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Combined analysis of charm-quark fragmentation-fraction measurements

Mykhailo Lisovyi[1], **Andrii Verbytskyi**[2,a], **Oleksandr Zenaiev**[3]

[1] Physikalisches Institut der Universität Heidelberg, Heidelberg, Germany
[2] Max-Planck-Institut für Physik, Munich, Germany
[3] DESY, Hamburg, Germany

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Abstract A summary of measurements of the fragmentation of charm quarks into a specific hadron is given. Measurements performed in photoproduction and deep inelastic scattering in $e^{\pm}p$, pp and e^+e^- collisions are compared, using up-to-date branching ratios. Within uncertainties, all measurements agree, supporting the hypothesis that fragmentation is independent of the specific production process. Averages of the fragmentation fractions over all measurements are presented. The average has significantly reduced uncertainties compared to individual measurements.

1 Introduction

The production of specific charm hadrons has been measured in different regimes and environments: in e^+e^- collisions at B-factories [1–8] and in Z decays [9–13], in $e^{\pm}p$ collisions in photoproduction (PHP) [14,15], deep inelastic scattering (DIS) [16–18] and in pp collisions [19–24].

The fragmentation process is soft and hence can not be calculated with the techniques of perturbative QCD (pQCD). Therefore, these measurements are a necessary ingredient for any QCD prediction of charm-hadron production. In this context, it is important to validate the hypothesis that fragmentation fractions are universal, i.e. independent of the hard production mechanism. Thus, once precisely measured in one experiment, they can be applied in any reaction. Another important check is that the sum of fragmentation fractions of all known weakly decaying charm hadrons is equal to unity, thus checking if all weakly decaying states are known.

To achieve these goals, a comparison of fragmentation-fraction measurements obtained in different production regimes is performed using a combination of individual measurements. Due to independent data sets and different detector types and constructions, the experimental statistical and systematic uncertainties in most cases can be treated as uncorrelated between measurements. However, a careful treatment of correlated uncertainties due to common usage of branching ratios and theory inputs is essential, as in many measurements these are one of the leading uncertainty sources. In the past several combinations of fragmentation-fraction data were performed with fewer inputs: the summary of the charm fragmentation fractions in e^+e^- at the Z resonance [25], the combination of e^+e^- measurements [26,27] as well as the combination of e^+e^- and $e^{\pm}p$ measurements [28]. Compared to those, the present analysis extends to a larger set of measurements, in particular the final measurement in PHP by the ZEUS experiment at HERA [15], the pp measurements from LHCb [19,20], ALICE [21–23], ATLAS [24] and the Λ_c^+ measurements from the BABAR experiment [8]. It uses the up-to-date branching-ratio values [25,29,30], treats correlations of branching-ratio uncertainties and recent theory predictions with reduced uncertainties [31,32] as input.

2 Combination of individual measurements

2.1 Update of input measurements to recent branching ratios

To make separate inputs consistent, the original measurements are corrected to the same up-to-date world averages of branching ratios of the charm-hadron decays, $\mathcal{B}$, summarised in Table 1. Most of the values were taken from Ref. [25]. The $\mathcal{B}(D^{*0} \to D^0\pi^0)$ was calculated from the two most precise measurements [29,30] of $\mathcal{B}(D^{*0} \to D^0\pi^0)/\mathcal{B}(D^{*0} \to D^0\gamma)$ assuming $\mathcal{B}(D^{*0} \to D^0\pi^0) + \mathcal{B}(D^{*0} \to D^0\gamma) = 1$.

[a] e-mail: andrii.verbytskyi@mpp.mpg.de

Table 1 Branching ratios used for calculations. The second uncertainty for the $\mathcal{B}(\Lambda_c^+)$ is the uncertainty of decay branching ratios of daughters. The numbers in the D_s^+ decay branching ratio indicate the used $|M(K^+K^+) - M(\phi(1020))|$ mass windows. For the experiments which measured combination of cross-sections and branching ratios, the values of branching ratios are not given

Decay	In this work (%)	In experiments (%)
$D^{*+} \to D^+$	32.30 ± 0.50 [25]	
$D^{*+} \to D^+\pi^0$	30.70 ± 0.50 [25]	30.70 ± 0.50, BELLE [7]
$D^{*+} \to D^0\pi^+$	67.70 ± 0.50 [25]	68.13 ± 1.40, ALEPH [11] 67.70 ± 0.50, ALICE [21–23]
		55.00 ± 4.00, ARGUS [3–5] 67.70 ± 0.50, ATLAS [24]
		67.70 ± 0.50, BELLE [7] 67.60 ± 0.50, H1 [18]
		67.70 ± 0.50, ZEUS [14–16,33] 67.70 ± 0.50, LHCb [20]
		68.30 ± 1.40, OPAL [9,10]
$D^{*0} \to D^0$	100.00 [29,30]	100.00, H1 [18] 100.00, ZEUS [16]
$D^{*0} \to D^0\pi^0$	64.94 ± 0.89 [29,30]	61.90 ± 2.90, BELLE [7]
$D_s^{*+} \to D_s^+\gamma$	93.50 ± 0.70 [25]	94.20 ± 0.70, BABAR [6,8]
$D^+ \to K^-\pi^+\pi^+$	9.46 ± 0.24 [25]	9.13 ± 0.19, ALICE [21–23] 7.70 ± 1.00, ARGUS [3–5]
		9.13 ± 0.19, ATLAS [24] 9.20 ± 0.60, BELLE [7]
		9.00 ± 0.60, H1 [18] 9.51 ± 0.34, ZEUS [16,33]
		9.20 ± 0.60, ZEUS [14] 9.13 ± 0.19, ZEUS [15]
		9.13 ± 0.19, LHCb [19,20]
$D^+ \to K^0\pi^+$	1.53 ± 0.06 [25]	
$D^0 \to K^-\pi^+$	3.93 ± 0.04 [25]	3.85 ± 0.09, ALEPH [11] 3.87 ± 0.05, ALICE [21–23]
		3.71 ± 0.25, ARGUS [3–5] 3.88 ± 0.05, ATLAS [24]
		3.80 ± 0.09, BELLE [7] 3.83 ± 0.09, H1 [18]
		3.80 ± 0.07, ZEUS [16,33] 3.80 ± 0.90, ZEUS [14]
		3.88 ± 0.05, ZEUS [15] 3.89 ± 0.05, LHCb [19,20]
$D^0 \to K^-\pi^+\pi^+\pi^-$	8.07 ± 0.23 [25]	
$D_s^+ \to K^{*0}(K^-\pi^+)K^+$	2.61 ± 0.09 [25]	
$D_s^+ \to \phi(K^+K^-)\pi^+$	2.27 ± 0.08 [25]	2.28 ± 0.12, ALICE [21,22] 2.16 ± 0.28, ZEUS [16]
		2.28 ± 0.12, ZEUS [15] 0.00, LHCb [19,20]
$D_s^+ \to \phi\pi^+$	4.50 ± 0.40 [25]	3.60 ± 0.90, ALEPH [11] 3.60 ± 0.90, BELLE [7]
		3.60 ± 0.90, H1 [18] 3.60 ± 0.90, ZEUS [14]
$D_s^+ \to K^+K^-\pi^+$, 7 MeV	1.83 ± 0.06 [25,34]	1.85 ± 0.11, ATLAS [24]
$D_s^+ \to K^+K^-\pi^+$, 20 MeV	2.23 ± 0.08 [25,34]	2.24 ± 0.13, LHCb [19,20]
$K^0 \to \pi^+\pi^-$	69.20 ± 0.05 [25]	
$K^{*0} \to K^+\pi^-$	66.67 [25]	66.67, DELPHI [11]
$\phi \to K^+K^-$	48.90 ± 0.50 [25]	49.10 ± 0.80, ALEPH [11] 49.00 ± 1.00, ARGUS [3–5]
		48.90 ± 0.50, BABAR [6,8] 49.10 ± 0.60, BELLE [7]
		49.00 ± 1.00, CLEO [1,2] 49.10 ± 0.80, DELPHI [11]
		49.20 ± 0.70, H1 [18] 49.10 ± 0.60, ZEUS [14]
		49.10 ± 0.80, OPAL [9,10]
$\Lambda^0 \to p\pi^+$	63.90 ± 0.50 [25]	
$\Lambda_c^+ \to pK^-\pi^+$	$6.84^{+0.32}_{-0.40}$ [25]	5.00 ± 1.30, BABAR [6,8] 5.00 ± 1.30, BELLE [7]
		5.00 ± 1.30, ZEUS [14,15,17] 5.00 ± 1.30, LHCb [19]
$\Lambda_c^+ \to \Lambda^0(p\pi^+)\pi^+$	$0.95 \pm 0.10 \pm 0.05$ [25]	0.68 ± 0.18, ZEUS [17]
$\Lambda_c^+ \to \Lambda^0\pi^+\pi^+\pi^-$	3.59 ± 0.28 [25]	
$\Lambda_c^+ \to \Lambda^0\pi^+$	1.46 ± 0.13 [25]	
$\Lambda_c^+ \to \Xi^-K^+\pi^+$	0.70 ± 0.08 [25]	
$\Lambda_c^+ \to pK^0$	3.21 ± 0.30 [25]	
$\Lambda_c^+ \to pK^0(\pi^+\pi^-)$	$1.09 \pm 0.06 \pm 0.11$ [25]	0.80 ± 0.21, ZEUS [17]
$\Lambda_c^+ \to pK^0\pi^+\pi^-$	3.50 ± 0.40 [25]	

2.2 Calculation of the fragmentation fractions

In this paper the charm-quark fragmentation fraction to a specific hadron H is defined as the production cross-section via charm quark divided by the production cross-section of the charm quark:

$$f(c \to H) = \sigma(H)/\sigma(c). \tag{1}$$

The charm hadrons produced in the decays of beauty hadrons are not considered. The Standard Model makes precise predictions for the total charm cross-section in e^+e^- collisions, therefore, for those processes it is possible to calculate $f(c \to H)$ according to Eq. (1). Sufficiently precise predictions for the charm-quark production in pp and $e^{\pm}p$ collisions are not available. However, it is possible to make an assumption that the sum of charm-quark fragmentation fractions to all known weakly decaying charm hadrons in the end of the fragmentation process is unity. Then the charm-quark fragmentation fraction to a specific hadron can be calculated as the ratio of the hadron-production cross-section over the sum of cross-sections of all known weakly decaying (w.d.) charm hadrons

$$f(c \to H) = \sigma(H)/\sum_{w.d.}\sigma(H). \tag{2}$$

To obtain the charm-quark fragmentation fractions according to Eq. (2), in addition to the production cross-sections of D mesons and Λ_c^+, it is necessary to know the cross-sections of the weakly decaying $\Xi_c^{+,0}$ and Ω_c^0 states. Those states are poorly studied, therefore as in Ref. [9] it is assumed that ratios of fragmentation fractions of charm and strange quarks into the corresponding baryons are similar, $f(c \to \Xi_c^+)/f(c \to \Lambda_c^+) = f(c \to \Xi_c^0)/f(c \to \Lambda_c^+) = f(s \to \Xi^-)/f(s \to \Lambda^0)$ and $f(c \to \Omega_c^0)/f(c \to \Lambda_c^+) = f(s \to \Omega^-)/f(s \to \Lambda^0)$. In this approach the sum of the production cross-sections of these states can be estimated as

$$\sigma(\Xi_c^+) + \sigma(\Xi_c^0) + \sigma(\Omega_c^0) = \lambda\sigma(\Lambda_c^+), \tag{3}$$

where we define

$$\lambda = 2\frac{f(s \to \Xi^-)}{f(s \to \Lambda^0)} + \frac{f(s \to \Omega^-)}{f(s \to \Lambda^0)} = 0.136 \pm 0.006. \tag{4}$$

The value of λ is calculated using the most precise set of s quark fragmentation fractions $f(s \to \Xi^-) = 0.0016 \pm 0.0003$, $f(s \to \Omega^-) = 0.0258 \pm 0.0010$ and $f(s \to \Lambda^0) = 0.3915 \pm 0.0065$ from Ref. [25] obtained at LEP. Hereby, the sum of production cross-sections of all weakly decaying states is

$$\sum_{w.d.}\sigma(H) = \sigma(D^0) + \sigma(D^+) + \sigma(D_s^+) + \sigma(\Lambda_c^+) + \lambda\sigma(\Lambda_c^+). \tag{5}$$

The fragmentation fractions calculated according to Eq. (1) for the e^+e^- collisions and Z decays allow an independent check that

$$S = f(c \to D^0) + f(c \to D^+) + f(c \to D_s^+) + f(c \to \Lambda_c^+) + \lambda f(c \to \Lambda_c^+) \tag{6}$$

is close to unity with sufficient accuracy.

2.3 Combination procedure

The combination of the measurements used in the present analysis is based on numerical χ^2 minimisation with respect to observables of interest. The numerical minimisation was performed with the MINUIT package [35] and the procedure for calculation of χ^2 itself is outlined below.

For a set of m measurements and corresponding expectation values calculated from n parameters, a column-vector of the residuals $R(1 \times m)$ is calculated as a difference of a measurement and the corresponding expectation. The covariance matrix, $V(m \times m)$, is calculated as

$$V_{ij} = U_i^2\delta_{ij} + \sum_k C_{j,k}C_{i,k},$$

where U_i stands for an uncorrelated uncertainty of ith residual, $C_{i,k}$ stands for the correlated uncertainty of source k of the ith measurement and the sum runs over all sources of correlated uncertainties. The χ^2 is then calculated as

$$\chi^2 = R^T V^{-1} R.$$

The correlated uncertainties are treated multiplicatively in the construction of the covariance matrix, i.e. the relative uncertainties are used to scale the corresponding expectation values instead of the measurement. This avoids the bias for normalisation uncertainties, such as branching ratio uncertainties, which are the main correlated uncertainties considered in the presented analysis. The statistical and uncorrelated systematic uncertainties are treated additively. Data sets and their systematic uncertainties are assumed to be independent between experiments. In addition, most of the measurements do not contain the information about a potential correlation between cross-section values for different charm hadrons. Therefore, in the following all experimental uncertainties are treated as uncorrelated, unless otherwise stated. Uncertainties on the combined values of the fragmentation fractions are determined using the Hessian method with the criterion $\Delta\chi^2 = 1$.[1]

[1] As an illustrative example, when for a given combination set-up the inputs are the m cross-section measurements. These data define the $m \times m$ covariance matrix V with the experimental statistical and systematic uncertainties contributing to the diagonal elements and the correlated uncertainties setting the off-diagonal elements and contributing

The evaluated total uncertainties on the free parameters comprise experimental, branching ratio and uncertainties of the λ parameter.

The combination of all the measurements is obtained imposing the normalisation constraint on the sum of all ground state hadrons by adding an additional "measurement" of S calculated from Eq. (6) with an uncertainty on λ and the corresponding prediction $S = 1$. In order to keep the main result with the normalisation constraint as model independent as possible, no theory inputs on the charm cross-section are used in such a combination, and the fragmentation fractions are calculated according to Eq. (5). For the same reason, any measurements that require theoretical inputs for conversion into cross-sections or fragmentation fractions and do not have these inputs in the original publications are also excluded from the main combination. However, such data are included in a more constrained combination. In the following, treatment of such measurements will be discussed case-by-case in the relevant sections.

The quantities commonly used as Monte Carlo generator parameters,

$$R_{u/d} = \frac{f(c\to c\bar{u})}{f(c\to c\bar{d})} \approx \frac{f(c\to D^0) - f(c\to D^{*+})\mathcal{B}_{D^{*+}\to D^0}}{f(c\to D^+) + f(c\to D^{*+})\mathcal{B}_{D^{*+}\to D^0}},$$

$$\gamma_s = \frac{2f(c\to c\bar{s})}{f(c\to c\bar{u}/\bar{d})}(J=0) \approx \frac{2f(c\to D_s^+)}{f(c\to D^+) + f(c\to D^0)},$$

$$\gamma_s^* = \frac{2f(c\to c\bar{s})}{f(c\to c\bar{u}/\bar{d})}(J=1) \approx \frac{2f(c\to D_s^{*+})}{f(c\to D^{*+}) + f(c\to D^{*0})}$$

and

$$P_V^d = \frac{f(c\to c\bar{u}/\bar{d})(J=1)}{f(c\to c\bar{u}/\bar{d})(J=0)} \approx \frac{f(c\to D^{*+}) + f(c\to D^{*0})}{f(c\to D^+) + f(c\to D^0)}$$

were calculated from the fit results with the full error propagation and taking into account the correlation between parameters.

3 Charm-quark fragmentation into hadrons in e^+e^- collisions

Measurements of charm-hadron-production cross-sections in e^+e^- collisions hadrons were based on the differential momentum spectrum $d\sigma(e^+e^- \to H)/dx_p$. The extrapolation to the total cross-section was made in the original papers using a theoretical fragmentation function (e.g. Bowler [36] or Peterson [37]).[2]

As mentioned before, the precise predictions of the total charm-production cross-section in e^+e^- allow calculation of the fragmentation fractions without constraints on the sum of fractions. This way the used hypothesis about the sum of fragmentation fractions (Eq. (6)) can be verified.

3.1 Charm-quark fragmentation fractions from measurements at B-factories

The B-factories provided many results on charm-hadron production around the Υ resonances, which can be used for the calculation of the charm-quark fragmentation fractions in hadrons (see Table 2). The results of the CLEO [1,2] and ARGUS [3–5] experiments are represented as a product of the charm-hadron cross-sections times decay branching ratios, $\sigma(e^+e^- \to H)\cdot\mathcal{B}(H \to \text{daughters})$. The BELLE experiment [7] provided measurements of $\sigma(e^+e^- \to H)$. The BABAR experiment [8] provided a measurement of an average number of $\Lambda_c^+ \to pK^-\pi^+$ decays per hadronic event

$$N^{q\bar{q}}_{\Lambda_c}\cdot\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+) = 2\frac{\sigma(e^+e^- \to \Lambda_c^+)}{\sigma(e^+e^- \to \text{hadrons})} \times\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+) = 2R_c\cdot f(c\to\Lambda_c^+)\cdot\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+),$$

where $R_c = \frac{\sigma(e^+e^-\to c\bar{c})}{\sigma(e^+e^-\to\text{hadrons})}$ is the average number of charm-quark pairs per hadronic event. A prediction of R_c is needed to use this measurement as an input, therefore, as discussed in Sect. 2.3, it is used only in the case of $\sigma(e^+e^- \to c\bar{c})$ fixed to a theoretical prediction.

For the calculation of the charm-quark fragmentation fractions a fit procedure is used as described in Sect. 2. The total charm-quark-production cross-section is calculated as described in Appendix A. The fit parameters are the fragmentation fractions. The obtained results are given in the middle column of Table 3. The sum of the charm-quark fragmentation fractions into weakly decaying states calculated according to Eq. (6), $S_\Upsilon = 0.9701 \pm 0.0284$, is consistent with unity.

Footnote 1 continued
to the total uncertainties on the diagonal. The correlated uncertainties considered are those related to λ in Eq. (5) and branching ratios. The residuals are obtained subtracting from the measurements cross-section expectations calculated from n free parameters in the fit, which could be fragmentation fractions, total charm cross-sections, kinematic factors, etc. The details of this calculation are outlined in the each section. With all these components at hand the χ^2 can be evaluated and iteratively numerically minimised with respect to the free parameters.

[2] A proper extrapolation procedure requires only the hadrons produced directly in fragmentation to be used in the fits. The hadrons produced in decays of excited charm hadrons should be treated separately. In many cases the limited precision of the measurements makes this requirement hard to follow and the decay part of the meson production is treated together with the fragmentation part. In the cases where the contribution of hadrons from decays is comparable with the contribution of the direct production in fragmentation, e.g. for D^0 and D^+, the joint treatment could bias the results.

Table 2 Measured cross-sections, σ, cross-sections times branching ratios, $\sigma \cdot \mathcal{B}$, and $N_H^{q\bar{q}} \cdot \mathcal{B}$(see text for explanation) for the production of the charm hadrons produced in e^+e^- collisions at centre-of-mass energies of $\sqrt{s} = 10.55$ GeV [1], $\sqrt{s} = 10.52$ GeV [2,7], $\sqrt{s} = 10.47$ GeV [3], $\sqrt{s} = 10.5$ GeV [4], $\sqrt{s} = 10.2$ GeV [5] and $\sqrt{s} = 10.54$ GeV [6,8]. The numbers are given as in the original publications with uncertainties related to branching ratios omitted. The first uncertainty is statistical and the second is systematic

Decay	CLEO $\sigma \cdot \mathcal{B}$, pb	ARGUS σ, pb	BABAR $\sigma \cdot \mathcal{B}$, pb	BELLE σ, pb
$D^{*+} \to D^0\pi^+$,				
$D^0 \to K^-\pi^+$	$17.0 \pm 1.5 \pm 1.4$ [1]	690 ± 80 [3]		$598 \pm 2 \pm 77$ [7]
$D^{*+} \to D^+\pi^0$,				
$D^+ \to K^-\pi^+\pi^+$				$590 \pm 5 \pm 78$ [7]
$D^{*+} \to D^0\pi$,				
$D^0 \to K^-\pi^-\pi^+\pi^+$	$33.0 \pm 3.0 \pm 1.8$ [1]			
$D^{*0} \to D^0\pi^0/\gamma$	$30.0 \pm 3.0 \pm 5.8$ [1]			
$D^{*0} \to D^0\pi^0$				$510 \pm 3 \pm 84$ [7]
$D^0 \to K^-\pi^+$	$52 \pm 5 \pm 4$ [1]	1180 ± 150 [3]		$1449 \pm 2 \pm 64$ [7]
$D^+ \to K^-\pi^+\pi^+$	$51 \pm 7 \pm 2$ [1]	650 ± 90 [3]		$654 \pm 1 \pm 36$ [7]
	$\sigma \cdot \mathcal{B}$, pb	$\sigma \cdot \mathcal{B}$, pb	$N_H^{q\bar{q}} \cdot \mathcal{B} \times 10^7$	σ, pb
$\Lambda_c^+ \to pK^-\pi^+$	$13.5 \pm 4.0 \pm 1.4$ [1]	$9.0 \pm 1.2 \pm 1.0$ [5]	$284 \pm 4 \pm 9$ [8]	$189 \pm 1 \pm 66$ [7]
$\Lambda_c^+ \to pK^-\pi^+$	$10.0 \pm 1.5 \pm 1.5$ [2]			
$\Lambda_c^+ \to K^0 p$	$4.6 \pm 0.6 \pm 0.8$ [2]			
$\Lambda_c^+ \to K^0 p\pi^+\pi^-$	$4.5 \pm 1.3 \pm 1.1$ [2]			
$\Lambda_c^+ \to \Lambda^0\pi^+$	$1.9 \pm 0.3 \pm 0.3$ [2]			
$\Lambda_c^+ \to \Lambda^0\pi^+\pi^+\pi^-$	$6.8 \pm 1.0 \pm 1.3$ [2]			
$\Lambda_c^+ \to \Xi^- K^+\pi^+$	$1.6 \pm 0.4 \pm 0.3$ [2]			
	$\sigma \cdot \mathcal{B}$, pb	$\sigma \cdot \mathcal{B}$, pb	$\sigma \cdot \mathcal{B}$, pb	σ, pb
$D_s^+ \to \phi\pi^+$	$7.2 \pm 1.9 \pm 1.0$ [1]	$7.5 \pm 0.8 \pm 0.7$ [4]	$7.55 \pm 0.20 \pm 0.34$ [6]	$231 \pm 2 \pm 92$ [7]
$D_s^* \to \phi\pi^+\gamma$			$5.8 \pm 0.7 \pm 0.5$ [6]	

The combination is also done according to Eq. (2) and imposing the constraint $S_\Upsilon - 1 = 0$, to be consistent with the definition used for $e^\pm p$ and pp data. The fit parameters are the fragmentation fractions and the total charm cross-section. The centre-of-mass energy dependence of the charm-quark cross-section is accounted for, according to formulae in Appendix A taking the total charm-quark cross-section at a centre-of-mass energy $\sqrt{s} = 10.5$ GeV as a reference. The results are given in Table 3 (right column). In this approach, the precise BABAR measurement of Λ_c^+ production [8] is not included in the combination since it requires usage of the R_c theoretical prediction. The latter has an influence on other fragmentation-fraction results.

3.2 Charm-quark fragmentation fractions from measurements at LEP

The LEP collider provided many results on the charm-hadron production. The most valuable for the studies of fragmentation are results obtained from Z decays. Most of those results are represented in the form of fraction of charm events multiplied by branching ratios $\frac{\Gamma(Z\to c\bar{c})}{\Gamma(Z\to \text{hadrons})}$ $f(c \to H) \cdot \mathcal{B}(H \to \text{daughters})$ (see Table 4). In addition, ALEPH [11], DELPHI [13] and OPAL [10] provided measurements of $f(c \to D^{*+})$ from the fits of fragmentation functions (see Table 4).

For the calculation of charm-quark fragmentation fractions, a fit procedure is used, as described in Sect. 2. The theoretically calculated value that is used, $\frac{\Gamma(Z\to c\bar{c})}{\Gamma(Z\to \text{hadrons})} = 0.17223 \pm 0.00001$ [32], is in agreement with the experimental world average 0.1721 ± 0.003 [25]. The fit parameters are the fragmentation fractions. The results are given in the middle column of Table 5. The sum of the charm-quark fragmentation fractions into weakly decaying states calculated according to Eq. (6), $S_Z = 0.9292 \pm 0.0261$, differs from unity by 2.7 standard deviations.

The combination is also done using Eq. (2), and imposing the constraint $S_Z - 1 = 0$, to be consistent with the definition

Table 3 Average of charm-quark fragmentation fractions in hadrons in e^+e^- collisions around $\sqrt{s} = 10.5$ GeV. The quantities S, $R_{u/d}$, P_V^d and γ_s were recalculated from the fit results taking into account correlation of fit parameters. The value of minimised χ^2 and the number degrees of freedom of the fit n_{dof} are given as well

	Fixed $\sigma(e^+e^- \to c)$	Constrained S
$f(c \to D^{*+})$	0.2470 ± 0.0137	0.2525 ± 0.0155
$f(c \to D^{*0})$	0.2241 ± 0.0304	0.2291 ± 0.0316
$f(c \to D_s^{*+})$	0.0532 ± 0.0082	0.0544 ± 0.0085
$f(c \to D^+)$	0.2639 ± 0.0139	0.2698 ± 0.0125
$f(c \to D^0)$	0.5772 ± 0.0241	0.5901 ± 0.0140
$f(c \to D_s^+)$	0.0691 ± 0.0045	0.0707 ± 0.0048
$f(c \to \Lambda_c^+)$	0.0526 ± 0.0031	0.0611 ± 0.0060
χ^2	19.2	17.0
n_{dof}	21	20
S	0.9701 ± 0.0284	1.0000 ± 0.0005
$R_{u/d}$	0.9508 ± 0.0752	0.9508 ± 0.0752
P_V^d	0.5601 ± 0.0432	0.5601 ± 0.0431
γ_s	0.1644 ± 0.0121	0.1644 ± 0.0121
γ_s^*	0.2257 ± 0.0385	0.2257 ± 0.0385

used for $e^\pm p$ and pp data. The fit parameters are the fragmentation fractions and the $\frac{\Gamma(Z \to c\bar{c})}{\Gamma(Z \to \text{hadrons})}$ ratio. The results, given in Table 5 (right column), are in good agreement with Ref. [27].

4 Charm-quark fragmentation into hadrons in $e^\pm p$ collisions

The charm-hadron-production cross-sections at HERA were measured in a restricted fiducial phase space. The extraction of the charm-quark fragmentation fractions requires a special treatment, as described in detail in Appendix B. The approach followed in this analysis is similar to the one originally used by the ZEUS collaboration [14].

4.1 Charm-quark fragmentation fractions from measurements in DIS

Charm-quark fragmentation fractions in DIS in $e^\pm p$ collisions are calculated from ZEUS and H1 measurements given in Table 6. For the calculation of charm-quark fragmentation fractions a fit procedure is used as it is described in Sect. 2. The free parameters in the fit are the charm fragmentation fractions and pairs of variables $\sigma(c)_i|_{i=1...3}$ and $\kappa_i|_{i=1...3}$ for each set of measurement. Here, $\sigma(c)_i$ is the total charm cross-section in $e^\pm p$, while κ_i is the kinematic factor for decays from higher states (see Appendix B). The parameter κ is fixed to one for the low-p_T measurements in Ref. [17] since the whole p_T kinematic space was covered. The present beauty contributions in Ref. [18] have small impact on the result and are neglected. The sum of charm fragmentation fractions $S_{ep\ \text{DIS}}$ is constrained to unity. The results of the averaging procedure are given in Table 7.

Table 4 LEP measurements of the products of the partial decay width of the Z into $c\bar{c}$ quark pairs, $\frac{\Gamma_{c\bar{c}}}{\Gamma_{\text{had}}}$, charm-hadron-production fractions, $f(c \to H)$, and corresponding branching ratios. The numbers are given as in the original publications with uncertainties related to branching ratios omitted. The first uncertainty is statistical and the second is systematical

Particle or decay	OPAL $\frac{\Gamma_{c\bar{c}}}{\Gamma_{\text{had}}} \cdot f(c \to H) \cdot \mathcal{B}$ (10^{-6})	ALEPH $\frac{\Gamma_{c\bar{c}}}{\Gamma_{\text{had}}} \cdot f(c \to H) \cdot \mathcal{B}$ (10^{-6})	DELPHI $\frac{\Gamma_{c\bar{c}}}{\Gamma_{\text{had}}} \cdot f(c \to H) \cdot \mathcal{B}$ (10^{-6})
$D_s^+ \to \phi(K^+K^-)\pi^+$	$560 \pm 150 \pm 70$ [9]	$352 \pm 57 \pm 21$ [11]	$765 \pm 69 \pm 37$ [12]
$D_s^+ \to K^{*0}(K\pi)K^+$			$624 \pm 122 \pm 73$ [12]
$D^0 \to K^-\pi^+$	$3890 \pm 270^{+260}_{-240}$ [9]	$3700 \pm 110 \pm 230$ [11]	$3570 \pm 100 \pm 146$ [12]
$D^+ \to K^-\pi^+\pi^+$	$3580 \pm 460^{+250}_{-310}$ [9]	$3680 \pm 120 \pm 200$ [11]	$3494 \pm 116 \pm 140$ [12]
$\Lambda_c^+ \to pK^-\pi^+$	$410 \pm 190 \pm 70$ [9]	$673 \pm 70 \pm 37$ [11]	$743 \pm 155 \pm 78$ [12]
$D^{*+} \to D^0(K^-\pi^+)\pi^+$	$1041 \pm 20 \pm 40$ [10]		$1089 \pm 27 \pm 39$ [12]
	$f(c \to H)$ (10^{-3})	$f(c \to H)$ (10^{-3})	$f(c \to H)$ (10^{-3})
D^{*+}	$222 \pm 14 \pm 14$ [10]	$233.3 \pm 10.2 \pm 8.41$ [11]	
	$f(c \to H) \cdot \mathcal{B}$ (10^{-3})	$f(c \to H) \cdot \mathcal{B}$ (10^{-3})	$f(c \to H) \cdot \mathcal{B}$ (10^{-3})
$D^{*+} \to D^0\pi^+$			$174 \pm 10 \pm 4.2$ [13]
$D_s^{*+} \to \phi(K^+K^-)\pi^+\gamma$		$69 \pm 18 \pm 7$ [11]	

Table 5 Average of charm-quark fragmentation fractions into hadrons in Z decays. The quantities S, $R_{u/d}$, P_V^d and γ_s are recalculated from the fit results taking into account correlation of fit parameters. The value of minimised χ^2 and the number degrees of freedom of the fit n_{dof} are given as well

	Fixed $\frac{\Gamma_{c\bar{c}}}{\Gamma_{\text{had}}}$	Constrained S
$f(c \to D^{*+})$	0.2369 ± 0.0064	0.2454 ± 0.0071
$f(c \to D_s^{*+})$	0.0545 ± 0.0144	0.0547 ± 0.0145
$f(c \to D^+)$	0.2267 ± 0.0100	0.2429 ± 0.0102
$f(c \to D^0)$	0.5470 ± 0.0215	0.5894 ± 0.0132
$f(c \to D_s^+)$	0.0925 ± 0.0082	0.0996 ± 0.0083
$f(c \to \Lambda_c^+)$	0.0555 ± 0.0065	0.0600 ± 0.0066
χ^2	6.7	7.8
n_{dof}	13	13
S	0.9292 ± 0.0261	1.0000 ± 0.0005
$R_{u/d}$	0.9987 ± 0.0627	1.0348 ± 0.0580
P_V^d	0.6119 ± 0.0185	0.6000 ± 0.0177
γ_s	0.2390 ± 0.0224	0.2394 ± 0.0223

Table 7 Average of charm-quark fragmentation fractions in $e^{\pm}p$ collisions in DIS. The quantities S, $R_{u/d}$, P_V^d and γ_s are recalculated from the fit results taking into account correlation of fit parameters. The value of minimised χ^2 and the number degrees of freedom of the fit n_{dof} are given as well

	Constrained S
$f(c \to D^{*+})$	0.2372 ± 0.0173
$f(c \to D^+)$	0.2170 ± 0.0203
$f(c \to D^0)$	0.6272 ± 0.0287
$f(c \to D_s^+)$	0.0945 ± 0.0124
$f(c \to \Lambda_c^+)$	0.0540 ± 0.0195
χ^2	1.7
n_{dof}	3
S	1.0000 ± 0.0004
$R_{u/d}$	1.2361 ± 0.1331
P_V^d	0.6282 ± 0.0440
γ_s	0.2240 ± 0.0320

The obtained fragmentation fractions are in agreement with those obtained in the original publications [16,18]. The uncertainties of the obtained results are somewhat larger because this analysis, contrary to those studies, relies only on DIS results, whereas the HERA DIS papers [16,18] used fragmentation fractions into Λ_c^+ measured at e^+e^- colliders.

4.2 Charm-quark fragmentation fractions from measurements in PHP

Charm-quark fragmentation fractions in PHP in $e^{\pm}p$ collisions were calculated from measurements of ZEUS collaboration and given in Table 8. For the update of the latest ZEUS measurement [15] to the decay branching ratios from Table 1, the measured fragmentation fractions are first transformed into total charm-hadron cross-sections according to the formulae in Appendix B and only then used in the calculations. In this procedure, the kinematic factor for decays from higher states, κ, is set to 1, since the total phase space is considered from the fragmentation fraction definition, and the $\sigma(c)$ value cancels out in the procedure.

For the calculation of charm-quark fragmentation fractions, a fit procedure is used as it is described in Sect. 2. The free parameters in the fit are the charm fragmentation fractions and pairs of variables $\sigma(c)_i|_{i=1...2}$ and $\kappa_i|_{i=1...2}$ for each set of measurement. Here, $\sigma(c)_i$ is the total charm cross-section in $e^{\pm}p$, while κ_i is the kinematic factor for decays from higher states (see Appendix B). The sum of charm fragmentation fractions $S_{ep\ \text{PHP}}$ is constrained to unity. The results of the averaging procedure are given in Table 9. The

Table 6 Measurements of charm-hadron-production cross-sections in DIS in $e^{\pm}p$ collisions. The numbers are given as in the original publications. The first uncertainty is statistical, the second is systematical and the third one corresponds to the branching ratio

Decay	ZEUS [16] σ, nb	ZEUS [17] σ, nb	H1 [18] σ, nb
$D^0 \to K^+\pi^-$	$7.34 \pm 0.36^{+0.35}_{-0.27} \pm 0.13$		$6.53 \pm 0.49^{+1.06}_{-1.30}$
$D^+ \to K^+\pi^+\pi^+$	$2.80 \pm 0.30^{+0.18}_{-0.14} \pm 0.10$		$2.16 \pm 0.19^{+0.46}_{-0.35}$
$D^+ \to K_s^0\pi^+$		$25.7 \pm 4.1^{+3.8}_{-5.2} \pm 0.8$	
$D_s^+ \to \phi(K^+K^-)\pi^+$	$1.27 \pm 0.16^{+0.11}_{-0.06}{}^{+0.19}_{-0.15}$		$1.67 \pm 0.41^{+0.54}_{-0.54}$
$D^{*+} \to D^0(K^-\pi^+)\pi^+$	$3.14 \pm 0.12^{+0.18}_{-0.15} \pm 0.06$		$2.90 \pm 0.20^{+0.58}_{-0.44}$
$\Lambda_c^+ \to \Lambda\pi^+$		$14.9 \pm 4.9^{+2.2}_{-2.6} \pm 3.9$	
$\Lambda_c^+ \to K_s^0 p$		$14.0 \pm 5.8^{+3.8}_{-3.3} \pm 3.7$	
$\Lambda_c^+(combination)$		$14.7 \pm 3.8^{+2.1}_{-2.2} \pm 3.9$	
$D^0 \to K^-\pi^+, no\ D^{*+}$	$1.78 \pm 0.08^{+0.12}_{-0.10} \pm 0.03$		

Table 8 Measurements of charm-hadron-production cross-sections and fragmentation fractions in photoproduction in $e^{\pm}p$ collisions. The numbers are given as in the original publications. The first uncertainty is statistical, the second is systematic and the third one corresponds to the branching ratio

Decay	ZEUS [14] σ, nb	ZEUS [15] $f(c \to H)$
$D^+ \to K^-\pi^+\pi^+$	$5.07 \pm 0.36^{+0.44+0.34}_{-0.23-0.30}$	$0.234 \pm 0.006^{+0.004+0.006}_{-0.006-0.008}$
$D^0 \to K^+\pi^-, no\ D^{*+}$	$8.49 \pm 0.44^{+0.47+0.20}_{-0.48-0.19}$	
$D^0 \to K^+\pi^-, with\ D^{*+}$	$2.65 \pm 0.08^{+0.11}_{-0.10} \pm 0.06$	$0.588 \pm 0.017^{+0.011+0.012}_{-0.006-0.018}$
$D_s^+ \to \phi(K^+K^-)\pi^+$	$2.37 \pm 0.20 \pm 0.20^{+0.72}_{-0.45}$	$0.088 \pm 0.006^{+0.002+0.005}_{-0.007-0.005}$
$\Lambda_c^+ \to pK^-\pi^+$	$3.59 \pm 0.66^{+0.54+1.15}_{-0.66-0.70}$	$0.079 \pm 0.013^{+0.005+0.024}_{-0.009-0.014}$
$D^{*+} \to D^0(K^-\pi^+)\pi^+$		$0.234 \pm 0.006^{+0.004+0.005}_{-0.004-0.007}$
${}^{\mathrm{add}}D^{*+} \to D^0$	$1.05 \pm 0.07^{+0.09}_{-0.04} \pm 0.03$	
${}^{\mathrm{kin}}D^{*+} \to D^0$	$4.97 \pm 0.14^{+0.23+0.13}_{-0.18-0.12}$	

obtained fragmentation fractions are in agreement with those obtained in the original publications [14,15].

5 Charm-quark fragmentation into hadrons in pp collisions

The ALICE experiment measured fiducial cross-sections of D_s^+ [22] and differential p_T cross-sections of D^0, D^+ and D^{*+} mesons [21,23] at $\sqrt{s} = 2.76$ TeV and $\sqrt{s} = 7$ TeV. With an integration of the differential cross-sections of D^0, D^+ and D^{*+} from Ref. [21] and D^0 from Ref. [23], a coherent set of measurements in the kinematic range $2 < p_T < 12$ GeV, $|y| < 0.5$ has been constructed (see Table 10) for the $\sqrt{s} = 2.76$ TeV and $\sqrt{s} = 7$ TeV. The LHCb experiment provided measurements of charm-hadron cross-sections at $\sqrt{s} = 7$ TeV [19] and at $\sqrt{s} = 13$ TeV [20]. The ATLAS experiment recently measured the production cross-sections of D^{*+}, D^+ and D_s^+ mesons at $\sqrt{s} = 7$ TeV [24] in the kinematic range $3.5 < p_T < 20$ GeV, $|\eta| < 2.1$.

The measurements together with the correlation matrix for LHCb $\sqrt{s} = 7$ TeV are given in Table 10.

For the calculation of charm-quark fragmentation fractions a fit procedure is used as it is described in Sect. 2. The free parameters in the fit are: the charm fragmentation fractions, the fiducial cross-sections for LHCb, ALICE and ATLAS measurements in kinematic regions given in Table 10 and corresponding κ parameters.

The constraint $S_{pp} - 1 = 0$ is imposed. As the Refs. [21, 23] do not provide detailed decomposition of the systematic uncertainties, for every bin all systematic uncertainties were conservatively assumed to be fully correlated. For all of these measurements we assume the statistical and systematic uncertainties uncorrelated and luminosity uncertainties – fully correlated within a set of measurements at a given value of $\sqrt{s}$.

A set of orthogonal fully correlated uncertainties was obtained from the covariance matrix of the $\sqrt{s} = 7$ TeV

Table 9 Average of charm-quark fragmentation fractions in hadrons in $e^{\pm}p$ collisions in photoproduction. The quantities S, $R_{u/d}$, P_V^d and γ_s are recalculated from the fit results taking into account correlation of fit parameters. The value of minimised χ^2 and the number degrees of freedom of the fit n_{dof} are given as well

	Constrained S
$f(c \to D^{*+})$	0.2345 ± 0.0081
$f(c \to D^+)$	0.2341 ± 0.0093
$f(c \to D^0)$	0.5991 ± 0.0126
$f(c \to D_s^+)$	0.0901 ± 0.0062
$f(c \to \Lambda_c^+)$	0.0675 ± 0.0106
χ^2	5.2
n_{dof}	4
S	1.0000 ± 0.0005
$R_{u/d}$	1.1209 ± 0.0545
P_V^d	0.5970 ± 0.0181
γ_s	0.2164 ± 0.0162

LHCb measurements with an eigenvector decomposition. The obtained uncertainties are later treated in the same way as other correlated sources in the combination. The Ref. [20] does not contain the correlation matrix for the measurements at $\sqrt{s} = 13$ TeV, therefore simplified correlations between measurements were calculated as follows. All of the measurements include 3.9 % fully correlated uncertainty related to luminosity included in the Ref. [20] to the systematic uncertainty. The systematic uncertainties also include the uncertainties on the branching ratios, which were treated correlated with other branching-ratio uncertainties. The remaining systematic uncertainty were treated as fully uncorrelated for different measurements with the same p_T cuts and fully correlated for the same measurements with different p_T cuts. The statistical uncertainties of $\sigma(D^0)_{p_T<8\,\mathrm{GeV}}$, $\sigma(D^+)_{p_T<8\,\mathrm{GeV}}$ were split in two parts, which correspond to $p_T < 1$ GeV and $p_T > 1$ GeV regions. The later were considered fully correlated to the statistical uncertainties

Table 10 Measurements of charm-hadron-production cross-sections in pp collisions. For the measurements from Ref. [19] the total uncertainty is given. For the rest of measurements the first uncertainty is statistical, the second is systematic. p_T is given in GeV

Decay	LHCb [19] 7 TeV $p_T \in [0, 8]$ $y \in [2, 4.5]$	Corr. (%)				LHCb [20] 13 TeV $p_T \in [1, 8]$ $y \in [2, 4.5]$	ALICE [23] 2.76 TeV $p_T \in [2, 12]$ $\|y\| < 0.5$	ALICE [21] 7 TeV $p_T \in [2, 12]$ $\|y\| < 0.5$	ATLAS [24] 7 TeV $p_T \in [3.5, 20]$ $\|\eta\| < 2.1$
	σ, μb	D^0	D^+	D^{*+}	D_s^+	σ, μb	σ, μb	σ, μb	σ, μb
$D^0 \to K\pi$	1661 ± 129					$2460 \pm 3 \pm 130$	$110 \pm 16^{+28}_{-34}$	$231 \pm 12^{+37}_{-56}$	
$D^+ \to K^-\pi^+\pi^+$	645 ± 74	76				$1000 \pm 3 \pm 110$	$47 \pm 9^{+10}_{-12}$	$81 \pm 7^{+23}_{-26}$	$328 \pm 16 \pm 27$
$D^{*+} \to D^0(K^-\pi^+)\pi^+$	677 ± 83	77	73			$460 \pm 13 \pm 100$	$59 \pm 14^{+13}_{-14}$	$104 \pm 6^{+17}_{-22}$	$331 \pm 18 \pm 28$
$D_s^+ \to \phi(K^+K^-)\pi^+$	197 ± 31	55	52	53		$880 \pm 5 \pm 140$		$53 \pm 12^{+13}_{-15}$[22]	$160 \pm 31 \pm 17$
$\Lambda_c^+ \to pK^-\pi^+$	233 ± 77	26	25	25	18				
						$p_T \in [0, 8]$			
$D^0 \to K\pi$						$3370 \pm 4 \pm 200$			
$D^+ \to K^-\pi^+\pi^+$						$1290 \pm 8 \pm 190$			

Table 11 Average of charm-quark fragmentation fractions in pp collisions. The quantities S, $R_{u/d}$, P_V^d and γ_s are recalculated from the fit results taking into account correlation of fit parameters. The value of minimised χ^2 and the number degrees of freedom of the fit n_{dof} are given as well

	Constrained S
$f(c \to D^{*+})$	0.2337 ± 0.0175
$f(c \to D^+)$	0.2274 ± 0.0128
$f(c \to D^0)$	0.6176 ± 0.0160
$f(c \to D_s^+)$	0.0824 ± 0.0084
$f(c \to \Lambda_c^+)$	0.0639 ± 0.0122
χ^2	6.9
n_{dof}	7
S	1.0000 ± 0.0005
$R_{u/d}$	1.1913 ± 0.1012
P_V^d	0.6059 ± 0.0306
γ_s	0.1951 ± 0.0216

of the $\sigma(D^0)_{1<p_T<8\,\text{GeV}}$ and $\sigma(D^+)_{1<p_T<8\,\text{GeV}}$ measurements.

For the LHCb and ATLAS measurements of the $D_s^+ \to K^+K^-\pi^+$ in the limited mass windows $M(K^+K^-)$ the following approach was used. The branching ratios were obtained from the integrals over the $M(K^+K^-)$ line shape that was parametrised as in Ref. [34] with the total $D_s^+ \to K^+K^-\pi^+$ signal normalised to $\mathcal{B}(D_s^+ \to K^+K^-\pi^+) = 5.45 \pm 0.17$ % [25]. The results are given in Table 1.

The results of the fit are reported in Table 11. In addition to the values of the fragmentation fractions, the fit delivers the inclusive charm-production cross sections in the corresponding fiducial regions, which have particular interest. Therefore, the values of these cross-sections obtained in the global combination with better precision are discussed below.

6 Selection of measurements for the extraction of fragmentation fractions

The selection of the measurements for the extraction of fragmentation fractions was done according a set of criteria explained below.

First, the selection is limited to the measurements obtained in the collisions of high energy particle beams as it assures an absence of possible matter effects and the charm quark production mechanism in these environments is well understood. The measurements of charm-hadron production in proton–meson, proton–nucleon and nucleon–nucleon collisions [38–43] were omitted as those provide results in very specific production environment and energy ranges which cannot be easily compared to the results in other experiments.

The second criteria of the selection is the precision of the measured quantities: the measurements in e^+e^- collisions with $\sqrt{s} = 12 - 90$ GeV from MARK-II [44], HRS [45–49], TPC [50], TASSO [51,52], JADE [53,54], VENUS [55] and some other experiments have very limited precision and are not used for the global combination.

The third criterion of the selection is the availability of sufficient measurements in the given physical environment needed for the extraction procedure. Several results on charm production in $e^{\pm}p$ collisions (e.g. Ref. [56]) and pp collisions (e.g. Refs. [42,43,57]) do not contain enough simultaneous measurements of hadron production and, therefore, cannot be treated independently and/or constrain the results of the combination.

Table 12 Average of charm-quark fragmentation fractions in hadrons. The quantities S, $R_{u/d}$, P_V^d and γ_s are recalculated from the fit results taking into account correlation of fit parameters. The value of minimised χ^2 and the number degrees of freedom of the fit n_{dof} are given as well

	Constrained S	Constrained S, fixed $\sigma(e^+e^- \to c)$, $\frac{\Gamma_{cc}}{\Gamma_{\text{had}}}$
$f(c \to D^{*+})$	0.2429 ± 0.0049	0.2386 ± 0.0046
$f(c \to D^{*0})$	0.2306 ± 0.0315	0.2250 ± 0.0299
$f(c \to D_s^{*+})$	0.0548 ± 0.0074	0.0537 ± 0.0072
$f(c \to D^+)$	0.2404 ± 0.0067	0.2439 ± 0.0067
$f(c \to D^0)$	0.6086 ± 0.0076	0.6141 ± 0.0073
$f(c \to D_s^+)$	0.0802 ± 0.0040	0.0797 ± 0.0040
$f(c \to \Lambda_c^+)$	0.0623 ± 0.0041	0.0549 ± 0.0026
χ^2	65.6	87.1
n_{dof}	64	67
S	1.0000 ± 0.0005	1.0000 ± 0.0004
$R_{u/d}$	1.0971 ± 0.0354	1.1164 ± 0.0354
P_V^d	0.5578 ± 0.0375	0.5403 ± 0.0355
γ_s	0.1890 ± 0.0103	0.1859 ± 0.0101
γ_s^*	0.2314 ± 0.0347	0.2316 ± 0.0346

7 The global combination

To check the consistency of the data from different production regimes and also to extract the charm-quark fragmentation fractions with high precision, all input measurements introduced in the previous sections are used together to produce a global combination. As discussed in Sect. 3.1, the Λ_c^+ measurement by the BABAR experiment [8] is not included while obtaining the combined result. The free parameters of the fit are the charm-quark fragmentation fractions and pairs of variables $\sigma(c)_i|_{i=1...5}$ and $\kappa_i|_{i=1...5}$ for three DIS and two PHP sets of measurements, $\frac{\Gamma_{c\bar{c}}}{\Gamma_{\text{had}}}$, $\sigma(e^+e^- \to c)$ at $\sqrt{s} = 10.5$ GeV, and the fiducial charm-quark cross-section and κ parameters in pp collisions, at $\sqrt{s} = 7$ TeV and $\sqrt{s} = 13$ TeV, corresponding to the phase space of the measurements. The constraint on the sum of the cross-sections of the weakly decaying charm states, S, is imposed in the combination, i.e. the prediction for the total charm cross-sections in e^+e^- collisions is not used, in order to minimise model dependence of the averaging procedure. The result of averaging e^+e^-, $e^\pm p$ and pp data, with the constraint $S = 1$ is presented in the middle column of Table 12 and is shown in Fig. 1. The correlations between the fitted parameters are given in Table 13. The input data are in very good agreement with $\chi^2/n_{\text{dof}} = 65.6/64$. The result of the combination has significantly reduced uncertainties compared to individual measurements.

As an alternative, the combination is also performed using both the constraint on S as well as theoretical predictions of charm production in e^+e^- collisions and Z decays, i.e. $\sigma(e^+e^- \to c)$ at $\sqrt{s} = 10.5$ GeV and $\frac{\Gamma_{c\bar{c}}}{\Gamma_{\text{had}}}$. This approach

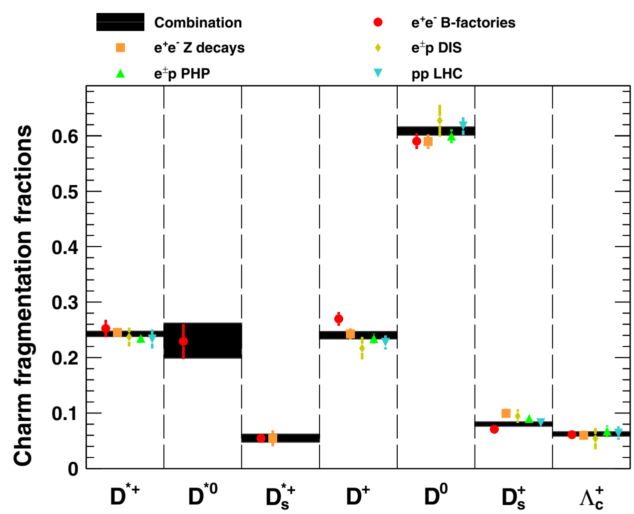

Fig. 1 The values of charm-quark fragmentation fractions, $f(c \to H)$, in different experiments with the S constraint. The global combination with the S constraint is shown with the *shaded band*. Averages of included data in different production regimes are shown with various *full symbols*

also allows to include the precise BABAR measurement of Λ_c^+ production [8] using the R_c calculation as described in Appendix A, which significantly affects the averaged value of $f(\Lambda_c^+)$. The result of the averaging procedure with this approach is given in the right column of Table 12 for completeness. The result is more model dependent than the default combination, but has a higher precision. At the same time, the result visibly differs from the result of the default procedure. This may partially be traced to the value $S_Z = 0.9292 \pm 0.0261$ for the accurate LEP measurements,

Table 13 Correlation of charm-quark fragmentation fractions from the fit with constrained S

	D^{*+}	D^{*0}	D_s^{*+}	D^+	D^0	D_s^+	Λ_c^+
D^{*+}	1.00	−0.02	−0.02	−0.08	0.19	−0.07	−0.12
D^{*0}	−0.02	1.00	0.02	−0.07	0.07	0.01	−0.01
D_s^{*+}	−0.02	0.02	1.00	−0.05	−0.07	0.23	−0.01
D^+	−0.08	−0.07	−0.05	1.00	−0.66	−0.19	−0.19
D^0	0.19	0.07	−0.07	−0.66	1.00	−0.32	−0.41
D_s^+	−0.07	0.01	0.23	−0.19	−0.32	1.00	−0.07
Λ_c^+	−0.12	−0.01	−0.01	−0.19	−0.41	−0.07	1.00

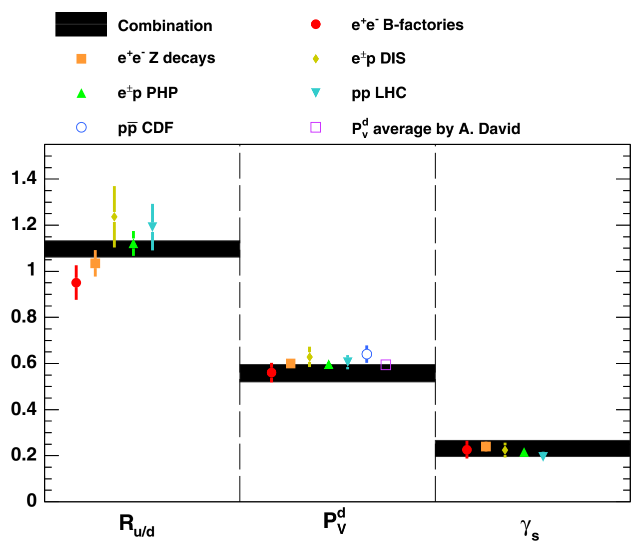

Fig. 2 The values of $R_{u/d}$, P_V^d and γ_s in different experiments with the S constraint. The global combination with the S constraint is shown with the *shaded band*. Combinations of included data in different production regimes are shown with *various full symbols*. Data that were not included in the combination [57,58] are shown with *open symbols*. Note, that the latter are quoted from the original papers, i.e. without correction to the up-to-date branching ratios and with no branching ratio uncertainty, if not given in the source

which differs markedly from 1 (see Sect. 3.2). This difference is also reflected in the larger χ^2/n_{dof} value compared to the default global combination. The difference in the $f(c \to \Lambda_c^+)$ precision is to a large extent due to inclusion of the precise BABAR data [8].

The extracted $R_{u/d}$, P_V^d and γ_s factors are provided in Table 12 and shown in Fig. 2. The combined data are also compared to recent measurements [57,58] that were not included in the combination. In particular, $R_{u/d} = 1.097 \pm 0.035$ is in fair agreement with the isospin invariance hypothesis $R_{u/d} = 1$ within 2.7 standard deviations. The values of the $\sigma(pp \to c)$ cross-sections, obtained in the global fit (see Table 14) are consistent with those obtained in the original analysis, but have significantly reduced uncertainties. The consistent treatment of the LHCb and ALICE measurements in the combination procedure allows unbiased calculation of the ratio of the inclusive fiducial charm-quark production cross-sections:

$$R_{7/2.76} = \frac{\sigma(pp \to c)_{7\,\text{TeV},2<p_T<12\,\text{GeV},|y|<0.5}}{\sigma(pp \to c)_{2.76\,\text{TeV},2<p_T<12\,\text{GeV},|y|<0.5}} = 1.89 \pm 0.66$$

and

$$R_{13/7} = \frac{\sigma(pp \to c)_{13\,\text{TeV},p_T<8\,\text{GeV},2<y<4.5}}{\sigma(pp \to c)_{7\,\text{TeV},p_T<8\,\text{GeV},2<y<4.5}} = 1.96 \pm 0.18.$$

The $R_{7/2.76}$ is compatible with the predictions in Ref. [59]. The $R_{13/7}$ value is visibly higher than the theoretical prediction $R_{13/7}(\text{theory}) = 1.39^{+0.12}_{-0.29}$ [60].

8 Excited states

In addition to the average fragmentation fractions for the ground, $L = 0$, states, some fragmentation fractions for the excited, $L = 1$ charm hadrons are calculated.

The measurements used for the calculations are shown in Table 15. The unpublished measurement of $f(c \to D_{s1}^+)$ from Ref. [61] was not used. The fragmentation fractions were not updated to the most recent branching ratios, as the difference between the used branching ratios and the newest is negligible in comparison to statistical and systematical uncertainties of the measurements, and is well below the given numerical precision of the individual measurements in Table 15.

The averages are calculated with an assumption of fully uncorrelated statistical and systematical uncertainties. The results of the averaging procedure are given in Table 16. The strangeness-suppression factor for $L = 1$, $J = 1^+$ charm mesons is calculated neglecting $D(2430)^0$ contribution and assuming D_1^+ is 1^+ state:

$$\gamma_{s1} \approx \frac{2f(c \to D_{s1}^+)}{f(c \to D_1^0) + f(c \to D_1^+)}.$$

Table 14 The values of the inclusive fiducial charm quark production cross-section, $\sigma(pp \to c)$, from the original publications and obtained in the global fit. The statistical, systematic and fragmentation uncertainties of the values from Refs. [19,20] were added in quadrature

$\sqrt{s}$, TeV	p_T range, GeV	y or η range	Fit result $\sigma(pp \to c)$, μb	Original $\sigma(pp \to c)$, μb
7	[0, 8]	$y \in [2, 4.5]$	2689 ± 203	2838 ± 268 [19]
13	[1, 8]	$y \in [2, 4.5]$	4174 ± 339	4300 ± 356 [20]
13	[0, 8]	$y \in [2, 4.5]$	5269 ± 293	5880 ± 482 [20]
2.76	[2, 12]	$\|y\| < 0.5$	229 ± 67	
7	[2, 12]	$\|y\| < 0.5$	434 ± 84	
7	[3.5, 20]	$\|\eta\| < 2.1$	1400 ± 141	

Table 15 Comparison of fragmentation-fraction-results of measurements of excited charm mesons

Particle	ZEUS $f(c \to H)$ (10^{-2})	OPAL $f(c \to H)$ (10^{-2})	ALEPH $f(c \to H)$ (10^{-2})
D_1^0	$2.9 \pm 0.5^{+0.5}_{-0.5}$ [62]	$2.1 \pm 0.7 \pm 0.3$ [63]	
	$3.5 \pm 0.4^{+0.4}_{-0.6}$ [33]		
D_2^{*0}	$3.9 \pm 0.9^{+0.8}_{-0.6}$ [62]	$5.2 \pm 2.2 \pm 1.3$ [63]	
	$3.8 \pm 0.7^{+0.5}_{-0.6}$ [33]		
D_1^+	$4.6 \pm 1.8^{+2.0}_{-0.3}$ [62]		
D_2^{*+}	$3.2 \pm 0.8^{+0.5}_{-0.2}$ [62]		
D_{s1}^+	$1.11 \pm 0.16^{+0.08}_{-0.10}$ [33]	$1.6 \pm 0.4 \pm 0.3$ [63]	$0.94 \pm 0.22 \pm 0.07$ [64]

Table 16 Average of charm-quark fragmentation fractions in excited charm mesons. The γ_{s1} quantity is calculated from the averaging results without taking into account correlations

	Average (10^{-2})
$f(c \to D_1^+)$	$4.60^{+2.69}_{-1.82}$
$f(c \to D_2^{*+})$	$3.20^{+0.94}_{-0.82}$
$f(c \to D_1^0)$	2.97 ± 0.38
$f(c \to D_2^{*0})$	3.94 ± 0.68
$f(c \to D_{s1}^+)$	1.09 ± 0.14
γ_{s1}	$28.7^{+8.0}_{-10.9}$

9 Summary

A summary of measurements of the fragmentation of charm quarks into a specific charm hadron is given. The analysis includes data collected in photoproduction and deep inelastic scattering in $e^{\pm}p$ collisions and well as e^+e^- and pp data. Measurements in different production regimes agree within uncertainties, supporting the hypothesis that fragmentation proceeds independent of the specific production process. Averages of the fragmentation fractions are presented. The global average has significantly reduced uncertainties compared to individual measurements. In addition, the hypothesis that the sum of fragmentation fractions of all known weakly decaying charm hadrons is equal to unity is checked to hold within 3 standard deviations using the e^+e^- data.

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Appendix A: Predictions for charm production at *B*-factories

The total cross-section of quark q production in e^+e^- collisions at energies $2M(q) \ll \sqrt{s} \ll M(Z)$ can be given as

$$\sigma(e^+e^- \to q) = 2\sigma(e^+e^- \to l^+l^-) \sum_{\text{colours}} v_q^2 r_q, \tag{7}$$

where v_q is the vector electromagnetic coupling of the quark q (i.e. charge), $r_q(s)$ are the correction coefficients with higher order QCD corrections and

$$\sigma(e^+e^- \to l^+l^-) = 4\alpha^2(s)\pi/3s \tag{8}$$

is the total cross-section of massless charged lepton pair production. In this work, the calculations of the $r_q(s)$ were done according to Ref. [31] at the reference energy of $\sqrt{s} = 10.5$ GeV and assuming the c quark is the heavy one. The constants used for the calculations in Eqs. (7) and (8) are the strong coupling $\alpha_s(\sqrt{s} = 10.5\,\text{GeV}) = 0.172$ [31], the $\overline{\text{MS}}$ charm-quark mass $m_c(\sqrt{s} = 10.5\,\text{GeV}) = 0.74\,\text{GeV}$ [31] and the electromagnetic coupling $\alpha(\sqrt{s} = 10.5\,\text{GeV}) = 1/132.0$ (calculated according to [65,66] as implemented in [67]). The uncertainties on the given values are negligible. The result of the calculations is $\sigma(e^+e^- \to c, \sqrt{s} = 10.5\,\text{GeV}) = 2399.23$ nb. To verify the calculations, Eq. (7) can be rewritten as

$$\sigma(e^+e^- \to c) = 2\sigma(e^+e^- \to l^+l^-)R_c R_{\text{had}},$$

where the quantities

$$R_{\text{had}} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to l^+l^-)} = \sum_{u,d,s,c}\sum_{\text{colours}} v_q^2 r_q$$
$$= 3.5239$$

and

$$R_c = \frac{\sigma(e^+e^- \to c\bar{c})}{\sigma(e^+e^- \to \text{hadrons})} = \frac{\sum_{\text{colours}} v_c^2 r_c}{\sum_{u,d,s,c}\sum_{\text{colours}} v_q^2 r_q}$$
$$= 0.4012$$

can be compared with the existing measurements and predictions. It was found that R_{had} is in agreement with the direct measurement from CLEO below $\sqrt{s} = 10.56\,\text{GeV}$ $R_{\text{had,CLEO}} = 3.591 \pm 0.003 \pm 0.067 \pm 0.049$ [68] and R_c is in agreement with the CLEO Monte-Carlo based estimation $R_{c,\text{CLEO}} = 0.37 \pm 0.05$ [1]. For all the theoretically calculated values, the uncertainties of calculations are negligible.

Appendix B: Extraction of the charm-quark fragmentation fractions from the measurements in the restricted phase space

Often the production cross-sections of charm hadrons are measured in some restricted (e.g. in transverse momentum and pseudorapidity) kinematical region v and cannot be used directly (i.e. without extrapolation to the full kinematical space) in the Eq. (2). To avoid the extrapolation and obtain unbiased charm-quark fragmentation fractions, the following approach is used.

The full cross-section of a charm hadron, $\sigma(H)_{\in v}$ can be split into cross-section of direct production $\sigma(H)_{\text{dir},\in v}$ and the contribution from the decays of heavier charm states H^*, $\sigma(H)_{\text{decays},\in v}$:

$$\begin{aligned}\sigma(H)_{\in v} &= \sigma(H)_{\text{dir},\in v} + \sigma(H)_{\text{decays},\in v}\\ &= \sigma(H)_{\text{dir},\in v} + \sum_{all\ H^*}\sigma(H^*)\mathcal{B}(H^* \to H)k_{H^*\to H},\end{aligned}$$

where $k_{H^*\to H} < 1$ is a fraction of $H^* \to H$ decays with H in v. The lack of experimental data in this analysis allows to consider only the heavier charm states that are giving the largest contribution to the total cross-sections. For this reason all D^0 and D^+ produced not in D^{*+} and D^{*0} decays are considered as produced directly. Because of similar kinematics of $D^* \to D$ decays it is also assumed that $k_{D^*\to D} = k$. The measurements on heavier charm baryons and charm strange mesons are absent, but all of those decay dominantly to Λ_c^+ and D_s^+ so we treat all Λ_c^+ and D_s^+ as produced directly. Within a common phase space, the impact of the different masses of the hadrons on the fragmentation process is neglected.

With the assumptions above, we have the equations:

$$\begin{cases}\sigma(D^+)_{\in v} &= \sigma(D^+)_{\text{dir},\in v} + k\sigma(D^{*+})_{\text{dir}}\mathcal{B}(D^{*+} \to D^+)\\ \sigma(D^0)_{\in v} &= \sigma(D^0)_{\text{dir},D^0\in v} + k\sigma(D^{*0})_{\text{dir}}\mathcal{B}(D^{*0} \to D^0)\\ & \quad +k\sigma(D^{*+})_{\text{dir}}\mathcal{B}(D^{*+} \to D^0)\\ \sigma(D_s^+)_{\in v} &= \sigma(D_s^+)_{\text{dir},\in v}\\ \sigma(\Lambda_c^+)_{\in v} &= \sigma(\Lambda_c^+)_{\text{dir},\in v}\\ \sigma(D^{*+})_{\in v} &= \sigma(D^{*+})_{\text{dir},\in v}\\ \sigma(D^{*0})_{\in v} &= \sigma(D^{*0})_{\text{dir},\in v}.\end{cases}$$

Assuming $\sigma(H)_{\text{dir},\in v} = \sigma(c)_{\in v} f(c \to H)_{\text{dir}}$ and introducing $\kappa = k\sigma(c)/\sigma(c)_{\in v}$ we have:

$$\begin{cases}\sigma(D^+)_{\in v} &= \sigma(c)_{\in v}(f(c \to D^+)_{\text{dir}}\\ & \quad +\kappa f(c \to D^{*+})_{\text{dir}}\mathcal{B}(D^{*+} \to D^+))\\ \sigma(D^0)_{\in v} &= \sigma(c)_{\in v}(f(c \to D^0)_{\text{dir}}\\ & \quad +\kappa f(c \to D^{*0})_{\text{dir}}\mathcal{B}(D^{*0} \to D^0)\\ & \quad +\kappa f(c \to D^{*+})_{\text{dir}}\mathcal{B}(D^{*+} \to D^0))\\ \sigma(D_s^+)_{\in v} &= \sigma(c)_{\in v} f(c \to D_s^+)_{\text{dir}}\\ \sigma(\Lambda_c^+)_{\in v} &= \sigma(c)_{\in v} f(c \to \Lambda_c^+)_{\text{dir}}\\ \sigma(D^{*+})_{\in v} &= \sigma(c)_{\in v} f(c \to D^{*+})_{\text{dir}}\\ \sigma(D^{*0})_{\in v} &= \sigma(c)_{\in v} f(c \to D^{*0})_{\text{dir}}.\end{cases}$$

In the full kinematical space:

$$\begin{cases}f(c \to D^+) &= f(c \to D^+)_{\text{dir}}\\ & \quad +f(c \to D^{*+})_{\text{dir}}\mathcal{B}(D^{*+} \to D^+)\\ f(c \to D^0) &= f(c \to D^0)_{\text{dir}}\\ & \quad +f(c \to D^{*0})_{\text{dir}}\mathcal{B}(D^{*0} \to D^0)\\ & \quad +f(c \to D^{*+})_{\text{dir}}\mathcal{B}(D^{*+} \to D^0)\\ f(c \to D_s^+) &= f(c \to D_s^+)_{\text{dir}}\\ f(c \to \Lambda_c^+) &= f(c \to \Lambda_c^+)_{\text{dir}}\\ f(c \to D^{*+}) &= f(c \to D^{*+})_{\text{dir}}\\ f(c \to D^{*0}) &= f(c \to D^{*0})_{\text{dir}}.\end{cases}$$

In general, to solve the system the measurements of D^{*0} production are needed. However, these can be avoided with an assumption of isospin invariance:

$$\frac{f(c \to D^+)_{\text{dir}}}{f(c \to D^0)_{\text{dir}}} = \frac{f(c \to D^{*+})_{\text{dir}}}{f(c \to D^{*0})_{\text{dir}}}.$$

The last two systems are the working equations for the calculation of the charm fragmentation fractions from the cross-section measurements in the restricted phase space.

References

1. CLEO Collaboration, D. Bortoletto et al., Charm production in nonresonant e^+e^- annihilations at $\sqrt{s} = 10.55$GeV. Phys. Rev. D. **37**, 1719 (1988)
2. CLEO Collaboration, P. Avery et al., Inclusive production of the charmed baryon Λ_c from e^+e^- annihilations at $\sqrt{s} = 10.55$GeV. Phys. Rev. D. **43**, 3599 (1991)
3. ARGUS Collaboration, H. Albrecht et al., Inclusive production of D^0, D^+ and $D^{*+}(2010)$ mesons in B-decays and nonresonant e^+e^- annihilation at 10.6GeV. Z. Phys. C. **52**, 353 (1991)
4. ARGUS Collaboration, H. Albrecht et al., Production of D_s^+ mesons in B decays and determination of $f(D_s^+)$. Z. Phys. C. **54**, 1 (1992)
5. ARGUS Collaboration, H. Albrecht et al., Observation of the charmed baryon Λ_c in e^+e^- annihilation at 10GeV. Phys. Lett. B. **207**, 109 (1988)
6. BaBar Collaboration, B. Aubert et al., Measurement of D_s^+ and D_s^{*+} production in B meson decays and from continuum e^+e^- annihilation at $\sqrt{s} = 10.6$GeV. Phys. Rev. D. **65**, 091104 (2002). arXiv:hep-ex/0201041
7. Belle Collaboration, R. Seuster et al., Charm hadrons from fragmentation and B decays in e^+e^- annihilation at $\sqrt{s} = 10.6$GeV. Phys. Rev. D. **73**, 032002 (2006). arXiv:hep-ex/0506068
8. BaBar Collaboration, B. Aubert et al., Inclusive Λ_c^+ production in e^+e^- annihilations at $\sqrt{s} = 10.54$GeV and in $\Upsilon(4S)$ decays. Phys. Rev. D. **75**, 012003 (2007). arXiv:hep-ex/0609004
9. OPAL Collaboration, G. Alexander et al., A study of charm hadron production in $Z \to c\bar{c}$ and $Z \to b\bar{b}$ decays at LEP. Z. Phys. C. **72**, 1 (1996)
10. OPAL Collaboration, K. Ackerstaff et al., Measurement of $f(c \to D^{*+}X)$, $f(b \to D^{*+}X)$ and $\Gamma_{c\bar{c}}/\Gamma_{\text{hadronic}}$ using $D^{*\pm}$ mesons. Eur. Phys. J. C. **1**, 439 (1998). arXiv:hep-ex/9708021
11. ALEPH Collaboration, R. Barate et al., Study of charm production in Z decays. Eur. Phys. J. C. **16**, 597 (2000). arXiv:hep-ex/9909032
12. DELPHI Collaboration, P. Abreu et al., Measurements of the Z partial decay width into $c\bar{c}$ and multiplicity of charm quarks per b decay. Eur. Phys. J. C. **12**, 225 (2000)
13. DELPHI Collaboration, P. Abreu et al., Determination of $P(c \to D^{*+})$ and $BR(c \to l^+)$ at LEP-I. Eur. Phys. J. C. **12**, 209 (2000)
14. ZEUS Collaboration, S. Chekanov et al., Measurement of charm fragmentation ratios and fractions in photoproduction at HERA. Eur. Phys. J. C. **44**, 351 (2005). arXiv:hep-ex/0508019
15. ZEUS Collaboration, H. Abramowicz et al., Measurement of charm fragmentation fractions in photoproduction at HERA. JHEP **1309**, 058 (2013). arXiv:1306.4862
16. ZEUS Collaboration, S. Chekanov et al., Measurement of D mesons production in deep inelastic scattering at HERA. JHEP **0707**, 074 (2007). arXiv:0704.3562
17. ZEUS Collaboration, H. Abramowicz et al., Measurement of D^+ and Λ_c^+ production in deep inelastic scattering at HERA. JHEP **1011**, 009 (2010). arXiv:1007.1945
18. H1 Collaboration, A. Aktas et al., Inclusive production of D^+, D^0, D_s^+ and D^{*+} mesons in deep inelastic scattering at HERA. Eur. Phys. J. C. **38**, 447 (2005). arXiv:hep-ex/0408149
19. LHCb Collaboration, R. Aaij et al., Prompt charm production in pp collisions at $\sqrt{s} = 7$TeV. Nucl. Phys. B. **871**, 1 (2013). arXiv:1302.2864
20. LHCb Collaboration, R. Aaij et al., Measurements of prompt charm production cross-sections in pp collisions at $\sqrt{s} = 13$ TeV. JHEP (2015, Submitted). arXiv:1510.01707
21. ALICE Collaboration, B. Abelev et al., Measurement of charm production at central rapidity in proton-proton collisions at $\sqrt{s} = 7$TeV. JHEP **1201**, 128 (2012). arXiv:1111.1553
22. ALICE Collaboration, B. Abelev et al., D_s^+ meson production at central rapidity in proton–proton collisions at $\sqrt{s} = 7$TeV. Phys. Lett .B. **718**, 279 (2012). arXiv:1208.1948
23. ALICE Collaboration, B. Abelev et al., Measurement of charm production at central rapidity in proton-proton collisions at $\sqrt{s} = 2.76$TeV. JHEP **1207**, 191 (2012). arXiv:1205.4007
24. ATLAS Collaboration, G. Aad et al., Measurement of $D^{*\pm}$, $D^\pm$ and $D_s^\pm$ meson production cross sections in pp collisions at $\sqrt{s} = 7$TeV with the ATLAS detector. Nucl. Phys. B. **907**, 717 (2016). arXiv:1512.02913
25. Particle Data Group, K.A. Olive et al., Review of particle physics. Chin. Phys. C. **38**, 090001 (2014 and 2015 update)
26. L. Gladilin, Charm hadron production fractions (1999). arXiv:hep-ex/9912064
27. L. Gladilin, Fragmentation fractions of c and b quarks into charmed hadrons at LEP. Eur. Phys. J. C. **75**, 19 (2015). arXiv:1404.3888
28. E. Lohrmann, A summary of charm hadron production fractions (2011). arXiv:1112.3757
29. BESIII Collaboration, M. Ablikim et al., Precision measurement of the D^{*0} decay branching fractions. Phys. Rev. D. **91**, 031101 (2015). arXiv:1412.4566
30. BaBar Collaboration, B. Aubert et al., Measurement of the branching ratios $\Gamma(D_s^{*+} \to D_s^+\pi^0)/\Gamma(D_s^{*+} \to D_s^+\gamma)$ and $\Gamma(D^{*0} \to D^0\pi^0)/\Gamma(D^{*0} \to D^0\gamma)$. Phys. Rev. D. **72**, 091101 (2005). arXiv:hep-ex/0508039
31. K.G. Chetyrkin, R.V. Harlander, J.H. Kuhn, Quartic mass corrections to R_{had} at order $\alpha^3(s)$. Nucl. Phys. B. **586**, 56 (2000). arXiv:hep-ph/0005139
32. A. Freitas, Higher-order electroweak corrections to the partial widths and branching ratios of the Z boson. JHEP **1404**, 070 (2014). arXiv:1401.2447
33. ZEUS Collaboration, S. Chekanov et al., Production of excited charm and charm-strange mesons at HERA. Eur. Phys. J. C. **60**, 25 (2009). arXiv:0807.1290
34. BaBar, P.del Amo Sanchez et al., Dalitz plot analysis of $D_s^+ \to K^+K^-\pi^+$. Phys. Rev. D. **83**, 052001 (2011). arXiv:1011.4190
35. F. James, M. Roos, Minuit: A system for function minimization and analysis of the parameter errors and correlations. Comput. Phys. Commun. **10**, 343 (1975)
36. M.G. Bowler, e^+e^- production of heavy quarks in the String Model. Z. Phys. C. **11**, 169 (1981)
37. C. Peterson, D. Schlatter, I. Schmitt, P.M. Zerwas, Scaling violations in inclusive e^+e^- annihilation spectra. Phys. Rev. D. **27**, 105 (1983)
38. E769 Collaboration, G.A. Alves et al., Forward cross-sections for production of D^+, D^0, D_s, D^{*+} and Λ_c in 250GeV $p^\pm$, $K^\pm$, and p-nucleon interactions. Phys. Rev. Lett. **77**, 2388 (1996)
39. ACCMOR Collaboration, S. Barlag et al., Production properties of D^0, D^+, D^{*+} and D_s^+ in 230GeV/c π^- and K^-Cu interactions. Z. Phys. C. **49**, 555 (1991)
40. ACCMOR Collaboration, S. Barlag et al., Production of the charmed baryon Λ_c^+ in π^-Cu and K^-Cu interactions at 230GeV. Phys. Lett .B. **247**, 113 (1990)

41. HERA-B Collaboration, I. Abt et al., Measurement of D^0, D^+, D_s^+ and D^{*+} production in fixed target 920GeV proton-nucleus collisions. Eur. Phys. J. C. **52**, 531 (2007). arXiv:0708.1443
42. STAR Collaboration, D. Tlusty et al., Open charm hadron production via hadronic decays at STAR. Nucl. Phys. A. **904–905**, 639c (2013). arXiv:1211.5995
43. STAR Collaboration, Z. Ye et al., Open charm hadron production in $p + p$, Au+Au and U+U collisions at STAR. Nucl. Phys. A. **931**, 520 (2014)
44. MARK-II Collaboration, J.M. Yelton et al., D^{*+} production in e^+e^- annihilation at 29GeV. Phys. Rev. Lett. **49**, 430 (1982)
45. HRS Collaboration, S.P. Ahlen et al., Inclusive D and D^* production in e^+e^- annihilation at 29GeV. Phys. Rev. Lett. **51**, 1147 (1983)
46. HRS Collaboration, M. Derrick et al., Charm quark production and fragmentation in e^+e^- annihilation at 29GeV. Phys. Lett. B. **146**, 261 (1984)
47. HRS Collaboration, M. Derrick et al., Production of ϕ and $F(1970) \to \phi\pi$ in e^+e^- Annihilation at 29GeV. Phys. Rev. Lett. **54**, 2568 (1985)
48. HRS Collaboration, E.H. Low et al., Production and fragmentation of the D^{*0} meson in e^+e^- annihilations. Phys. Lett. B. **183**, 232 (1987)
49. HRS Collaboration, P.S. Baringer et al., Production cross-section and electroweak asymmetry of D^* and D mesons produced in e^+e^- annihilations at 29GeV. Phys. Lett. B. **206**, 551 (1988)
50. TPC/Two Gamma Collaboration, H. Aihara et al., Charged D^* meson production in e^+e^- annihilation at $\sqrt{s} = 29$GeV. Phys. Rev. D. **34**, 1945 (1986)
51. TASSO Collaboration, M. Althoff et al., Observation of F meson production in high-energy e^+e^- annihilation. Phys. Lett. B. **136**, 130 (1984)
52. TASSO Collaboration, W. Braunschweig et al., Production and decay of charmed mesons in e^+e^- annihilation at $\sqrt{s} > 28$GeV. Z. Phys. C. **44**, 365 (1989)
53. JADE Collaboration, W. Bartel et al., Charged D^* production in e^+e^- annihilation. Phys. Lett. B. **146**, 121 (1984)
54. JADE Collaboration, W. Bartel et al., Inclusive neutral D^* production and limits on F^* production in e^+e^- annihilation at PETRA. Phys. Lett. B. **161**, 197 (1985)
55. VENUS Collaboration, F. Hinode et al., A study of charged D^* production in e^+e^- annihilation at an average center-of-mass energy of 58GeV. Phys. Lett. B. **313**, 245 (1993)
56. ZEUS Collaboration, S. Chekanov et al., Measurement of D^+ and D^0 production in deep inelastic scattering using a lifetime tag at HERA. Eur. Phys. J. C. **63**, 171 (2009). arXiv:0812.3775
57. CDF Collaboration, D. Acosta et al., Measurement of prompt charm meson production cross sections in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$TeV. Phys. Rev. Lett. **91**, 241804 (2003). arXiv:hep-ex/0307080
58. A. David, Is the fragmentation of charm quarks into D mesons described by heavy quark effective theory? Phys. Lett. B. **644**, 224 (2007)
59. R. Averbeck et al., Reference heavy flavour cross sections in pp collisions at $\sqrt{s} = 2.76$TeV, using a pQCD-driven $\sqrt{s}$-scaling of ALICE measurements at $\sqrt{s} = 7$TeV (2011). arXiv:1107.3243
60. R. Gauld, J. Rojo, L. Rottoli and J. Talbert, Charm production in the forward region: constraints on the small-x gluon and backgrounds for neutrino astronomy (2015). arXiv:1506.08025
61. A. Verbytskyi, Production of the excited charm mesons D_1 and D_2^* at HERA, PhD, Universität Hamburg, DESY-THESIS-2013-006, 2013
62. ZEUS Collaboration, H. Abramowicz et al., Production of the excited charm mesons D_1 and D_2^* at HERA. Nucl. Phys. B. **866**, 229 (2013). arXiv:1208.4468
63. OPAL Collaboration, K. Ackerstaff et al., Production of P-wave charm and charm-strange mesons in hadronic Z decays. Z. Phys. C. **76**, 425 (1997)
64. ALEPH Collaboration, A. Heister et al., Production of $D^{**}(s)$ mesons in hadronic Z decays. Phys. Lett. B. **526**, 34 (2002). arXiv:hep-ex/0112010
65. G. Altarelli, R. Kleiss, C. Verzegnassi, Z physics at LEP-I, vol. 3, Event generators and software, pp. 129–131 (1989)
66. H. Burkhardt et al., Uncertainties in the hadronic contribution to the QED vacuum polarization. Z. Phys. C. **43**, 497 (1989)
67. B.W. Harris, J. Smith, Charm quark and $D^{*\pm}$ cross-sections in deeply inelastic scattering at HERA. Phys. Rev. D. **57**, 2806 (1998). arXiv:hep-ph/9706334
68. CLEO Collaboration, D. Besson et al., Measurement of the total hadronic cross section in e^+e^- annihilations below 10.56GeV. Phys. Rev. D. **76**, 072008 (2007). arXiv:0706.2813