1. The membrane, owing to the fact that there is no glazed cap to the tube, goes right up to the ends.*

2. There is no necessity to have pressure-tight joints.

3. Greater speed in working; the actual experiment takes from 2 to 3 hours only.

4. The form of tube used is such that it will withstand very high pressures—it may be of interest to mention that one of the tubes and membrane stood a pressure of 120 atmospheres without apparent harm.

We publish this preliminary notice as it will be some time before the experiments can be continued—a new apparatus has to be cast, and new porcelain tubes are required. We hope, by means of the new apparatus, to reduce greatly the guard-ring leak.

We are glad to avail ourselves of this opportunity to thank Mr. W. C. D. Whetham for the kindly interest he has taken in the research and Mr. H. Darwin for designing the pressure apparatus.

"Colours in Metal Glasses and in Metallic Films." By J. C. MAXWELL GARNETT, B.A., Trinity College, Cambridge. Communicated by Professor LARMOR, Sec. R.S. Received April 19,—Read June 2, 1904.

(Abstract.)

The first part of the paper is devoted to coloured glasses. The phenomena which it seeks to explain were observed by Siedentopf and Zsigmondy.† Expressions are first obtained for the electric vector of the light scattered from a small metal sphere when a train of plane polarised light falls upon it, the investigation following Lord Rayleigh.‡ By means of these expressions it is proved, from the diagrams and statements given by Siedentopf and Zsigmondy, that the metal particles which they observed in gold glass are spherical in shape when the diameters are less than 10^{-5} cm. The fact that such particles are spherical throws light on the manner in which metals crystallise out of solution, the particles taking first a spherical form under the action of surface tension, and later, when they become too large for the forces of surface tension to overcome the crystallic forces, becoming amenable to the latter. Mr. G. T. Beilby has previously arrived at similar conclusions.§

An investigation into the optical properties of a transparent medium

* *Cf.* Adie, *loc. cit.*

† 'Ann. der Phys.,' January, 1903.

‡ 'Phil. Mag.,' vol. 44, 1897, and 'Collected Papers,' vol. 4, p. 305.

§ 'Brit. Assoc. Report,' Southport, 1903.

containing metal spherules, so that the average distance between two neighbouring spheres is considerably less than a wave-length of light, is next undertaken. It is shown that every such medium has a perfectly definite colour by transmitted light, depending only on the optical constants of the metal of which the spheres are made, on the refractive index of the substance in which they are embedded, and on the quantity of metal, but not on the size or distance apart of the spheres.

The intensity of the absorption of light of each colour is proportioned to μ, the volume of metal per unit volume of medium. It is calculated, by means of the metal constants given by Drude,* that, with glass of refractive index 1·56, gold glass is more red than yellow, silver glass a little more yellow than red, copper glass considerably more red than yellow, and "potassium-sodium" glass much more blue than yellow, provided always that the average distance between two neighbouring particles of metal in the glass be considerably less than one wave-length; in which case, as stated above, the particles must be spherical. Metal glasses for which this provision is satisfied will be called "regular."

It is next proved that the presence of metal spheres accounts for the optical properties of regular gold ruby glass, and that the irregularities in the effects of colour and polarisation, sometimes exhibited by gold glasses, are due either to excessive distance between adjacent gold particles or to excessive size of such particles—the latter, however, involving the former.

Experiments are described, proving that this regular colour can be produced in a colourless metal glass, containing the metal in solution (which is the state in the manufacture of gold or copper ruby glass before the second heating) by the β-radiation from radium. Thus, a piece of clear gold glass and a piece of clear soda glass were exposed to the emanation for two days, when the gold glass had acquired an unmistakable pink tint, while the soda glass had turned an intense blue-violet.

In the second part of the paper, the optical properties of media built up out of metal spheres as before, but now so that the volume of metal may have any value between zero and unity, instead of remaining very small, as in metal glasses, are investigated. The changes of colour, and the final change to almost complete transparency, observed by Mr. G. T. Beilby† in gold and silver films, are accounted for. Explanations are also given of the changes of colour on heating, observed by Professor R. W. Wood‡, in potassium and sodium films deposited on the insides of exhausted glass bulbs. The

* 'Phys. Zeitschrift,' January, 1900.

† 'Roy. Soc. Proc.,' vol. 72, 1903, p. 226.

‡ 'Phil. Mag.,' 1902, p. 396.

increase in strength of colour, which was generally observed in the light transmitted through these films when the plane of polarisation of obliquely incident light was changed from that of incidence to a perpendicular position is accounted for.

In Part III some evidence is brought to show that the allotropic silvers obtained by Carey Lea* are particular cases of the media which have been considered in the second part.

"The General Theory of Integration." By W. H. Young, Sc.D., St. Peter's College, Cambridge. Communicated by Dr. E. W. Hobson, F.R.S. Received April 23,—Read May 19, 1904.

(Abstract.)

The paper begins with a recapitulation of the well-known definitions of integration and of upper and lower integration (intégral par excès, par défaut; oberes, unteres Integral). The theorem on which the Darboux definition of upper (lower) integration is founded is stated and proved in the following form :—

Given any small positive quantity e_1, we can determine a positive quantity e, such that, if the fundamental segment S be divided up in any manner into a finite number of intervals, then, provided only the length of each interval is less than e, the upper summation of any function over these intervals differs by less than e_1 from a definite limiting value (the upper integral).

Next follows a discussion as to whether it is admissible to adopt a more general mode of division of the fundamental segment than that used by Riemann, Darboux and other writers, when forming summations (upper, lower summations), defining as limit the integral (upper, lower integral), of a function over the fundamental segment. It is shown by examples first that the restriction as to the finiteness of the number of intervals into which the fundamental segment is divided cannot be removed without limitations; but that it can be removed, provided the content of the intervals is always equal to that of the fundamental segment. Secondly it is shown that the error introduced by taking the summation over an infinite number of intervals whose content is less than that of the fundamental segment, is not in general corrected by adding to the summation the content of the points external to the intervals multiplied by corresponding value (upper, lower limit) of the function. Similarly it is shown that the more

* 'Amer. Journ. of Science,' 1886.