component will be favored energetically. This is because each pair of quarks in the singlet are in an antisymmetric color state, thus minimizing the contribution of C_3 in Eq. (4).

In order to make the mathematical procedure clear, we review some properties of mixed representations. The permutation properties of a function of A, B, and C may be specified completely from the behavior of the function under general products of the two transpositions (AB) and (AC). The behavior of the

two mixed-symmetry components λ and ρ is²

$$(AB)\lambda = \lambda, (AC)\lambda = -\frac{1}{2}\lambda - \frac{1}{2}\sqrt{3}\rho, \quad (5a)$$

$$(AB)\rho = -\rho, (AC)\rho = \frac{1}{2}\rho - \frac{1}{2}\sqrt{3}\lambda. \quad (5b)$$

The two variables $\vec{\lambda}_{AB}$ and $\vec{\rho}_{AB}$ of Eqs. (1a) and (1b) are such mixed-symmetry components; the subscript AB will be suppressed in the rest of the paper.

For each orbital wave function RL of Eq. (3), and for both U_S and U_λ , we will compute the magnitudes of the components of URL that are completely symmetric, and of λ mixed symmetry. These correspond to a color singlet and a color octet, respectively.

We consider first the orbital wave function $\phi_k = RL_k$. Since ϕ is symmetric under (AB), it is a linear combination of a symmetric and a λ mixed state, i. e. ,

$$\phi = a\phi_S + (1 - a^2)^{\frac{1}{2}}\phi_\lambda, \quad (6)$$

where ϕ , ϕ_S , and ϕ_λ are normalized. We define the matrix element M_k for the k state by

$$M_k = \langle \phi_k, (AC)\phi_k \rangle. \quad (7)$$

It follows from Eq. (5a) and the orthogonality of ϕ_S , ϕ_λ , and ϕ_ρ that

$$M = a^2 - \frac{1}{2}(1 - a^2). \quad (8)$$

Next we consider the products of SU(6) and orbital wave functions $\chi = U\phi$. If $\chi = U_\lambda\phi_\lambda$, evaluation of $\langle \chi, (AC)\chi \rangle$ from Eq. (5a) leads to the value $\frac{1}{4}$. Using an equation analogous to Eq. (8), one can write $U_\lambda\phi_\lambda = (1/\sqrt{2})(\chi_S + \chi_\lambda)$, where U_λ , ϕ_λ , χ_S , and χ_λ are normalized. Therefore, if ϕ is in the form of Eq. (6), the probabilities that $U_S\phi$ and $U_\lambda\phi$ are symmetric (and consequently in a color singlet) are a^2 and $\frac{1}{2}(1 - a^2)$, respectively. It follows from Eq. (8) that the symmetric SU(6) representation 56 is favored over the 70 if and only if M is positive.

We will evaluate M for harmonic oscillator wave functions, using the usual dimensionless variables, determined from the λ (long-range) force constant.

The orbital wave function is

$$\phi_k = N_k H_k(\lambda) e^{-\frac{1}{2}\lambda^2} e^{-\frac{1}{2}D\rho^2}, \quad (9)$$

where N_k is a normalization constant, H_k is a three-dimensional Hermite polynomial, and $D > 1$, since the ρ wave function is of comparatively short range.

For convenience we define M'_k by $M_k = M'_k f_k(D)$, where $f_k(1) = 1$. We first evaluate M' by setting $D = 1$, in which case the exponent $-\frac{1}{2}(\lambda^2 + \rho^2)$ is invariant to the permutation (AC). The Hermite polynomial H may be written as a sum of polynomials h of the form

$$h = N' \lambda_x^{n_x} \lambda_y^{n_y} \lambda_z^{n_z} + O(\ell),$$

where N' is a constant, $n = n_x + n_y + n_z$ is the order of the energy level, and $O(\ell)$ is a lower-order polynomial in λ_i .

We consider the quantity (AC) h . Since (AC) leaves $\lambda^2 + \rho^2$ invariant, this transposition leaves the quantum number $n(\lambda) + n(\rho)$ unchanged when $D = 1$.

Therefore, when (AC) is applied to h , the lower-order term (AC) $O(\ell)$ is determined from the leading term of the polynomial. From these facts and Eq. (5a),

(AC) $\phi_k = [(-\frac{1}{2})^n \phi_k + \text{orthogonal states}]$, when $D = 1$. Hence,

$$\langle \phi_k, (AC)\phi_k \rangle = M = (-\frac{1}{2})^n f_k(D). \quad (10)$$

Since the sign of M determines the preferred $SU(6)$ representation, we see that if $f_k(D)$ is positive, the representations 56 and 70 are favored for even and odd oscillator levels, respectively.

The evaluation of $f_k(D)$ for a specific oscillator state is straightforward, but tedious. It is convenient to define a function $\Delta_k(D)$ by the formula

$$f_k(D) = \left[\frac{16D}{(3D+1)(D+3)} \right]^{\left(\frac{3}{2} + n\right)} [1 + \Delta_k(D)] .$$

A calculation shows that for every energy level n , the function $\Delta(D)$ is zero when $\ell = n$, where ℓ is the orbital angular momentum. In the cases ($n = 2$, $\ell = 0$) and ($n = 3$, $\ell = 1$), Δ may be written $\Delta = \kappa (D^2 - 1)^2 / D^2$, where the constant κ is (9/128) and (99/640) in the (2,0) and (3,1) cases, respectively.

Since $f_k(D)$ is positive, its inclusion does not change the predicted relative ordering of the 56 and 70 representations. Furthermore, if D is on the order of 2 or so, and n is not large, $f(D)$ is appreciable, so the expected difference between the 56 and 70 energies may be large enough so that the unfavored state does not appear physically. On the other hand, $|M|$ is small when n is large, primarily because of the $(-\frac{1}{2})^n$ factor in Eq. (10). Consequently, we predict that for sufficiently high quark-model level, the 56 and 70 should both appear.

Since the sign of M depends primarily on the parity of the wave function $L(\lambda)$, the use of oscillator wave functions is not crucial to the result. A more detailed treatment of this color-symmetry breaking effect, including the case when the symmetry breaking is relatively small, will be published elsewhere.

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