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Combating the Dimensionality of Nonlinear MIMO Amplifier Predistortion by Basis Pursuit

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***Abstract*—A general description of nonlinear dynamic MIMO systems, given by Volterra series, has significantly larger complexity than SISO systems. Modeling and predistortion of MIMO amplifiers consequently become unfeasible due to the large number of basis functions. We have designed digital predistorters for a MIMO amplifier using a basis pursuit method for reducing model complexity. This method reduces the numerical problems that appear in MIMO Volterra predistorters due to the large number of basis functions. The number of basis functions was reduced from 1402 to 220 in a 2x2 MIMO amplifier and from 127 to 13 in the corresponding SISO case. Reducing the number of basis functions caused an increase of approximately 1 dB of model error and adjacent channel power ratio.**

Keywords*—Digital pre distortion (DPD), MIMO power amplifiers, LASSO, basis pursuit, power amplifier linearization.

I. Introduction

Digital pre-distortion (DPD) is one of the most commonly used techniques to linearize radio frequency amplifiers. The objective of DPD is to invert the nonlinear function of the amplifier, such that the cascade connection of predistorter and power amplifier is a linear function. Considerable amount of research has been dedicated to the predistortion of single-input single-output (SISO) amplifiers. However, there is a growing interest in amplifiers handling multiple signals simultaneously e.g., concurrent dual-band amplifiers [1] and multiple-input multiple-output (MIMO) amplifiers [2], [3].

Digital predistortion of both concurrent dual band [1] and MIMO amplifiers [2], [3] have been studied previously. The former was limited to low nonlinear order, while for the latter, the predistorter models used were subsets of the MIMO Volterra series.

Amplifiers are weakly nonlinear and can be described by Volterra series. A Volterra series is a complete description of a nonlinear dynamic system, but has the drawback of large numbers of basis functions, which limits its practical use. In the Volterra series of MIMO systems, the number of basis functions is even larger compared to SISO systems. The increase in the number of basis functions for MIMO systems is not proportional to the number of input/output signals. The increased number of basis functions is due to 'cross-kernels' which contain combinations of different input signals due to linear and nonlinear cross-talk. These cross-kernels have lower symmetry properties than self-kernels [4] which requires more basis functions. As a consequence, the use of MIMO Volterra series exacerbates the need for model reduction techniques when compared to SISO series.

Sparse estimation techniques have attracted the attention since they can be used to discriminate between regressors and to trade-off model complexity and accuracy [5]. Such properties make them candidates for modeling and predistortion of SISO power amplifiers [6], [7]. In this paper, a basis pursuit approach is used to design MIMO Volterra predistorters applying sparse estimation methods. Previous work with basis pursuit for finding the behavioral amplifier models [6] or predistorter models [7] have dealt with the SISO case; we consider the MIMO case. The work in [8] prunes a polynomial model for a concurrent dual band amplifier. However, the pruning technique is in earlier stages of development and there is no proof of convergence or optimality. In this work, we employ a sparse estimation technique which is mature and optimal for the problem to solve [9]. In previous studies the initial set of basis functions has been memory polynomials [6], [8] or lower order Volterra models [7]. In contrast to these works, we use a complete Volterra model as the initial set since the inverse of the nonlinear dynamic system is in general more complex than the direct system [10], which suggests the use of the complete Volterra series.

II. Theory

A. Volterra Model

A $K \times K$ nonlinear dynamic MIMO system can be described as K MISO (multiple input single ouput) systems [11]. The Volterra series for a MIMO system with complex baseband signals is described as [4]:

$$
\begin{aligned}
y_i(n) &= y_i^{(1)}(n) + y_i^{(3)}(n) + y_i^{(5)}(n) + \dots \\
y_i^{(1)}(n) &= \sum_{k_1=1}^{K} \sum_{m_1=0}^{\infty} h_{k_1}(m_1) x_{k_1}(n-m_1), \\
y_i^{(3)}(n) &= \sum_{k_1,k_2,k_3=1}^{K} \sum_{m_1,m_2,m_3=0}^{\infty} h_{k_1,k_2,k_3}(m_1,m_2,m_3) \dots \\
&\quad x_{k_1}(n-m_1) x_{k_2}(n-m_2) x_{k_3}^*(n-m_3), \\
y_i^{(5)}(n) &= \sum_{k_1,\dots,k_5=1}^{K} \sum_{m_1,\dots,m_5=0}^{\infty} h_{k_1,\dots,k_5}(m_1,\dots,m_5) \dots \\
&\quad x_{k_1}(n-m_1) x_{k_2}(n-m_2) x_{k_3}(n-m_3) \dots \\
&\quad x_{k_4}^*(n-m_4) x_{k_5}^*(n-m_5),
\end{aligned} \tag{1}
$$

where $i = 1, \dots, K$, and $x_i(n)$, and $y_i(n)$ denote the complex-valued baseband input and output signals, respectively, on the

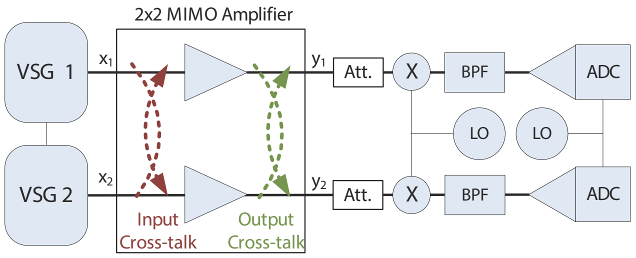

Fig. 1. Measurement system used to perform the experiments of a 2x2 MIMO power amplifier.

i-th port at n-th instance. Further, the $y_i^{(p)}(n)$ represents the contribution of the p-th nonlinear order at the i-th received signal, and $\{h_{k_1,\ldots}(m_1,\ldots)\}$ are the Volterra kernels. Equation (1) not only contains self-kernels i.e., $x_1(n)x_1(n)x_1^*(n-1)$, it also includes cross-kernels. By cross-kernels, we refer to the interaction of signals coming from different channels, e.g., $x_1(n)x_1(n)x_2^*(n-2)$. This is one of the key differences between MIMO and SISO Volterra series, as MIMO requires the information of all signals appearing in all channels.

B. Metrics for evaluation

The performance of the predistorter is evaluated in terms of normalized mean-square error (NMSE) and the adjacent channel power leakage ratio (ACPR) given by [12]:

$$\text{NMSE} = \frac{\int \Phi_e(f)\,df}{\int \Phi_y(f)\,df}, \tag{2}$$

where $\Phi_y(f)$ is the power spectrum of the measured output signal $y(n)$. The integration is carried out across the complete available bandwidth. The ACPR is computed as [12]:

$$\text{ACPR} = \frac{\int_{\text{adj. ch.}} \Phi_y(f)\,df}{\int_{\text{ch.}} \Phi_y(f)\,df}, \tag{3}$$

where the numerator integral is made over the adjacent channel with the largest amount of power and the denominator integrates over the channel band.

III. Experimental

A. Test setup

The measurement system depicted in Fig. 1 is composed of 2 R&S SMBV100A vector signal generators (VSGs), a downconvertion chain formed with 2 mixers, bandpass filters and dual channel ADQ 214 SP Devices analog-to-digital converter (ADC). The generators have baseband coherency and RF coherency. The coherency is needed to have full control of the signal at the RF level, required in a DPD scheme. The setup is automated, with a personal computer, allowing full control of the excitation signals.

For the MIMO case, the device under test (DUT) was formed of two ZVE-8G+ power amplifiers sandwiched by two coupling stages of 20 dB at its input and output. The coupling stages were made using dual couplers connecting both lines causing input and output cross-talks. Two different multi tone signals, x_1 and x_2, each with 4.8 MHz bandwidth and 20000 samples were created in baseband, upconverted to 1.8 GHz and applied to the MIMO amplifier.

TABLE I. Number of basis functions in a SISO and 2x2 MIMO system described by Volterra series (1)

Nonlinear order	Memory used	# basis (SISO)	# basis (2x2 MIMO)
1	$m_1 \leq 5$	6	12
3	$m_{1,2,3} \leq 3$	40	240
5	$m_{1,2,\ldots,5} \leq 2$	60	720
7	$m_{1,2,\ldots,7} \leq 1$	20	400
9	$m_{1,2,\ldots,9} = 0$	1	30
		Total = 127	Total = 1402

For the SISO case, the setup only uses a single amplifier, without cross-talks, and a single path to the acquisition (cf. Fig. 1).

B. System Identification

We denote the complex-valued i-th baseband input signal of the MIMO amplifier $\mathbf{x}_i = [x_i(1), x_i(2), \ldots, x_i(N)]^T$, with T being the transpose operator. N the number of samples available. The j-th output is denoted $\mathbf{y}_j = [y_j(1), y_j(2), \ldots, y_j(N)]^T$. Using this notation, the Volterra model in (1) can be written as:

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi} & 0 & \ldots & 0 \\ 0 & \mathbf{\Phi} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \mathbf{\Phi} \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_K \end{bmatrix}$$

where $\mathbf{w}_i$ denotes the vector of model parameters, corresponding to the Volterra kernels that we wish to estimate, and $\mathbf{\Phi} = f(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K)$ is the regression matrix, whose columns are basis functions of the MIMO Volterra series. For SISO systems, this reduces to $\mathbf{\Phi} = f(\mathbf{x}_1)$. Note that in MIMO Volterra, the regression matrix $\mathbf{\Phi}$ is formed using all input signals, as described by (1), which predicts the presence of the cross kernels in the basis functions.

To estimate a predistorter, the input and output signals are interchanged, following the principle of the indirect learning architecture, in which the post-distorter function is the same as the predistorter [13]. Finally, a least square (LS) technique is used to compute the model parameters,

$$\hat{\boldsymbol{\theta}}_i = (\mathbf{\Psi}^H \mathbf{\Psi})^{-1} \mathbf{\Psi}^H \mathbf{x}_i. \tag{4}$$

where H denotes the Hermitian operator, $\mathbf{\Psi} = f(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_K)$ contains the basis functions of the post-distorter and $\hat{\boldsymbol{\theta}}_i$ are the parameters of the post-distorter. This process yields models that are *dense* i.e. all parameters have some weight in the model output. Hence, MIMO Volterra predistortion is computationally expensive or unfeasible, since it employs a large set of parameters, limiting its applicability to lower nonlinear order and memory depth.

1) Sparse Estimation: To seek for an efficient (minimal) set of basis functions, we use a basis pursuit approach using *sparse* estimation techniques in the post-distorter estimation problem. These methods produce sparse models in which many parameters are forced to be zero while retaining the modeling performance. The most significant basis functions are then used. The LS technique can then be used for identifying the parameters of the sparse model.

The sparse technique solve the post-distorter problem given by [14]:

$$\begin{aligned} \underset{\{\boldsymbol{\theta}_i\}_i}{\text{minimize}} \quad & \sum_{i=1}^{K} \|\mathbf{x}_i - \boldsymbol{\Psi}\boldsymbol{\theta}_i\|_2 \,, \\ \text{subject to} \quad & \sum_{n=1}^{N} R_n |\theta_i(n)| \leq \gamma_i, \qquad i = 1, \ldots, K. \end{aligned} \tag{5}$$

Here $\|\cdot\|_2$ denotes the ℓ_2 norm, γ_i is a trade-off parameter which is described later, $\theta_i(n)$ is the n-th parameter of the vector $\boldsymbol{\theta}_i$. R_n is a scalar normalizing factor required since the parameters in the Volterra series have different scales of magnitude. R_n is set as the sample variance of the n-th column of the regression matrix $\boldsymbol{\Psi}$ [14],

$$R_n = \frac{1}{N} \boldsymbol{\varphi_n}^H \boldsymbol{\varphi_n},$$

where $\boldsymbol{\varphi_n}$ denotes the n-th column of the matrix $\boldsymbol{\Psi}$. Such a normalization is recommended for basis selection [14]. The matrix $\boldsymbol{\Psi}$ is constructed using the Volterra model (1) with the nonlinear orders and memory depths given in Table I. Such settings are chosen to include the linear and nonlinear dynamics of the MIMO and SISO amplifiers. The problem in (5) can be solved reliably using convex solvers [9]. The constants γ_i are used to trade-off sparsity against model fitting.

Once the problem in (5) is solved, the basis are discriminated by its amplitude contribution. Hence, basis functions whose parameters are lower than 10^{-4} are eliminated. However, the selection of this threshold is not critical as the amplitude of the parameters varies by orders of magnitude; different thresholds do not affect the results significantly.

IV. Results

Cross-talk is an important difference between MIMO and SISO amplifiers. The cross correlation can be used as a rough metric for determining cross-talk levels between several ports of a MIMO amplifier. The maximum of cross correlation of the j-th output and the i-th input in a MIMO amplifier is a measure of the cross-talk between these ports. However, this method can only be used when the correlation of the input signals is low, or preferably when they are orthogonal, $R_{x_i x_j} = 0$. This metric for the 2×2 MIMO amplifier resulted in a cross-talk of 17 dB, which is expected from 2 coupling stages of 20 dB each.

The Paretto bound presented in Fig. 2 shows a trade-off between complexity and accuracy for both output signals in the 2x2 MIMO amplifier; similar trends were found for the SISO case. The curves in Fig. 2 are encountered solving (5) for a set of values of γ_i. As shown in Fig. 2, a reasonable good model performance can be achieved at a fraction of the total number of basis functions required in the original Volterra model. Also shown in Fig. 2 is the performance of a linear and a full Volterra post-distorter computed by employing only a LS technique.

Table II shows the number of basis functions of the Volterra models in the SISO and MIMO cases and the reduced counterparts. The number of basis functions in the full MIMO model is more than an order of magnitude larger than of the full SISO model, this is also indicated in Table I and illustrates the need of model reduction techniques.

TABLE II. Comparison of the SISO and MIMO predistorters

Model	# of basis	NMSE	ACPR
2×2 Full MIMO Volterra of Table I	1402	-	-
2×2 reduced MIMO Volterra	220	-42 / -43 dB	-52 / -54 dB
Full SISO Volterra of Table I	127	-44 dB	-54 dB
Reduced SISO Volterra	13	-43 dB	-55 dB

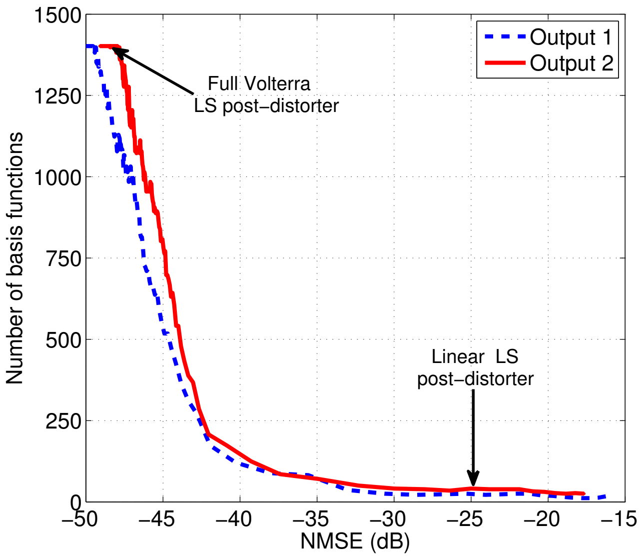

Fig. 2. Paretto bound for the NMSE and Number of parameters of the post distorter function in (5)

Using Fig. 2, we select the predistorter model such that NMSE ≤ -42 dB, which gives a set of 220 basis functions required. The dimension of the estimation problem is reduced by a factor of $1402/220 \approx 6.37$. For the SISO system, the reduction is $127/13 \approx 9.7$. Table II indicates the performances of the predistorters. For the SISO case the reduction in number of basis functions does not degrade the model performance. In the MIMO case, the reduced model, performs to acceptable levels but, the full Volterra (cf. to Table I) could not be evaluated due to numerical instability.

Fig. 3 shows the condition numbers of the regression matrices for the two systems studied, a SISO and a 2×2 MIMO as a function of the nonlinear order considered. Fig. 3 shows that the condition number of the full Volterra systems is large, which may cause numerical instabilities. The reduced models (which keep some terms of 9-th nonlinear order) have lower condition numbers which gives better numerical properties when using LS techniques.

Using the selected set of basis functions in the reduced models, we identify a predistorter using LS techniques. An iterative process is applied to compute the predistorter parameters [12]. Fig. 4 shows the power spectral density of the two inputs and outputs of the amplifier when using the reduced set of basis selected in the 2 × 2 MIMO amplifier. Clearly the amount of leakage to adjacent channels in both outputs is reduced substantially. For the MIMO amplifier, the reduction

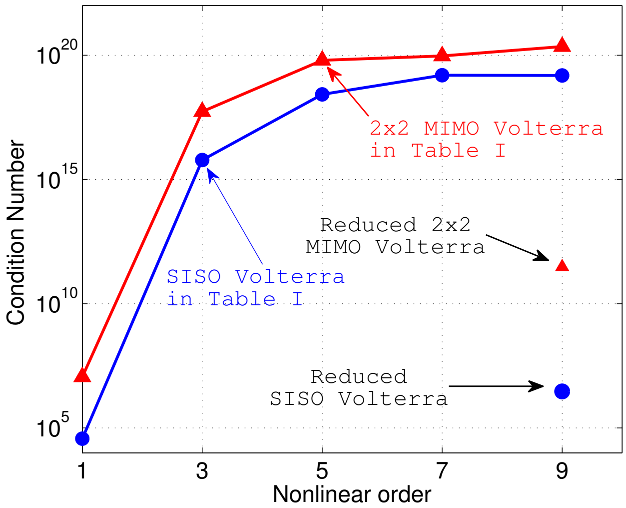

Fig. 3. Condition number of the regression matrices in the LS techinque.

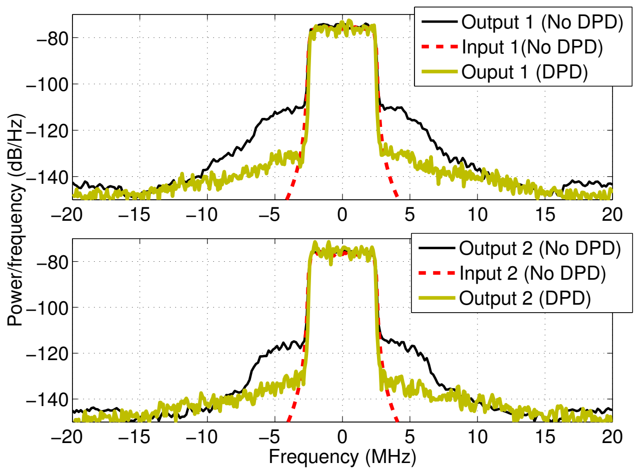

Fig. 4. Power spectral density of the inputs and outputs of the 2x2 MIMO amplifier

in ACPR measured in both channels was in excess of 20 dB. The corresponding reduction in the SISO case was in excess of 22 dB.

V. Discussions

This paper compares SISO and MIMO Volterra models when used for predistortion of amplifiers. The number of basis functions in MIMO Volterra series is larger than SISO Volterra series; this, besides increasing the complexity may cause numerical instabilities when computing the predistorter using linear LS techniques.

A basis pursuit approach is outlined in this paper to design MIMO Volterra predistorters. This yields predistorters which combat the numerical instabilities of the MIMO Volterra series, and have lower model complexity. Hence, the basis pursuit employed here is a suitable model reduction technique for predistortion of MIMO amplifiers.

NMSE and ACPR evaluation of the reduced MIMO Volterra predistorters show that such models perform to acceptable levels and are competitive candidates for predistortion of MIMO amplifiers.

In future research we plan to investigate the effect of basis pursuit on DUTs with only input or only output crosstalk. We also plan to investigate the effect on predistorter performance from different initial sets of basis functions e.g., various memory polynomials.

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