IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received December 13, 2019, accepted December 30, 2019, date of publication January 3, 2020, date of current version January 23, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2963887

Combined Heat and Power Economic Emission Dispatch Using Hybrid NSGA II-MOPSO Algorithm Incorporating an Effective Constraint Handling Mechanism

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ABSTRACT This research work proposes a synergistic hybrid metaheuristic algorithm a merger of Nondominated Sorting Genetic Algorithm II and Multiobjective Particle Swarm Optimization algorithm for solving the highly complicated combined heat and power economic emission dispatch problem to operate the power system economically and to reduce the impact of environmental pollution. During the iteration, based on ranking, the population is divided into two halves. The exploration is carried out by Nondominated Sorting Genetic Algorithm II using the upper half of the population. The modification of Multiobjective Particle Swarm Optimization to effectively exploit the lower half of the population is done by increasing the personal learning coefficient, decreasing the global learning coefficient and by using an adaptive mutation operator. To satisfy the linear, nonlinear constraints, and to ensure the populations always lie in the Feasible Operating Region of the cogeneration plant, an effective constraint handling mechanism is developed. The proposed hybrid algorithm with an effective constraint handling mechanism enhances the searching capability by effective information interchange. The algorithm is applied to standard test functions and test systems while considering the valve point effects of the thermal plants, transmission power losses, bounds of the units and feasible operating region of the cogeneration units. The hybrid algorithm can obtain a well spread and diverse Pareto optimal solution and also can converge to the actual Pareto optimal front faster than some of the existing algorithms. The statistical analysis reveals that the proposed hybrid algorithm is a viable alternative to solve this complicated and vital problem.

INDEX TERMS Air pollution, genetic algorithms, heuristic algorithms, particle swarm optimization, power generation economics, statistical analysis.

I. INTRODUCTION

Utmost energy is wasted in the form of heat when fossil fuel is burned to produce electricity in thermal power stations [1]. The efficiency of the thermal power stations can be vastly improved by integrating Cogeneration or Combined Heat and Power (CHP) plants to the existing power system. The CHP plants can produce power using a variety of fuels and also have the ability to recover and reuse the heat, which would have been generally wasted during power generation. The overall efficiency of the energy conversion process can be increased from 60% to as great as 80% by utilizing a CHP plant to produce power and heat simultaneously [2], [3]. The increased focus on sustainability has led many countries to integrate CHP units into their existing power system. This process will result in saving precious resources and also will lead to the economic operation of the power system [4].

Electricity production by fossil fuels results in the emission of Sulphur dioxide (SO_2), Nitrogen oxides (NO_x) and other greenhouse gases [5]. These gases pollute the air, produce acid rain, and are also the major contributors to global warming. Also, they cause various health-related issues in human beings, harmful effects on biodiversity, and ecosystem. All electric companies are forced to minimize their emissions to reduce air pollution and their harmful effects [6].

The associate editor coordinating the review of this manuscript and approving it for publication was Jagdish Chand Bansal.

In the next section, the literature survey and the solution techniques available to solve the multi objective Combined Economic Emission Dispatch (CEED), single objective Combined Heat and Power Economic Dispatch (CHPED), and multiobjective Combined Heat and Power Economic Emission Dispatch (CHPEED) are carried out. Out of these four formulations, the most challenging problem to solve is the multiobjective CHPEED problem due to the conflicting objectives, consideration of the power-heat dependency constraint of the CHP units and due to the presence of linear, nonlinear equality and inequality operational constraints. The focus of this research is to find the trade-off solutions of a highly complex and challenging multiobjective CHPEED problem to reduce the fuel costs and emission levels of the power system.

A. LITERATURE REVIEW

The economic dispatch problem aims to find the optimal schedule of the generators to minimize the fuel cost of power generation subject to power balance constraint and other operational constraints. The formulation of ED to economically operate the power system is available in [7]. To simultaneously minimize the fuel cost and the emissions from the power plants, a biobjective CEED problem is formulated. These objectives not only provide considerable economic benefits and but also reduce the harmful effects of the pollutant gases [5]. There are two ways to tackle this highly nonlinear CEED problem with nonconvex cost functions. One way is to convert the biobjective, multidimensional, and highly constrained problem into a single objective problem and then solve using a potent stochastic algorithm. Another way to tackle the problem is to use multiobjective algorithms and to obtain Pareto Optimal (PO) solutions or trade-off solutions since it is not possible to obtain one unique optimal solution satisfying the conflicting objectives.

This biobjective CEED optimization problem is converted to a single objective problem using penalty factors and then solved using various potent metaheuristic algorithms and their variants such as improved artificial bee colony algorithm (ABC) in [8], global particle swarm optimization (GPSO) in [9], the chaotic improved harmony search algorithm in [10], flower pollination algorithm in [11], biogeography based optimization in [12], the gravitational search algorithm in (GSA) [13], the stochastic fractal search algorithm in [14], the symbiotic organism search algorithm for multi area power system in [15], fluid mechanism inspired algorithm in [16], and the lightning flash algorithm in [17].

Each of the metaheuristic algorithms used for optimization has its own merits and demerits. To overcome the demerits and to enhance the merits of the individual algorithm, hybrid single objective algorithms are available in the literature to solve this complicated CEED problem. These hybrid algorithms combine either two different metaheuristic algorithms or combine a metaheuristic algorithm with a local search technique to find the trade-off solutions of the CEED problem. The artificial bee colony (ABC) algorithm is combined with simulated annealing (SA) in [18], two different metaheuristic algorithms PSO and firefly algorithm is combined in [19], hybrid firefly and bat algorithm in [20], hybrid PSO-GSA in [21] and other nonconventional methods such as artificial neural network in [22] have been employed in the literature to solve the CEED problem. These algorithms provide only one compromise solution in a single run. These algorithms have to be run multiple times to obtain compromise solutions. One way to overcome this drawback is to use the highly efficient multiobjective algorithms to produce trade-off solutions in a single run.

The Pareto optimal curve for the combined economic emission dispatch problems is obtained using multiobjective algorithms such as fuzzy dominance based bacterial foraging algorithm in [23], multiobjective scatter search approach in [24], multiobjective quasi-oppositional teaching-learning based optimization in [25], the multiobjective backtracking search algorithm in [26], a robust multiobjective opposition based greedy heuristic search with adaptive parameters in [27], Nondominated Sorting Genetic Algorithm II (NSGA II) and modified NSGA II in [28], multiobjective particle swarm optimization (MOPSO) in [29], multiobjective differential evolution MODE in [30], multiobjective harmony search in [31], and multiobjective bat algorithm in [32]. The holistic review on solution strategies for CEED problem is available in [33]. The integration of CHP units into the power system is not considered in all these literature reviewed so far.

The integration of cogeneration units into ED problem converts the ED problem into a CHPED problem. This CHPED problem is a highly nonlinear and complex problem to solve since in addition to the linear and nonlinear constraints of ED problem, the feasible operating region (FOR) constraint of the CHP units must also be satisfied. To obtain the solution of the CHPED problem is challenging due to the interdependence of heat and power generation of the CHP unit, and it requires highly efficient algorithms to obtain the optimal solution. The metaheuristic algorithms such as gravitational search algorithm in [34], grey wolf optimization in [35], improved genetic algorithm (GA) in [36], [37], modified PSO in [38], Cuckoo optimization in [39], [40], civilized swarm optimization in [41], exchange market algorithm in [42], the differential algorithm in [43], bee colony optimization in [44], [45], artificial immune system algorithm in [46], oppositional teaching learning based optimization in [47], the harmony search algorithm in [48], [49] and its variants in [50], the squirrel search algorithm in [51], group search optimization in [52] and other methods such as Lagrangian relaxation in [53], benders decomposition approach in [54] are used in literature to obtain the optimal solution of the CHPED problem. The hybrid methods that have been successfully employed to solve the CHPED problem are the hybrid bat and ABC algorithm in [55], hybrid harmony search and PSO algorithm in [56], hybrid harmony search algorithm and Nelder-Mead numerical method in [57]. The survey of the metaheuristic optimization algorithms to solve CHPED problem along with the quality of the solution and computational performance of each algorithm is available in [58].

The multiobjective, multidimensional CHPEED problem is very challenging to solve because of the consideration of nonsmooth, nonconvex, nonlinear, nondifferential fuel cost function. Solving conflicting objectives in a multiobjective problem generates trade-off solutions called as PO solutions [59]. The PO solution obtained from CHPEED problem has to lie in the Feasible Operating Region (FOR) of the CHP unit. The Pareto Optimal Front (POF) of the multiobjective CHPEED problem by considering reserve constraints is obtained using probabilistic mutation enhanced firefly algorithm in [60]. The computationally efficient NSGAII to solve multiobjective problems is proposed in [61]. The main features of this algorithm are the fast nondominated sorting, efficient elite preserving strategy, preserving diversity, and spread among PO solutions. The author in [62] has successfully employed NSGA II to obtain the PO solutions of the CHPEED problem, but the discussion of the constraint handling mechanism applied to maintain the optimal solution in the FOR of the CHP units is not available. The authors in [63] propose a deterministic model for CHPEED, and this model is solved using time-varying acceleration multiobjective particle swarm optimization to obtain the trade-off solutions by simultaneously minimizing the conflicting economic and emission objectives. The authors in [64] have employed a θ-dominance-based evolutionary algorithm to find multiple trade-off solutions and then using fuzzy c-means clustering to identify the best compromise solution.

Since computational models have their unique features and characteristics, it is common to conceive of incorporating unique computational multiobjective models into a hybrid model to solve challenging real world problems. The processes of hybridization of two different multiobjective algorithms are complex, needs skill and creativity. The authors in [65] propose a hybrid framework to combine the evolutionary multiobjective optimization algorithm. These hybrid algorithms can improve the shortcomings of the multiobjective heuristics that solve many practical problems [65].

The exploitation phase of the hybrid multiobjective framework in [65] is carried out by a gradient based sequential quadratic programming method by converting the multiobjective problem into a single objective problem using Achievement Scalarizing Function (ASF). This method can be applied only to continuously differential functions. The hybrid framework has employed a local search algorithm based on the characteristic of the problem to be solved. The CHPEED problem solved in this paper is highly nonlinear, nondifferential, and nonconvex. It is challenging to convert this CHPEED problem into a single objective problem. The conversion of the multiobjective problem into a single objective problem increases the complexity and computational burden of the hybrid algorithm. The consideration of the prohibited zones constraint makes the CHPEED problem discontinuous. So the method proposed in [65] cannot be applied to this challenging CHPEED problem.

By incorporating Pareto dominance into PSO the authors in [66], have proposed an efficient Multi Objective Particle Swarm Optimization (MOPSO) algorithm to solve multiobjective optimization problems (MOOP). The performance of the MOPSO algorithm is tested using typical test functions. In [67], hybrid multiobjective GA-PSO is applied to solve the associate rule mining problem. In [68] and [69] the CEED problem is solved using a hybrid NSGAII-MOPSO algorithm and hybrid MOPSO-Differential Evolution (DE) algorithm, respectively. The results obtained by these hybrid models which are available in literature imply that the hybrid multiobjective frameworks are potent, can interchange information inside the model, can do parallel processing, can enhance the searching capabilities and can also produce more favourable performance than any single computational multiobjective model. The detailed literature survey carried out indicates there is no hybrid multiobjective metaheuristic still available to solve the highly complicated CHPEED problem. The development of the hybrid multiobjective metaheuristic to solve the CHPEED problem is the primary motivation for this research work. In this paper, a hybrid algorithm which is the synergistic combination of NSGA II [61] and MOPSO [70] is employed to solve the highly challenging CHPEED problem.

The process of exploitation and exploration are different in NSGAII and MOPSO algorithms. In addition to using crossover and mutation operators, NSGA II also uses the principle of elitism, fast nondominated sorting, and crowding distance calculations to enhance the spread of the solutions and to preserve the diversity of the PO solutions. The crowded comparisons can restrict the convergence of the NSGA II algorithm. The particles of the MOPSO do not utilize genetic operators, and their information sharing mechanism is different compared to NSGAII algorithm. The particles search the space by updating their velocity and inertia weight. To guide the flight of the particles, MOPSO selects a leader from the PO solutions stored in an external memory called repository. For complicated problems, the MOPSO tends to get trapped in local optima, and this can be avoided by adaptively updating the parameters of MOPSO.

A hybrid multiobjective algorithm requires a compromise between exploitation and exploration tasks to avoid trapping of global solutions in local optima. The rationale of the proposed hybrid model is to improve the overall search mechanism of the hybrid algorithm by combining NSGA II and MOPSO, which use different ways to explore/exploit the search space. Exploration phase in this algorithm is carried out by NSGA II using the best upper half population. The NSGAII searches every part of the solution space to have a proper assessment of the global solution. In this algorithm, the exploitation is carried out by the MOPSO using lower half population. By using enhanced mutation operator, increasing the personal learning coefficient c1 and decreasing the global learning coefficient c2 of MOPSO algorithm makes it an efficient local search algorithm. In the hybrid algorithm, the MOPSO carries a local search to improve the available solution by examining for better solutions in their neighbourhood.

By using an effective method for exploring the search space, this hybrid algorithm can improve the compromise among the exploitation and exploration tasks to find the best promising solutions. These compromise solution must satisfy the FOR constraint of CHP units and other linear, nonlinear constraints only if an effective constraint handling mechanism is employed.

The constraint handling mechanisms often used in literature due to their simplicity are the static, dynamic, adaptive, and stochastic penalty functions [71]. Extraction of information is not possible in the strategy which rejects individuals who do not satisfy the constraints, but this strategy is not suitable for a discontinuous search space. For multiobjective optimization problems distance measures and adaptive penalty functions are used in the [71].

The development of the constraint handling mechanism is vital for the hybrid algorithm. During the generation of a new population or modifying the existing population during crossover and mutation, the constraint handling mechanism must ensure the solutions satisfy the bounds. In addition to satisfying the constraints and bounds, the constraint handling mechanism developed for the hybrid algorithm to solve the CHPEED problem must ensure the solutions also lie in the FOR of the CHP units. An efficient constraint handling mechanism is proposed in this research work to ensure the optimal solutions always lie within the feasible operating region of the CHP units, to satisfy constraints and bounds.

B. CONTRIBUTION OF THE PAPER

The main contributions of the paper are the following:

1) The No Free Lunch theorem [72] states there is always a possibility of proposing new algorithms for solving optimization problems and the fact that there is scope for developing a new hybrid multiobjective framework to solve the CHPEED problem made the author propose a hybrid NSGAII-MOPSO algorithm to solve the highly complex and very challenging CHPEED problem for energy conservation and reduction of pollutant gases.
2) The novelty of the work lies in the synergistic combination of the NSGAII and MOPSO algorithm to solve CHPEED problem, population evolution in the algorithm, search mechanism of the algorithm, and the archive updating mechanism. During the implementation of the hybrid algorithm, the exploration of the algorithm is carried out by NSGA II using the best upper half population and the exploitation is carried out by an enhanced MOPSO using lower half population.
3) The development of an efficient constraint handling mechanism to make sure the solutions at any stage of the algorithm remain in the FOR of cogeneration units and to satisfy the bounds. The proposed constraint handling mechanisms get rid of using ASF or penalty factors typically used in literature to regulate the solutions.
4) The consideration of transmission losses in power balance equality constraint makes the generation and modification of the population for a CHPEED problem difficult. The proposed constraint handling mechanism ensures the power balance equality is always satisfied by considering the transmission losses in the system.

C. ORGANIZATION OF THE PAPER

The organization of the paper is as follows. The subsequent section describes the mathematical model of the CHPEED. The discussion of the proposed efficient constraint handling mechanism is in section III. Elucidation of the hybrid algorithm is in section IV. In section V, the discussions on the results obtained by this algorithm are carried out. Finally, section VI concludes the paper.

II. COMBINED HEAT AND POWER ECONOMIC EMISSION DISPATCH

The CHPEED problem has two conflicting objectives. Obtaining the optimal heat generation and power generation schedule from a list of available power generating unit, CHP units, and heat only units is the foremost objective. The secondary objective is to minimize air pollution from these units. The optimal schedule obtained should reduce the total production cost and must also satisfy the heat demand, the power demand of the system, several operational and physical constraints.

Operating the system with minimum fuel cost results in increased emission, and it is not feasible to only minimize the emission from the plants since it increases fuel cost. These two conflicting objectives must be simultaneously minimized, taking into account the FOR of the cogeneration units. This section describes the mathematical formulation of the CHPEED problem. The objective of this paper is to find the diverse set of PO solutions of the CHPEED problem, which minimize the two conflicting objectives and also to satisfy the constraints.

A. OBJECTIVE FUNCTIONS AND CONSTRAINTS OF CHPEED PROBLEM

A Multiobjective Optimization Problem (MOOP) simultaneously minimizes the m objective of the vector x in the feasible region $\mathcal{D}$, and the general mathematical model is represented below [59]:

$$\mathit{Minimize}\, f(x) = \{f_1(x), f_2(x), \cdots f_m(x)\}, \quad x \in \mathcal{D} \tag{1}$$

where $f(x)$ is the vector of the objective functions and the mapping of the decision variable x into objective space $f_i = \Re^n \rightarrow \Re$ is given. The scalar decision variables are represented by $f_i(x), i = 1, 2, \cdots, m$. The j inequality constraints and the k equality constraints make the n-dimensional decision variable x to lie in a feasible region D i.e.

$$\mathcal{D} = \left\{x : g_j(x) \le 0, h_k(x) = 0, j = 1, 2, \cdots J; \; k = 1, 2, \cdots, K\right\} \tag{2}$$

The variable x can be represented as

$$x = [x_1, x_2, x_3, \cdots, x_n]^T \tag{3}$$

where T denotes the transposition vector. The decision variable x is limited to take a value within the upper bound x_i^{max} and the lower bound x_i^{min}. These bounds are called the decision space [59]. In the CHPEED problem, there are two objective functions i.e. $m = 2$. The formulation of the CHPEED is given below:

$$Minimize\, f(x) = \{f_1(x), f_2(x)\}, \quad x \in \mathcal{D} \tag{4}$$

to satisfy heat and power balance equality constraints, inequality constraints to represent the FOR of CHP units, and the limits of the variable x. The function $f_1(x)$ minimizes the total fuel cost function, and the function $f_2(x)$ minimizes the emissions from the power plants, cogeneration units and heat only units. In a CHPEED problem, the decision variable x is given by

$$x = \left[P_1, \cdots, P_{Np}, O_1, \cdots O_{Nc}, H_1, \cdots H_{Nc}, T_1, \cdots T_{Nh}\right]^T \tag{5}$$

where Np is the number of units producing only power, Nc is the number of cogeneration units and Nh is the number of heat only units. P_i is the power generation of the i^{th} unit producing only power, O_i is the power produced by the i^{th} cogeneration unit, H_i is the heat generated by the i^{th} cogeneration unit and T_i is the heat generated by the i^{th} heat only unit.

1) OBJECTIVES

a: FUEL COST

The function $f_1(x)$ represents the total fuel cost of the system integrated with CHP plants.

$$\begin{aligned} f_1(x) &= \sum_{i=1}^{Np} C_{pi}(P_i) + \sum_{i=1}^{Nc} C_{ci}(O_i, H_i) + \sum_{i=1}^{Nh} C_{hi}(T_i) \\ &= \sum_{i=1}^{Np} \left[a_i P_i^2 + b_i P_i + c_i + \left| d_i sin\left\{e_i\left(P_i^{min} - P_i\right)\right\}\right|\right] \\ &\quad + \sum_{i=1}^{Nc} \left[\alpha_i O_i^2 + \beta_i O_i + \gamma_i + \delta_i H_i^2 + \varepsilon_i H_i + \zeta_i O_i H_i\right] \\ &\quad + \sum_{i=1}^{Nh} \left[\eta_i T_i^2 + \theta_i T_i + \lambda_i\right] \end{aligned} \tag{6}$$

The fuel cost of the i^{th} power only unit with valve point effect, i^{th} cogeneration unit, and i^{th} heat only unit are given by C_{pi}, C_{ci}, and C_{hi} respectively in (6). The coefficients a_i, b_i, c_i represent the fuel cost coefficients of the i^{th} generator. Opening of steam valves in a thermal power plant creates a sudden increase in the observed losses, and this phenomenon is called valve point loading. This loading is modeled as ripples in the cost function using sine terms [73]. The coefficients d_i and e_i are the coefficients used to incorporate the valve point effect of the generator i. This valve point effect makes the fuel cost function highly nonlinear and discontinuous. $\alpha_i, \beta_i, \gamma_i, \delta_i, \varepsilon_i, \zeta_i$ are the coefficients of the i^{th} CHP unit. $\eta_i, \theta_i, \lambda_i$ are the cost coefficients of the i^{th} heat only unit.

b: EMISSIONS

The function $f_2(x)$ in (7) represents the total emission levels of the system integrated with CHP.

$$\begin{aligned} f_2(x) &= \sum_{i=1}^{Np} E_{pi}(P_i) + \sum_{i=1}^{Nc} E_{ci}(O_i, H_i) + \sum_{i=1}^{Nh} E_{hi}(T_i) \\ &= \sum_{i=1}^{Np} \left[k_i P_i^2 + l_i P_i + m_i + n_i exp^{(q_i P_i)}\right] \\ &\quad + \sum_{i=1}^{Nc} \mu_i O_i + \sum_{i=1}^{Nh} \xi_i T_i \end{aligned} \tag{7}$$

The emission levels of the i^{th} power only unit, i^{th} cogeneration unit, and i^{th} heat only unit are given by E_{pi}, E_{ci} and E_{hi} respectively in (7). k_i, l_i, m_i, n_i, q_i are the emission coefficients of the generator i. μ_i is the emission coefficient of the i^{th} CHP unit. ξ_i is the emission coefficients of the i^{th} heat only unit.

The fuel cost function has quadratic terms, which make it nonlinear. In addition to this term, there is also sine terms used in fuel cost equation and exponential terms used in emission level equation, which makes the fuel cost function highly nonlinear. Since the fuel cost function has discontinuous gradients, it is nonsmooth, and since it is discontinuous, it is also nondifferential. These objectives are subject to the equality constraints of power and heat production, the inequality constraints of the CHP units, and the bounds of the decision variables.

2) CONSTRAINTS

a: POWER BALANCE EQUALITY CONSTRAINT

The power balance equality constraint given by (8), balances the power produced by Np power only units and Nc CHP units with the sum of the total power demand Pd in the system and the total transmission power loss Pl in the system. Pl is the transmission power loss represented by the B-loss coefficients, as shown in (9).

$$h_1(x) = \sum_{i=1}^{Np} P_i + \sum_{i=1}^{Nc} O_i - Pd - Pl = 0 \tag{8}$$

The calculation of active power loss Pl for the power network integrated with CHP plants is by using B-loss coefficients given by (9)

$$\begin{aligned} Pl &= \sum_{i=1}^{Np}\sum_{j=1}^{Np} P_i B_{ij} P_j + \sum_{i=1}^{Np}\sum_{j=1}^{Nc} P_i B_{ij} O_j + \sum_{i=1}^{Nc}\sum_{j=1}^{Nc} O_i B_{ij} O_j \\ &\quad + \sum_{i=1}^{Np} B_{0i} P_i + \sum_{i=1}^{Nc} B_{0i} O_i + B_{00} \end{aligned} \tag{9}$$

B_{ij} is the transmission loss coefficients of the transmission lines connecting the buses i and j.

b: HEAT BALANCE EQUALITY CONSTRAINT

The heat balance equality constraint given by (10) balances the heat produced by Nc CHP units and Nh heat only units with the total heat demand Hd of the system.

$$h_2(x) = \sum_{i=1}^{Nc} H_i + \sum_{i=1}^{Nh} T_i - Hd = 0 \tag{10}$$

c: INEQUALITY CONSTRAINT OF CHP UNITS

The modelling of the interdependency between the power and heat produced by the cogeneration units as inequality constraints is given by (11) to (14). These inequality constraints are satisfied by the constraint handling mechanism proposed in section III.

$$g_1(x) = P_i - P_i^{max}(H_i) \leq 0; \quad i\epsilon 1, 2, \cdots, Nc \tag{11}$$
$$g_2(x) = P_i^{min}(H_i) - P_i \leq 0; \quad i\epsilon 1, 2, \cdots, Nc \tag{12}$$
$$g_3(x) = H_i - H_i^{max}(P_i) \leq 0; \quad i\epsilon 1, 2, \cdots, Nc \tag{13}$$
$$g_4(x) = H_i^{min}(P_i) - H_i \leq 0; \quad i\epsilon 1, 2, \cdots, Nc \tag{14}$$

d: BOUNDS OF VARIABLES P AND H

$$P_i^{min} \leq P_i \leq P_i^{max}; \quad i\epsilon 1, 2, \cdots, Np \tag{15}$$
$$H_i^{min} \leq H_i \leq H_i^{max}; \quad i\epsilon 1, 2, \cdots, Nh \tag{16}$$

The power generated by each unit i should lie within limits given by the minimum limit P_i^{min} and maximum limit P_i^{max}, as shown in (15). The heat output of the i^{th} heat only unit should lie within its limits given by the minimum limit H_i^{min} and maximum limit H_i^{max}, as shown in (16). The next section describes how to fix the bounds for CHP units.

3) FEASIBLE OPERATING REGIONS (FOR) OF THE COGENERATION UNITS

The heat production capacity of a CHP unit depends on the power production capacity and vice versa. The Fig.1 shows the FOR of a general CHP unit. For example, the coordinates of the red point shown in Fig.1 is (h_4, o_3). The inequality constraints are formed by finding the coordinates of each line from Fig.1. The formulation of inequality constraints to alleviate the complexity arising due to this mutual dependency of power and heat of the cogeneration units is carried out in this section. The constraint handling mechanism described in section III uses these inequality constraints. If the power produced by the CHP unit i is O_i, then based on the value of O_i the heat generation of the unit H_i can be calculated using (17).

$$H_i = h_i^{min} - rand * \left(h_i^{min} - h_i^{max}\right); \quad i = 1, 2, \cdots, Nc \tag{17}$$

In (17) $rand$ is a random number between 0 and 1. Referring to Fig.1 and also based on the value of O_i, the procedure to calculate h_i^{min} and h_i^{max} used in (17) is shown below

- If $O_i = o_4$ then $h_i^{min} = 0$; $h_i^{max} = h_2$
- If $o_3 < O_i < o_4$ then $h_i^{min} = 0$; $h_i^{max} = \frac{h_4 - h_2}{o_3 - o_4}(O_i - o_4) + h_2$
- If $O_i = o_3$ then $h_i^{min} = h_4$; $h_i^{max} = h_4$

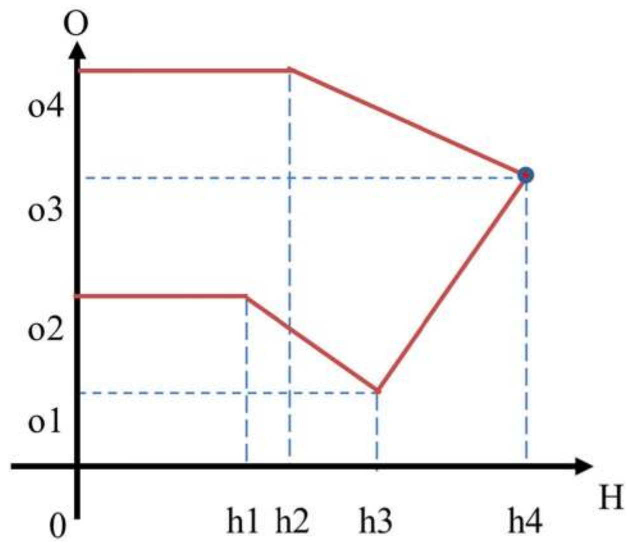

FIGURE 1. Heat power feasible operation region of a CHP.

- If $o_2 < O_i < o_3$ then $h_i^{min} = 0$; $h_i^{max} = \frac{h_3 - h_4}{o_1 - o_3}(O_i - o_3) + h_4$
- If $O_i = o_2$ then $h_i^{min} = 0$; $h_i^{max} = h_1$
- If $o_1 < O_i < o_2$ then
$$h_i^{min} = \frac{h_3 - h_1}{o_1 - o_2}(O_i - o_2) + h_1$$
$$h_i^{max} = \frac{h_3 - h_4}{o_1 - o_3}(O_i - o_3) + h_4$$
- If $O_i = o_1$ then $h_i^{min} = h_3$; $h_i^{max} = h_3$

B. FORMULATION

The biobjective CHPEED problem can be formulated as

$$Minimize\, f = [f_1(x), f_2(x)] \tag{18}$$

The objective of the CHPEED problem is to obtain the PO solution vector x^* that simultaneously minimizes the conflicting objectives given by (6) and (7) while satisfying the real power balance equality constraint given by (8), heat balance constraint given by (10), inequality constraints given by (11) to (14) and the bounds given by (15) and (16).

The essential goals of the CHPEED problem are

1. Find a diverse set of nondominated solutions which lie on the POF.
2. Find a diverse set of PO solutions to represent the entire range of the POF.

III. CONSTRAINT HANDLING MECHANISM

It is imperative to use an effective constraint handling mechanism to improve the quality of solutions in a metaheuristic algorithm. While generating a new solution or while modifying an existing solution, the constraint handling mechanism must ensure the population lies within the bounds and FOR of the CHP units. The consideration of transmission losses makes this even more challenging. However, the constraint handling mechanism proposed in this paper deftly handles this challenging task. Instead of using slack variables, random numbers are generated and used in the proposed algorithm to enhance the quality and diversity of the multiobjective algorithm. The three phases of the proposed constraint handling mechanism are:

Phase 1: The first seven steps of the constraint handling mechanism make sure the power balance constraint is satisfied.

Phase 2: The next two steps of the constraint handling mechanism make sure the heat generation of the cogeneration units lie within the FOR of the unit.

Phase 3: The next two steps of the constraint handling mechanism make sure the heat balance constraint given by (10) is satisfied.

The algorithm of the constraint handling mechanism is given below:

Step 1: Generate a random number k in the range of $[1, Np + Nc]$. If the value of $k \leq Np$, then generate P_i; $i = 1, 2, \cdots, Np$ and O_i; $i = 1, 2, \cdots, Nc$ using (19) and (20). If the value of $k > Np$, then generate P_i; $i = 1, 2, \cdots, Np$ and O_i; $i = 1, 2, \cdots, Nc$ using (21) and (22).

$$P_i = P_i^{min} - rand * \left(P_i^{min} - P_i^{max}\right); \quad i \in \alpha_n \tag{19}$$

$$O_i = O_i^{min} - rand * \left(O_i^{min} - O_i^{max}\right); \quad i = 1, 2, \cdots, Nc \tag{20}$$

$$P_i = P_i^{min} - rand * \left(P_i^{min} - P_i^{max}\right); \quad i = 1, 2, \cdots, Np \tag{21}$$

$$O_i = O_i^{min} - rand * \left(O_i^{min} - O_i^{max}\right); \quad i \in \alpha_n \tag{22}$$

Here *rand* is a uniformly distributed number randomly generated in the range of [0, 1]. If the value of $k \leq Np$, then the set α_n comprises integers in the range $[1, Np]$ excluding k. If the value of $k > Np$, then generate a number $m = k - Np$ and the set α_n includes the integers in the range $[1, Nc]$ excluding m.

Step 2: If the value of $k \leq Np$, the value of the k^{th} variable of the candidate solution P_k is obtained by subtracting the total system demand Pd from the total power generation from the power only and CHP units given by $\sum_{i\in\alpha_n} P_i + \sum_{i=1}^{Nc} O_i$. If the value of $k > Np$, the value of the k^{th} variable of the candidate solution O_k is obtained by subtracting the total system demand Pd from the total power generation from the power only and CHP units given by $\sum_{j\in\alpha_n} O_j + \sum_{i=1}^{Np} P_i$. If the value of P_k or O_k lie outside its bounds, then they are set equal to the respective bounds.

Step 3: Calculate the residue PRd by subtracting the total system demand Pd from the total power generation from the power only and CHP units $\sum_{i=1}^{Np} P_i + \sum_{i=1}^{Nc} O_i$. If $|PRd| < tol$, then go to step 5; otherwise go to step 4. Here, tol is the tolerance for demand set as $0.001\,p.u.$ This step ensures the balance of power generation and demands without power transmission loss.

Step 4: Generate a random number k which lies between $[1, Np+Nc]$. If the value of $k \leq Np$, then the set α_n comprises integers in the range $[1, Np]$ excluding k, If the value of $k > Np$, then generate a number $m = k - Np$ and the set α_n comprises integers in the range $[1, Nc]$ excluding m. Go to step 2.

Step 5: If the value of $k \leq Np$, then the value of the k^{th} variable of the candidate solution P_k is obtained by finding the roots of the quadratic equation (23) else if the value of $k > Np$, the value of the k^{th} variable O_k is obtained by finding the roots of the quadratic equation (24).

$$\begin{aligned} &B_{kk}P_k^2 + \left(2\sum_{i\in\alpha_n} B_{ki}P_i + 2\sum_{j=1}^{Nc} B_{kj}O_j + B_{0k} - 1\right) P_k \\ &+ \left(Pd + \sum_{i\in\alpha_n}\sum_{j\in\alpha_n} P_iB_{ij}P_j + \sum_{i\in\alpha_n}\sum_{j=1}^{Nc} P_iB_{ij}O_j \right. \\ &+ \sum_{i\in\alpha_n} B_{0i}P_i + \sum_{j=1}^{Nc} B_{0j}O_j - \sum_{i\in\alpha_n} P_i \\ &\left. - \sum_{j=1}^{Nc} O_j + B_{00}\right) = 0 \end{aligned} \tag{23}$$

$$\begin{aligned} &B_{kk}O_k^2 + \left(2\sum_{i=1}^{Np} B_{ki}P_i + 2\sum_{j\in\alpha_n} B_{kj}O_j + B_{0k} - 1\right) O_k \\ &+ \left(Pd + \sum_{i=1}^{Np}\sum_{j=1}^{Nc} P_iB_{ij}P_j + \sum_{i=1}^{Np}\sum_{j\in\alpha_n} P_iB_{ij}O_j \right. \\ &+ \sum_{j\in\alpha_n} B_{0j}P_j + \sum_{i=1}^{Np} B_{0i}P_i - \sum_{j\in\alpha_n} O_j \\ &\left. - \sum_{i=1}^{Np} P_i + B_{00}\right) = 0 \end{aligned} \tag{24}$$

Solving (23) or (24) results in two roots, out of which only one root is selected as the value of the candidate solution P_k or O_k. If roots of the equation (23) or (24) lie inside the bounds, then the root that has the least value is chosen. If only one root lies inside the bound, select this root as the value of the candidate solution and also discard the other root. If both the roots lie outside the bounds the value of P_k or O_k is set equal to P_k^{min} or O_k^{min} respectively.

Step 6: Calculate the residue PRd by subtracting the total system demand Pd and the transmission loss Pl from the total power generation from the power only and CHP units $\sum_{i=1}^{Np} P_i + \sum_{i=1}^{Nc} O_i$. If $|PRd| < tol$, then go to step 8; else go to step 7. Here, tol is the tolerance for demand set as $0.001\,p.u.$ This step ensures the power balance equation (8) is satisfied.

Step 7: Generate a random number k which lies between $[1, Np+Nc]$. If the value of $k \leq Np$, then the set α_n comprises of integers in the range $[1, Np]$ excluding k, if the value of $k > Np$, then generate a number $m = k - Np$ and the set α_n comprises of integers in the range $[1, Nc]$ excluding m. Go to step 5.

Step 8: The variable H_i, $i = 1, 2, \cdots Nc$ is generated based on the value of O_i, $i = 1, 2, \cdots Nc$ using (17). The h_i^{min} and h_i^{max} used in (17) is found out corresponding to the value O_i based on the procedure discussed in section II.A.3.

Step 9: Generate a random number k which lies between $[1, Nc + Nh]$. If the value of $k \leq Nc$, then generate

$H_i; i = 1, 2, \cdots, Nc$ and $T_i; i = 1, 2, \cdots, Nh$ using the equations (25) and (26). If the value of $k > Nc$, then generate $H_i; i = 1, 2, \cdots, Nc$ and $T_i; i = 1, 2, \cdots, Nh$ using the equations (27) and (28).

$$H_i = h_i^{min} - rand * \left(h_i^{min} - h_i^{max}\right); \quad i \in \beta_n \tag{25}$$

$$T_i = T_i^{min} - rand * \left(T_i^{min} - T_i^{max}\right); \quad i = 1, 2, \cdots, Nh \tag{26}$$

$$H_i = h_i^{min} - rand * \left(h_i^{min} - h_i^{max}\right); \quad i = 1, 2, \cdots, Nc \tag{27}$$

$$T_i = T_i^{min} - rand * \left(T_i^{min} - T_i^{max}\right); \quad i \in \beta_n \tag{28}$$

Here *rand* is a uniformly distributed number generated randomly in the range of [0, 1]. If the value of $k \leq Nc$, then the set β_n comprises of integers that lie within $[1, Nc]$ excluding k. If the value of $k > Nc$, then generate a number $m = k - Nc$ and the set β_n comprises of integers in the range $[1, Nh]$ excluding m.

Step 10: If the value of $k \leq Nc$, then obtain the value of the k^{th} variable of the candidate solution H_k by subtracting the total heat demand Hd from the total heat generation of the cogeneration units and heat only units given by $\sum_{i \in \beta_n} H_i + \sum_{i=1}^{Nh} T_i$. If the value of $k > Nc$, obtain the value of the k^{th} variable of the candidate solution T_k by subtracting the total heat demand Hd from the total heat generation from the CHP units and heat only units given by $\sum_{j \in \beta_n} T_j + \sum_{i=1}^{Nc} H_i$. If the value of H_k lie outside the bounds given by the variables h_k^{min} and h_k^{max} determined in step 8, then they are set equal to the respective bounds. If the value of T_k lie outside its bounds, then they are set equal to its bounds.

Step 11: Calculate the residue HRd by subtracting the total heat demand Hd from the total heat generation from the cogeneration units and heat only units $\sum_{i=1}^{Nc} H_i + \sum_{i=1}^{Nh} T_i$. If $|HRd| < tol$, then go to step 12; else go to step 9. Here, tol is the tolerance for heat demand set as 0.001 $p.u.$ This step ensures that the heat balance equation given by (9) is satisfied.

Step 12: Stop the constraint handling procedure.

The flowchart of the constraint handling mechanism is illustrated in Fig.2.

IV. HYBRID NSGA II - MOPSO ALGORITHM FOR SOLVING CHPEED PROBLEM

The computationally efficient NSGAII algorithm to solve multiobjective optimization problems is proposed in [61]. This algorithm is widely used in literature to solve multiobjective optimization problems due to its features of elitism, fast nondominated sorting, and the crowding distance operator which enhances the diversity and the spread of the PO solutions.

The well-known and highly efficient PSO algorithm which was widely used in literature to solve single objective optimization problems was extended to solve multiobjective optimization problems in [70]. This extended version of PSO called MOPSO incorporates Pareto dominance into PSO, uses a leader (a swarm particle that guides other particles), stores a historical record of the dominated solution in an external repository and also employs an optional mutation operator.

Hybrid single objective optimization methods to solve complex problems integrate the desirable features of the different algorithm to find an optimal solution in a challenging search space. Even though many multiobjective heuristics have been used to solve many practical problems, they have their shortcomings, and hybrid multiobjective algorithms can reduce the shortcomings [65]. Hybrid multiobjective optimization methods are now becoming popular, and the general framework to combine the evolutionary multiobjective optimization algorithm is proposed in [65]. This framework uses an achievement scalarizing function to convert the multiobjective problem to a single objective problem during the iteration. This existing technique cannot be used to solve the multiobjective CHPEED problem.

In this research work, the two stochastic multiobjective optimization algorithms NSGA II and MOPSO is creatively combined to solve the highly nonlinear and complex CHPEED problem. The search processes in these two algorithms are different. The NSGAII uses the principle of elitism, sorting, and crowding distance calculations to enhance the spread of the solutions and to preserve the diversity of the PO Solutions. The crowded comparisons can restrict the convergence of the NSGA II algorithm. The particles of the MOPSO do not utilize genetic operators, and their information-sharing mechanism is different when compared to NSGA II. In MOPSO, a non-dominant solution called leader is used to guide the other particles. The particles search the space by updating their velocity and inertia weight. For complicated problems, the MOPSO tends to get trapped in local optima, and this can be avoided by continuously updating the parameters of MOPSO.

The rationale of the proposed hybrid model is to improve the overall search mechanism of the hybrid algorithm by combining NSGA II and MOPSO, which use different ways to explore/exploit the search space. The trapping of solutions in local optima can be avoided by having a compromise between exploitation and exploration done by the hybrid algorithm. To avoid premature convergence and to obtain well spread POF, the whole population is split into two half, based on the ranking generated by the non-domination fronts, the best half of the population are improved by NSGA II algorithm while the other half of the population are considered as swarm particles and are optimized by MOPSO to make them converge around the best possible solutions.

Exploration phase in this algorithm is carried out by fast and elitist NSGA II algorithm using the best upper half population. This exploration provides the hybrid algorithm with a reasonable assessment of global solutions. In this algorithm, the exploitation is carried out by the MOPSO using the lower half of the population. The MOPSO examines for better solutions in their neighborhood by improving the orientation of the lower-ranked particles towards a global solution. MOPSO

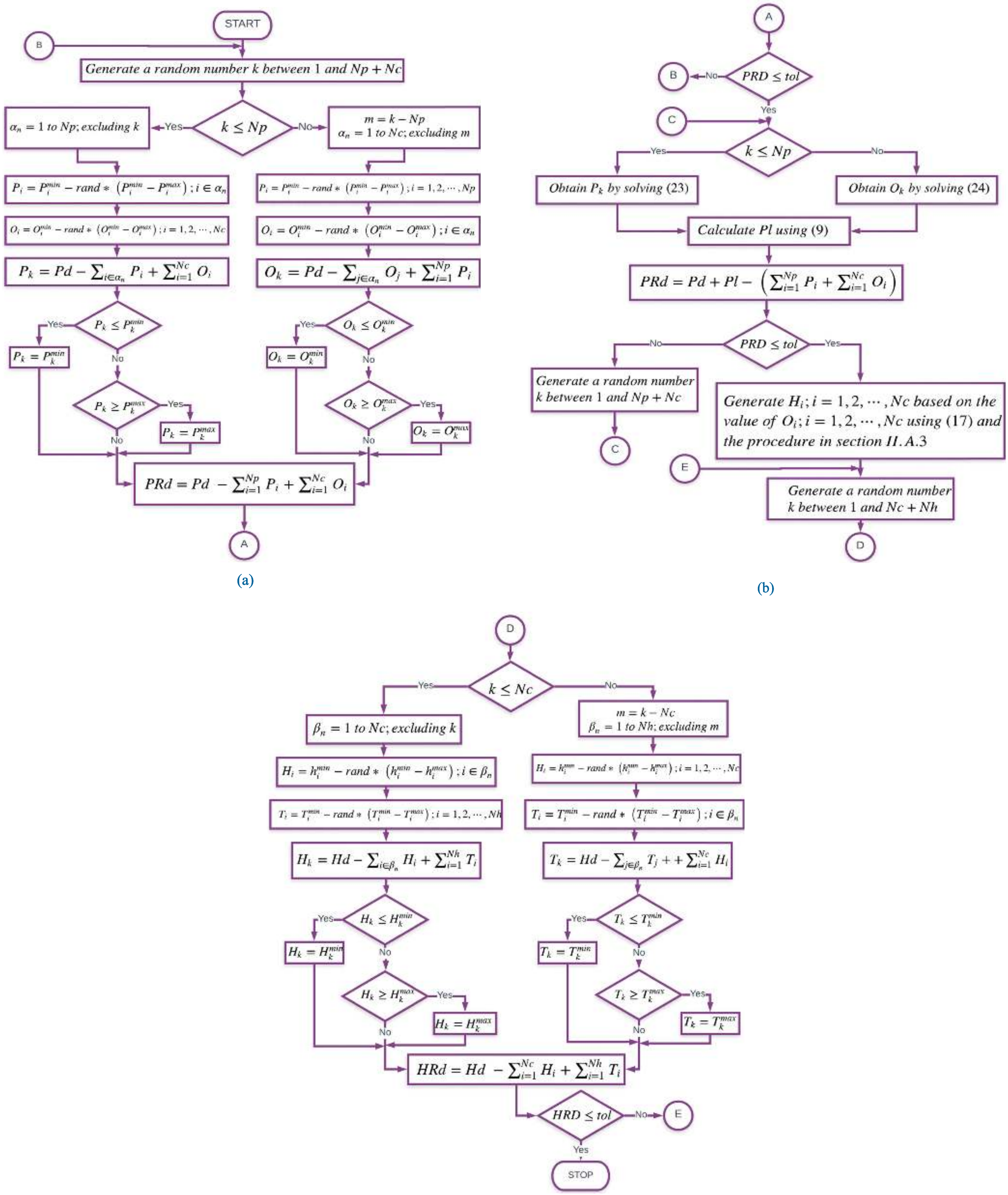

FIGURE 2. **The flowchart of the proposed constraint handling mechanism.**

algorithm is used as an efficient local search procedure by using enhanced mutation operator [66], by increasing the personal learning coefficient $c1$ and by decreasing global learning coefficient $c2$ of MOPSO algorithm. By using an effective method for exploring the search space, this hybrid algorithm can improve the compromise among the exploitation and exploration tasks to find the best promising solutions. The novelty of the work lies in the overall search mechanism of the hybrid algorithm, the evolution of the population, and the archive updating mechanism, as shown in Fig. 3.

This proposed hybrid framework maintains a balance between exploration done by NSGA II and exploitation carried out by enhanced MOPSO algorithm. The purpose of the mutation rate is to improve the searching capability of MOPSO [66]. In the MOPSO algorithm, the adaptive parameter acts on the entire population to examine the better solutions in the search space and on the full range of decision variables, but in the proposed hybrid algorithm the mutation rate *mu* acts only on the lower half of the population. It examines better solutions in the entire lower half of the population.

NSGA II uses an external archive F1, and MOPSO uses a repository. At the end of each iteration, the nondominated solution in F1 and the repository are combined and then sorted to be stored in F1. The external archive is filled by solutions of different nondominated fronts, one at a time until the list is full. The filling starts with the first nondominated front of class one and then continues with the solutions of the second nondominated front, and so on [59]. When the list F1 is full, the algorithm deletes the remaining nondominated fronts. When the last front is being considered the points with the highest diversity are chosen by using the crowding distance valves. The hybrid NSGA II-MOPSO algorithm for solving the CHPEED problem is shown in Fig.3.

V. CASE STUDY

The effectiveness and validity of the proposed hybrid algorithm are tested using two standard test functions and two typical test systems widely used in literature. Test function I and II are obtained from the literature [74] and [75], respectively. In [66] these two test functions are used to compare the performance of MOPSO with other multi-objective algorithms such as the Micro genetic algorithm (microGA), the NSGA II, and Pareto Archived Evolution Strategy (PAES). Reference [66] shows the full POF obtained for each of this test function. In section A and section B, the two test functions are solved using the proposed hybrid NSGAII-MOPSO algorithm and the Pareto front obtained with the proposed hybrid algorithm is compared with the POF obtained in [66]. The main idea here is to check if the hybrid algorithm can find the actual Pareto optimal front for these test functions and also to explain the working of the algorithm. Then this hybrid algorithm is applied to CHPEED test system I [63] and test system II [62] to find its ability to solve the CHPEED problem. Table 1 shows the parameters for the algorithms. Unless specified in the case study, the values of the parameters are set, as shown in Table 1. The selection of the values for the parameter was based on guidelines available in [66], and in some test cases, these parameters were empirically determined based on the complexity of the test system adopted. The hybrid algorithm is implemented using MATLAB 8 on an H.P Pavilion Laptop, 1.80 GHZ, Intel $i7$ processor, 16 GB RAM with WINDOWS 10 operating system. The NSGA II algorithm, MOPSO algorithm, and Hybrid NSGA II-MOPSO algorithm are applied to each of the two typical test systems. The optimal dispatch obtained from the proposed hybrid algorithm is then compared among themselves and with the results reported in the literature.

TABLE 1. The parameters of the NSGAII, MOPSO and NSGAII-MOPSO algorithm.

NSGAII	MOPSO	Hybrid NSGAII-MOPSO
$nPop = 100$	$nPop = 150$	$nPop = 100$
$MaxIt = 100$	$MaxIt = 2000$	$MaxIt = 100$
$pCrossover = 0.7$	$nRep = 100$	$pCrossover = 0.7$
$pMutation = 0.4$	$w = 0.5$	$pMutation = 0.4$
$mr = 0.02$	$wdamp = 0.99$	$mr = 0.02$
	$c1 = 1, c2 = 2$	$c1 = 4, c2 = 1$
	$alpha = 0.1$	$alpha = 0.1$
	$beta = 2$	$beta = 2$
	$gamma = 2$	$gamma = 2$
	$mu = 0.1$	$nRep = 50$
		$w = 0.5$
		$wdamp = 0.99$
		$mu = 0.1$

A. TEST FUNCTION I

This test function was proposed in [74] and used in [66] to compare the performance of MOPSO with other multiobjective algorithms such as the Micro genetic algorithm (microGA), the NSGA II, and Pareto Archived Evolution Strategy (PAES). The multiobjective function used in [66] is a maximization function. It has two objectives $f_1(x, y), f_2(x, y)$, which are functions of x, y and three inequality constraints. Since CHPEED is a multiobjective minimization problem and in this section, we are testing the proposed hybrid algorithm to solve CHPEED, the objective function of the test function I is modified to a minimization function by adding a minus sign as shown below:

$$Minimize\ F = (-f_1(x, y), -f_2(x, y)),\ where$$
$$f_1(x, y) = -x^2 + y, f_2(x, y) = 0.5x + y + 1 \quad (29)$$

subject to

$$0 \geq \frac{1}{6}x + y - \frac{13}{2}, 0 \geq \frac{1}{2}x + y - \frac{15}{2},$$
$$0 \geq 5x + y - 30\ and\ x, y \geq 0 \quad (30)$$

The limits for the variables are $0 \leq x \leq 10^{20}; -10^{20} \leq y \leq 7$. The total number of function evaluation is set to 120. Fig. 4 shows the POF and the extreme points produced by the hybrid algorithm. By looking at POF, it is easy to notice that this hybrid algorithm can produce the actual POF for this test function and this can also be verified with results available in [66] after adding a negative sign to the PO solutions.

Fig.5 shows the number of population affected due to the choice of the mutation rate used in the MOPSO stage of the hybrid algorithm. From the figure, we can see that the entire lower half of the population is affected at the start of the

Step 1: Specify the parameters for the CHPEED problem

- The total demand of the power system Pd and the total heat demand Hd
- Fuel cost and emission coefficients for each power only, cogeneration, CHP and heat only unit.
- B matrix coefficients for transmission loss calculations
- Number of decision variables $nVar$
- Lower bounds of the decision variables $VarMin$
- Upper bounds of the decision variables $VarMax$

Step 2: Specify the parameters for NSGA II Algorithm

- Population Size $nPop$
- Maximum number of iteration $MaxIt$
- Crossover Percentage $pCrossover$
- Mutation Percentage $pMutation$
- Mutation rate mu
- Mutation step size $sigma = 0.1 \times (VarMax - VarMin)$

Step3: Specify the parameters for MOPSO Algorithm

- Repository size $nRep$
- Inertia Weight w and Inertia Weight damping rate $wdamp$
- Personal learning coefficient $c1$ and global learning coefficient $c2$
- Number of grids per dimension $nGrid$
- Inflation Rate $alpha$, leader selection pressure $beta$, Deletion selection pressure $gamma$
- Mutation rate mu

Step 4: Initialize Population

- Generate a random population of size $nPop$ using steps 1 to 10 provided in constraint handling mechanism available in section III

Step 5: Evaluate the objective functions of CHPEED problem

- For each population evaluate fuel cost objective $f_1(x)$, and the emission level objective function $f_2(x)$.

Step 6: Perform Non Domination Sorting

Step 7: Sort each population based on nondominated fronts, calculate crowding distance for each nondomination level and rank the population based on crowding distance.

Step 8:

For each generation do

Step 8a: (Start of NSGAII Algorithm)

- Using the upper half of the population create offspring population
  - Tournament selection and simulated binary crossover (SBX)
  - Generate a random number k in the range of $[1, N_p + N_c]$ and apply the steps 2 to 10 of the constraint handling mechanism available in section III.
  - Apply mutation.
  - Generate a random number k in the range of $[1, N_p + N_c]$ and apply the steps 2 to 10 of the constraint handling mechanism available in section III.
- Evaluate fuel cost objective $f_1(x)$, and the emission level objective function $f_2(x)$.
- Merge the parent and offspring population.
- Perform fast nondomination sorting
- Calculate crowding distance and rank population based on nondomination fronts.
- Select solutions
  - Each front is filled in ascending order.
  - Last front is filled with descending order of crowding distance.
- Store the nondominated solutions in list F_1.
- Plot the nondominated solutions in list F_1.

Step 8b: (Start of MOPSO Algorithm)

- Position and cost of the particle are initialized from the lower half of the population.
- Store the values of the particles as their personal best $pBest$.
- Determine domination for the particles
  - Initialize external repository rep.
  - Create grid and find grid index.
- **For each particle do**
  - Select leader for external repository.
  - Update the speed of the particle.
  - Generate a random number k in the range of $[1, N_p + N_c]$ and apply the steps 2 to 10 of the constraint handling mechanism available in section III.
  - Evaluate fuel cost objective $f_1(x)$, and the emission level objective function $f_2(x)$.
  - Apply mutation and calculate new solutions.
  - Generate a random number k in the range of $[1, N_p + N_c]$ and apply the steps 2 to 10 of the constraint handling mechanism available in section III.
  - Determine domination.
  - Update $pBest$.
- **End For**
- Add nondominated solutions to repository.
- Determine domination of new repository members.
- Keep only the nondominated members in the repository.
- Update grid and grid index.
- Modify inertia weight.

Step 8c:

- Create a new set of particles half the size of $nPop$ and fill it with nondominated solution in the external repository followed by the $pBest$.
- Combine the population of NSGAII and the new set of particles of the MOPSO.
- Perform fast nondomination sorting.
- Calculate crowding distance and rank the population based on nondominated fronts.
- Truncate and divide the population into two halves.
- Increment generation count.

End For

Step 9: Stop Algorithm when it reaches $MaxIt$.

FIGURE 3. Hybrid NSGA II-MOPSO algorithm for solving CHPEED problem.

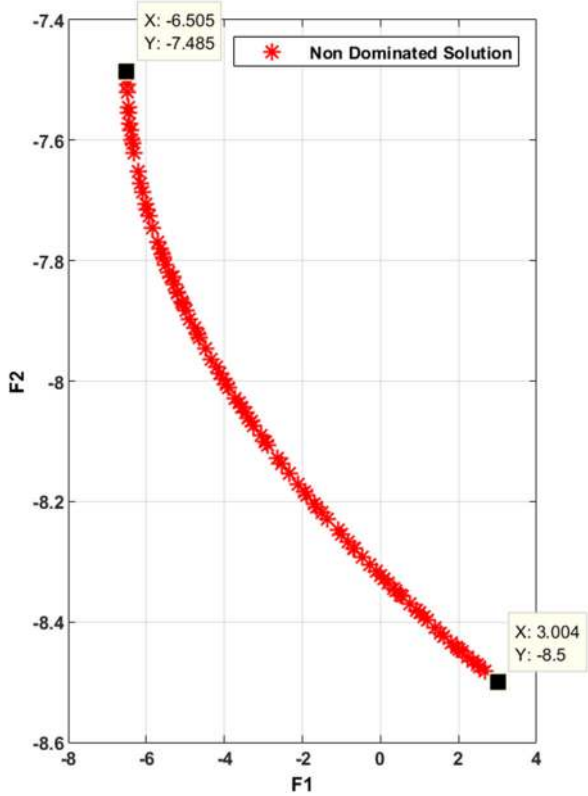

FIGURE 4. Pareto front produced by the proposed hybrid algorithm for test function I.

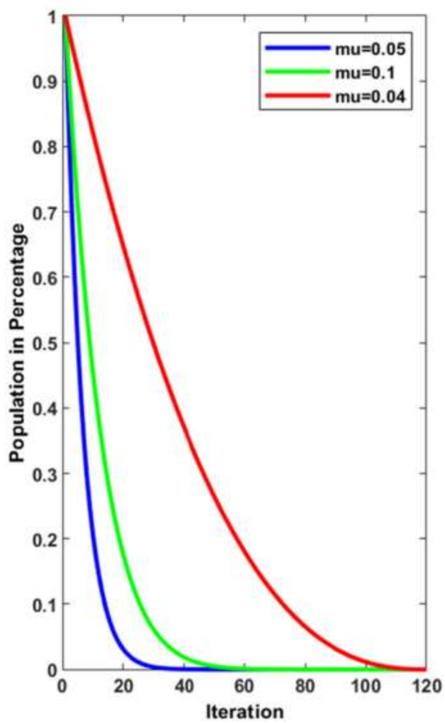

FIGURE 5. Behaviour of the mutation operator for various values of mu.

iteration and narrows down over time using a nonlinear characteristic. When $mu = 0.05$ the mutation operation does not affect the population after 30 iterations but when $mu = 0.4$ it affects up to 110 iterations. This adaptive mutation operator

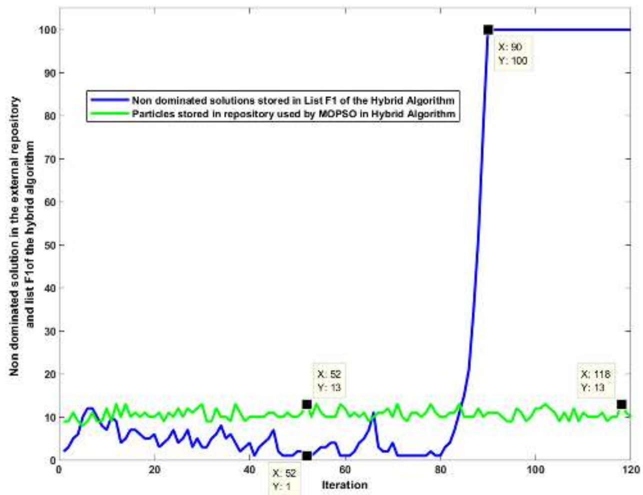

FIGURE 6. The non-dominated solution stored in external repository and in list F1at the end of each iteration of the hybrid NSGA II-MOPSO algorithm for test system I. The size of list F1is 100 and repository size is 50.

is utilized in the hybrid algorithm to enhance the searching capability of the MOPSO in the hybrid algorithm.

Fig. 6 shows the number of nondominated particles in the list F1 of the hybrid algorithm and also it shows the nondominated particles stored in the external repository of the MOPSO stage of the hybrid algorithm. In this test function, the search space is too vast, and for the list to be full, it nearly takes 90 iterations.

The size of the external repository was set to 50 and as seen from the graph shown in Fig. 6 the external repository used by MOPSO was never full, and the maximum non dominated particles found from the lower half of the population was only 13 at iteration number 52 and 118. At iteration number 52, the one nondominated solution found by NSGA II from the upper half of the population dominates the 13 nondominated solutions found by MOPSO from the lower half of the population, and so the list F1 has only one non-dominated solution. The maximum number of non-dominated particle stored in list F1 fluctuates until it reaches 100. Once it exceeds 100, these nondominated particles are ranked, and only 100 non-dominated solutions are retained. The Fig. 7 is same as Fig. 6 the only difference is the size of the external repository was set to 5 instead of 50. As seen from the graph, the repository was full from the beginning to the end of the iteration, but the particles stored were truncated when they exceed 5.

B. TEST FUNCTION II

This function was proposed in [75] and used in [66] to compare the performance of MOPSO with other multiobjective algorithms. This test function has two objectives f_1 and f_2 which are functions of the variables x_1, x_2. The mathematical model of the test function II is given below:

$$Minimize\, f_1(x_1) = x_1 \tag{31}$$

$$Minimize\, f_2(x_1, x_2) = g(x_2).h(x_1, x_2) \tag{32}$$

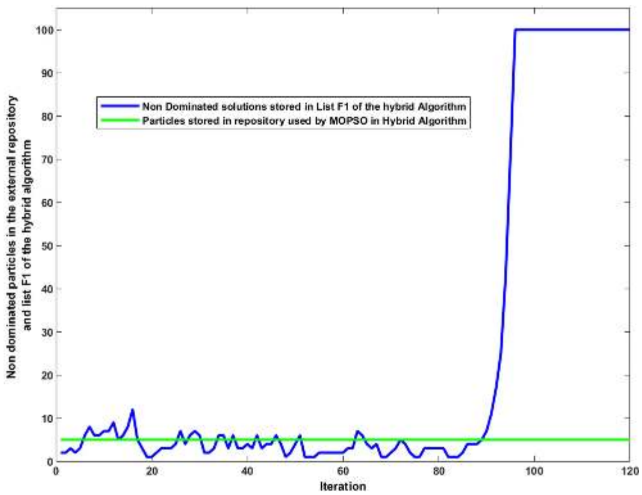

FIGURE 7. The non-dominated solution stored in external repository and in list F1 at end of each iteration of the hybrid NSGA II-MOPSO algorithm for test system I. The size of list F1is 100 and repository size is 5.

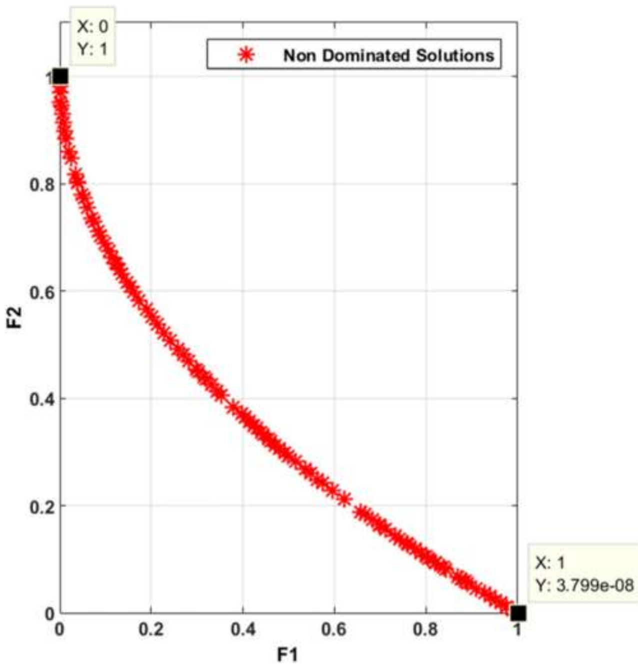

FIGURE 8. Pareto Front produced by the hybrid NSGAII-MOPSO algorithm for test function II.

where

$$g(x_2) = 11 + x_2^2 - 10.cos(2\pi x_2) \tag{33}$$

$$h(x_1, x_2) = \begin{cases} 1 - \sqrt{\dfrac{f_1(x_1)}{g(x_2)}}, & \text{if } f_1(x_1) \leq g(x_2) \\ 0, & \text{otherwise} \end{cases} \tag{34}$$

The range of the decision variables are $0 \leq x_1 \leq 1, -30 \leq x_2 \leq 30$. Fig. 8 shows the Pareto optimal front and the extreme points produced by the hybrid algorithm. By looking at the Pareto optimal front, it is easy to notice that this hybrid algorithm can produce the actual Pareto optimal front

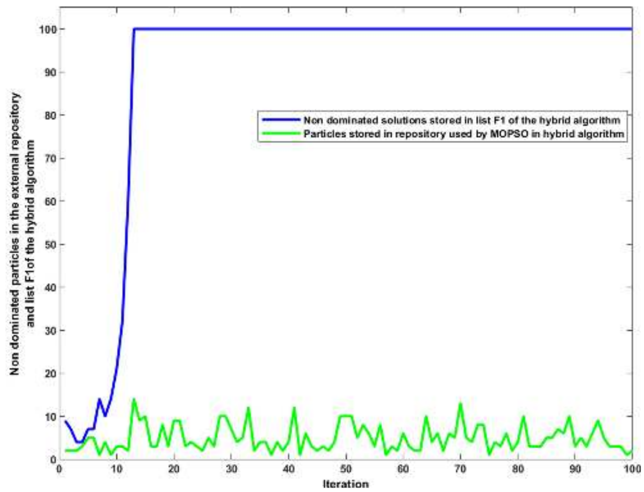

FIGURE 9. The non-dominated solution stored in external repository and in list F1of the hybrid NSGA II -MOPSO algorithm for test system II. The size of list F1is 100 and repository size is 50.

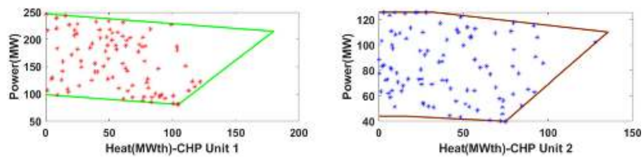

FIGURE 10. Initial solution plotted in FOR of the CHP units.

for this test function, as shown in [66]. Fig. 9 shows the number of nondominated particles in the list F1 of the hybrid algorithm and also it shows the nondominated particles stored in the external repository of the MOPSO stage of the hybrid algorithm. In this test function, the search space is not vast, and the list is full by the 15$^{\text{th}}$ iteration.

C. TEST SYSTEM 1

The test system I has 4 power only units, 2 CHP Units, and 1 heat only unit. The system data for the test system I is given in appendix A. The data contains the quadratic fuel cost equations with valve point effect and emission equations. The B matrix transmission loss coefficients is available in [63]. Fig 16. and Fig. 17 in the appendix depicts the FOR of the two CHP units. The heat demand and power demand of this test system are 150 $MWth$ and 600 MW respectively. The initial solution generated using the steps given in section 3 for the two CHP units are shown in Fig. 10. It is evident from Fig. 10 that the initial solution covers the entire FOR of the CHP units. Fig. 11 shows the POF obtained using the NSGA II algorithm. Table 2 shows the extreme points of the POF obtained for the best fuel cost in 50 trials and the time of execution of the NSGA II algorithm.

Fig. 12 shows the POF found using the MOPSO algorithm. Table 2 provides the extreme points of the POF obtained for the best fuel cost in 50 trials and the execution time of the MOPSO algorithm. Table 1 provides the parameters of the hybrid algorithm. Table 2 provides the extreme points of

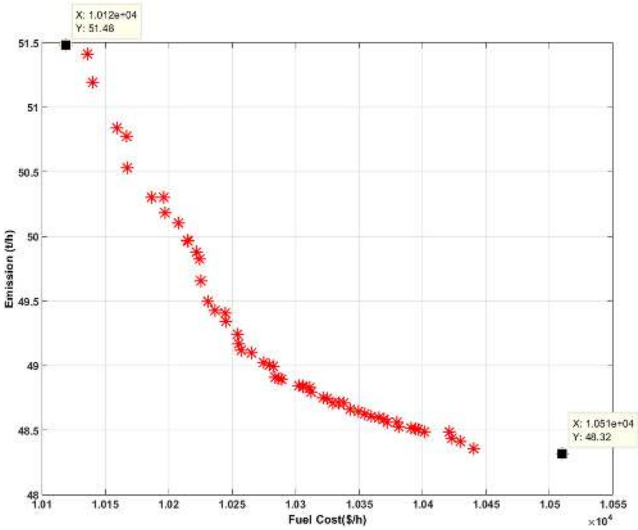

FIGURE 11. Pareto optimal curve obtained using NSGA II.

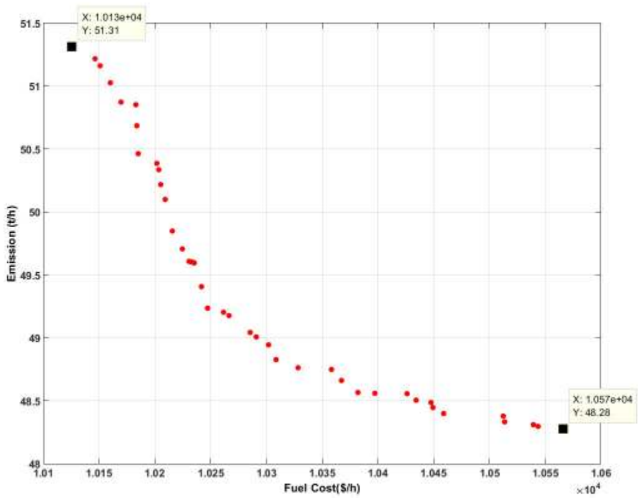

FIGURE 12. Pareto optimal curve obtained using MOPSO.

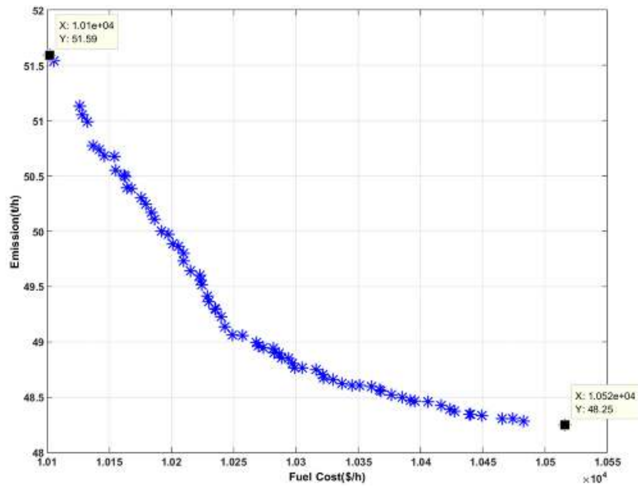

FIGURE 13. Pareto optimal curve obtained using Hybrid NSGA II-MOPSO for test system I.

the POF obtained for best fuel cost and execution time of the proposed hybrid algorithm.

Only 49 solutions filled the repository of the MOPSO algorithm and Fig. 13 shows the POF found using hybrid NSGAII-MOPSO algorithm. Table 2 shows the extreme points of the POF and the execution time of the NSGAII-MOPSO algorithm. Table 2 also provides the best fuel cost and emission obtained using each algorithm.

Table 2 provides the extreme points of the POF, the minimum fuel cost and minimum emission found by the NSGA II, MOPSO, and the proposed hybrid algorithm. The proposed hybrid algorithm with the efficient constraint handling mechanism produces better results when compared to results directly quoted from [63]. The improvement level in the percentage of the extreme points of the PO is obtained using (35) [76]. The performance improvement in the hybrid method in saving the fuel costs compared to NSGAII, MOPSO, and TVAC-PSO is 0.168%, 0.237%, and 0.746% respectively. The performance improvement in the hybrid method in the reduction of emission levels compared to NSGAII, MOPSO, and TVAC-PSO is 0.149%, 0.062%, and 2.38% respectively. The proposed method has obtained less fuel cost compared to NSGAII, MOPSO, and TVAC-PSO by 17\$/h, 24\$/h, and 76\$/h respectively. The proposed method has obtained fewer emission levels compared to NSGAII, MOPSO, and TVAC-PSO by 0.072 t/h, 0.03 t/h, and 1.176 t/h respectively.

$$\begin{aligned} &\mathit{Improvement\ Percentage} \\ &= \frac{\mathit{result\ of\ compared\ method} - \mathit{result\ of\ hybrid\ method}}{\mathit{result\ of\ compared\ method}} \times 100 \end{aligned} \tag{35}$$

The NSGA II algorithm is the fastest in terms of convergence, but it is not able to produce a well spread and diverse optimal solution, as shown in Fig. 11 when compared to the POF produced by the hybrid algorithm shown in Fig. 13. The MOPSO algorithm is the slowest but can produce better extreme points than NSGA II but the POF produced by MOPSO shown in Fig. 12 is not well spread when compared to NSGA II or hybrid algorithm. The NSGA II algorithm is faster than the hybrid algorithm, but the hybrid algorithm can produce well spread and diverse PO solution, produce the minimum fuel cost and minimum emission when compared to NSGA II and other algorithms. The reason for the better performance of the hybrid algorithm is mainly because of the local search carried out by the MOPSO in the hybrid algorithm and the effective constraint handling mechanism.

1) STATISTICAL ANALYSIS

The criteria's that are used to evaluate the performance of multiobjective optimization in general and the usage of these criteria's concerning CHPEED problems are found in literature [35], [69], [77]–[82]. Due to the stochastic nature of

TABLE 2. Comparison of optimal solution of the extreme points of the POF and convergence time using NSGA II, MOPSO, TVAC-PSO [63] and hybrid NSGA II-MOPSO for test system I.

Method	NSGA II		MOPSO		TVAC-PSO [63]		Hybrid NSGA II-MOPSO	
	Best Fuel Cost	Best Emission	Best Fuel Cost	Best Emission	Best Fuel Cost	Best Emission	Best Fuel Cost	Best Emission
$P_1(MW)$	47.406	75.000	51.033	75.000	60.743	75.000	46.4075	75.0000
$P_2(MW)$	97.393	103.07	95.914	113.76	92.594	116.34	98.8986	112.179
$P_3(MW)$	113.43	130.25	111.27	124.83	108.67	120.92	112.312	125.948
$P_4(MW)$	208.98	165.84	209.13	161.17	205.68	164.74	209.307	161.138
$O_1(MW)$	93.433	86.384	93.324	85.295	92.813	81.990	93.7756	86.4419
$O_2(MW)$	40.066	40.166	40.022	40.651	40.132	41.653	40.0000	40.0000
$H_1(MWth)$	32.701	75.812	32.502	81.836	35.251	98.969	29.5995	74.3466
$H_2(MWth)$	74.786	74.077	75.012	66.477	73.040	50.579	75.0000	75.0000
$T_1(MWth)$	42.512	0.1106	42.487	1.6872	41.708	0.4513	45.4005	0.6534
$Total\ Power(MW)$	600.702	600.708	600.700	600.707	600.642	600.638	600.702	600.708
$Total\ Heat\ (MWth)$	150	150	150	150	150	150	150	150
$Fuel\ Cost\ (\$/h) \times 10^4$	**1.0119**	1.0510	**1.0126**	1.0566	**1.0178**	1.0668	**1.0102**	1.0516
$Emissions\ (t/h)$	51.483	**48.319**	51.314	**48.277**	51.378	**49.423**	51.594	**48.247**
$Time\ Taken\ (s)$	84.881		192.45		N/A		123.12	

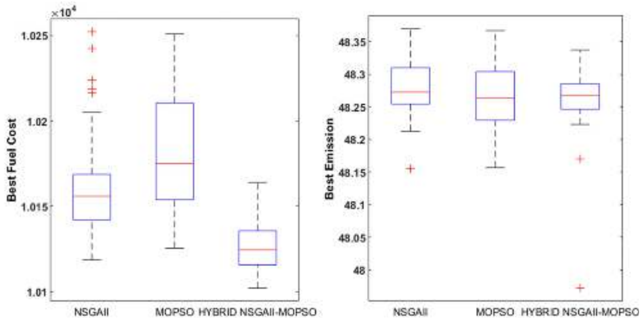

FIGURE 14. Box-Whisker plot for the results obtained by each algorithm in 50 trials for test system I.

the evolutionary algorithm, it is very important to provide the statistical significance of the results and in this section the quality of the obtained solutions is analyzed using various criteria's.

a: BOX-WHISKER PLOT

By the repetition of each algorithm for 50 trials and during each trial, the best fuel cost and emission objectives are observed to plot the box-whisker plot. The Fig.14 portrays the comparison of the median, extreme values, the spread of the best solutions, and the unusual observations of the data set, the distribution outlines for each algorithm by box and whisker plot. Table 3 provides the quantitate analysis of the results. From Fig. 14, it's evident that the hybrid approach can produce solutions which always remain closer to the best-obtained value in each trial; the median is smallest for the proposed hybrid method. Table 3 indicates the proposed hybrid algorithm obtains the minimum fuel cost (shown in bold) and minimum emissions (shown in bold).

b: HYPER-VOLUME (HV)

Hyper-volume for two objectives is a very well-known indicator which measures the area dominated by the PO solutions.

TABLE 3. Quantitative analysis of the results obtained by each algorithm in 50 trials.

	Fuel Cost				
	Max.	**Min.**	**Median**	**Outliers**	**Std. Dev.**
NSGAII	10252.07	10118.72	10155.97	10252.07 10242.23 10223.95 10218.49 10216.24	30.6574
MOPSO	10250.85	10125.6	10175.045	-	34.5086
Hybrid NSGA II-MOPSO	10163.99	**10102.2**	**10124.63**	-	15.5455
	Emission				
	Max.	**Min.**	**Median**	**Outliers**	**Std. Dev.**
NSGAII	48.3695	48.1556	48.2725	48.1556	0.0434
MOPSO	48.3667	48.1568	**48.2633**	-	0.0513
Hybrid NSGA II-MOPSO	48.337	**47.9724**	48.2674	48.1705 47.9724	0.0772

The coordinates of the reference point chosen to measure the HV is (10600, 52). Table 4 provides the normalized HV measure for the POF obtained for each algorithm using the program developed in [82]. Table 4 shows that the highest HV is obtained by the proposed method, which indicates the POF produced by the proposed method is better than the NSGA II or the MOPSO algorithms.

c: SPACING

The spacing indicator measures the spread of the solutions throughout the nondominated front. A value of zero indicates that the solutions are equally spaced. Table 5 indicates that the proposed hybrid NSGA II-MOPSO algorithm can obtain the least value of the spacing measure compared to the other algorithms. The minimal value of the spacing measure indicates the equal spacing of the solutions in the POF for the proposed algorithm.

TABLE 4. Results of Hyper Volume Metric for NSGAII,MOPSO, and proposed hybrid NSGAII-MOPSO algorithms.

HV OBTAINED FOR THE BEST FUEL COST		
NSGAII	MOPSO	Hybrid NSGA II-MOPSO
0.0178	0.0180	**0.0184**
HV OBTAINED FOR THE BEST EMISSION		
NSGAII	MOPSO	Hybrid NSGA II-MOPSO
0.0206	0.0151	**0.0209**

TABLE 5. Results of spacing Metric for NSGAII,MOPSO, and proposed hybrid NSGAII-MOPSO algorithms.

SPACING METRIC FOR THE BEST FUEL COST		
NSGAII	MOPSO	Hybrid NSGA II-MOPSO
$5.585e^{-4}$	$1e^{-3}$	$4.754e^{-4}$
SPACING METRIC FOR THE BEST EMISSION		
NSGAII	MOPSO	Hybrid NSGA II-MOPSO
$4.828e^{-4}$	$9.345e^{-4}$	$3.696e^{-4}$

TABLE 6. MHD for the extreme solutions obtained from each algorithm.

Modified Hausdorff Distance (MHD) –Best Economic dispatch			
	NSGAII	**MOPSO**	**TVAC-PSO** [63]
Hybrid NSGA II-MOPSO	1.1317	1.6029	3.5799
Modified Hausdorff Distance (MHD) –Best Emission dispatch			
	NSGAII	**MOPSO**	**TVAC-PSO** [63]
Hybrid NSGA II-MOPSO	2.2179	2.0042	4.6893

d: MODIFIED HAUSDORFF DISTANCE (MHD)

Hausdorff Distance (HD) is a mutual proximity measure which measures the maximum distance of a set to the nearest point in the other set. Among nearly 20 distance measures based on Hausdorff distance, the MHD is the best and robust for measuring distances of two objects based on their edge points (in CHPEED problem the distance between the extreme solution obtained from two algorithms) is proved in [80]. Table 6 shows the comparison of the MHD for the best economic and best emission solution obtained by the different algorithm. The significant distance between the extreme solutions obtained by the proposed method with the other algorithms is inferred from Table 6.

e: SOLUTION QUALITY

In every trial, the proposed hybrid algorithm can produce Pareto solutions very close to the best-attained solution, and the Box-Whisker plot indicates this. The Box-Whisker plot also indicates that the hybrid algorithm can produce better extreme solutions than other algorithms. The normalized hyper volume indicator proves the Pareto front obtained from the hybrid algorithm is the best when compared to other algorithms. The spacing metric provides the distribution of the solution in the Pareto optimal front. The least value for the hybrid algorithm, when compared to other algorithms, indicates the smooth and uniform distribution of the solutions in the Pareto optimal front. The MHD measure also indicates the significant distance between the extreme solutions obtained by the hybrid algorithm compared to other algorithms. The statistical analysis carried out in this section proves the effectiveness of the proposed hybrid algorithm with efficient constraint handling mechanism is a viable alternative to solve the highly complex CHPEED problem. Further it also able to produce Pareto-optimal front which is well-distributed over the trade-off curve and outperforms the existing algorithms.

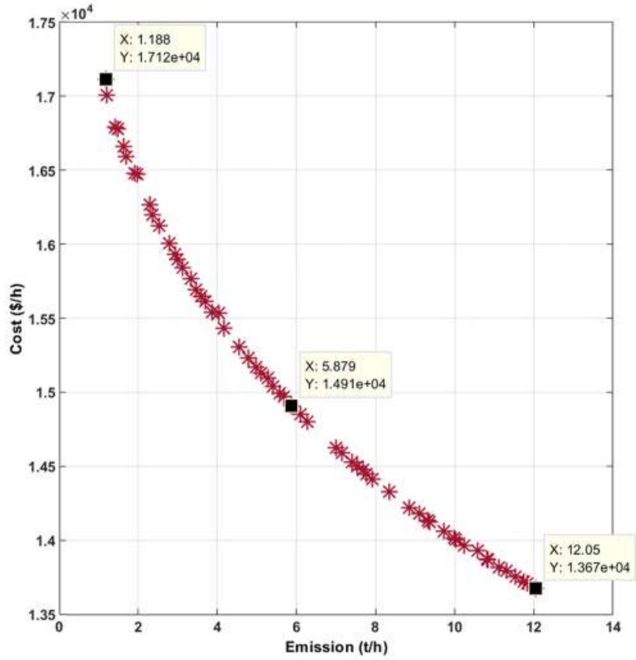

FIGURE 15. Pareto optimal curve obtained using hybrid NSGA II-MOPSO for test system II.

D. TEST SYSTEM 2

The previous section establishes the effectiveness of the hybrid algorithm to solve CHPEED problem. In this section, the hybrid NSGA II-MOPSO algorithm is applied to solve a test system which consists of 1 power only unit, 3 CHP units, and 1 heat only unit. Appendix B provides the fuel cost equations and emission equations. In this test system, the transmission losses are not considered. The power and heat demand of this test system is 300 MW and 150 MWth respectively. Fig. 17, Fig. 18, and Fig. 19 in the appendix shows the FOR of the three CHP units. Fig. 15 shows the Pareto front obtained using the hybrid algorithm. Fig.15 also shows that the hybrid algorithm can produce well spread and diverse solutions.

Table 7 shows the best comprise solution obtained using the proposed algorithm and the execution time of the hybrid algorithm. The results obtained in [62] are also available in Table 7 for comparison. The proposed method can reduce fuel cost of the compromise solution by 99$/h compared to NSGA II and by 55$/h compared to SPEA2. The improvement percentage of the proposed method in reducing the fuel cost compared to NSGA II and SPEA2 is 0.6596%

TABLE 7. Comparison of best comprise solution and time taken for convergence using hybrid NSGA II-MOPSO for test system II with NSGA II and SPEA 2 in [62].

Method	**NSGAII** [62]	**SPEA 2** [62]	**Hybrid NSGA II-MOPSO**
$P_1(MW)$	93.9044	96.4846	92.4930
$O_1(MW)$	72.8298	71.1705	70.3707
$O_2(MW)$	43.3448	44.5018	32.1959
$O_3(MW)$	89.9210	87.8431	104.9404
$H_1(MWth)$	84.9250	84.7660	81.4378
$H_2(MWth)$	22.6032	10.2186	25.5919
$H_3(MW)$	2.6268	17.9054	0.0297
$T_1(MWth)$	39.8449	37.1100	42.9406
$Total\ Power(MW)$	300	300	300
$Total\ Heat\ (MWth)$	150	150	150
$Fuel\ Cost\ (\$/h) \times 10^4$	1.5008	1.4964	**1.4909**
$Emissions\ (t/h)$	6.0563	6.3667	**5.8794**
$Time\ Taken(s)$	7.3627	50.0416	**7.2424**

TABLE 8. Comparison of optimal solution of the extreme points of the Pareto front using hybrid NSGA II-MOPSO for test system II and RCGA in [62].

Method	**RCGA** [62]		**Hybrid NSGA II-MOPSO**	
$P_1(MW)$	134.9904	39.2000	135.000	35.000
$O_1(MW)$	49.9525	125.8000	40.2760	109.4862
$O_2(MW)$	25.0827	45.0000	19.7240	53.1560
$O_3(MW)$	89.9744	90.0000	105.000	102.3578
$H_1(MWth)$	73.5089	32.3998	72.9971	124.3056
$H_2(MWth)$	35.8519	55.0000	37.9461	23.6797
$H_3(MW)$	1.2916	24.9999	0.0000	1.0991
$T_1(MWth)$	39.3476	37.6002	39.0568	0.9156
$Total\ Power(MW)$	300	300	300	300
$Total\ Heat\ (MWth)$	150	150	150	150
$Fuel\ Cost\ (\$/h) \times 10^4$	**1.3776**	1.7048	**1.3673**	1.7118
$Emissions\ (t/h)$	12.0647	**1.4460**	12.0547	**1.1883**

and 0.3675% respectively. The proposed method can reduce emission levels of the compromise solution by 0.1769 t/h and 0.4873 t/h, respectively. The improvement percentage of the proposed method in reducing the emission levels compared to NSGA II and SPEA2 is 2.92% and 7.653%, respectively. From the analysis of the results in Table 7, it is clear that the proposed hybrid method outperforms the existing method and also has the least convergence time.

Table 8 shows the best fuel cost and best emission obtained from the proposed method and the comparison of the results obtained in [62]. From the results shown in bold in Table 8, it is evident that the proposed hybrid method can produce better solutions than the existing method. The proposed method can reduce the fuel cost of the extreme solution and emission levels compared to RCGA by 103 $/h and 0.2577 t/h, respectively. The improvement percentage of the proposed method in reducing the fuel cost and emission levels compared to RCGA is 0.75% and 17.82%, respectively. The proposed hybrid method shows a significant reduction in fuel cost and emission levels compared to existing RCGA method. There are strong indications such as better compromise solutions, better extreme solutions, and faster searchability, which lead to a conclusion of the superiority of the hybrid NSGAII-MOPSO over existing methods. Even though most of the improvement percentage is less than 3%; however, fuel cost saving and emission level reduction for 24h a day, 8760h in a year is highly considerable.

VI. CONCLUSION AND FUTURE WORK

The hybrid NSGA II-MOPSO algorithm with effective constraint handling mechanism is developed and tested with standard test functions and widely used test systems to assess the overall efficiency of the hybrid algorithm. PO solutions obtained by the hybrid algorithm are compared with the results available in the literature. For the test system I, the performance improvement in the hybrid method in saving the fuel costs and reducing the emission levels compared to existing methods is as high as 0.746% and 2.38%, respectively. For the test system II, the performance improvement in the compromise solution is in saving the fuel costs and reducing the emission levels compared to existing methods is as high as 99$/h and 0.4873 t/h, respectively. The result improvements compared to existing methods are significant and it is observed that the proposed hybrid algorithm can converge to produce well diverse and widespread solutions along with better extreme solutions due to its effective searching capability. The statistical analysis indicates the quality of the solutions obtained by the proposed method is better than the existing methods. As a result, the hybrid NSGAII-MOPSO can be a viable alternative for solving the CHPEED problem and can considerably save fuel cost and reduce emission levels. As future work, the complexity analysis using big O notation will be carried out and also effective tuning of parameters, sensitivity analysis of the parameters, and its impact on the solution will be analysed.

APPENDIXES

APPENDIX A

Cost and Emission function of each unit of test system 1

Power-Only Units

$$C_{p1}(P_1) = 0.008P_1^2 + 2P_1 + 25 + \left|100sin\left\{0.042\left(P_1^{min} - P_1\right)\right\}\right| \$; \quad 10 \le P_1 \le 75MW \tag{A.1}$$

$$E_{p1}(P_1) = 10^{-4} \times \left(6.490P_1^2 - 2.777P_1 + 4.091\right) + 2 \times 10^{-4} \times exp\,(0.02857P_1)\,Kg \tag{A.2}$$

$$C_{p2}(P_2) = 0.003P_2^2 + 1.8P_2 + 60 + \left|140sin\left\{0.04\left(P_2^{min} - P_2\right)\right\}\right| \$ \quad 20 \le P_2 \le 125MW \tag{A.3}$$

$$E_{p2}(P_2) = 10^{-4} \times \left(5.638P_2^2 - 3.0235P_2 + 2.534\right) + 5 \times 10^{-4} \times exp\,(0.03333P_2)\,Kg \tag{A.4}$$

$$C_{p3}(P_3) = 0.0012P_3^2 + 2.1P_2 + 100 + \left|160sin\left\{0.038\left(P_3^{min} - P_3\right)\right\}\right| \$; \quad 0 \le P_2 \le 175MW \tag{A.5}$$

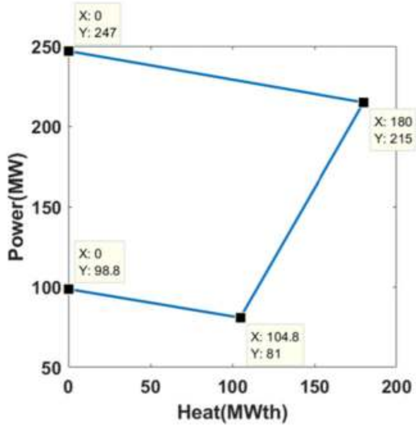

FIGURE 16. Heat-Power feasible operating region for the CHP unit 1 of test system I.

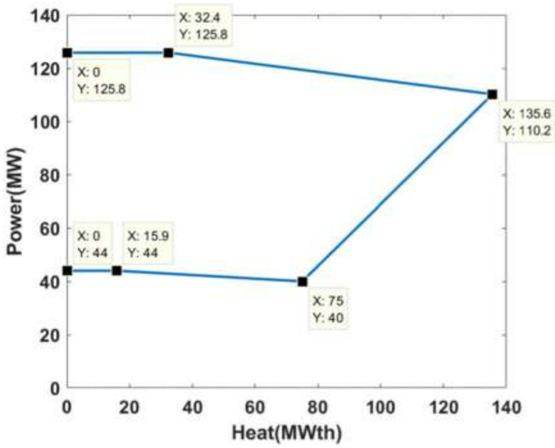

FIGURE 17. Heat-power feasible operating region for the CHP unit 2 of test system I.

$$E_{p3}(P_3) = 10^{-4} \times \left(4.586P_3^2 - 2.547P_3 + 4.258\right) + 1 \times 10^{-6} \times exp(0.08P_3)\,Kg \quad (A.6)$$

$$C_{p4}(P_4) = 0.001P_4^2 + 2P_4 + 120 + \left|180sin\left\{0.037\left(P_4^{min} - P_4\right)\right\}\right| \$; \quad 40 \leq P_2 \leq 250MW \quad (A.7)$$

$$E_{p4}(P_4) = 10^{-4} \times \left(3.38P_4^2 - 1.775P_3 + 5.326\right) + 2 \times 10^{-3} \times exp(0.02P_4)\,Kg \quad (A.8)$$

CHP Unit

$$C_{c1}(O_1, H_1) = 0.0345O_1^2 + 14.5O_1 + 2650 + 0.03H_1^2 + 4.2H_1 + 0.031O_1H_1\$ \quad (A.9)$$

$$E_{c1}(O_1, H_1) = 16.5 \times 10^{-6}O_1Kg \quad (A.10)$$

$$C_{c2}(O_2, H_2) = 0.0435O_2^2 + 36O_2 + 1250 + 0.027H_2^2 + 0.6H_2 + 0.011O_2H_2\$ \quad (A.11)$$

$$E_{c2}(O_2, H_2) = 16.5 \times 10^{-6}O_2Kg \quad (A.12)$$

Feasible Operation Region of the CHP Units

The FOR of the CHP units 1 and 2 are illustrated in Fig.16 and Fig.17 respectively. The inequality constraint associated with the FOR of unit 1 is given by (A.13) and for unit 2 is provided by (A.14) and (A.15).

$$\frac{32}{180}H_1 + O_1 - 247 \leq 0; \quad \frac{134}{75.2}H_1 - O_1 - \frac{7952}{75.2} \leq 0; \; -\frac{17.8}{104.8}H_1 - O_1 + 98.8 \leq \quad (A.13)$$

$$O_2 = 125.8 \Rightarrow 0 \leq H_2 \leq 32.4; \quad O_2 = 44 \Rightarrow 0 \leq H_2 \leq 15.9 \quad (A.14)$$

$$\frac{15.6}{103.2}H_2 + O_2 - \frac{13488}{103.2} \leq 0; \; \frac{70.2}{60.6}H_2 - O_2 - \frac{2841}{60.6} \leq 0; \; -\frac{4}{59.1}H_2 - O_2 + \frac{2664}{59.1} \leq 0 \quad (A.15)$$

Heat Only Units

$$C_{h1}(T_1) = 0.038T_1^2 + 2.0109T_1 + 950\$; \quad 0 \leq T_1 \leq 2695.2MWth \quad (A.16)$$

$$E_{h1}(T_1) = 1.8 \times 10^{-5}T_1Kg \quad (A.17)$$

APPENDIX B

Cost and Emission function of each unit of test system 2

Power-Only Units

$$C_{p1}(P_1) = 0.000115P_1^3 + 0.00172P_1^2 + 7.699P_1 + 254.8863\$; \; 35 \leq P_1 \leq 135MW \quad (B.1)$$

$$E_{p1}(P_1) = 10^{-4} \times \left(6.490P_1^2 - 5.554P_1 + 4.091\right) + 2 \times 10^{-4} \times exp(0.02857P_1)\,Kg \quad (B.2)$$

CHP Units

$$C_{c1}(O_1, H_1) = 0.0435O_1^2 + 36O_1 + 1250 + 0.027H_1^2 + 0.6H_1 + 0.011O_1H_1\$ \quad (B.3)$$

$$E_{c1}(O_1, H_1) = 0.00165O_1Kg \quad (B.4)$$

$$C_{c2}(O_2, H_2) = 0.1035O_2^2 + 34.5O_2 + 2650 + 0.025H_2^2 + 2.203H_2 + 0.051O_2H_2\$ \quad (B.5)$$

$$E_{c2}(O_2, H_2) = 0.0022O_2Kg \quad (B.6)$$

$$C_{c3}(O_3, H_3) = 0.072O_3^2 + 20O_3 + 1565 + 0.02H_3^2 + 2.3H_3 + 0.04O_3H_3\$ \quad (B.7)$$

$$E_{c3}(O_3, H_3) = 0.0011O_3Kg \quad (B.8)$$

Feasible Operation Region of the CHP Units

Fig 17 shows the FOR of the CHP unit 1. The inequality constraint for this unit is the same as (A.14) and (A.15) after H_2 and O_2 in the equation is replaced by H_1 and O_1 respectively. The FOR of the CHP unit 2 of the test system2 is provided in Fig.18 and the inequality constraints for this unit is provided by (B.9).

$$\frac{15}{55}H_2 + O_2 - 60 \leq 0; \; \frac{35}{15}H_2 - O_2 - \frac{1250}{15} \leq 0; \; -\frac{10}{40}H_2 - O_2 + 20 \leq 0 \quad (B.9)$$

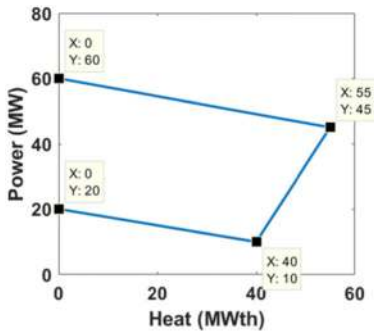

FIGURE 18. Heat-power feasible operating region for the CHP unit 2 of test system 2.

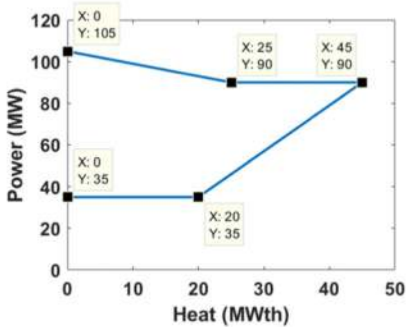

FIGURE 19. Heat-power feasible operating region for the CHP unit 3 of test system 2.

The FOR of the CHP unit 3 of the test system 2 is shown in Fig.19 and the inequality constraints for this unit is given by (B.10) and (B.11).

$$O_3 = 90 \Rightarrow 25 \leq H_3 \leq 45;$$
$$O_3 = 35 \Rightarrow 0 \leq H_3 \leq 20(B.10)$$
$$\frac{15}{25}H_3 + O_3 - 105 \leq 0; \; \frac{55}{25}H_3 - O_3 - 9 \leq 0 \tag{B.10}$$

Heat Only Units

$$C_{h1}(T_1) = 0.038T_1^2 + 2.0109T_1 + 950\$; \quad 0 \leq T_1 \leq 60\,MWth \tag{B.11}$$
$$E_{h1}(T_1) = 0.0017T_1Kg \tag{B.12}$$

ACKNOWLEDGMENT

The author would like to thank the facilities provided by the authorities of Royal Commission of Jubail and Yanbu, and the management of Jubail Industrial College to carry out this research.

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