# Combined Head and Eye Tracking System for Dynamic Testing of the Vestibular System 

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#### Abstract

We present a combined head-eye tracking system suitable for use with free head movement during natural activities. This system provides an integrated head and eye position measurement while allowing for a large range of head movement (approx 1.8 m of head translation is tolerated). Six degrees of freedom of head motion and two degrees of freedom of eye motion are measured by the system. The system was designed to be useful for the evaluation of the vestibulo-ocular reflex (VOR). The VOR generates compensatory eye movements in order to stabilize gaze during linear or rotational motion of the head. Current clinical and basic research evaluation of the VOR has used a restricted range of head motion, mainly low-frequency, yaw rotation. An integrated eye-head tracking system such as the one presented here allows the VOR response to linear and angular head motion to be studied in a more physiologically relevant manner. Two examples of the utility of the integrated head and eye tracking system in evaluating the vestibular response to linear and angular motion are presented.


## I. Introduction

DURING natural activities the visual, vestibular, and motor-kinesthetic systems interact to maintain stable gaze upon objects of interest. The vestibulo-ocular reflex (VOR) generates compensatory eye movements in response to head motion detected by the vestibular sense organs located in the inner ear. The oculomotor response to angular head movement is called the angular VOR (AVOR or VOR) and has been demonstrated for rotation in yaw, pitch, and roll. An oculomotor response to linear acceleration has been described for acceleration along the interaural axis [1], [2], spinal axis [3], and nasal-occipital axis [4] and has been called the linear VOR (LVOR). The VOR is crucial to the maintenance of gaze stability and visual acuity [5]. Patients who have lost their vestibular systems suffer from illusory motion of the seen world (oscillopsia) during head motion and may have difficulty recognizing familiar faces while walking (for an illustrative account see [6]). These deficits illustrate that the VOR plays a crucial role during natural activities such as

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locomotion. During locomotion, the head is subjected to highfrequency head perturbations of up to 15 Hz [7]. Although the VOR seems specialized for high frequency, the reflex has typically been studied with nonphysiological stimuli such as low frequency or constant velocity rotation.
In order to study the VOR under natural conditions, both head and eye movements must be measured accurately during free head motion. Current systems for tracking head and eye movements have serious limitations for use during free head movements [8]. For example, use of the magnetic scleral search coil [9], is precluded during free head motion by the requirement to remain in the center of the magnetic field generated by the field coil. Use of this technique is also limited by the inability to make measurements of linear motion of the head. Electrooculography (EOG), the standard clinical technique, is limited by poor sensitivity, poor vertical and binocular measurements, artifacts introduced by muscle action potential, and electrical noise introduced by electrode movement and other physiological and environmental sources [10]. Due to these limitations, most studies of eye movements have been performed with the subject's head stationary, often "chair-fixed." Little is known about the role that eye movements play during natural activities such as locomotion [11].

## II. Description of Integrated Head and Eye Tracking System

To address the above concerns an integrated eye-head tracking system has been developed (see Fig. 1) which allows for free head and body motion over a range of 1.8 m in any direction ( 5.8 cubic meters of space). The combined head-eye tracking device is based upon a video eye tracking system (VTS) (Model 2020, EL-MAR Inc., Toronto, Ont., Canada; [8], [12]) and a magnetic head tracking system [Flock of Birds Position and Orientation Measurement System, Ascension Technology Corp., Burlington, VT].

## A. Eye Tracking Subsystem

Binocular horizontal and vertical eye position estimates are derived from the relative positions of multiple corneal reflections and the center of the pupil [13]. By using two eye landmarks (corneal reflections and pupil center) whose relative position are invariant under translation, this instrument is able to estimate the angular position of the eye independently of lateral motion of the video system relative to the head. This is of critical importance since perfect mechanical stabilization


Fig. 1. A subject wearing the integrated head and eye tracking system is shown. The head tracker transmitter is located above the subject. The head tracker receiver is a small cube mounted to the goggle assembly (arrow).
of the instrument to the head is impossible during dynamic head movements.

The optical components of the VTS are mounted on a light weight (total system weight 190 g ) eye glasses frame. The system is binocular and two mirror-image optical systems are mounted on each side of the eyeglasses frame. The corneal reflections are generated by illumination with two infrared LED's mounted to the glasses frame. These LED's also serve to illuminate the pupil. Use of infrared (IR) light allows for invisible illumination of the eye. The use of multiple corneal reflections extends the linear range of the system by ensuring that one corneal reflection is always visible on the spherical surface of the cornea even with eccentric gaze [10]. The images of the pupil and corneal reflections are reflected off of an IR mirror positioned in front of the subject's eye and directed to the cameras. This mirror is transparent to visible light and thus does not interfere with normal vision.

The video image is sampled by a custom charge-coupled device (CCD) array that allows images to be sampled at 120 Hz . Images from the CCD camera are processed in real time to obtain estimates of the corneal reflection and pupil center locations. Precise estimation of the corneal reflex is based on techniques described by Eizenman et al [14]. The estimate of the pupil center and pupil size is based on algorithms described by Landau [15].

The eye tracking subsystem has been characterized recently and compared with the magnetic scleral search coil data recorded simultaneously [8]. This study reported excellent
agreement between the VTS estimates and search coil data with somewhat higher noise levels in the VTS estimates. The resolution of the VTS is less than $0.1^{\circ}$ with a range of $\pm 40^{\circ}$ and $\pm 30^{\circ}$ in the horizontal and vertical directions respectively [8]. Typical noise in eye velocity records during pursuit tasks was greater than that of the search coil and was approximately $4-5 \%$. Calibration of the eye tracker can be performed using a calibration procedure requiring $2-3 \mathrm{~min}$ [12]. The noise spectra of the eye tracker was found to be flat throughout the 60Hz bandwidth at a typical value of $8 \times 10^{-4} \operatorname{deg}^{2} / \mathrm{Hz}$ with no significant spectral peaks [16].

## B. Head Tracking Subsystem

Six degrees of freedom of head motion are measured by a magnetic position transducer mounted to the eye-tracker goggle assembly. The small $(25.4 \mathrm{~mm} \times 25.4 \mathrm{~mm} \times 20.3$ mm ), lightweight transducer is mounted on a strap over the top of the head close to the center of head rotation for azimuth rotation. A fixed transmitting device radiates a pulsed magnetic field in which the head mounted receiver is immersed. The field is sensed by the receiver and processed by a microprocessor to provide three-dimensional (3-D) position information as well as head elevation, azimuth and roll angles. The head tracker provides absolute angular and translational position measurements and does not require calibration for each subject. The head tracker can operate with multiple receivers allowing for measurement of other important parameters such as hand position in hand-eye coordination studies. Measurement quality is compromised by low-frequency electromagnetic interference (EMI) and the presence of large metal objects in the field. To ensure performance, the transmitter and receiver were located away from sources of EMI such as monitors and power supplies.

The head tracker [17] has a translational linear range of $\pm 91 \mathrm{~cm}$ and an angular measurement linear range of $\pm 180^{\circ}$ in azimuth and roll, and $\pm 90^{\circ}$ in roll. The output power of the head tracker transmitter is adaptively changed in order to maximize signal-to-noise (SNR) ratio without saturating the receiver. Within 24 cm , adaptive gain control increases transmitter power as the distance between transmitter and receiver increases such that SNR is relatively constant. Outside this range SNR deteriorates as the magnetic field strength decreases with distance. With the receiver within 24 cm of the transmitter, the angular resolution of the system is less than $0.1^{\circ}$ root mean square (rms) and the translational resolution is less than 0.75 mm (at 48 and 60 cm the azimuth resolution is $0.75^{\circ}$ and $1.78^{\circ} \mathrm{rms}$, respectively). The head tracker was designed to make accurate position measurements over the same $60-\mathrm{Hz}$ bandwidth as the eye tracker. The noise spectra of the head tracker was found to be flat throughout the 60Hz bandwidth at a typical value of $6 \times 10^{-3} \operatorname{deg}^{2} / \mathrm{Hz}$ with no significant spectral peaks [16].

## C. Evaluation of the Integrated System

Data from the head and eye trackers is synchronized by a hardware control signal that triggers the video frames of the eye tracker in synchrony with the pulsed magnetic field of
the head tracker. Data from the eye tracker is also delayed by 8.3 ms in order to match the delays of the two subsystems. The dynamic performance of the combined head-eye tracking system was investigated by calculating gaze angle and the gain and phase of the VOR during smooth head oscillations and during active head turns. These results were compared with the results of studies using the magnetic scleral search coil technique (the "gold standard").
VOR Gain and Phase: VOR gain and phase during active self-paced head movements were estimated using power spectral analysis. The target for the experiment was a lit LED located centrally at a distance of 1.2 m . Head and eye velocities were obtained from the position estimates using a five-point differentiator ( $30-\mathrm{Hz}$ bandwidth). Saccadic eye movements, blinks, and artifact (for example interference of partially closed eyelids with the pupil boundary, see also [8], [10], and [13] for artifact associated with video-based eye tracking methods) were identified and removed. The removed intervals were interpolated through with a least squares quadratic fit [18]. Power spectra of head motion $P_{H H}(\omega)$, eye movements $P_{E E}(\omega)$, and cross-spectral density $P_{H E}(\omega)$ (the Fourier transform of the cross-correlation) of the eye and head sequences were calculated using Welch's averaged periodogram ( 1024 points, 512 -point overlap) with a Hanning window for sidelobe reduction [19]. We have also used the magnitude squared coherence (MSC) function, $C_{H E}(\omega)=P_{H E}^{2}(\omega) / P_{H H}(\omega) P_{E E}(\omega)$, to define the coherence between the two data sequences. A high degree of coherence is indicative of a strong linear relation between the two signals [20]. From the MSC, approximate $95 \%$ confidence intervals for gain and phase as provided by Jenkins and Watts [21] were computed and used to distinguish spectral peaks from spurious spikes.

VOR gain and phase estimates were computed at the highest power spectral components where coherence was always higher than 0.98 . VOR gain and phase estimates for three subjects are shown in Fig. 2 where gain is always close to one and phase is near $180^{\circ}$ (defined as $0^{\circ}$ by convention). This is in agreement with studies which indicate that VOR gain and phase while fixating a space-fixed stationary target (visually enhanced VOR) are nearly perfect until frequencies higher than 5 Hz [7] and [22]. Since our target was located at a distance of 1.2 m an increase in measured VOR gain is to be expected [23] and ideal VOR gain for our experiments is approximately 1.1 . When this gain enhancement is considered, the gains (range $0.97-1.16$ ) in Fig. 2 are comparable with the 0.83-1.09 for horizontal and vertical head shaking during fixation at optical infinity (ideal gain of 1.0 ) reported by Grossman et al. [7] and to the results of Collewjin et al. [22]

## D. Point of Regard Estimation

The point of regard is the intersection of the line of sight with the surface on which the stimulus is presented. In the integrated system, the head tracker measures the location and orientation of the head with respect to an earth-fixed transmitter frame of reference. The eye tracker measures the orientation of the eyes with respect to a head fixed frame


Fig. 2. VOR gain and phase from spectral analysis of the ocular response to rapid active head turns in three subjects.
of reference. These measurements can be combined using transformation matrices as described in the Appendix in order to obtain the origin and orientation of the line of sight in the original transmitter frame of reference. The point of regard is then obtained by finding the intersection of this line with the stimulus surface.

The dynamic performance of the combined head-eye tracking system was investigated by calculation of gaze angle and point of regard during self-paced rhythmic head oscillations and random, vigorous free head movements in two normal subjects. Point of regard estimation was studied at two target distances ( 82 cm and 222 cm ) to investigate the effects of target distance on the point of regard estimate.

Since the head and eye do not rotate around the same center of rotation, the eye is translated during head rotation. This translation becomes significant for point of regard estimation with close targets. The relative location of the center of rotation of the eye with respect to the head mounted head tracker receiver varies for each subject as a result of anatomical considerations. For each subject, the distance from the receiver to the apex of the cornea was measured. The point of regard was computed off-line according to the


Fig. 3. The point of regard estimate is shown as a $Y-Z$ plot of the point of regard estimates (shown with error bars) for several fixation targets, (*) during smooth head oscillations at 222 cm .
calculations outlined in the Appendix and compared to the target position.

During active head oscillations, the stability of the point of regard estimate was measured for fixation of a central target at a distance of 222 cm . For attempts at smooth vertical, horizontal, or depth translations (which always had associated rotational components) the standard deviation of the point of regard was typically less than $1.63 \mathrm{~cm}\left(0.42^{\circ}\right.$ at 222 cm , Fig. 3). The head movements had peak to peak oscillations ranging from $6.7-14.6^{\circ}$ and the predominant frequencies of the oscillations ranged from $0.35-0.82 \mathrm{~Hz}$. The relatively small deviations in gaze during these oscillations indicate that gaze stability was maintained throughout these trials. Ferman et al. [24] have used a scleral search coil to measure gaze stability during low frequency, low amplitude ( $0.67 \mathrm{~Hz}, 10^{\circ}$ ) head oscillations and have reported a standard deviation of gaze angular position of approximately 16 min of arc for fixation of a distant target. The increased variability in our results can be attributed to larger variances in eye-position and head-position estimates or intersubject variability.
Similar head oscillations at the close distance $(82 \mathrm{~cm})$ introduced an average gaze deviation of $1 \mathrm{~cm}\left(0.69^{\circ}\right)$ during fixation of the central target. The additional variance for close targets can be attributed to errors in measurement of the location of the center of rotation of the eye relative to the head tracker receiver. Since the eyes are not located at the center of head rotation, any rotation of the head also entails translation of the eye relative to visual targets [23]. For targets at optical infinity this translation does not require any compensatory movement. For near targets (our stimulus), however, this translation becomes significant and compensatory eye movements are required for stable gaze [23] and [25]. In order to calculate the effect of translation on point
of regard at close target distances, the distance between the center of rotation of the eye and the head tracker receiver must be measured. Errors in these measurements could contribute to additional instability.

During vigorous random head movements, measured gaze stability declines and the distribution of point of regard estimates spreads out. This was accompanied by subjective reports of difficulty in maintaining fixation. During unrestrained head movement at a distance of 82 cm , the standard deviation of gaze is less than 2 cm (approx. 1.39 ${ }^{\circ}$ [16].

## III. Applications

As an illustration of the utility of the integrated system, the next section discusses two applications of this system. The two studies describe how this system may be used to investigate the compensatory vestibulo-ocular responses to angular (angular VOR) and linear movement of the head (linear VOR). The first application is in a study involving clinical testing of the angular VOR. The second investigates the role of the otolith-ocular response during changing translational forces.

## A. Evaluation of the VOR Following Unilateral Peripheral Lesion

Identification of the side of a vestibular lesion is a key clinical determination which must be made in the diagnosis of the patient with dizziness. Loss of peripheral vestibular function in one ear results in asymmetry in vestibular response. Tests to lateralize a vestibular lesion rely on detecting this asymmetry. The adaptive capabilities of the VOR make detection of a unilateral lesion with low-frequency rotational testing difficult [26].

Halmagyi and Curthoys [27] have described a clinical test for canal paresis based on the response to rapid passive head turns. In this test the subject's head is grasped firmly and rapidly turned to one side. If the VOR gain is adequate, the eye will rotate opposite to the head and maintain stable gaze. If inadequate, visible catch-up saccades (quick refixation movements) will be required to maintain gaze.

Using the integrated system we studied the response to rapid head movements in normal subjects $(N=6)$ and patients after compensation from therapeutic surgical unilateral vestibular ablation $(N=3)$. Rapid, passive, horizontal head turns ( $10-20^{\circ},>150^{\circ} / \mathrm{s}^{2}$ peak velocity, $>1500^{\circ} / \mathrm{s}^{2}$ peak acceleration) were delivered manually while the subject fixated on an LED at 1.2 m . Timing and direction of the passive head movements were randomized to eliminate prediction. Blinks, saccades or artifact were identified and removed from the analysis.

In normal subjects the eye velocity during the first 100 ms of the head turn was highly correlated ( $r>0.97$ ) to the head velocity and no nonlinearities can be seen. Gain in these normal subjects is symmetrical for both directions of head motion [see Fig. 4(a)] and is always close to or above unity (mean 1.03). As in Halmagyi's work the neurectomy patients we studied show a marked reduction in gain (mean 0.35 ) during rotation toward the side of lesion. Compensatory eye movements are very small and catch-up saccades (quick re-fixation movements) are typically required. This reduction


Fig. 4. (a) The response to rapid passive head turns in a normal subject and (b) the asymmetric response after a right hemilabyrinthectomy.
in gain is apparent at even low head velocity [see Fig. 4(b)]. Rotation toward the intact side elicits eye movements with a normal or slightly reduced gain in the three patients tested (mean 0.95).
Following vestibular nerve ablation, a deficit exists in the response to rapid head turns toward the side of lesion [27, this study]. This persistent deficit occurs despite near complete recovery from static symptoms such as spontaneous nystagmus, recovery of low-frequency VOR gain symmetry, no vertigo, and a return to an active lifestyle.

During locomotion, high-frequency, moderate velocity $\left(200^{\circ} / \mathrm{s}\right)$ vibrations can be transmitted to the head during heel spike [7]. These events are similar to the types of perturbations used in our experiment. The response to rapid, unpredictable
head perturbations such as those in this study may prove to be a better functional indicator of VOR performance than low-frequency rotation. Rapid passive head turns provide a means of testing the high-frequency VOR and may be useful as an adjunct to low-frequency testing.

## B. Evaluation of the Vestibulo-Ocular Motor Response Under Changing Force Environment

The ability of a pilot to perceive important visual cues, either from the external world or from flight deck instruments, can be degraded by factors that impair either the quality of the retinal image or the transduction process of the image by the sensory cells of the retina. In flight, vibration, angular motion and translation are common causes of destabilization of the retinal image, and can, in certain circumstances, be of sufficient severity to prevent the pilot from reading the instrument. Using the integrated head and eye tracking system, we studied the oculomotor response of subjects in upright and supine position under linear translation as induced by counter-rotation. This type of movement simulates the hovering capability of a helicopter and the proposed translational capability of next generation fighter aircraft.

The motion device is a man-rated small centrifuge $(1.8 \mathrm{~m}$ in radius) driven by a motor of maximum torque ( $12.2 \mathrm{~N}-\mathrm{m}$ ) to produce the rotating force environment. At the end of the centrifuge arm is a secondary turntable ( 0.75 m in radius) housing the seat. The secondary turntable always revolved at the same rate as the main centrifuge but in the opposite direction and hence was not subjected to angular velocity. During rotation, the subject either sat upright or lay with head supine at the end of the arm (Fig. 5). When the subject was supine, the subject experienced linear accelerations along both the interaural and dorsoventral body axes. To measure eye and head movements during rotation in the centrifuge, the eyetracker and head-tracker receiver were mounted on a helmet (Fig. 5).
Experiments with six subjects [28] show that each change in direction of the acceleration stimulus was accompanied by a change in the slow component of horizontal and vertical eye velocity. The time course of these changes was such that the reversals in the direction of the slow components lagged behind the reversals in the direction of the acceleration (Fig. 6). The horizontal and vertical nystagmus induced in the supine subject is primarily an otolith mediated reflex since only a linear acceleration vector with a component that is rotated in the transverse plane of the head was evident and there was no angular acceleration involved.

In this application, we have shown how the integrated system can be used in the investigation of oculomotor response under translation as induced by counter-rotation. The ability of the system to measure linear head movements allows it to be used for similar experiments under various flight maneuvers in a flight simulator.

## IV. Discussion

The system described in this paper is based upon a precision eye tracker and head tracker each with a resolution of several


Fig. 5. A centrifuge with an eye-head measurement system for studies of the otolith organs. Subjects rotating in the centrifuge experience pure linear acceleration (no angular acceleration to stimulate the semicircular canals). The subject's eye movements are measured by a helmet-mounted video eye-tracking system (EL-MAR Inc.-Model 2020). The subject's head movements are measured by a magnetic head-tracking system (Ascension Inc.-Flock of Birds). The inset shows a close-up of the integrated system.
minutes of arc. This level of precision is normally sufficient for the measurement of combined head and eye movements. The instrument is more precise than the standard clinical EOG technique and is comparable to the magnetic scleral search coil technique [8]. The current device does not support torsional eye movement monitoring although this is possible with video techniques [29] and [30].

The system allows a wide range of natural head motion, which makes it suitable for use during natural activities. The system is light weight and incurs minimal discomfort or interference with movement. The linear range of the eye tracker includes the range of eye movements seen during most natural unrestrained head and eye movements [5]. The ability to track six degrees of head motion and two degrees of eye rotation allows for real time calculation of line of sight in 3-D space. Human beings have the ability to make precise and rapid head and eye movements making line of sight tracking suitable for control applications. This is useful in applications where the hands and feet are required for other tasks or when operator overload is a concern; a prime example being aircraft weapons targeting systems [10].

The ability to precisely measure head and eye movements during natural activities has potentially useful clinical and basic research applications. During natural motion the head and body experience linear motion as well as rotation. The role that the otolith driven LVOR plays during every day activities or during highly demanding activities such as flying a high performance aircraft is poorly understood. The ability to measure linear head motion easily suggests that the system may be useful in measurement of the LVOR response [25]. The ability to move the head freely over a large linear range suggests that the LVOR could be measured with rather simple stimuli for clinical or basic research evaluation of the reflex.
In the clinic, the vestibulo-ocular reflex is often evaluated in the diagnosis of patients with dizziness, vertigo, ataxia, and hearing loss. Current testing utilizes low-frequency rotation (e.g., 0.1 Hz ) of a subject in a horizontal plane. During natural movements such as locomotion, however, the head is subjected to rapid, unpredictable transient head perturbations with frequency components of up to 15 Hz [7]. The long latency and poor high-frequency response of visual or cervical reflexes make them poorly suited for responding to these


Fig. 6. Horizontal and vertical eye movements of a subject attempting to maintain stable fixation in darkness while rotating in a centrifuge ( 20 rotations per minute). The top two traces show horizontal eye movements and the centrifuge's horizontal linear acceleration. Notice the correspondence between the phases of linear acceleration and deceleration of the centrifuge and the direction of the slow phases (smooth, low velocity portions) of the eye movements. The bottom two traces show vertical eye movements and the centrifuge's vertical linear acceleration.
perturbations [5]. Leigh and Brandt [11] have argued that a primary physiological role of VOR is to compensate for the high-frequency perturbations encountered during locomotion. Patients may show complete recovery of low-frequency VOR response but complain of poor vision and oscillopsia during locomotion. The ability to study the VOR with more physiologically relevant stimuli may allow the development of tests of VOR function that correlate well with the patient's symptoms. Current clinical vestibular equipment is highly specialized, bulky and requires a dedicated laboratory. The ability to measure the VOR with a portable system such as the one described above might allow quantitative evaluation of the VOR in the physician's office.

## Appendix <br> Point of Regard Estimation

Three-dimensional motion of rigid bodies can be described by homogeneous transformations between frames of reference fixed to objects of interest. Homogeneous transformations combine the operations of translation and rotation into a single matrix multiplication. This approach is a standard method for describing the kinematics of 3-D systems, particularly in robotics [31], machine vision, aircraft attitude control, and computer graphics modeling. Halswanter [32] has provided a recent review of the use of these techniques for eye movement kinematics.

Head position and orientation is measured with respect to an earth fixed frame of reference designated frame 0 (Fig. 7). For our studies, this frame of reference is attached to the transmitter of the magnetic head-tracker. The head tracker transmitter is located above the head and with the same orientation as the head mounted receiver when the head is in the primary position. The second frame of reference (frame 1 ,


Fig. 7. Head and eye position are referred to frame 0 , an earth fixed frame of reference. The head tracker measures the position $\left(d_{0}^{1}\right)$ and orientation in frame 0 coordinates of a head mounted receiver fixed to frame 1 . The orientation of an eye fixed frame, frame 2 , in the orbit (i.e., in frame 1 coordinates) is measured by the eye tracker. The eye position, $d_{1}^{2}$, in frame 1 (i.e., head coordinates) is determined by the anatomy of the subject and mounting location of the head tracker receiver. The equivalent of frame 2 for the left eye is not shown. Head orientation is measured for azimuth rotation about a dorsal-ventral axis $(z)$, elevation about an interaural axis $(y)$ or roll about a nasal-occipital axis ( $x$ ); head position is measured for translation along $x, y$, or $z$. A right-handed coordinate system for measurement of eye position is established through the center of rotation of the eye with the $x$ axis along the line of sight, the $y$ axis is horizontal and the $z$ axis vertical. The directions of the azimuth and elevation rotations are positive for rightwards and upwards, respectively.
see Fig. 7) is fixed to the head tracker receiver. The orientation and translation of this frame of reference are measured by the head tracker. The right-handed coordinate system chosen for head position and orientation measurements is shown in Fig. 7. Any head position and orientation can be described as combination of rotations in azimuth $\left(\theta_{h}\right)$, elevation $\left(\phi_{h}\right)$, and roll $\left(\psi_{h}\right)$ plus a translation of the head $\left(x_{h}, y_{h}, z_{h}\right)$. Since rotation is not commutative [31] the order of the rotations must be specified. In the system used in this Appendix, the head is first rotated $\theta_{h}$ (azimuth) around the $z$-axis, then rotated $\phi_{h}$ (elevation) around the resulting $y$-axis and then $\psi_{h}$ (roll) around the resulting $x$ axis. The two frames of reference will coincide when the position is $(0,0,0)$ and the azimuth, elevation, and roll are $\left(0^{\circ}, 0^{\circ}, 0^{\circ}\right)$.

The third frame of reference (frame 2, see Fig. 7) is fixed to the center of rotation of the eye and is at a fixed distance
$\left(x_{d}, y_{d}, z_{d}\right)$ from the head fixed, head tracker receiver. This distance is constant but differs for each subject based on anatomical characteristics. For study of the VOR it has been suggested that the same coordinate conventions be maintained for head and eye orientation measurements [4]. In order to be compatible with the frame of reference for head tracker measurements we have defined the eye axes as $x_{2}, y_{2}$, and $z_{2}$ for the primary, horizontal, and vertical axes, respectively (see Fig. 7). Since we do not measure ocular torsion this right-handed system can be used for measurements of eye movements without modifying the directions of azimuthal and elevation rotation from a conventional left-handed Fick's system (e.g., [9]). The position of the eyes relative to the primary position can be described in terms of a sequence of successive rotations about the principle axes. These rotations are $\theta_{e}$ around the $z_{2}$ axis (horizontal, yaw, or azimuth rotation) and $\phi_{e}$ around the $y_{2}$ axis (vertical, pitch, or elevation rotation). Torsional rotation or roll of $\psi_{e}$ around the $x_{2}$ axis can also occur but does not reorient the line of sight.

Measurement of eye and head position and orientation allows for estimation of the point of regard. To measure the point of regard the intersection of the line of sight with the stimulus plane must be determined. The eye tracker allows measurement of the line of sight in a head fixed frame of reference. A simple set of transformations must be performed to determine the line of sight in an earth fixed (i.e., $x_{0}, y_{0}, z_{0}$ ) frame of reference.

The first transformation is from the base frame (frame 0 ) to the head fixed frame (frame 1) and is described by the transformation

$$
H_{0}^{1}=\left[\begin{array}{cc}
R_{0}^{1} & \boldsymbol{d}_{0}^{1} \\
\langle 0\rangle_{1 \times 3} & 1
\end{array}\right] .
$$

This homogeneous transformation consists of a rotation $R_{0}^{1}$ and a translation $\boldsymbol{d}_{0}^{1}$. Both the rotation matrix $(3 \times 3)$ and the translation vector $(3 \times 1)$ are provided by the head tracker. The translation vector is obtained from the linear position of the head tracker receiver

$$
\boldsymbol{d}_{0}^{1}=\left[\begin{array}{l}
x_{h} \\
y_{h} \\
z_{h}
\end{array}\right]
$$

The rotation matrix shown at the bottom of the page is obtained from the azimuth $\theta_{h}$, elevation $\phi_{h}$, and roll $\psi_{h}$ [31].

The second transformation is from the head frame to the eye frame and is described by

$$
H_{1}^{2}=\left[\begin{array}{cc}
R_{1}^{2} & d_{1}^{2} \\
\langle 0\rangle_{1 \times 3} & 1
\end{array}\right]
$$

where

$$
\boldsymbol{d}_{0}^{1}=\left[\begin{array}{l}
x_{d} \\
y_{d} \\
z_{d}
\end{array}\right]
$$

is the distance from the head tracker receiver to the center of rotation of the eye in frame 1 coordinates (the head frame) and must be measured for each subject. The orientation of the eye in frame 1 coordinates is obtained from the eye tracker estimates of azimuth and elevation.

Since the direction of the line of sight in the head frame of reference (frame 1) can be obtained from the unit direction vector along the $x$ axis of the eye, we are only concerned with the first column $\hat{\boldsymbol{\imath}}_{1}^{2}$ of the rotation matrix

$$
R_{1}^{2}=\left[\begin{array}{lll}
\hat{\mathbf{i}}_{1}^{2} & \hat{\boldsymbol{\jmath}}_{1}^{2} & \hat{\boldsymbol{k}}_{1}^{2}
\end{array}\right]
$$

or more explicitly

$$
\hat{\mathbf{\imath}}_{1}^{2}=\left[\begin{array}{c}
\cos \left(\theta_{c}\right) \cos \left(\phi_{e}\right) \\
\sin \left(\theta_{e}\right) \cos \left(\phi_{e}\right) \\
-\sin \left(\phi_{e}\right)
\end{array}\right]
$$

where $\theta_{e}$ and $\phi_{e}$ are the eye tracker azimuth and elevation angles.

The overall transformation from the transmitter frame to the eye frame can be described by multiplying the two transformation matrices. The order of the multiplication is determined by the fact that the eye tracker measures of eye position are made with respect to the head frame of reference.

$$
\begin{aligned}
T_{0}^{2} & =H_{0}^{1} H_{1}^{2} \\
& =\left[\begin{array}{cc}
R_{0}^{2} & \boldsymbol{d}_{0}^{2} \\
\langle 0\rangle_{1 \times 3} & 1
\end{array}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
R_{0}^{2} & =R_{0}^{1} R_{1}^{2} \\
\boldsymbol{d}_{0}^{2} & =\boldsymbol{d}_{0}^{1}+R_{0}^{1} \boldsymbol{d}_{1}^{2} \\
& =\left[\begin{array}{l}
x_{e} \\
y_{e} \\
z_{e}
\end{array}\right]
\end{aligned}
$$

The line of sight (l.o.s.) is along the $x$ axis in the eye coordinate frame and is described by the unit direction vector corresponding to the $x$ axis of the eye frame (frame 2 ) in earth fixed (frame 0) coordinates

$$
\begin{aligned}
\text { l.o.s. } & =\hat{\mathbf{\imath}}_{0}^{2} \\
& =R_{0}^{1} \hat{\mathbf{i}}_{1}^{2} \\
& =R_{0}^{1}\left[\begin{array}{c}
\cos \left(\theta_{e}\right) \\
\cos \left(\phi_{e}\right) \\
\sin \left(\theta_{e}\right) \\
-\cos \left(\phi_{e}\right) \\
-\sin \left(\phi_{e}\right)
\end{array}\right] \\
& =\left[\begin{array}{c}
S x_{\text {los }} \\
S y_{l o s} \\
S z_{l o s}
\end{array}\right]
\end{aligned}
$$

$$
R_{0}^{1}=\left[\begin{array}{ccc}
\cos \left(\phi_{h}\right) \cos \left(\theta_{h}\right) & -\cos \left(\psi_{h}\right) \sin \left(\theta_{h}\right)+\sin \left(\psi_{h}\right) \sin \left(\phi_{h}\right) \cos \left(\theta_{h}\right) & \sin \left(\psi_{h}\right) \sin \left(\theta_{h}\right)+\cos \left(\psi_{h}\right) \sin \left(\phi_{h}\right) \cos \left(\theta_{h}\right) \\
\cos \left(\phi_{h}\right) \sin \left(\theta_{h}\right) & \cos \left(\psi_{h}\right) \cos \left(\theta_{h}\right)+\sin \left(\psi_{h}\right) \sin \left(\phi_{h}\right) \sin \left(\theta_{h}\right) & -\sin \left(\psi_{h}\right) \cos \left(\theta_{h}\right)+\cos \left(\psi_{h}\right) \sin \left(\phi_{h}\right) \sin \left(\theta_{h}\right) \\
-\sin \left(\phi_{h}\right) & \cos \left(\phi_{h}\right) \sin \left(\psi_{h}\right) & \cos \left(\phi_{h}\right) \cos \left(\psi_{h}\right)
\end{array}\right]
$$

The location of the center of rotation of the eye $\left(\boldsymbol{d}_{0}^{2}\right)$ is given by the overall translation ( $x_{e}, y_{e}, z_{e}$ ) from the transmitter origin to the origin of the eye frame. From these expressions, the equation of a line originating at the center of rotation of the globe along the direction of the line of sight is

$$
\begin{aligned}
\frac{x-x_{e}}{S x_{\mathrm{los}}} & =\frac{y-y_{e}}{S y_{\mathrm{los}}} \\
& =\frac{z-z_{e}}{S z_{\mathrm{los}}}
\end{aligned}
$$

If we attempt to find the point of regard of a subject on a flat screen (i.e., the intersection of the l.o.s. with a plane) located in the frontal plane of the transmitter frame (coplanar to the $y z$ plane) at a distance $x=d$, we obtain the following:

$$
\begin{aligned}
x & =d \\
y & =\frac{S y_{\mathrm{los}}}{S x_{\mathrm{los}}}\left(d-x_{e}\right)+y_{e} \\
z & =\frac{S z_{\mathrm{los}}}{S x_{\mathrm{los}}}\left(d-x_{e}\right)+z_{e}
\end{aligned}
$$

which is the point of regard in the transmitter frame of reference.

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