The solution of this set of equations is

$b = 0\cdot540814\ldots, \quad c = -0\cdot5, \quad d_0 = -0\cdot061224\ldots,$
$d_1 = 0\cdot061224\ldots, \quad d_2 = 0\cdot061224\ldots.$

The values of u_1 and u_2 calculated from these are each 0·12448 ... and the central value is 0·13804. The approximation again has precisely the symmetry of the analytical solution, and the error has been reduced to 1·04 per cent.

Applying the Hermite method to this case, we find that the approximation is *exactly* the same as that given by the spline!

Concluding remarks

These simple experiments show that the method is potentially useful. Work is proceeding on an examination of more complicated cases, and on an error analysis which may, possibly, give us an *a priori* estimate of the size of interval necessary to achieve a prescribed accuracy. The gain to be achieved by *un*equal intervals could also be explored.

As regards the relative merit of the two methods, the dominant consideration may well be that, with a larger number of intervals, the number of equations in the Hermite method, $2(n + 1)$, is considerably greater than that in the spline method, $n + 3$. Moreover, the matrix of the equation for the Hermite method, although a band-matrix, has a more complex structure, and in the forward elimination more than one multiplier is required per equation.

Book Review

Combinatorial methods in the theory of stochastic processes, by LAJOS TAKAÇS, 1967; 262 pp. (London: *Wiley*, 96s.)

It is not obvious, even to those with some knowledge of combinatorial probability or of stochastic processes, what a book of this title might be expected to contain. Except in its origin the combinatorial aspect has little to do with combinations, partitions and so on of finite collections of objects. It is more related to combinatorial methods used in the study of fluctuations of sums of independent random variables. These methods appeared originally in the work of Sparre-Anderson and Spitzer in the middle nineteen-fifties.

The author's combinatorial methods reduce in fact to basically one method which depends on his very interesting theorem stated on page 1. At this point no reference to the proof is given but fortunately for the reader the proof does appear a few pages later. After this, the book is mostly a series of machine-like analytical applications of this theorem to a class of problems in stochastic processes.

This basic theorem is about non-decreasing functions whose derivative vanishes almost everywhere. It is essentially combinatorial in nature and in Chapter 1 the author demonstrates that it is a generalisation of the classical ballot theorem due to Bertrand in 1887, which in its most elementary form answers the following problem. In a ballot with two candidates, candidate A obtains a votes and B obtains b votes where b does not exceed a; what is the probability that throughout the counting candidate A leads candidate B? (The answer is the ratio of $a - b$ to $a + b$.) A probabilist will recognise the connection of this problem with the random walk, and it is this connection which is generalised and exploited by the author.

If one studies the probability theory of queues, storage systems and insurance risk (these are well established fields in applied probability theory) one frequently meets the following stochastic process: it has independent increments, its sample functions are discontinuous and linear with constant slope between successive discontinuities whose signs are opposite to that of the slope, that is the sample functions have a 'saw-tooth' appearance. Perhaps the best known of these is the Takaçs process (after the author) or virtual waiting time process in the theory of queues. In discrete time the corresponding process is a rather special kind of random walk with discrete steps which may assume values among the non-negative integers and minus one.

With relentless vigour the author exploits the application of his basic theorem to problems of the distribution of the supremum and infimum of these processes. These problems arise naturally in the theory of queues, storage and insurance risk.

The final chapter concerning order statistics and statistical problems of tests of the Kolmogorov–Smirnov type, is in a slightly different vein from the remainder of the book. Another extension of the ballot theorem is given and used to obtain many results in this field.

Much of the material is collected from the author's own published results of recent years. The bibliography is really excellent. But in the opinion of this reviewer the book is open to serious criticism in the matter of style and general design. It takes a major effort of concentration to find out what this book is about. While not imputing this motive to the author the impression given is that his main concern is to set out his mathematical thoughts in the most economical way and to leave the reader to his own devices. Theorems and proofs follow in an overpowering succession with little explanation of direction or line of reasoning. For the author the outcome is surely self-defeating, at any rate if his intention is to reach a wide audience. For only the most dedicated and patient of readers will not be put off by the unattractive style. Another point to the author's credit is that there are many exercises all of which have detailed solutions into which he has clearly put a great deal of effort.

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