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ORIGINAL ARTICLE

Combined Earth orientation parameters based on homogeneous and continuous VLBI and GPS data

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Abstract The CONT02 campaign is of great interest for studies combining very long baseline interferometry (VLBI) with other space-geodetic techniques, because of the continuously available VLBI observations over 2 weeks in October 2002 from a homogeneous network. Especially, the combination with the Global Positioning System (GPS) offers a broad spectrum of common parameters. We combined station coordinates, Earth orientation parameters (EOPs) and troposphere parameters consistently in one solution using technique-specific datum-free normal equation systems. In this paper, we focus on the analyses concerning the EOPs, whereas the comparison and combination of the troposphere parameters and station coordinates are covered in a companion paper in *Journal of Geodesy*. In order to demonstrate the potential of the VLBI and GPS space-geodetic techniques, we chose a sub-daily resolution for polar motion (PM) and universal time (UT). A consequence of this solution set-up is the presence of a one-to-one correlation between the nutation angles and a retrograde diurnal signal in PM. The Bernese GPS Software used for the combination provides a constraining approach to handle this singularity. Simulation studies involving both nutation offsets and rates helped to get a deeper understanding of this singularity. With a rigorous combination of UT1–UTC and length of day (LOD) from VLBI and GPS, we showed that such a combination works very well and does not suffer from the systematic effects present in the GPS-derived LOD values. By means of wavelet analyses and the formal errors of the estimates, we explain this important result. The same holds for the combination of nutation offsets and rates. The local geodetic ties between GPS and VLBI antennas play an essential role within the inter-technique combination. Several studies already revealed non-negligible discrepancies between the terrestrial measurements and the space-geodetic solutions. We demonstrate to what extent these discrepancies propagate into the combined EOP solution.

Keywords CONT02 · Rigorous combination · Sub-daily Earth rotation · Nutation · Local ties

1 Introduction

The International VLBI (very long baseline interferometry) Service for Geodesy and Astrometry (IVS) scheduled the so-called CONT02 campaign for October 16–31, 2002 (Thomas and MacMillan 2003). CONT02

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follows the tradition of episodic VLBI campaigns with continuous observations, much in contrast to the regular VLBI sessions named R1 and R4 that take place twice a week and last only 24 hours (see the IVS observing program at http://ivscc.gsfc.nasa.gov/program/index.html). In view of combining VLBI with other space-geodetic techniques, the availability of continuous observations is the major argument for using the CONT02 data instead of the regular IVS sessions. A second advantage is that the identical network configuration is maintained during the whole time span, whereas the networks of the regular R1 and R4 IVS sessions change among sessions. Therefore, the geodetic datum can be realized more consistently for the CONT02 sessions leading to better results for the Earth orientation parameters (EOPs).

A broad spectrum of common parameters exists for a combination of VLBI and GPS: station coordinates, EOPs and troposphere parameters. In this context, we distinguish between the expression *Earth rotation parameters (ERPs)* comprising the two pole components and UT1–UTC only, whereas the term *Earth orientation parameters (EOPs)* includes the two nutation angles $\Delta\varepsilon$ and $\Delta\psi$ as well. All parameters were set up consistently in our VLBI and GPS single-technique least-squares (LS) adjustments so that the subsequent combination can be considered rigorous and, thus, the approach is comparable to that applied by Yaya (2002).

In order to achieve the overall consistency, all a priori models in the VLBI and GPS processing software packages used for these studies have been adapted. This level of consistency cannot be reached at present with the VLBI and GPS solutions provided by the IVS and the International GNSS (global navigation satellite systems) Service (IGS) as, apart from the International Earth Rotation and Reference Systems Service (IERS) Conventions (McCarthy and Petit 2004), each Analysis Center can use different models for, e.g., ocean tide loading or pole tide corrections. Discrepancies in the a priori models show up in the estimated parameters so that a rigorous combination based on the officially available solutions is not yet possible.

Besides the homogeneous VLBI and GPS solutions, our studies go one step further than any combination done so far within the IERS because we include troposphere parameters. More details about our studies devoted to the troposphere parameters are in a companion paper by M. Krügel et al. (submitted). In the paper at hand, we concentrate on the EOPs, although the results stem from a solution where the terrestrial reference frame (TRF) and the troposphere parameters have been treated together. Altogether, three topics about the EOPs will be addressed:

1. the special situation of estimating sub-daily ERPs together with nutation,
2. the consistent combination of all five EOPs,
3. the role of local geodetic ties for the combined EOP results.

A special characteristic of our analyses is the sub-daily resolution chosen for the ERPs in order to demonstrate the capability of the space-geodetic techniques to estimate such parameters. Furthermore, for the validation of daily estimated values, only time-series derived from the space-geodetic techniques themselves (e.g., IERS-C04, IERS Bulletin A; see Gambis 2004) or from geophysical fluids, such as atmospheric angular momentum (AAM) and oceanic angular momentum (OAM), can be used for comparisons.

The problems related to these data sets are well known: the IERS-C04 series is not consistent anymore with the ITRF2000 (Gambis 2004), and the AAM and OAM series are mostly based on models whose accuracy is not clear. Contrary to this situation, the IERS Conventions 2003 (McCarthy and Petit 2004) provide a sub-daily model that is based on satellite altimetry data, so that this model is well suited for an independent validation of the sub-daily ERP estimates from VLBI and GPS.

The drawback of estimating sub-daily ERP and nutation angles simultaneously is the presence of a one-to-one correlation between the nutation angles and a retrograde diurnal term in the PM. In the case of GPS (and other satellite techniques), three additional degrees of freedom, i.e. a common rotation of all orbital planes is involved (Hefty et al. 2000). Several studies are devoted to the determination of diurnal and sub-diurnal terms in PM and UT from VLBI (e.g., Herring and Dong 1994) or from satellite laser ranging (SLR) (e.g, Watkins and Eanes 1994).

In contrast to these studies, the EOPs are modeled by a piecewise linear function in our processing strategy, instead of explicitly setting up unknowns for the diurnal and sub-diurnal terms of interest. However, a retrograde diurnal term is implicitly contained in the estimated PM time-series. In order to handle its correlation with the nutation angles (and orbital elements), a special constraint described by Brockmann (1997) is applied. The present paper takes a closer look at this constraint to assess whether the length of the time-series and the temporal resolution of the estimated pole coordinates influence the results. The correlation becomes even more complicated in the case of also estimating nutation rates, and we will show how we can avoid this additional singularity in our solutions.

Regarding the EOPs, we combined the contributions of VLBI and GPS for all five components rigorously. This means that not only PM coordinates including their time derivatives were combined, but also the UT1–UTC time-series and the nutation angles results from combining the contributions of GPS and VLBI. Although the IERS is approaching a simultaneous combination of the TRF together with the EOPs, the inclusion of GPS in time-series of UT1–UTC is thought to be problematic (Ray et al. 2005; Gross et al. 1998; Gross 2000) and the officially available GPS solutions do not even contain the nutation angles. Thus, up to now, a consistent combination has been performed only by Yaya (2002), where all space-geodetic techniques were analyzed with the same software package and the resulting normal equations were subsequently combined, and by Andersen (2000), where all observations were analyzed within one step.

2 Processing and combination strategy

2.1 Single-technique normal equation systems

In a first step, the observations of each space-geodetic technique were analyzed separately and datum-free unconstrained daily normal equations were generated. Altogether, eight VLBI stations participated in CONT02, namely, Algonquin Park, Fairbanks (Gilmore Creek), Hartebeesthoek, Kokee Park (Kauai), Ny-Alesund, Onsala, Westford, and Wettzell (see Fig. 1 in M. Krügel et al. submitted). Additionally, observations of a GPS network consisting of 153 stations were analyzed, including those co-located with the eight VLBI stations.

The VLBI data (observed in sessions starting and ending at 18:00 h UTC) were concatenated into daily files starting and ending at 0:00 h UTC so as to be in accordance with the GPS data sets. Then, only the fully available days of the concatenated VLBI observations were used for the studies, i.e., October 17–30, 2002. The whole VLBI analysis was performed at Deutsches Geodätisches Forschungsinstitut (DGFI), using the OCCAM software version 6.0 (Titov et al. 2004).

For generating datum-free daily GPS normal equations in analogy to the VLBI analysis the Bernese GPS Software version 5.0 (Hugentobler et al. 2005) was used. It is important to mention that no constraint has been applied to the GPS orbital parameters, in order to allow for a successful combination with VLBI for the time-series of UT1–UTC and the nutation angles. This is an essential pre-condition, because as a consequence of deficiencies in modeling the satellite orbits over longer time spans, the UT and nutation time-series derived from GPS show a drift. Thus, if the orbital elements are constrained, the drifting time-series contained in the satellite-only solution is not free anymore to adapt to the combined drift-free values mainly given by VLBI.

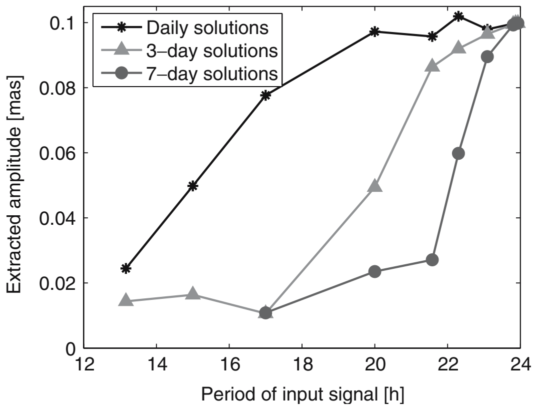


Fig. 1 Amplitude of the retrograde diurnal PM signal that fits best to the artificial sub-daily input signals (0.1 mas). Three different solution lengths are used in order to demonstrate the dependence on the length of the time-series

Regarding the analysis of the VLBI and GPS data, it must be emphasized that both software packages were prepared in such a way that identical a priori models and identical parameterizations for all parameters common to both techniques were used. This means, concerning the EOPs, not only the same a priori model was used, but also the procedure to interpolate the tabulated daily a priori values to the requested epoch was unified. These adaptations are crucial for a rigorous combination based on normal equations, and ensure that the results are identical to those that would be obtained by a combination at the observation level.

In detail, all EOPs were set up as piecewise linear functions with an hourly resolution for the ERPs and a 14-day resolution for the two nutation angles $\Delta\varepsilon$ and $\Delta\psi$. The piecewise linear parameterization as a polygon with functional values at the interval boundaries includes the same information as if offset and drift parameters were set up for each interval. However, the number of parameters is almost doubled for the latter parameterization and, furthermore, it does not automatically guarantee the continuity at the interval boundaries because each pair of offset and drift is estimated independently from those of the other intervals. For details about the parameterization of the station coordinates and the troposphere parameters we refer to the companion paper by M. Krügel et al. (submitted).

2.2 Inter-technique combination

Of the set of parameters common to VLBI and GPS, the EOPs should be identical for all techniques so that they can simply be stacked in an inter-technique combination. In contrast, the station-specific parameters (coordinates and troposphere parameters) can be combined only for co-located sites and only if additional information is available about the difference caused by non-identical reference points. In the case of combining station coordinates, this implies that the so-called local ties, i.e., 3D coordinate differences between the reference points of the space-geodetic techniques determined by local geodetic surveys are required. More information about the local tie values for the eight co-locations available for CONT02 is given in our companion paper (M. Krügel et al., submitted). Therein, the studies concerning the quality of the local ties are included as well.

The datum for the combined solution is realized by a no-net-rotation (NNR) condition using a subset of IGS core stations (see Heflin 2003) and the local ties are added as pseudo-observations with a standard deviation of 0.1 mm in order to integrate the VLBI network into the GPS polyhedron (also see M. Krügel et al., submitted). The origin is given solely by GPS and the scale is a weighted mean of the GPS and VLBI scale. Details about our method of combining the troposphere parameters can be found in M. Krügel et al. (submitted).

One crucial point in the combination is the weighting of the single-technique normal equations. Helmert transformations that allow for three translations, three rotations and a scaling factor between the daily solutions and the 14-day technique-specific solution were performed and the resulting daily coordinate residuals after these transformations are used to compute the technique-specific repeatability of station coordinates. Finally, the ratio between the technique-specific repeatabilities was used as the relative weighting factor between GPS and VLBI.

3 Studying the correlations between retrograde diurnal PM and nutation

3.1 Theoretical background

The problem of a one-to-one correlation between an exact retrograde diurnal term of PM, an offset in the two nutation angles, and — in the case of satellite-geodetic techniques — a rotation of the orbital planes, must be addressed if PM is to be estimated with a sub-daily resolution. The theoretical background was described by Moritz and Mueller (1987) and aims at the determination of diurnal signals in PM using space-geodetic techniques to treat this problem. Substitutional for the analysis of VLBI, SLR, GPS or satellite data in general, we refer to Herring and Dong (1994), Watkins and Eanes (1994), Hefty et al. (2000) or Gambis (1986), respectively. In the following studies, we concentrate on the singularity between nutation parameters and retrograde diurnal PM. For further details about the correlations between EOPs and orbital parameters, we refer to Rothacher et al. (1999).

The singularity is purely mathematical and is present in the estimation, although we do not explicitly estimate specific periods of PM. However, if PM is set-up as a polygon with sub-daily resolution, this polygon contains a retrograde diurnal term that is one-to-one correlated with a long-term nutation term. Generally speaking, the correlation occurs due to the fact that the transformation from an inertial reference frame to a TRF (and vice versa) is conventionally carried out using five angles instead of the three that would be sufficient.

Neglecting the precession of the Earth yields the following transformation matrix $\boldsymbol{M}$, where $\boldsymbol{R}$ stands for a 3D rotation matrix around the axis indicated by the index and the angle given in brackets:

$$\begin{aligned}\boldsymbol{M} = {} & \boldsymbol{R}_2(-x_P)\cdot\boldsymbol{R}_1(-y_P)\cdot\boldsymbol{R}_3(\theta)\cdot\boldsymbol{R}_1(-\varepsilon_0-\Delta\varepsilon)\cdot\\ & \boldsymbol{R}_3(-\Delta\psi)\cdot\boldsymbol{R}_1(\varepsilon_0)\end{aligned} \tag{1}$$

Five angles are used to describe the time-dependent transformation Eq. (1), namely, the two pole coordinates x_P and y_P, the Greenwich true sidereal time (GAST) θ, the nutation in obliquity $\Delta\varepsilon$ and the nutation in longitude $\Delta\psi$. The mean obliquity of the ecliptic ε_0 is constant.

In order to derive a mathematical description of the relationship between nutation angles and PM, the matrix $\boldsymbol{M}$ according to Eq. (1) was split into one matrix containing only PM and GAST, and a second matrix containing only GAST, the nutation angles and the mean obliquity. Due to three unknown angles, both matrices can fully describe a 3D similarity transformation. Equating them and assuming small angles for all quantities (except for the mean obliquity ε_0 and GAST) yield the relationship between nutation offsets and PM that we are looking for:

$$x_P = -\Delta\psi\cdot\sin\varepsilon_0\cdot\cos\theta - \Delta\varepsilon\cdot\sin\theta \tag{2a}$$

$$y_P = -\Delta\psi\cdot\sin\varepsilon_0\cdot\sin\theta + \Delta\varepsilon\cdot\cos\theta \tag{2b}$$

Due to the angle θ in Eq. (2), the PM signal is diurnal, and as the x_P component precedes the y_P component by 90°, it is a retrograde signal. The PM signal corresponding to offsets in the nutation angles $\Delta\varepsilon$ and $\Delta\psi$ as described by Eq. (2) has an amplitude of

$$C_{xy} = \sqrt{\Delta\varepsilon^2 + \Delta\psi^2\cdot\sin^2\varepsilon_0} \tag{3}$$

Going one step further and investigating how a linear drift in $\Delta\dot{\varepsilon}$ and $\Delta\dot{\psi}$ corresponds to a signal in the PM reveals the expressions:

$$x_P = -\Delta\dot{\psi} \cdot \sin\varepsilon_0 \cdot \Delta t \cdot \cos\theta - \Delta\dot{\varepsilon} \cdot \Delta t \cdot \sin\theta \tag{4a}$$

$$y_P = -\Delta\dot{\psi} \cdot \sin\varepsilon_0 \cdot \Delta t \cdot \sin\theta + \Delta\dot{\varepsilon} \cdot \Delta t \cdot \cos\theta \tag{4b}$$

It becomes evident that a nutation rate is identical to a retrograde diurnal signal in PM with an amplitude linearly increasing with time:

$$C_{xy}(\Delta t) = \Delta t \cdot \sqrt{\Delta\dot{\varepsilon}^2 + \Delta\dot{\psi}^2 \cdot \sin^2\varepsilon_0} \tag{5}$$

To summarize the theoretical considerations, if the nutation angles are estimated together with PM using a sub-daily resolution, there will be two types of singularity present in the solution (see Eqs. 2 and 4).

In order to remedy these singularities, a special constraint is applied to the Bernese GPS Software, which prevents retrograde diurnal terms in PM by constraining the amplitude of any retrograde diurnal signal (independent of the phase) to zero. This means that the estimated PM time-series does not contain a retrograde diurnal term, because this term is equivalent to a constant nutation offset for the entire time span considered and, thus, will appear in the nutation estimates.

The formalism of this constraint and the mathematical background is documented in Brockmann (1997). In the following, the mechanism and the limitation of the constraint are discussed by means of simulated GPS data and real VLBI observations.

3.2 Simulation studies for GPS

Theoretically, only an exactly diurnal signal in PM can be fully expressed as a nutation offset. However, in practice, the constraint does not only affect the exact retrograde diurnal term but also adjacent retrograde terms. The reason for this behavior must be seen in the limited time-span covered by the data set that is analyzed.

To fully decorrelate two signals with similar periods T_0 and $T_0 - \Delta T$, the phases of both signals have to differ by at least 2π at the end of the time interval T_S considered. This requirement leads to the bandwidth ΔT of periods that cannot be fully decorrelated:

$$\Delta T \leq \frac{T_0^2}{T_S + T_0} \tag{6}$$

In the case that the period T_0 is one sidereal day ($T_0 = 23.934$ h), this implies that all retrograde signals that differ by less than ΔT from a diurnal period are still affected by the constraint, at least partly. The values for ΔT together with the period that can still be distinguished unambiguously from a diurnal signal are summarized in Table 1 for different lengths T_S of time-series.

Table 1 Periods $T_0 - \Delta T$ that can be fully decorrelated from a diurnal signal (period T_0) depending on the length of the time-series T_S according to Eq. (6)

T_S (days)	ΔT	$T_0 - \delta T$ (h)
1	11.9505 h	11.9835
3	5.9712 h	17.9628
7	2.9845 h	20.9495
14	1.5915 h	22.3425
28	0.8231 h	23.1109
365	3.91 min	23.8688

The impact of the constraint on the bandwidth of the affected retrograde terms was studied using simulated GPS observations for the time span of CONT02. The commonly used sub-daily model IERS2003 was applied to generate the GPS observations. Subsequently, several solutions were computed, where only the pole coordinates and the nutation angles were estimated and where the retrograde diurnal constraint mentioned above was applied. For each solution, exactly one artificial sub-daily retrograde term was added to the IERS2003 model and this extended model was introduced as an a priori model. The artificial signals that were tested had periods between half a day and exactly one sidereal day, each with an amplitude of 0.1 mas.

For a solution with length T_S of one sidereal day, Table 1 allows the conclusion that only signals with periods below 11.98 h are not affected by the blocking constraint. Signals with periods between 11.98 and 23.934 h are partly blocked. The retrograde diurnal part implicitly contained in the artificial signal is thereby suppressed by the constraint, i.e., the retrograde diurnal term that best fits (in a LS sense) this artificial signal is removed (and converted into a nutation offset), and only the remaining part is left in the estimated PM time-series. The more the period of the artificial signal deviates from a diurnal term, the smaller the amplitude of the best-fitting diurnal signal will be, and — theoretically — if the difference in period exceeds ΔT listed in Table 1, the amplitude should become zero.

Figure 1 shows this evolution of the amplitudes for 1-day, 3-day and 7-day solutions for different artificial sub-daily signals used as an a priori model. The results are in good agreement with the theoretical values given in Table 1. It has to be mentioned, however, that about 10% of the artificial sub-daily input signal remains affected by the constraint even if both periods should be decorrelated according to theory (see Eq. 6). The reason for this behavior is not fully clear yet.

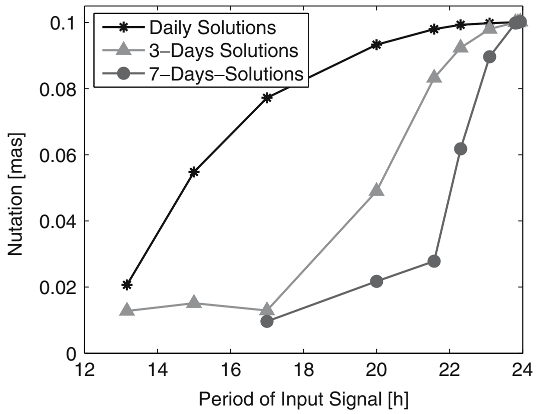


Fig. 2 Estimated nutation offset if retrograde PM terms of different periods (each with an amplitude of 0.1 mas) have been introduced as an a priori model

Figure 2 displays the total nutation offsets computed from the estimated $\Delta\varepsilon$ and $\Delta\psi$ according to Eq. (3) for the solutions mentioned above. Comparing the results with theory (Table 1), the same conclusions as above can be drawn concerning the effect of the constraint. Furthermore, the similarity between Figs. 1 and 2 confirms the one-to-one correspondence between an offset in the nutation angles and a retrograde diurnal term in the PM.

The simulation results shown above originate from solutions with a very high temporal resolution (15 min) of PM estimates. However, from further analyses with a lower temporal resolution (1 and 4 h; typical for operational time-series), it turned out that the bandwidth of the retrograde diurnal constraint only depends on the length of the time-series, not on the temporal resolution of the estimated parameters. Figure 3 demonstrates this fact by means of daily solutions showing the estimated residual PM exemplarily in the case of an artificial a priori signal with a 15-h period. The only difference that can be recognized in Fig. 3 is that seven values (in the case of a 4-h resolution) cannot represent a periodic signal as accurately as 25 or even 97 values.

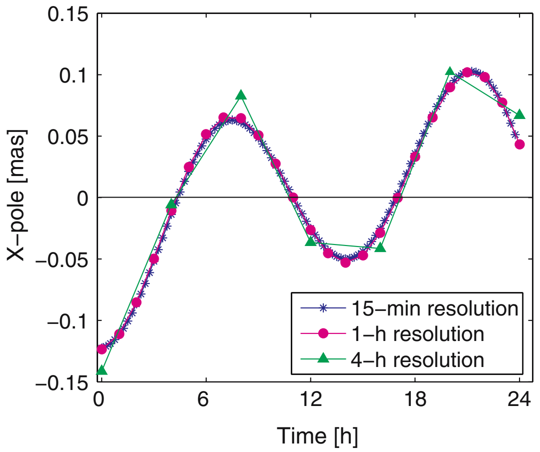


Fig. 3 Estimated PM (x_P component) for different temporal resolutions for an artificial sub-daily signal with a period of 15 h

If nutation rates are estimated in addition to the ERPs, an additional one-to-one correlation with a retrograde diurnal signal of increasing amplitude according to Eq. (5) has to be handled. Consequently, a retrograde PM term with a mean amplitude is suppressed by the constraint, corresponding to the mean nutation offset over the time-span considered, but, again, the PM estimates remaining after applying the constraint are equal to the difference between a retrograde diurnal signal with linearly increasing amplitude (corresponding to a nutation rate) and the best-fitting retrograde diurnal signal with constant amplitude (corresponding to the mean nutation offset). All three signals are shown in Fig. 4 exemplarily for the x_P coordinate of a 14-day solution after introducing a rate of 1 mas per 14 days for both nutation angles. Applying Eq. (5), these rates lead to an offset of 1.077 mas after 14 days.

3.3 Verification of the singularity using real VLBI data

The consequence of the singularity between nutation and retrograde diurnal PM can easily be demonstrated by using real VLBI data because there is no further singularity with orbital parameters as would be the case for GPS. For this purpose, we computed three types of VLBI solutions: in the first solution, the retrograde constraint was not applied. The second and third solutions were derived by applying the retrograde diurnal

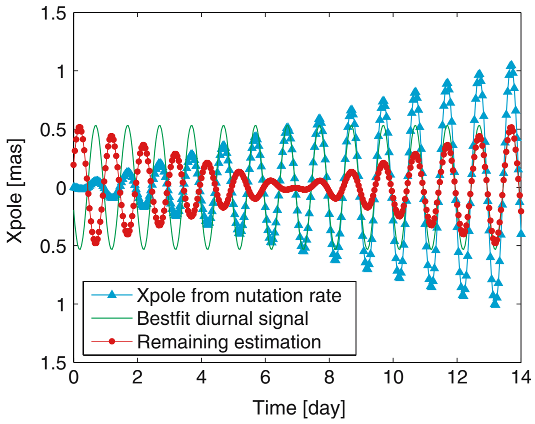


Fig. 4 X_P coordinates estimated simultaneously with nutation rates: x_P directly correlated with the nutation rate has a linearly increasing amplitude (*light grey triangles*), the retrograde diurnal constraint blocks a signal with constant amplitude (*black line*) and the remaining PM estimates are the difference between the two former time-series (*dark grey circles*)

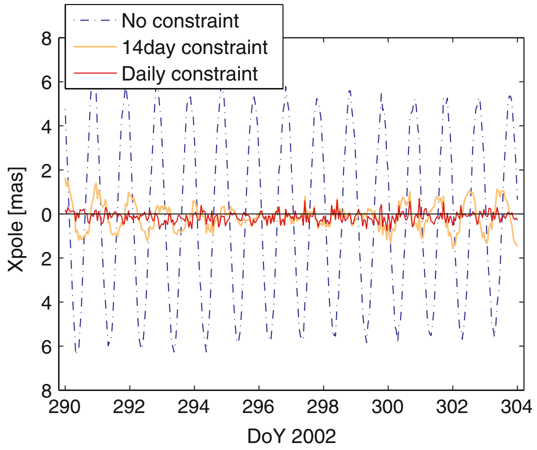


Fig. 5 VLBI-derived hourly PM estimates (x_P) compared to IERS-C04 / IERS2003 with different handling of the retrograde diurnal constraint: the singularity with the nutation angles is fully present if no constraint is applied (*dotted line*). If the retrograde diurnal constraint is applied for the 14-day time span, the singularity with the nutation rate still has an impact (*grey curve*), whereas the singularity is removed if the constraint is applied on a daily basis (*black curve*). In all solutions, the nutation is estimated as a linear function with one interval only

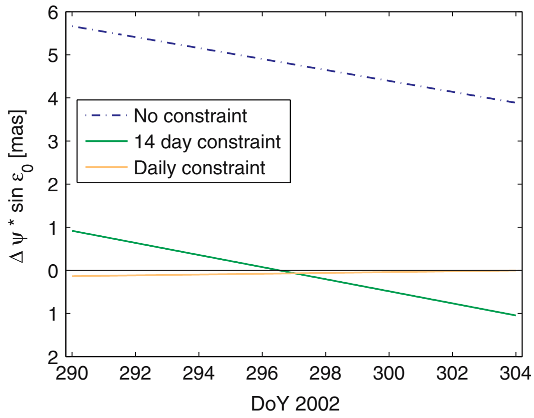


Fig. 6 Nutation in longitude estimated by VLBI (correction to the IAU2000 model) with different handling of the retrograde diurnal constraint: the singularity with the nutation angles is fully present if no constraint is applied (*dotted line*). If the retrograde diurnal constraint is applied for the 14-day time span, the singularity with the nutation rate still has an impact (*dark grey curve*), whereas the singularity is removed if the constraint is applied on a daily basis (*light grey curve*)

constraint, once on the 14-day normal equation and once on a daily basis.

In the first solution, the singularity is fully present and the main signal contained in the resulting x_P coordinates shown in Fig. 5 is a daily signal with more-or-less constant amplitude. The results for y_P (not shown) look similar. Thus, the PM estimates mainly suffer from an offset in the nutation angles due to the correlation. This behavior is confirmed by the results for the nutation angles (see Fig. 6, exemplarily for the nutation in longitude). They are shifted by about 2.776 mas in $\Delta\varepsilon$ and 4.932 mas in $\Delta\psi \sin\varepsilon_0$ compared to the second solution, where the retrograde constraint was applied on the 14-day normal equation. Applying Eq. (3), this shift results in an amplitude of 5.660 mas, which is in good agreement with the results for the PM displayed in Fig. 5.

Comparing the first two solutions, it becomes obvious that the singularity causing the large retrograde diurnal term in the pole coordinates and the corresponding nutation offsets is remedied by the constraint. However, a signal with varying amplitude is still present in the pole coordinates (x_P, y_P) and the size of the amplitude of about 1.6 mas at the beginning and the end is in quite good agreement with the nutation rates estimated in this solution (about 2.6 mas per 14 days computed according to Eq. (5), i.e., 1.3 mas compared with the maximum PM amplitude). Together with the theoretical considerations above, this leads to the conclusion that the singularity caused by a linear drift in the nutation is still present.

Since the basic normal equation systems were generated on a daily basis, a third solution type was derived with the constraint already applied on a daily basis. If the studies concerning the bandwidth of the constraint's influence are taken into account (Table 1), one may argue that it would be better to apply the constraint on the longest possible time span, i.e., 14 days in our case, because then a minor part of the sub-daily spectrum would be blocked. However, the results for PM displayed in Fig. 5 suggest that the singularity involving nutation rates is better remedied by applying the constraint on a daily basis.

Due to the retrograde constraint, VLBI is capable of correctly determining the mean nutation offset. In the case of applying the constraint on the 14-day normal equation, only the mean offset over 14 days is correctly determined (see DoY 297 in Fig. 6), but the singularity concerning nutation rates is still present, so that the separation of the estimated corrections into a PM signal and a nutation rate is arbitrary, except that it has to obey the requirements of the LS adjustment. Applying the constraint on a daily basis requires that the mean nutation offset is given correctly for each day by VLBI so that the nutation rate over 14 days is implicitly also known. Hence, there is no singularity left.

4 Combination results

The results of analyzing real VLBI and GPS data of the CONT02 campaign will be summarized in the sequel. Based on the theoretical studies presented in Sect. 3, the constraint preventing the singularity between nutation and retrograde diurnal PM has been applied on a daily basis.

4.1 Polar motion

The hourly estimates of PM are compared with the IERS-C04 series (Gambis 2004) with a sub-daily model according to the IERS Conventions 2003 (McCarthy and Petit 2004). Figure 7a demonstrates that the x_P coordinates (estimated without applying any constraints except for blocking a retrograde diurnal signal) agree very well with the IERS-C04 and IERS2003 models with mean offsets of 0.083, −0.136 and 0.063 mas for the GPS, VLBI and combined solution, respectively.

The very good agreement of the space-technique solutions with the independent IERS2003 model is confirmed by the root mean square (RMS) values of the comparisons listed in Table 2. Regarding the y_P coordinates (Fig. 7b), the agreement is comparably good if the offsets are neglected. Two types of offsets are present in the y_P estimates: the GPS and the combined solution are shifted by a similar amount (GPS = 0.149 mas, combination = 0.159 mas), whereas the VLBI solution shows a larger offset of about 0.350 mas.

Table 2 RMS of the comparison between the estimated ERPs and the a priori model IERS-C04 / IERS2003 (mean offset removed; a linear drift was additionally removed from the GPS-only UT1–UTC time-series)

	GPS	VLBI	Combined
X-pole [mas]	0.143	0.237	0.120
Y-pole [mas]	0.143	0.244	0.130
UT1–UTC (ms)	0.021	0.015	0.011

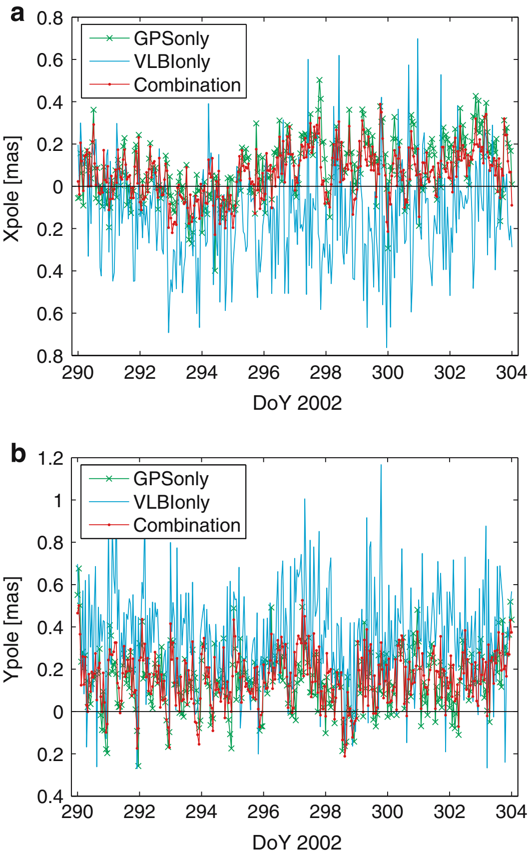


Fig. 7 Hourly PM estimates compared to IERS-C04 / IERS2003 derived from GPS, VLBI and the combined solution (retrograde constraint applied on a daily basis): **a** x_P component, **b** y_P component

Since the "IERS Analysis Campaign to Align EOPs to ITRF2000/ICRF" (see IERS Message No. 19) and the "IERS SINEX Combination Campaign" (see IERS Message No. 27), it is well known that the *y*-pole of IERS-C04 is not consistent with the currently used ITRF2000 reference frame. All analyses in the framework of these two IERS campaigns consistently revealed a bias of about 0.2 mas (e.g., Dill and Rothacher 2003; Thaller and Rothacher 2003).

The significantly larger offset in the y_P component of the VLBI-only solution can be explained mainly by the realization of the geodetic datum. This aspect will be addressed in more detail in Sect. 5.

4.2 Universal time

The combination of UT1–UTC and LOD from VLBI and GPS is often thought to be extremely difficult, or at least not as straightforward as for the pole coordinates (e.g., Ray et al. 2005; Gross et al. 1998). Since satellite orbits have to be estimated in global GPS solutions, the orientation of the Earth in space given by the nutation angles and the diurnal rotation UT1–UTC is not accessible in an absolute sense. However, faster changes in the Earth orientation in space (mainly the corresponding rates) can be determined by GPS quite well.

Due to our method of representing the EOP by a piecewise linear function with functional values at the interval boundaries, UT1–UTC is estimated. This is possible for GPS under the pre-condition that at least one functional value has to be fixed to its a priori value. Starting from this fixed value, the time-series of UT1–UTC estimates is then drifting away due to systematic errors in the modeling of the satellite orbits (see Gross et al. (1998)).

In Thaller et al. (2005), it has already been reported that our GPS solution for UT1–UTC is drifting by nearly 0.7 ms within 14 days, whereas the VLBI time-series does not show any significant offset or drift compared to IERS-C04 or IERS2003. Nevertheless, if a linear trend

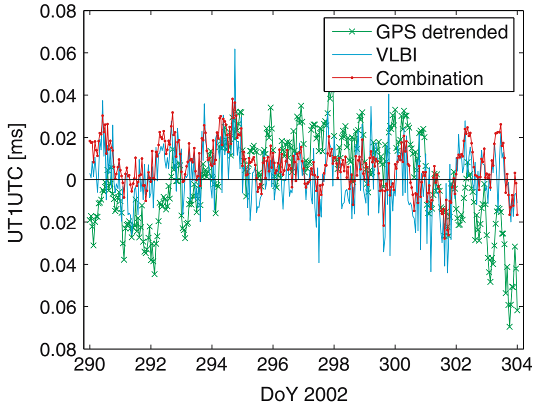


Fig. 8 Hourly UT1–UTC estimates compared to IERS-C04 / IERS2003 derived from GPS, VLBI and the combined solution. A linear drift of about 0.7 mas per 14 days has been removed from the GPS-only time-series as the integration of the estimated LOD values over 14 days evokes a large systematic drift because the satellite orbits cannot be modelled stable enough over such long time spans

is removed from the GPS time-series, the major part of the error due to the drifting orbits is corrected (see Fig. 8). However, a linear trend is only a rough approximation for the behavior of the GPS time-series so that the RMS is still larger than that of the VLBI time-series by a factor of about 1.5 (see Table 2). However, both series have the same order of magnitude agreement with IERS-C04 and IERS2003.

Figure 8 demonstrates that the combined solution follows the VLBI-only solution without removing linear or any other fractions from the GPS contribution. Furthermore, the scatter of the combination is reduced compared to the VLBI-only solution (Table 2). These two facts lead to the conclusion that the potential of VLBI is not deteriorated by the satellite techniques, but quite the contrary: the capability of GPS to deliver very stable rates even leads to an improvement compared to the VLBI-only UT1–UTC time-series.

A wavelet analysis of the estimated UT1–UTC time-series gives additional evidence that VLBI and GPS complement each other. The Morlet wavelet (see, e.g., Goupillaud et al. 1984) was used to analyze our UT1–UTC series. Regarding the periods from 14 days down to about 2 days, Fig. 9 shows that VLBI contributes stability and longer periods to the combined solution, whereas contributions to the sub-daily periods are small (Fig. 10). For this part of the spectrum, GPS contributes more than VLBI since the number of VLBI observations does not permit a strong determination of such high frequencies. For periods around 1 day, both techniques are able to contribute information of about the same quality.

A further explanation for the successful combination can be given by means of the formal errors of the UT1–UTC estimates displayed in Fig. 11. In the same way as the formal errors of the GPS-derived values rapidly increase with time, their weight within the

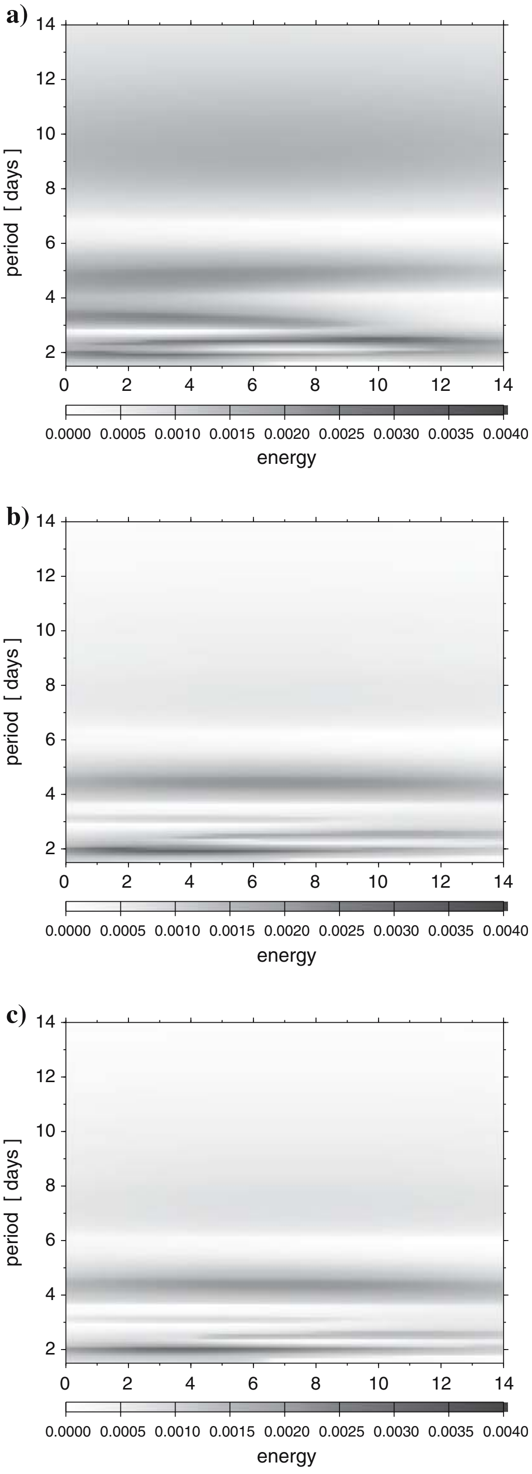
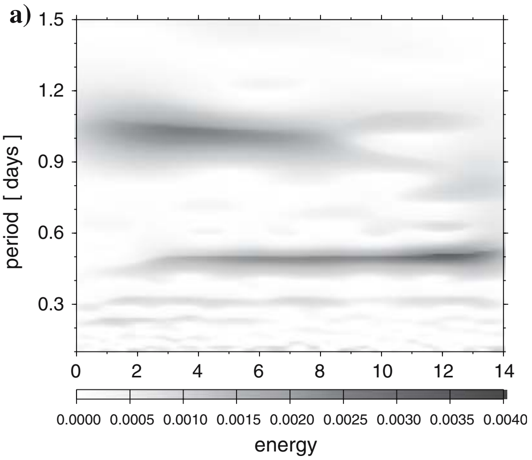
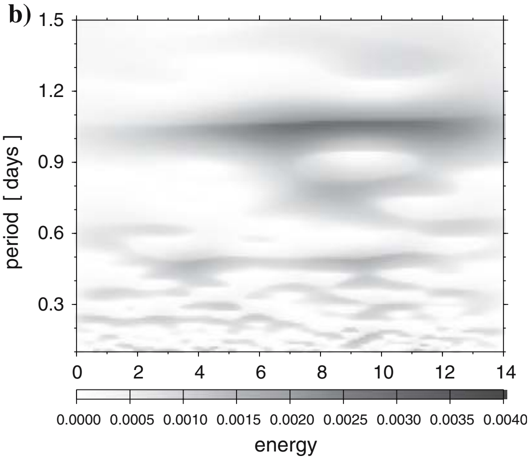


Fig. 9 Morlet wavelet analysis of the hourly UT1–UTC estimates for periods longer than about 2 days [unit for the energy is (s)]: **a** GPS-only, **b** VLBI-only, **c** combined solution





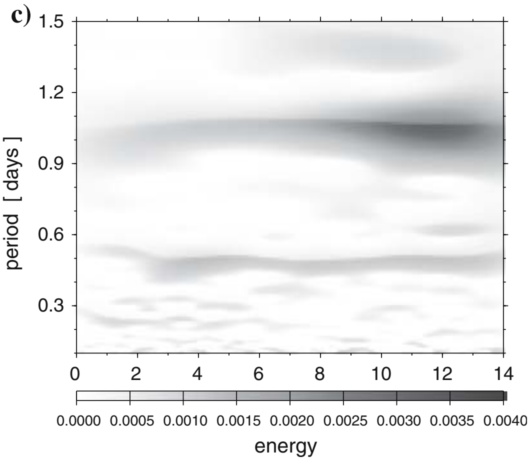


Fig. 10 Morlet wavelet analysis of the hourly UT1–UTC estimates for periods shorter than 1.5 days [unit for the energy is (s)]: **a** GPS-only, **b** VLBI-only, **c** combined solution

combination decreases. This behavior is in contrast to the VLBI solution, where the formal errors are nearly constant over the entire time span. As this performance is conserved in the combination the weighting applied to the input normal equations of the single techniques seems to be reasonable in view of a good determination of UT1–UTC.

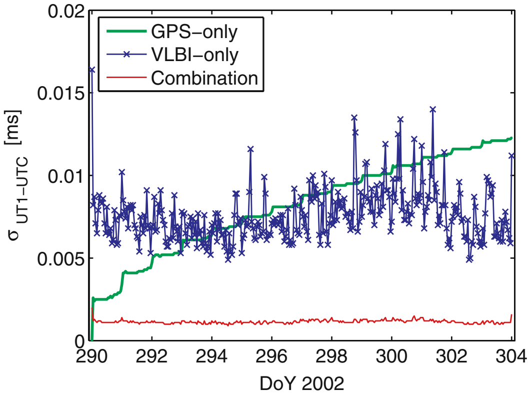


Fig. 11 Formal errors of hourly UT1–UTC estimates (ms) from GPS, VLBI and the combined solution

When interpreting the absolute level of the formal errors, it should be kept in mind that the total number of observations (and consequently the degrees of freedom) within the combination is dominated by the huge amount of GPS observations (several orders of magnitude more than VLBI). Therefore, the decrease of the formal errors for the combination compared to VLBI must be dedicated to the formalism of LS adjustment, and solely the temporal behavior of the formal errors should be compared.

4.3 Nutation

The above statements for UT regarding the different capabilities of VLBI and GPS also hold for the nutation angles $\Delta\varepsilon$ and $\Delta\psi$. The results for the single-technique solutions and for the combination are summarized in Table 3. For the GPS solution, the first value has to be kept fixed on the a priori nutation model. Only the second value can be estimated freely, whereas in the cases of VLBI and combined solutions, both values are estimated without any constraint.

Nevertheless, GPS shows large drifts for the nutation angles: after 14 days, a difference of 1.629 mas for $\Delta\varepsilon$ and −0.740 mas for $\Delta\psi \sin\varepsilon_0$ compared to the a priori model. The drifts derived from VLBI are only about 0.144 and 0.284 mas within 14 days for $\Delta\varepsilon$ and $\Delta\psi \sin\varepsilon_0$, respectively. However, the combined solution shows a rather small drift (0.021 mas for $\Delta\varepsilon$ and 0.108 mas for $\Delta\psi \sin\varepsilon_0$ within 14 days) whose size is comparable to that derived from VLBI. The small shift of the combined solution compared to the VLBI-only solution is caused by the slightly different datum realization: the NNR condition in the combination is based on a subset of GPS stations only, whereas the VLBI network is

Table 3 Nutation angles (mas) and their formal errors (mas) estimated by GPS, VLBI and the combined solution for the starting and the ending epoch of the 14-day CONT02 campaign

	$\Delta\psi\sin\varepsilon_0$		$\Delta\varepsilon$	
	DoY 290	DoY 304	DoY 290	DoY 304
GPS only	Fixed on a priori	−0.740 ± 0.233	Fixed on a priori	+1.629 ± 0.194
VLBI only	−0.207 ± 0.079	+0.077 ± 0.079	−0.188 ± 0.077	−0.044 ± 0.076
Combination	−0.152 ± 0.004	−0.044 ± 0.004	−0.121 ± 0.004	−0.100 ± 0.004

appended by the local ties. A similar situation occurs for the UT1–UTC time-series (cf. Sect. 5).

Nevertheless, we can conclude that the nutation estimates are not biased by the satellite techniques if the orbits are kept free. The reason, again, must be seen in the large formal errors listed in Table 3 and consequently in the small weight of the GPS estimates within the combination over longer time spans. As already mentioned for UT (see Sect. 4.2), only the temporal behavior of the formal errors should be interpreted rather than their absolute level.

Table 4 Mean offsets of the UT1–UTC time-series with respect to IERS-C04 / IERS2003, demonstrating the influence of the local ties (LT). The LT for Fairbanks and Westford have not been applied to the solutions with only six LTs

Solution type	Mean offset (ms)
VLBI only	0.001
Combination (all 8 LTs)	0.014
VLBI constrained to GPS + all 8 LTs	0.015
Combination (6 LTs)	0.006
VLBI constrained to GPS + 6 LTs	0.007

5 Influence of local ties on EOP results

The connection of the TRFs given by the individual space-geodetic techniques is done by applying the local ties (see Sect. 2.2). Since several studies have shown that often they do not fit well to the space-geodetic solutions (Angermann 2004; Ray et al. 2005), the question arises whether the combined EOP solution is sensitive to the selected sub-set of local ties.

The most critical parameters in our approach are UT1–UTC and the nutation angles because the NNR condition for the combination is realized via a GPS sub-network of about 90 stations, whereas the VLBI network itself is appended only by the local ties and is not contained in the realization of the geodetic datum (see Sect. 2.2). However, solely VLBI has access to UT1–UTC and nutation in an absolute sense. Consequently, we evaluated the impact of the local ties by looking at those parameters.

Table 4 demonstrates convincingly what happens if all eight local ties are introduced into the combination: the UT1–UTC time-series is shifted by 0.013 ms compared to the VLBI-only solution. We assumed that this shift might mainly be caused by the different underlying datum definitions. In order to demonstrate whether the shift in UT1–UTC is due to wrong local ties or a misorientation between the GPS and the VLBI part in the ITRF2000 solution, another VLBI solution was computed by tightly constraining all eight stations to the coordinates of the GPS reference points (given in ITRF2000), corrected with the appropriate local tie values instead of directly using the VLBI coordinates. Although the VLBI normal equations are processed only, this procedure simulates the situation of the combined solution using all eight local ties. The resulting UT1–UTC time-series with a nearly identical offset of 0.015 ms confirms our assumption that the bias must be dedicated to the different a priori reference frames used for the NNR condition (see Table 4).

Earlier analyses (Altamimi et al. 2002; Thaller et al. 2005) and the second part of our studies (M. Krügel et al., submitted) revealed that the local ties for Fairbanks and Westford do not fit very well to the space-geodetic techniques. Hence, we did the same studies for solutions ignoring these two local ties and the mean offset of UT1–UTC compared to IERS-C04 and IERS2003 could be reduced to 0.006 ms (see Table 4).

Further tests showed that it is difficult to reduce the offset to that of the VLBI-only solution by ignoring additional local ties. Therefore, the remaining offset of 0.005 ms has to be explained by a slightly different orientation of the VLBI and GPS part of the ITRF2000 network, rather than by the influence of wrong local ties. Any further reduction of local ties introduced in the combination is accompanied by a less stable reference frame for the VLBI stations manifested in a degraded coordinate repeatability. Therefore, we decided to use all remaining six local ties for our final solution (the results have been presented in Sect. 4).

Additionally, the experiment of constraining the VLBI solution on the GPS coordinates corrected by the local ties demonstrates that the offset in the y_P component visible in the VLBI-only solution (Fig. 7b) can be explained by the underlying a priori reference frame used for the NNR condition. The offset in the VLBI-only solution of 0.350 mas is reduced to 0.193 mas if the GPS coordinates together with the six "good" local ties are used for the realization of the VLBI reference frame. This offset agrees much better with the GPS-only solution and the combined solution (see Sect. 4.1).

6 Conclusions

The CONT02 campaign delivered a valuable set of homogeneous and continuous high-quality VLBI observations. Due to the adaptation of our VLBI and GPS software packages with respect to identical models and parameterizations, we have consistent normal equation systems at hand. Such a high consistency is unique, as this is not the case for the solutions officially available from the technique-specific IGS and IVS Analysis Centers. The homogeneity allows us to rigorously combine both techniques without any model discrepancies that may propagate into the estimated parameters and cause a distortion of the combined solution.

Moreover, the homogeneity of the VLBI and GPS normal equations is surely one major reason that the combination yields time-series of PM and UT1–UTC that are more stable than any of the single-technique solutions.

Especially the combination of UT1–UTC and LOD delivered by VLBI and GPS revealed that both techniques perfectly complement each other. Thus, a combination is really worthwhile and yields reasonable results of high quality. Under the pre-condition that the orbits of the GPS satellites are free to be aligned to the VLBI-determined UT1–UTC time-series, the combined solution is not disturbed by the systematic drifts in the GPS orbits. Furthermore, the capability of GPS to contribute very stable short-term information stabilizing the VLBI-only time-series is fully exploited. We envisage to improve these results by similar investigations using longer time-series of homogeneous GPS and VLBI normal equations.

In principle, the conclusions drawn above concerning the combination of UT1–UTC and LOD could also be checked for the other satellite-geodetic techniques, but this was not done so far since a DORIS (Doppler orbitography and radiopositioning integrated by satellite) solution stemming from a perfectly adapted software package was not available. SLR does not even have the capability to determine hourly ERPs, but in the case of estimating daily ERPs, SLR can contribute to the results. Studies concerning the formal errors of daily UT1–UTC estimates for CONT02 confirm the conclusions drawn for GPS (Thaller et al. 2006).

The hourly resolution chosen for PM and UT1–UTC demonstrates the potential of GPS, VLBI and their combination to deliver time-series of ERP with such a high resolution.

Regarding the one-to-one correlation between the nutation angles and a retrograde diurnal signal in the PM, our studies helped to get a better understanding on how to deal with this singularity. It was demonstrated that not only the correlation concerning nutation offsets can be handled with the available constraint but also the correlation involving nutation rates. However, the dependence on the length of the time-series has to be kept in mind if special signals are to be detected in the resulting time-series of PM.

The sensitivity of the combined EOPs to the selection of local ties was demonstrated. Furthermore, in the companion paper (M. Krügel et al., submitted), we show the sensitivity of the combined troposphere parameters to discrepancies between the local ties and the space-geodetic TRFs. Both studies emphasize the request formulated by Altamimi et al. (2005) in view of the International Association of Geodesy (IAG) project GGOS (Global Geodetic Observing System) that the maintenance of co-location sites needs more attention as they are highly important for the quality of combined geodetic solutions.

Finally, the succeeding continuous VLBI campaign of CONT02, named CONT05, already took place with very promising first results (Thomas and MacMillan 2006). The estimation of EOPs should benefit especially from the enlarged network (11 stations altogether) with an improved global distribution. We recommend that the studies similar to those presented here should be performed using the CONT05 data set.

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