

# Colors in Context: A Pragmatic Neural Model for Grounded Language Understanding

Will Monroe,<sup>1</sup> Robert X.D. Hawkins,<sup>2</sup> Noah D. Goodman,<sup>1,2</sup> and Christopher Potts<sup>3</sup>

Departments of <sup>1</sup>Computer Science, <sup>2</sup>Psychology, and <sup>3</sup>Linguistics

Stanford University, Stanford, CA 94305

wmonroe4@cs.stanford.edu, {rxdh, ngoodman, cgpotts}@stanford.edu

## Abstract

We present a model of pragmatic referring expression interpretation in a grounded communication task (identifying colors from descriptions) that draws upon predictions from two recurrent neural network classifiers, a speaker and a listener, unified by a recursive pragmatic reasoning framework. Experiments show that this combined pragmatic model interprets color descriptions more accurately than the classifiers from which it is built, and that much of this improvement results from combining the speaker and listener perspectives. We observe that pragmatic reasoning helps primarily in the hardest cases: when the model must distinguish very similar colors, or when few utterances adequately express the target color. Our findings make use of a newly-collected corpus of human utterances in color reference games, which exhibit a variety of pragmatic behaviors. We also show that the embedded speaker model reproduces many of these pragmatic behaviors.

## 1 Introduction

Human communication is *situated*. In using language, we are sensitive to context and our interlocutors' expectations, both when choosing our utterances (as speakers) and when interpreting the utterances we hear (as listeners). Visual referring tasks exercise this complex process of grounding, in the environment and in our mental models of each other, and thus provide a valuable test-bed for computational models of production and comprehension.

Table 1 illustrates the situated nature of reference understanding with descriptions of colors from a





	Context	Utterance
1.		darker blue
2.		Purple
3.		blue
4.		blue

Table 1: Examples of color reference in context, taken from our corpus. The target color is boxed. The speaker's description is shaped not only by this target, but also by the other context colors and their relationships.

task-oriented dialogue corpus we introduce in this paper. In these dialogues, the speaker is trying to identify their (privately assigned) target color for the listener. In context 1, the comparative *darker* implicitly refers to both the target (boxed) and one of the other colors. In contexts 2 and 3, the target color is the same, but the distractors led the speaker to choose different basic color terms. In context 4, *blue* is a pragmatic choice even though two colors are shades of blue, because the interlocutors assume about each other that they find the target color a more prototypical representative of blue and would prefer other descriptions (*teal, cyan*) for the middle color. The fact that *blue* appears in three of these four cases highlights the flexibility and context dependence of color descriptions.

In this paper, we present a scalable, learned model of pragmatic language understanding. The model is built around a version of the Rational Speech Acts (RSA) model (Frank and Goodman, 2012; Goodman and Frank, 2016), in which agents reason recur-

sively about each other’s expectations and intentions to communicate more effectively than literal semantic agents could. In most work on RSA, the literal semantic agents use fixed message sets and stipulated grammars, which is a barrier to experiments in linguistically complex domains. In our formulation, the literal semantic agents are recurrent neural networks (RNNs) that produce and interpret color descriptions in context. These models are learned from data and scale easily to large datasets containing diverse utterances. The RSA recursion is then defined in terms of these base agents: the *pragmatic speaker* produces utterances based on a literal RNN listener (Andreas and Klein, 2016), and the *pragmatic listener* interprets utterances based on the pragmatic speaker’s behavior.

We focus on accuracy in a listener task (i.e., at language understanding). However, our most successful model integrates speaker and listener perspectives, combining predictions made by a system trained to understand color descriptions and one trained to produce them.

We evaluate this model with a new, psycholinguistically motivated corpus of real-time, dyadic reference games in which the referents are patches of color. Our task is fundamentally the same as that of Baumgaertner et al. (2012), but the corpus we release is larger by several orders of magnitude, consisting of 948 complete games with 53,365 utterances produced by human participants paired into dyads on the web. The linguistic behavior of the players exhibits many of the intricacies of language in general, including not just the context dependence and cognitive complexity discussed above, but also compositionality, vagueness, and ambiguity. While many previous data sets feature descriptions of individual colors (Cook et al., 2005; Munroe, 2010; Kawakami et al., 2016), situating colors in a communicative context elicits greater variety in language use, including negations, comparatives, superlatives, metaphor, and shared associations.

Experiments on the data in our corpus show that this combined pragmatic model improves accuracy in interpreting human-produced descriptions over the basic RNN listener alone. We find that the largest improvement over the single RNN comes from blending it with an RNN trained to perform the speaker task, despite the fact that a model based

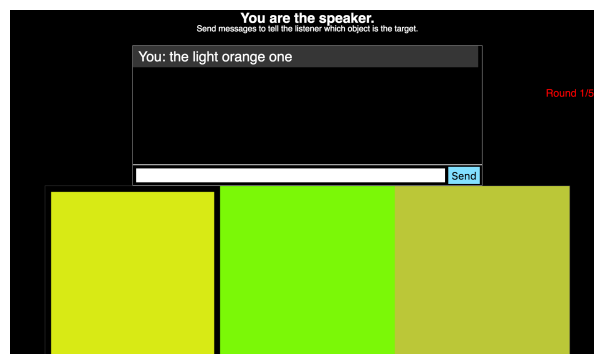


Figure 1: Example trial in corpus collection task, from speaker’s perspective. The target color (boxed) was presented among two distractors on a neutral background.

only on this speaker RNN performs poorly on its own. Pragmatic reasoning on top of the listener RNN alone also yields improvements, which moreover come primarily in the hardest cases: 1) contexts with colors that are very similar, thus requiring the interpretation of descriptions that convey fine distinctions; and 2) target colors that most referring expressions fail to identify, whether due to a lack of adequate descriptive terms or a consistent bias against the color in the RNN listener.

## 2 Task and data collection

We evaluate our agents on a task of language understanding in a dyadic reference game (Rosenberg and Cohen, 1964; Krauss and Weinheimer, 1964; Paetzel et al., 2014). Unlike traditional natural language processing tasks, in which participants provide impartial judgements of language in isolation, reference games embed language use in a goal-oriented communicative context (Clark, 1996; Tanenhaus and Brown-Schmidt, 2008). Since they offer the simplest experimental setup where many pragmatic and discourse-level phenomena emerge, these games have been used widely in cognitive science to study topics like common ground and conventionalization (Clark and Wilkes-Gibbs, 1986), referential domains (Brown-Schmidt and Tanenhaus, 2008), perspective-taking (Hanna et al., 2003), and overinformativeness (Koolen et al., 2011).

To obtain a corpus of natural color reference data across varying contexts, we recruited 967 unique participants from Amazon Mechanical Turk to play 1,059 games of 50 rounds each, using the open-

source framework of Hawkins (2015). Participants were sorted into dyads, randomly assigned the role of speaker or listener, and placed in a game environment containing a chat box and an array of three color patches (Figure 1). On each round, one of the three colors was chosen to be the target and highlighted for the speaker. They were instructed to communicate this information to the listener, who could then click on one of the colors to advance to the next trial. Both participants were free to use the chat box at any point.

To ensure a range of difficulty, we randomly interspersed an equal number of trials from three different conditions: 1) *close*, where colors were all within a distance of  $\theta$  from one another but still perceptible,<sup>1</sup> 2) *split*, where one distractor was within a distance of  $\theta$  of the target, but the other distractor was farther than  $\theta$ , and 3) *far*, where all colors were farther than  $\theta$  from one another. Colors were rejection sampled uniformly from RGB (red, green, blue) space to meet these constraints.

After excluding extremely long messages,<sup>2</sup> incomplete games, and games whose participants self-reported confusion about the instructions or non-native English proficiency, we were left with a corpus of 53,365 speaker utterances across 46,994 rounds in 948 games. The three conditions are equally represented, with 15,519 *close* trials, 15,693 *split* trials, and 15,782 *far* trials. Participants were allowed to play more than once, but the modal number of games played per participant was one (75%). The modal number of messages sent per round was also one (90%). We release the filtered corpus we used throughout our analyses alongside the raw, pre-filter data collected from these experiments (see Footnote 11).

### 3 Behavioral results

Our corpus was developed not only to facilitate the development of models for grounded language un-

<sup>1</sup>We used the most recent CIEDE standard to measure color differences, which is calibrated to human vision (Sharma et al., 2005). All distances were constrained to be larger than a lower bound of  $\epsilon = 5$  to ensure perceptible differences, and we used a threshold value of  $\theta = 20$  to create conditions.

<sup>2</sup>Specifically, we set a length criterion at  $4\sigma$  of the mean number of words per message (about 14 words, in our case), excluding 627 utterances. These often included meta-commentary about the game rather than color terms.

derstanding, but also to provide a richer picture of human pragmatic communication. The collection effort was thus structured like a large-scale behavioral experiment, closely following experimental designs like those of Clark and Wilkes-Gibbs (1986). This paves the way to assessing our model not solely based on the listener’s classification accuracy, but also in terms of how qualitative features of the speaker’s production compare to that of our human participants. Thus, the current section briefly reviews some novel findings from the human corpus that we use to inform our model assessment.

#### 3.1 Listener behavior

Since color reference is a difficult task even for humans, we compared listener accuracy across conditions to calibrate our expectations about model performance. While participants’ accuracy was close to ceiling (97%) on the *far* condition, they made significantly more errors on the *split* (90%) and *close* (83%) conditions (see Figure 4).

#### 3.2 Speaker behavior

For ease of comparison to computational results, we focus on five metrics capturing different aspects of pragmatic behavior displayed by both human and artificial speakers in our task (Table 2). In all cases, we report test statistics from a mixed-effects regression including condition as a fixed effect and game ID as a random effect; except where noted, all test statistics reported correspond to  $p$ -values  $< 10^{-4}$  and have been omitted for readability.

**Words and characters** We expect human speakers to be more verbose in *split* and *close* contexts than *far* contexts; the shortest, simplest color terms for the target may also apply to one or both distractors, thus incentivizing the speaker to use more lengthy descriptions to fully distinguish it. Indeed, even if they *know* enough simple color terms to distinguish all the colors lexically, they might be unsure their listeners will and so resort to modifiers anyway. To assess this hypothesis, we counted the average number of words and characters per message. Compared to the baseline *far* context, participants used significantly more words both in the *split* context ( $t = 45.85$ ) and the *close* context ( $t = 73.06$ ). Similar results hold for the character metric.

	human			$S_0$			$S_1$		
	far	split	close	far	split	close	far	split	close
# Chars	7.8	12.3	14.9	9.0	12.8	16.6	9.0	12.8	16.4
# Words	1.7	2.7	3.3	2.0	2.8	3.7	2.0	2.8	3.7
% Comparatives	1.7	14.2	12.8	3.6	8.8	13.1	4.2	9.0	13.7
% High Specificity	7.0	7.6	7.4	6.4	8.4	7.6	6.8	7.9	7.5
% Negatives	2.8	10.0	12.9	4.8	8.9	13.3	4.4	8.5	14.1
% Superlatives	2.2	6.1	16.7	4.7	9.7	17.2	4.8	10.3	16.6

Table 2: Corpus statistics and statistics of samples from artificial speakers (rates per utterance).  $S_0$ : RNN speaker;  $S_1$ : pragmatic speaker derived from RNN listener (see Section 4.3). The human and artificial speakers show many of the same correlations between language use and context type.

**Comparatives and superlatives** As noted in Section 1, comparative morphology implicitly encodes a dependence on the context; a speaker who refers to the target color as *the darker blue* is presupposing that there is another (lighter) blue in the context. Similarly, superlatives like *the bluest one* or *the lightest one* presuppose that all the colors can be compared along a specific semantic dimension. We thus expect to see this morphology more often where two or more of the colors are comparable in this way. To test this, we used the Stanford CoreNLP part-of-speech tagger (Toutanova et al., 2003) to mark the presence or absence of comparatives (JJR or RBR) and superlatives (JJS or RBS) for each message.

We found two related patterns across conditions. First, participants were significantly more likely to use both comparatives ( $z = 37.39$ ) and superlatives ( $z = 31.32$ ) when one or more distractors were close to the target. Second, we found evidence of an asymmetry in the use of these constructions across the *split* and *close* contexts. Comparatives were used significantly more often in the *split* context ( $z = 4.4$ ), where only one distractor was close to the target, while superlatives were much more likely to be used in the *close* condition ( $z = 32.72$ ).<sup>3</sup>

**Negatives** In our referential contexts, negation is likely to play a role similar to that of comparatives: a phrase like *not the red or blue one* singles out the third color, and *blue but not bright blue* achieves a more nuanced kind of comparison. Thus, as with

comparatives, we expect negation to be more likely where one or more distractors are close to the target. To test this, we counted occurrences of the string ‘not’ (by far the most frequent negation in the corpus). Compared to the baseline *far* context, we found that participants were more likely to use negative constructions when one ( $z = 27.36$ ) or both ( $z = 34.32$ ) distractors were close to the target.

**WordNet specificity** We expect speakers to prefer basic color terms wherever they suffice to achieve the communicative goal, since such terms are most likely to succeed with the widest range of listeners. Thus, a speaker might choose *blue* even for a clear periwinkle color. However, as the colors get closer together, the basic terms become too ambiguous, and thus the risk of specific terms becomes worthwhile (though lengthy descriptions might be a safer strategy, as discussed above). To evaluate this idea, we use WordNet (Fellbaum, 1998) to derive a specificity hierarchy for color terms, and we hypothesized that *split* or *close* conditions will tend to lead speakers to go lower in this hierarchy.

For each message, we transformed adjectives into their closest noun forms (e.g. ‘reddish’ → ‘red’), filtered to include only nouns with ‘color’ in their hypernym paths, calculated the depth of the hypernym path of each color word, and took the maximum depth occurring in a message. For instance, the message “deep magenta, purple with some pink” received a score of 9. It has three color terms: “purple” and “pink,” which have the basic-level depth of 7, and “magenta,” which is a highly specific color term with a depth of 9. Finally, because there weren’t meaningful differences between words at depths of

<sup>3</sup>We used Helmert coding to test these specific patterns: the first regression coefficient compares the ‘far’ condition to the mean of the other two conditions, and the second regression coefficient compares the ‘split’ condition to the ‘close’ condition.

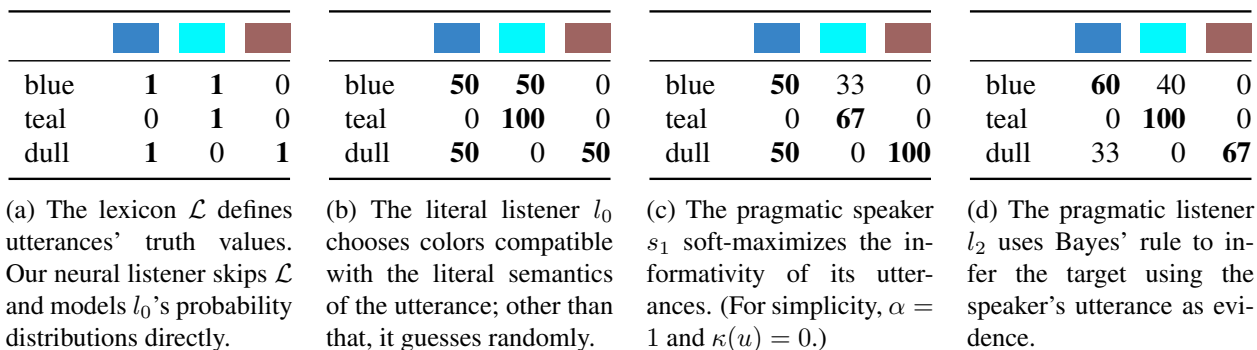


Figure 2: The basic RSA model applied to a reference task (literal semantics and alternative utterances simplified for demonstration). (b)-(d) show conditional probabilities (%).

8 (“rose”, “teal”) and 9 (“tan,” “taupe”), we conducted our analyses on a binary variable thresholded to distinguish “high specificity” messages with a depth greater than 7. We found a small but reliable increase in the likelihood of “high specificity” messages from human speakers in the *split* ( $z = 2.84, p = 0.005$ ) and *close* ( $z = 2.33, p = 0.02$ ) contexts, compared to the baseline *far* context.

## 4 Models

We first define the basic RSA model as applied to the color reference games introduced in Section 2; an example is shown in Figure 2.

**Listener-based listener** The starting point of RSA is a model of a *literal listener*:

$$l_0(t | u, \mathcal{L}) \propto \mathcal{L}(u, t)P(t) \quad (1)$$

where  $t$  is a color in the context set  $C$ ,  $u$  is a message drawn from a set of possible utterances  $U$ ,  $P$  is a prior over colors, and  $\mathcal{L}(u, t)$  is a semantic interpretation function that takes the value 1 if  $u$  is true of  $t$ , else 0. Figure 2a shows the values of  $\mathcal{L}$  defined for a very simple context in which  $U = \{blue, teal, dull\}$ , and  $C = \{\text{blue square}, \text{teal square}, \text{dull square}\}$ ; Figure 2b shows the corresponding literal listener  $l_0$  if the prior  $P$  over colors is flat. (In our scalable extension, we will substitute a neural network model for  $l_0$ , bypassing  $\mathcal{L}$  and allowing for non-binary semantic judgments.)

RSA postulates a model of a *pragmatic speaker* (Figure 2c) that behaves according to a distribution that soft-maximizes a utility function rewarding informativity and penalizing cost:

$$s_1(u | t, \mathcal{L}) \propto e^{\alpha \log(l_0(t|u, \mathcal{L})) - \kappa(u)} \quad (2)$$

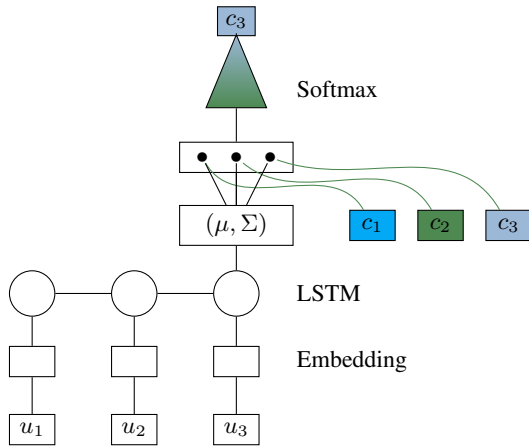
Here,  $\kappa$  is a real-valued cost function on utterances, and  $\alpha \in [0, \infty)$  is an inverse temperature parameter governing the “rationality” of the speaker model. A large  $\alpha$  means the pragmatic speaker is expected to choose the most informative utterance (minus cost) consistently; a small  $\alpha$  means the speaker is modeled as choosing suboptimal utterances frequently.

Finally, a *pragmatic listener* (Figure 2d) interprets utterances by reasoning about the behavior of the pragmatic speaker:

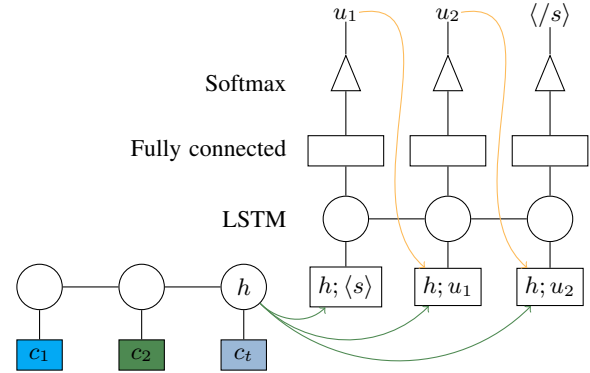
$$l_2(t | u, \mathcal{L}) \propto s_1(u | t, \mathcal{L})P(t) \quad (3)$$

The  $\alpha$  parameter of the speaker indirectly affects the listener’s interpretations: the more reliably the speaker chooses the optimal utterance for a referent, the more the listener will take deviations from the optimum as a signal to choose a different referent.

The most important feature of this model is that the pragmatic listener  $l_2$  reasons not about the semantic interpretation function  $\mathcal{L}$  directly, but rather about a speaker who reasons about a listener who reasons about  $\mathcal{L}$  directly. The back-and-forth nature of this interpretive process mirrors that of conversational implicature (Grice, 1975) and reflects more general ideas from Bayesian cognitive modeling (Tenenbaum et al., 2011). The model and its variants have been shown to capture a wide range of pragmatic phenomena in a cognitively realistic manner (Goodman and Stuhlmüller, 2013; Smith et al., 2013; Kao et al., 2014; Bergen et al., 2016), and the central Bayesian calculation has proven useful in a variety of communicative domains (Tellex et al., 2014; Vogel et al., 2013).



(a) The  $L_0$  agent processes tokens  $u_i$  of a color description  $u$  sequentially. The final representation is transformed into a Gaussian distribution in color space, which is used to score the context colors  $c_1 \dots c_3$ .



(b) The  $S_0$  agent processes the target color  $c_t$  in context and produces tokens  $u_i$  of a color description sequentially. Each step in production is conditioned by the context representation  $h$  and the previous word produced.

Figure 3: The neural base speaker and listener agents.

**Speaker-based listener** The definitions of  $s_1$  (2) and  $l_2$  (3) give a general method of deriving a speaker from a listener and vice versa. This suggests an alternative formulation of a pragmatic listener, starting from a literal speaker:

$$s_0(u | t, \mathcal{L}) \propto \mathcal{L}(u, t) e^{-\kappa(u)} \quad (4)$$

$$l_1(t | u, \mathcal{L}) \propto s_0(u | t, \mathcal{L}) P(t) \quad (5)$$

Here, it is the speaker that reasons about the semantics, while the listener reasons about this speaker.

Both of these versions of RSA pose problems with scalability, stemming from the set of messages  $U$  and the interpretation function  $\mathcal{L}$ . In most versions of RSA, these are specified by hand (but see Monroe and Potts 2015). This presents a serious practical obstacle to applying RSA to large data sets containing realistic utterances. The set  $U$  also raises a more fundamental issue: if this set is not finite (as one would expect from a compositional grammar), then in general there is no exact way to normalize the  $s_1$  scores, since the denominator must sum over all messages. The same problem applies to  $s_0$ , unless  $\mathcal{L}$  factorizes in an unrealistically clean way.

Over the next few subsections, we overcome these obstacles by replacing  $l_0$  and  $s_0$  with RNN-based listener agents, denoted with capital letters:  $L_0$ ,  $S_0$ . We use the  $S_0$  agent both as a base model for a pragmatic listener analogous to  $l_1$  in (5) and to acquire

sample utterances for approximating the normalization required in defining the  $s_1$  agent in (2).

#### 4.1 Base listener

Our base listener agent  $L_0$  (Figure 3a) is an LSTM encoder model that predicts a Gaussian distribution over colors in a transformed representation space. The input words are embedded in a 100-dimensional vector space. Word embeddings are initialized to random normally-distributed vectors ( $\mu = 0$ ,  $\sigma = 0.01$ ) and trained. The sequence of word vectors is used as input to an LSTM with 100-dimensional hidden state, and a linear transformation is applied to the output representation to produce the parameters  $\mu$  and  $\Sigma$  of a quadratic form<sup>4</sup>

$$\text{score}(f) = -(f - \mu)^T \Sigma (f - \mu)$$

where  $f$  is a vector representation of a color. Each color is represented in its simplest form as a three-dimensional vector in RGB space. These RGB vectors are then Fourier-transformed as in Monroe et al. (2016) to obtain the representation  $f$ .

The values of  $\text{score}(f)$  for each of the  $K$  context colors are normalized in log space to produce a probability distribution over the context colors. We denote this distribution by  $L_0(t | u, C; \theta)$ , where  $\theta$

<sup>4</sup>The quadratic form is not guaranteed to be negative definite and thus define a Gaussian; however, it is for  $> 95\%$  of inputs. The distribution over context colors is well-defined regardless.

represents the vector of parameters that define the trained model.

## 4.2 Base speaker

We also employ an LSTM-based speaker model  $S_0(u | t, C; \phi)$ . This speaker serves two purposes: 1) it is used to define a pragmatic listener akin to  $l_1$  in (5), and 2) it provides samples of alternative utterances for each context, to avoid enumerating the intractably large space of possible utterances.

The speaker model consists of an LSTM context encoder and an LSTM description decoder (Figure 3b). In this model, the colors of the context  $c_i \in C$  are transformed into Fourier representation space, and the sequence of color representations is passed through an LSTM with 100-dimensional hidden state. The context is reordered to place the target color last, minimizing the length of dependence between the most important input color and the output (Sutskever et al., 2014) and eliminating the need to represent the index of the target separately. The final cell state of this recurrent neural network is concatenated with a 100-dimensional embedding for the previous token output at each step of decoding. The resulting vector is input along with the previous cell state to the LSTM cell, and an affine transformation and softmax function are applied to the output to produce a probability distribution predicting the following token of the description. The model is substantively similar to well-known models for image caption generation (Karpathy and Fei-Fei, 2015; Vinyals et al., 2015), which use the output of a convolutional neural network as the representation of an input image and provide this representation to the RNN as an initial state or first word (we represent the context using a second RNN and concatenate the context representation onto each input word vector).

## 4.3 Pragmatic agents

Using the above base agents, we define a pragmatic speaker  $S_1$  and a pragmatic listener  $L_2$ :

$$S_1(u | t, C; \theta) = \frac{L_0(t | u, C; \theta)^\alpha}{\sum_{u'} L_0(t | u', C; \theta)^\alpha} \quad (6)$$

$$L_2(t | u, C; \theta) = \frac{S_1(u | t, C; \theta)}{\sum_{t'} S_1(u | t', C; \theta)} \quad (7)$$

These definitions mirror those in (2) and (3) above, with  $\mathcal{L}$  replaced by the learned weights  $\theta$ .

Just as in (2), the denominator in (6) should consist of a sum over the entire set of potential utterances, which is exponentially large in the maximum utterance length and might not even be finite. As mentioned in Section 4.2, we limit this search by taking  $m$  samples from  $S_0(u | i, C; \phi)$  for each target index  $i$ , adding the actual utterance from the testing example, and taking the resulting multiset as the universe of possible utterances, weighted towards frequently-sampled utterances.<sup>5</sup> Taking a number of samples from  $S_0$  for each referent in the context gives the pragmatic listener a variety of informative alternative utterances to consider when interpreting the true input description. We have found that  $m$  can be small; in our experiments, it is set to 8.

To reduce the noise resulting from the stochastically chosen alternative utterance sets, we also perform this alternative-set sampling  $n$  times and average the resulting probabilities in the final  $L_2$  output. We again choose  $n = 8$  as a satisfactory compromise between effectiveness and computation time.

**Blending with a speaker-based agent** A second pragmatic listener  $L_1$  can be formed in a similar way, analogous to  $l_1$  in (5):

$$L_1(t | u, C; \phi) = \frac{S_0(u | t, C; \phi)}{\sum_{t'} S_0(u | t', C; \phi)} \quad (8)$$

We expect  $L_1$  to be less accurate than  $L_0$  or  $L_2$ , because it is performing a listener task using only the outputs of a model trained for a speaker task. However, this difference in training objective can also give the model strengths that complement those of the two listener-based agents. One might also expect a realistic model of human language interpretation to lie somewhere between the “reflex” interpretations of the neural base listener and the “reasoned” interpretations of one of the pragmatic models. This has an intuitive justification in people’s uncertainty about whether their interlocutors are speaking pragmatically: “should I read more into that statement, or take it at face value?” We therefore also evaluate models defined as a weighted average of  $L_0$

<sup>5</sup>An alternative would be to enforce uniqueness within the alternative set, keeping it a true set as in the basic RSA formulation; this could be done with rejection sampling or beam search for the highest-scoring speaker utterances. We found that doing so with rejection sampling hurt model performance somewhat, so we did not pursue the more complex beam search approach.



and each of  $L_1$  and  $L_2$ , as well as an “ensemble” model that combines all of these agents. Specifically, we consider the following blends of neural base models and pragmatic models, with  $\mathbf{L}_i$  abbreviating  $L_i(t \mid u, C; \theta, \phi)$  for convenience:

$$\mathbf{L}_a \propto \mathbf{L}_0^{\beta_a} \cdot \mathbf{L}_1^{1-\beta_a} \quad (9)$$

$$\mathbf{L}_b \propto \mathbf{L}_0^{\beta_b} \cdot \mathbf{L}_2^{1-\beta_b} \quad (10)$$

$$\mathbf{L}_e \propto \mathbf{L}_a^\gamma \cdot \mathbf{L}_b^{1-\gamma} \quad (11)$$

The hyperparameters in the exponents allow tuning the blend of each pair of models—e.g., overriding the neural model with the pragmatic reasoning in  $L_b$ . The value of the weights  $\beta_a$ ,  $\beta_b$ , and  $\gamma$  can be any real number; however, we find that good values of these weights lie in the range  $[-1, 1]$ . As an example, setting  $\beta_b = 0$  makes the blended model  $L_b$  equivalent to the pragmatic model  $L_2$ ;  $\beta_b = 1$  ignores the pragmatic reasoning and uses the base model  $L_0$ ’s outputs; and  $\beta_b = -1$  “subtracts” the base model from the pragmatic model (in log probability space) to yield a “hyperpragmatic” model.

#### 4.4 Training

We split our corpus into approximately equal train/dev/test sets (15,665 train trials, 15,670 dev, 15,659 test), ensuring that trials from the same dyad are present in only one split. We preprocess the data by 1) lowercasing; 2) tokenizing by splitting off punctuation as well as the endings *-er*, *-est*, and *-ish*;<sup>6</sup> and 3) replacing tokens that appear once or not at all in the training split<sup>7</sup> with  $\langle \text{unk} \rangle$ . We also remove listener utterances and concatenate speaker utterances on the same context. We leave handling of interactive dialogue to future work (Section 8).

We use ADADELTA (Zeiler, 2012) and Adam (Kingma and Ba, 2014), adaptive variants of stochastic gradient descent (SGD), to train listener and speaker models. The choice of optimization algorithm and learning rate for each model were tuned with grid search on a held-out tuning set consisting of 3,500 contexts.<sup>8</sup> We also use a fine-grained

<sup>6</sup>We only apply this heuristic ending segmentation for the listener; the speaker is trained to produce words with these endings unsegmented, to avoid segmentation inconsistencies when passing speaker samples as alternative utterances to the listener.

<sup>7</sup>1.13% of training tokens, 1.99% of dev/test.

<sup>8</sup>For  $L_0$ : ADADELTA, learning rate  $\eta = 0.2$ ; for  $S_0$ : Adam, learning rate  $\alpha = 0.004$ .

grid search on this tuning set to determine the values of the pragmatic reasoning parameters  $\alpha$ ,  $\beta$ , and  $\gamma$ . In our final ensemble  $L_e$ , we use  $\alpha = 0.544$ , base weights  $\beta_a = 0.492$  and  $\beta_b = -0.15$ , and a final blending weight  $\gamma = 0.491$ . It is noteworthy that the optimal value of  $\beta_b$  from grid search is *negative*. The effect of this is to amplify the difference between  $L_0$  and  $L_2$ : the listener-based pragmatic model, evidently, is not quite pragmatic enough.

## 5 Model results

### 5.1 Speaker behavior

To compare human behavior with the behavior of our embedded speaker models, we performed the same behavioral analysis done in Section 3.2. Results from this analysis are included alongside the human results in Table 2. Our pragmatic speaker model  $S_1$  did not differ qualitatively from our base speaker  $S_0$  on any of the metrics, so we only summarize results for humans and the pragmatic model.

**Words and characters** We found human speakers to be more verbose when colors were closer together, in both number of words and number of characters. As Table 2 shows, our  $S_1$  agent shows the same increase in utterance length in the *split* ( $t = 18.07$ ) and *close* ( $t = 35.77$ ) contexts compared to the *far* contexts.

**Comparatives and superlatives** Humans used more comparatives and superlatives when colors were closer together; however, comparatives were preferred in the *split* contexts, superlatives in the *close* contexts. Our pragmatic speaker shows the first of these two patterns, producing more comparatives ( $z = 14.45$ ) and superlatives ( $z = 16$ ) in the *split* or *close* conditions than in the baseline *far* condition. It does not, however, capture the peak in comparative use in the *split* condition. This suggests that our model is simulating the human strategy at some level, but that more subtle patterns require further attention.

**Negations** Humans used more negations when the colors were closer together. Our pragmatic speaker’s use of negation shows the same relationship to the context ( $z = 8.55$  and  $z = 16.61$ , respectively).



model	accuracy (%)	perplexity
$L_0$	83.30	1.73
$L_1 = L(S_0)$	80.51	1.59
$L_2 = L(S(L_0))$	83.95	1.51
$L_a = L_0 \cdot L_1$	84.72	1.47
$L_b = L_0 \cdot L_2$	83.98	1.50
$L_e = L_a \cdot L_b$	<b>84.84</b>	<b>1.45</b>
human	90.40	
<hr/>		
$L_0$	85.08	1.62
$L_e$	<b>86.98</b>	<b>1.39</b>
human	91.08	

Table 3: Accuracy and perplexity of the base and pragmatic listeners and various blends (weighted averages, denoted  $A \cdot B$ ). Top: dev set; bottom: test set.

**WordNet specificity** Humans used more “high specificity” words (by WordNet hypernymy depth) when the colors were closer together. Our pragmatic speaker showed a similar effect ( $z = 2.65, p = 0.008$  and  $z = 2.1, p = 0.036$ , respectively).

## 5.2 Listener accuracy

Table 3 shows the accuracy and perplexity of the base listener  $L_0$ , the pragmatic listeners  $L_1$  and  $L_2$ , and the blended models  $L_a$ ,  $L_b$ , and  $L_e$  at resolving the human-written color references. Accuracy differences are significant<sup>9</sup> for all pairs except  $L_2/L_b$  and  $L_a/L_e$ . As we expected, the speaker-based  $L_1$  alone performs the worst of all the models. However, blending it with  $L_0$  doesn’t drag down  $L_0$ ’s performance but rather produces a considerable improvement compared to both of the original models, consistent with our expectation that the listener-based and speaker-based models have complementary strengths.

We observe that  $L_2$  significantly outperforms its own base model  $L_0$ , showing that pragmatic reasoning on its own contributes positively. Blending the pragmatic models with the base listener also improves over both individually, although not significantly in the case of  $L_b$  over  $L_2$ . Finally, the most effective listener combines both pragmatic models with the base listener. Plotting the number of ex-

<sup>9</sup> $p < 0.012$ , approximate permutation test (Padó, 2006) with Bonferroni correction, 10,000 samples.

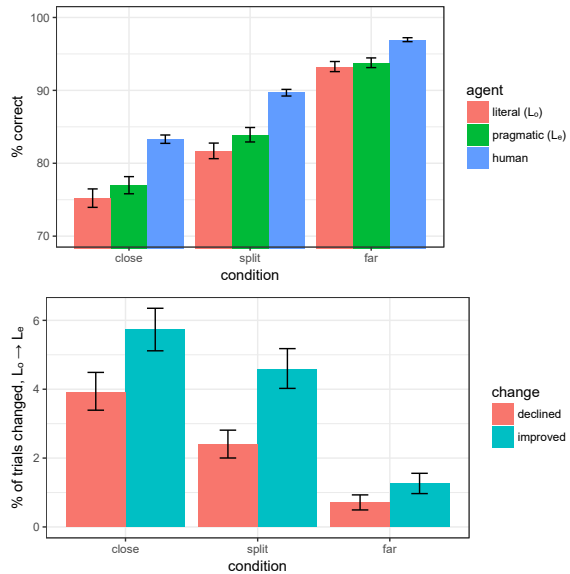


Figure 4: Human and model reference game performance (top) and fraction of examples improved and declined from  $L_0$  to  $L_e$  (bottom) on the dev set, by condition.

amples changed by condition on the dev set (Figure 4) reveals that the primary gain from including the pragmatic models is in the *close* and *split* conditions, when the model has to distinguish highly similar colors and often cannot rely only on basic color terms. On the test set, the final ensemble improves significantly<sup>10</sup> over the base model on both metrics.




## 6 Model analysis

Examining the full probability tables for various dev set examples offers insight into the value of each model in isolation and how they complement each other when blended together. In particular, we see that the listener-based ( $L_2$ ) and speaker-based ( $L_1$ ) pragmatic listeners each overcome a different kind of “blind spot” in the neural base listener’s understanding ability.




First, we inspect examples in which  $L_2$  is superior to  $L_0$ . In most of these examples, the alternative utterances sampled from  $S_0$  for one of the referents  $i$  fail to identify their intended referent to  $L_0$ . The pragmatic listener interprets this to mean that referent  $i$  is inherently difficult to refer to, and it compensates by increasing referent  $i$ ’s probability.

This is beneficial when  $i$  is the true target. The




<sup>10</sup> $p < 0.001$ , approximate permutation test, 10,000 samples.

$L_0$			
<b>blue</b>	9	<b>91</b>	<1
true blue	<b>11</b>	<b>89</b>	<1
light blue	<1	> <b>99</b>	<1
brightest	<1	> <b>99</b>	<1
bright blue	<1	> <b>99</b>	<1
red	<1	1	<b>99</b>
purple	<1	2	<b>98</b>



  

$S_1$			
<b>blue</b>	41	19	<1
true blue	<b>47</b>	19	<1
light blue	<b>5</b>	<b>20</b>	<1
brightest	<1	<b>20</b>	<1
bright blue	2	<b>20</b>	<1
red	1	2	<b>50</b>
purple	5	1	<b>50</b>


  

$L_2$			
<b>blue</b>	<b>68</b>	32	<1
$S_0$	<i>5.71</i>	<i>7.63</i>	<i>0.01</i>
$L_1$	43	<b>57</b>	<1
$L_a$	<b>50</b>	<b>50</b>	<1
$L_b$	<b>68</b>	32	<1
$L_e$	<b>59</b>	41	<1

$L_0$			
<b>drab green not the bluer one</b>	<1	<1	> <b>99</b>
gray	<b>96</b>	4	<1
blue dull green	<b>24</b>	<b>76</b>	<1
blue	<1	> <b>99</b>	<1
bluish	<1	> <b>99</b>	<1
green	4	1	<b>95</b>
yellow	<1	<1	> <b>99</b>

$S_1$			
<b>drab green not the bluer one</b>	1	<1	<b>34</b>
gray	<b>58</b>	5	<1
blue dull green	<b>27</b>	28	<1
blue	2	<b>32</b>	<1
bluish	1	<b>32</b>	<1
green	10	3	<b>33</b>
yellow	<1	<1	<b>34</b>




$L_2$			
<b>drab green not the bluer one</b>	5	<1	<b>95</b>
$S_0 (\times 10^{-9})$	<b>5.85</b>	<i>0.38</i>	< <i>0.01</i>
$L_1$	<b>94</b>	6	<1
$L_a$	<b>92</b>	6	2
$L_b$	8	1	<b>91</b>
$L_e$	<b>63</b>	6	32

Figure 5: Conditional probabilities (%) of all agents for two dev set examples. The target color is boxed, and the human utterances (*blue*, *drab green not the bluer one*) are **bolded**. Boxed cells for alternative utterances indicate the intended target; largest probabilities are in **bold**.  $S_0$  probabilities (*italics*) are normalized across all utterances. Sample sizes are reduced to save space; here,  $m = 2$  and  $n = 1$  (see Section 4.3).

left column of Figure 5 shows one such example: a context consisting of a somewhat prototypical blue, a bright cyan, and a purple-tinged brown, with the utterance *blue*. The base listener interprets this as referring to the cyan with 91% probability, perhaps due to the extreme saturation of the cyan maximally activating certain parts of the neural network. However, when the pragmatic model takes samples from  $S_0$  to probe the space of alternative utterances, it becomes apparent that indicating the more ordinary blue to the listener is difficult: for the utterances chosen by  $S_0$  intending this referent (*true blue*, *light blue*), the listener also chooses the cyan with >89%

confidence.

Pragmatic reasoning overcomes this difficulty. Only two utterances in the alternative set (the actual utterance *blue* and the sampled alternative *true blue*) result in any appreciable probability mass on the true target, so the pragmatic listener’s model of the speaker predicts that the speaker would usually choose one of these two utterances for the prototypical blue. However, if the target were the cyan, the speaker would have many good options. Therefore, the fact that the speaker chose *blue* is interpreted as evidence for the true target. This mirrors the back-and-forth reasoning behind the definition of conver-

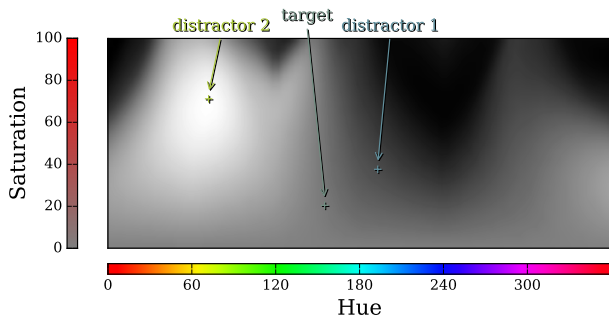


Figure 6:  $L_0$ 's log marginal probability density, marginalizing over  $V$  (value) in HSV space, of color conditioned on the utterance *drab green not the bluer one*. White regions have higher probability. Labeled colors are the three colors from the right column of Figure 5.

sational implicature (Grice, 1975).

This reasoning can be harmful when  $i$  is one of the distractors: the pragmatic listener is then in danger of overweighting the distractor and incorrectly choosing it. This is a likely reason for the small performance difference between  $L_0$  and  $L_2$ . Still, the fact that  $L_2$  is more accurate overall, in addition to the negative value of  $\beta_b$  discovered in grid search, suggests that the pragmatic reasoning provides value on its own.

However, the final performance improves greatly when we incorporate both listener-based and speaker-based agents. To explain this improvement, we examine examples in which both listener-based agents  $L_0$  and  $L_2$  give the wrong answer but are overridden by the speaker-based  $L_1$  to produce the correct referent. The discrepancy between the two kinds of models in many of these examples can be explained by the fact that the speaker takes the context as input, while the listener does not. The listener is thus asked to predict a region of color space from the utterance *a priori*, while the speaker can take into account relationships between the context colors in scoring utterances.

The right column of Figure 5 shows an example of this. The context contains a grayish green (the target), a grayish blue-green (“distractor 1”), and a yellowish green (“distractor 2”). The utterance from the human speaker is *drab green not the bluer one*, presumably intending *drab* to exclude the brighter yellowish green. However, the  $L_0$  listener must choose a region of color space to predict based on the utter-

ance alone, without seeing the other context colors.

Figure 6 shows a visualization of the listener’s prediction. The figure is a heatmap of the probability density output by the listener, as a function of hue and saturation in HSV (hue, saturation, value) space. We use HSV here, rather than the RGB coordinate system used by the model, because the semantic constraints are more clearly expressed in terms of hue and saturation components: the color should be *drab* (low-saturation) and *green* (near 120 on the hue spectrum) but not *blue* (near 240 in hue). The utterance does not constrain the value (roughly, brightness–darkness) component, so we sum over this component to summarize the 3-dimensional distribution in 2 dimensions.

The  $L_0$  model correctly interprets all of these constraints: it gives higher probability to low-saturation colors and greens, while avoiding bluer colors. However, the result is a probability distribution nearly centered at distractor 2, the brighter green. In fact, if we were not comparing it to the other colors in the context, distractor 2 would be a very good example of a drab green that is not bluish.

The speaker  $S_0$ , however, produces utterances conditioned on the context; it has successfully learned that *drab* would be more likely as a description of the grayish green than as a description of the yellowish one in this context. The speaker-based listener  $L_1$  therefore predicts the true target, with greater confidence than  $L_0$  or  $L_2$ . This prediction results in the blends  $L_a$  and  $L_e$  preferring the true target, allowing the speaker’s perspective to override the listener’s.

## 7 Related work

Prior work combining machine learning with probabilistic pragmatic reasoning models has largely focused on the speaker side, i.e., generation. Golland et al. (2010) develop a pragmatic speaker model,  $S(L_0)$ , that reasons about log-linear listeners trained on human utterances containing spatial references in virtual-world environments. Tellex et al. (2014) apply a similar technique, under the name *inverse semantics*, to create a robot that can informatively ask humans for assistance in accomplishing tasks. Meo et al. (2014) evaluate a model of color description generation (McMahan and Stone, 2015) on the

color reference data of Baumgaertner et al. (2012) by creating an  $L(S_0)$  listener. Monroe and Potts (2015) implement an end-to-end trained  $S(L(S_0))$  model for referring expression generation in a reference game task. Many of these models require enumerating the set of possible utterances for each context, which is infeasible when utterances are as varied as those in our dataset.

The closest work to ours that we are aware of is that of Andreas and Klein (2016), who also combine neural speaker and listener models in a reference game setting. They propose a pragmatic speaker,  $S(L_0)$ , sampling from a neural  $S_0$  model to limit the search space and regularize the model toward human-like utterances. We show these techniques help in listener (understanding) tasks as well. Approaching pragmatics from the listener side requires either inverting the pragmatic reasoning (i.e., deriving a listener from a speaker), or adding another step of recursive reasoning, yielding a two-level derived pragmatic model  $L(S(L_0))$ . We show both approaches contribute to an effective listener.

## 8 Conclusion

In this paper, we present a newly-collected corpus of color descriptions from reference games, and we show that a pragmatic reasoning agent incorporating neural listener and speaker models interprets color descriptions in context better than the listener alone.

The separation of referent and utterance representation in our base speaker and listener models in principle allows easy substitution of referents other than colors (for example, images), although the performance of the listener agents could be limited by the representation of utterance semantics as a Gaussian distribution in referent representation space. Our pragmatic agents also rely on the ability to enumerate the set of possible referents. Avoiding this enumeration, as would be necessary in tasks with intractably large referent spaces, is a challenging theoretical problem for RSA-like models.

Another important next step is to pursue multi-turn dialogue. As noted in Section 2, both participants in our reference game task could use the chat window at any point, and more than half of dyads had at least one two-way interaction. Dialogue agents are more challenging to model than

isolated speakers and listeners, requiring long-term planning, remembering previous utterances, and (for the listener) deciding when to ask for clarification or commit to a referent (Lewis, 1979; Brown and Yule, 1983; Clark, 1996; Roberts, 1996). We release our dataset<sup>11</sup> with the expectation that others may find interest in these challenges as well.

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<sup>11</sup><https://cocolab.stanford.edu/datasets/colors.html>

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