Numerical results: Figs. 2 and 3 show the IBC results for the scattering problem of a 2D perfect-conducting rectangular cylinder which is illuminated by a plan TM wave with incident angle $\varphi = 0°$. The cross-section dimensions of the cylinder are $1\lambda \times 2\lambda$. The convergence process for IBC coefficients $A(\vec{r}_i, \vec{r}_j)$ and induced current $J(\vec{r}')$ are clearly shown in Figs. 2 and 3. It can be seen that the iterative process is very quick to converge. Usually a steady solution for the induced current can be obtained after three or four iterative processes. Fig. 4 shows the final solutions of induced current distributions for the scattering problems of a 2D perfectly-conducting circular cylinder ($r = 2\lambda$ and $\varphi = 0°$) and rectangular cylinder ($3\lambda \times 1\lambda$ and $\varphi = 30°$).

In all these calculations, the conformal FD meshes have three layers, and the node step is 0.05λ. The first initial values for induced current are all set as 1. Of course, other values can be used with no effect on the final results. To check the validity of IBC, we calculate the corresponding MoM solutions and MEI solutions, which are given accompanying the IBC solutions for comparison. It can be seen that the solutions of these three methods agree very well with each other.

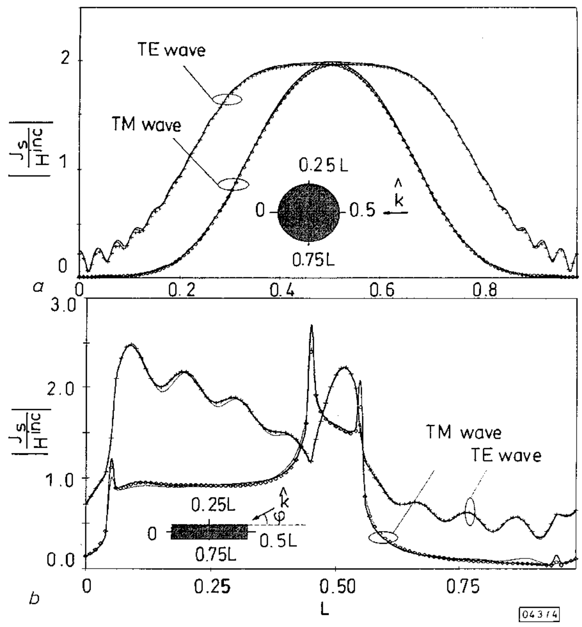

Fig. 4 *Induced currents on 2D cylinder scatterers*

a Circular cylinder: $r = 2\lambda$ and $\varphi = 0°$
b Rectangular cylinder: $3\lambda \times 1\lambda$ and $\varphi = 30°$
—— by MoM
——— by MEI
◇, + this Letter

Conclusion: IBC is a novel truncation boundary condition for solving electromagnetic scattering problems by the FD method. It is simple in concept, easily applied and quick to converge. Since there is no limitation on the two points involved in the equation, eqns. 1 and 2 can be applied to the node at an arbitrary position. Therefore, we can truncate the mesh very close to the object's surface, which results in a reduction in the number of the unknowns and a saving in computer memory. Three general cases have been successfully solved by applying IBC.

23 January 1997
Electronics Letters Online No: 19970356

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Combined LMS/F algorithm

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Indexing terms: Least mean squares methods, Adaptive filters

A new adaptive filter algorithm has been developed that combines the benefits of the least mean square (LMS) and least mean fourth (LMF) methods. This algorithm, called LMS/F, outperforms the standard LMS algorithm judging either constant convergence rate or constant misadjustment. While LMF outperforms LMS for certain noise profiles, its stability cannot be guaranteed for known input signals even for very small step sizes. However, both LMS and LMS/F have good stability properties and LMS/F only adds a few more computations per iteration compared to LMS. Simulations of a non-stationary system identification problem demonstrate the performance benefits of the LMS/F algorithm.

Introduction: The least mean square algorithm (LMS) has been used for many years to adapt filter structures in such problems as system identification, equalisation and interference cancellation [1]. Not surprisingly, many researchers have studied methods for improving the convergence rate of LMS without dramatically increasing the complexity of computation. Some of the first studies considered simple variable-step algorithms in which the step size is slowly decreased against time but such simple procedures fail for non-stationary inputs. The least mean fourth (LMF) algorithm, developed by Walach and Widrow [6], optimised a criterion of the error raised to the fourth power instead of the more usual square power used for LMS. Though LMF has been shown to outperform LMS in certain situations, LMF is hampered by the difficulty of setting a stable step size parameter. In practice, higher order power filters can quickly become unstable unless an extremely small step size is used [6]. In an attempt to address the above problems, least mean mixed-norm adaptive filtering [4] and the LMS+F algorithm [2] have been designed. Though these algorithms also combine LMS and LMF, they result in systems whose stability is still a function of the unknown plant for system identification. In this Letter, we develop the combined LMS/F algorithm as a method to improve the performance of the LMS adaptive filter without sacrificing the simplicity and stability properties of LMS. Many other methods have been suggested for adapting the step size but these require much more complicated formulations, and more computation than LMS/F [3, 5]. For example, the VS (variable step) algorithm [3] keeps track of sign changes of $\epsilon_k x_k$, and increases the step size until sign changes become frequent and decreases the step size if there are frequent sign changes. In fact, VS faces some problems such as setting appropriate values for six adjustable parameters including the maximum and minimum step sizes. If these values are not set properly, a smaller minimum step size might not decrease the misadjustment, but only degrade the convergence rate of the VS algorithm.

Combined LMS/F: The weight update equation for the LMS/F algorithm is

$$W_{k+1} = W_k + 2\mu \frac{\epsilon_k^3}{\epsilon_k^2 + V_{th}} X_k \tag{1}$$

where W_k represents the adaptive weight values at iteration k, ϵ_k is the system error and X_k is a vector of the last $L + 1$ samples of the input signal. The positive threshold V_{th} provides a mechanism to trade off faster convergence and lower misadjustment. When $\epsilon_k^2 \gg Vth$, the weight update reduces to the standard LMS algorithm with a step size of μ. When $V_{th} \gg \epsilon_k^2$, eqn. 1 behaves like the least mean fourth (LMF) algorithm with a step size of μ/V_{th}. This gives the combined benefits of a large step size LMS for fast convergence and small step size LMF for low misadjustment. The algorithm automatically changes between the two types of behaviour based on the current error measurement. The choice of step size μ is subject to exactly the same restrictions as for LMS-that is, μ must be less than the reciprocal of the largest eigenvalue of the autocorrelation matrix of the input signal [1]. The simplicity of setting the step size is a marked difference from algorithms such as LMF [6] and LMS+F [2] where a stable step size parameter depends on parameters from the unknown system, the initial value of the weights as well as the characteristics of the input signal. Once a proper value of μ is chosen for LMS/F, no value of the positive threshold V_{th} can make the update unstable.

We can integrate eqn. 1 to determine the error criterion that is minimised by the LMS/F update rule at each iteration:

$$J_k = \epsilon_k^2 - V_{th} \ln(\epsilon_k^2 + V_{th}) \qquad (2)$$

As we would expect, a Taylor series analysis of eqn. 2 shows that when $|\epsilon_k|$ is small $J_k \simeq \epsilon_k^4/(2V_{th})$ (LMF) and when $|\epsilon_k|$ is large $J_k \simeq \epsilon_k^2$ (LMS).

In fact, by setting $\mu_{lms} = 0.45\mu$ (where μ_{lms} and μ are the step sizes of LMS and LMS/F, respectively) and $V_{th} = 5E[n_k^2]$, we could make both the algorithms have the same misadjustment when the plant noise is Gaussian distributed.

To deal with non-stationary problems, an automatically adjusted V_{th} is proposed. Since we usually do not have any prior knowledge of the unknown plant noise n_k, the expected value of $5\epsilon_k^2$ may be considered as a way to estimate the value $5E[n_k^2]$. Since $5E[|\epsilon_k^2|]$ is sensitive to large error variations when the adaptive process is not in steady state, $5E[|\epsilon_k|]^2$ is used to estimate V_{th} in the following non-stationary identification problems in order to add robustness to outliers.

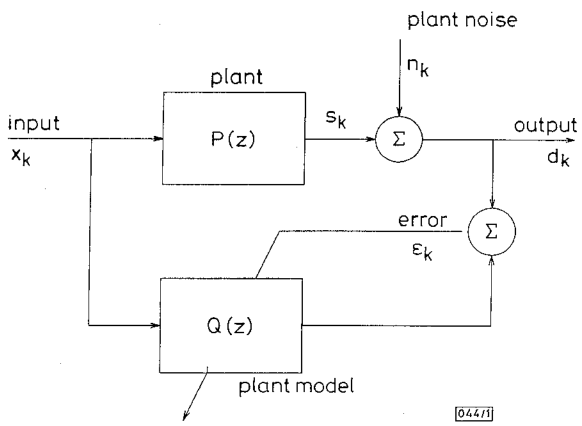

Fig. 1 *Block diagram for system identification of unknown plant P(z) with adaptive filter given by Q(z)*

Simulation results: Fig. 1 depicts a non-stationary system identification problem to show the performance of LMS/F compared to both LMS and VS [3]. For the first 1000 iterations, the non-stationary unknown plant transfer functions is given as $P(z) = 1 + 10z^{-1} + 20z^{-2} + 30z^{-3} + 20z^{-4} + 10z^{-5} + z^{-6}$ while the next 1000 iterations the unknown system was changed to $P(z) = 1 - 10z^{-1} + 20z^{-2} - 30z^{-3} + 20z^{-4} - 10z^{-5} + z^{-6}$. The adaptive filter is given by $Q(z) = W_0 + W_1z^{-1} + W_2z^{-2} + W_3z^{-3} + W_4z^{-4} + W_5z^{-5} + W_6z^{-6}$ where W_n denotes the free weights to be determined and they are initially set to zero. The standard deviation of the white Gaussian noise n_k abruptly switches from 2.5 to 3.5 at iteration 1001. Also, $\mu = 0.009$, $\mu_{lms} = 0.45\mu$ and V_{th} is set using the automatic adaptation formula

$$\sigma_{k+1} = \lambda\sigma_k + (1-\lambda)|\epsilon_k| \qquad (3)$$

$$(V_{th})_{k+1} = 5(\sigma_{k+1})^2 \qquad (4)$$

where λ is set equal to 0.9995 and σ_k is a standard IIR estimator for $E[|\epsilon_k|]$. Fig. 2 shows that LMS/F has a faster convergence rate and better misadjustment rate than both LMS and VS [3] for this non-stationary system identification problem. Comparison against other algorithms (e.g. LMS+F [2]) are not shown since their stability properties cannot be guaranteed.

Since both LMS and LMS/F use only one step size for each dimension, they are slowed by large eigenvalue spread problems. However LMS/F can be generalised to provide different step sizes for each dimension to solve such problems. Since the white Gaussian noise in this example has an eigenvalue spread of one for any number of dimensions, eigenvalue spread is not an issue for this problem.

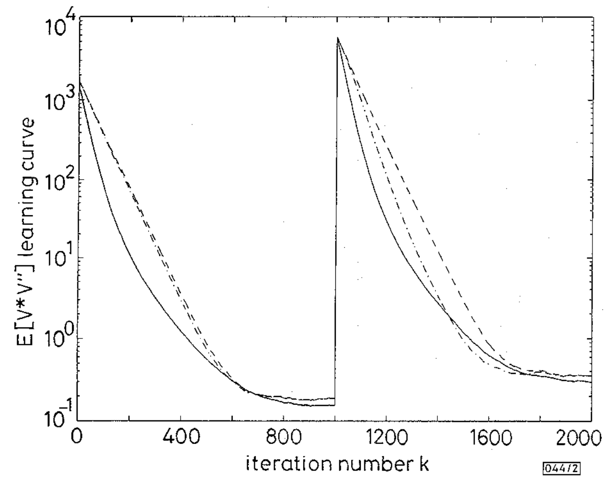

Fig. 2 *Comparison of adaptive threshold LMS/F against LMS for non-stationary system identification problem*

For both LMS and LMS/F, $\mu = 0.009$, $\mu_{lms} = 0.45\mu$
One version of VS algorithm is shown for comparison purposes with optimised parameters $\mu_{max} = 0.009$, $\mu_{min} = 0.0041$, $m_0 = 2$, $m_1 = 3$ and $\alpha = 1.01$
- - - - LMS
— · — · variable step
——— LMS/F

Conclusion: This LMS/F algorithm has been developed and shown to outperform LMS and VS judging either constant convergence rate or constant misadjustment. Unlike many other attempts at speeding the convergence rate, the choice of step size to guarantee stability for LMS/F is exactly the same choice as for LMS. Finally, LMS/F is still simple enough for many 8-bit microcontroller and even analogue hardware implementations.

Acknowledgments: This work was supported by an NSF CAREER award #MIP-9502307.

6 February 1997
Electronics Letters Online No: 19970311

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