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COMBINED SOLUTION FOR LOW DEGREE LONGITUDE HARMONICS OF GRAVITY FROM 12 AND 24 HOUR SATELLITES

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CARL A. WAGNER

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Carl A. Wagner
Special Projects Branch
Goddard Space Flight Center
Greenbelt, Maryland

Abstract

Over 600 near stationary orbits of five high altitude communications satellites have been examined for the strong effects on them of low degree earth gravity harmonics. The accelerated drift of these 12 and 24 hour orbits, due to the earth's anomalous potential, has been measured at 50 longitude locations. From these accelerations, 5 low degree earth gravity harmonics which resonate with these orbits have been calculated (10 gravity coefficients). The best determined harmonic is felt to be H_{22}, which dominates the accelerated drift of the three 24 hour satellites (SYNCOMS 2, 3, and Early Bird) and the two 12 hour satellites (Molniya 1 and Cosmos 41). The constants found for this harmonic are (Normalized): $10^6 C_{22} = 2.40 \pm .03$, and $10^6 S_{22} = -1.43 \pm .03$. The other four harmonics in decreasing order of their estimated strength of absolute determination are: H_{32}, H_{44}, H_{33} and H_{31}. Their normalized coefficients are found to be: $10^6 C_{32} = 0.69 \pm 0.20$, $10^6 S_{32} = -0.53 \pm 0.20$, $10^6 C_{44} = 0.02$, $10^6 S_{44} = 0.70$, $10^6 C_{33} = 0.16$, $10^6 S_{33} = 1.10$, $10^6 C_{31} = -0.42$, and $10^6 S_{31} = -1.58$ The measured drift accelerations themselves have been compared to accelerations calculated from a number of recently published satellite and combined satellite-surface gravity geoids. The "best" geoid representing these accelerations is a 1967 S.A.O. determination of Walter Kohnlein.

INTRODUCTION

The refinement of knowledge about the gravitational field of the earth is still a subject of great interest to geophysicists. The unique contribution of satellite geodesy in this effort is well recognized. That contribution is principally to the accurate detection of large scale anomalies in the field.

The first major contributions in satellite geodesy have been towards the precise definition of the axisymmetric, or zonal, part of the potential, down to about the 10th degree harmonic. This definition has been principally due to the work of Kozai in Japan [1] and King-Hele's group in England. [2, 3] These early efforts have been successful because of the ready availability of many different satellites whose orbits and orbit planes are strongly rotated over long periods of time by the zonal components of the field. Because of the averaging effect of the rotation of the earth, no such easily observed effects, due to the non-zonal part of the potential, are obtained from satellites, except from those in so called resonant orbits. These are the orbits whose periods are commensurate with the earth's rotation period.

Yet, in spite of the difficulties of extracting non-zonal geodetic information from observing small short period variations in the great majority of satellite orbits, much progress has already been made in this area too. It is generally recognized that the large scale geoid features due to non-zonal gravity anomalies are now fairly well established from independent and parallel studies of general satellite orbits at the Smithsonian, [4] Johns Hopkins, [5] the Naval Weapons Laboratory, [6] and by Bill Kaula. [7] Nevertheless, our knowledge of the details of the non-zonal field is still much less secure than that of the zonal field. In particular, while the discrimination of the zonal field has been accomplished (to a fairly high degree now) by dynamic satellite geodesy alone, the equivalent discrimination of the much more complex non-zonal field has required and will continue to require many different sources of accurate information and methods of analysis.

Among these important supplementary sources of information are the strong long term perturbations of the special resonant orbit satellites. Unfortunately, except for the class of eccentric 24-hour satellites, these cannot tell us about the whole non-zonal field. But the limited parts they do measure, they measure exceedingly well because of the amplification possible on a resonant orbit.

Many of the recent non-zonal satellite and combined satellite-surface gravity geoids already incorporate some resonant orbit information. In this report, I will present acceleration data reduced from long term observations on five high altitude satellites in resonant orbits. This data is strongly relevant to the solution for the terms of low degree in the longitude (non-zonal) gravity field of the earth.

OBSERVATION DATA AND CONDITION EQUATIONS

In Table 1 is listed 50 reduced long term longitude accelerations of the five 24 and 12-hour communications satellites 1963 - 31A (Syncom 2), 1964 - 47A (Syncom 3), 1965 - 28A (Early Bird), 1964 - 49D (Cosmos 41), and 1965 - 30A (Molniya 1). The longitude acceleration measured ($\ddot{\lambda}$) is essentially that of the nearly stationary ascending equator crossings of these commensurate orbit satellites. In those cases where the geographical drift rates of the ground tracks were appreciable, the accelerations were derived by differentiating a smoothing function best fit to drift rate data. Details of the theory governing the drift of these deeply resonant orbits, under the influence of the earth's longitude harmonics, are to be found in papers by Allan,[8, 9] Wagner,[10, 11] Gedeon et. al.,[12] and in Kaula's book.[13] The 24-hour data summarized in Table 1 comes directly from a recent analysis by Wagner.[14] The 12-hour acceleration data has been derived from the gravity results of a recent study of the drift of the Russian communication satellites.[11]

The definition of the earth's gravitational potential I use comes from Kaula's book:[13]

$$V = \sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} \frac{\mu a_e^{\ell}}{r^{\ell+1}} P_{\ell m}(\sin\phi)\left[C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda\right] , \qquad (1)$$

where μ is the Gaussian gravity constant of the earty (3.9860×10^5 km^3/sec^2), r is the distance to the satellite from the earth's center of mass, a_e is the mean equatorial radius of the earth, ϕ is the geocentric latitude and λ the geographic longitude of the satellite, (elsewhere λ stands for the satellites ascending equator crossing longitude) and the $P_{\ell m}$'s are associated LeGendre functions. The non-dimensional gravity constants $C_{\ell m}$ and $S_{\ell m}$ in this spherical harmonic series are unnormalized constants. From the above definition, $C_{00} = 1$, $C_{10} = C_{11} = S_{11} = 0$; and $C_{21}, S_{21} \doteq 0$, since the North Pole (the latitude reference axis) is very nearly a principal axis of inertia for the earth.

In reporting numerical values of gravity coefficients in this series, it has become conventional to report normalized values $\overline{C}$, $\overline{S}$ related to the unnormalized coefficients of (1) by:

$$\overline{C}_{\ell m}, \overline{S}_{\ell m} = \left[N_{\ell m}\right]\left[C_{\ell m}, S_{\ell m}\right] , \qquad (2)$$

where:

$$N_{\ell m} = \left[\frac{2(2\ell + 1)(\ell - m)!}{(\ell + m)!}\right]^{-1/2}, \qquad (2a)$$

for m $\neq$ 0, or the longitude terms. The normalized coefficients are better indeces of the true magnitude of physical effects than the unnormalized ones.

From the theory of resonant orbit perturbations,[10, 11] the observations in Table 1 are related to the relevant longitude harmonics of the potential by (for the 24-hour data, zero eccentricity orbits):

$$\ddot{\lambda} = 12\pi^2 \sum_{\ell - m \text{ even}} C_{\ell m}\left[F_{\ell m} \sin m\lambda\right] + S_{\ell m}\left[-F_{\ell m} \cos m\lambda\right],$$

$$\text{radians/sidereal day}^2 . \qquad (3)$$

where:

$$F_{22} = \frac{6}{a^2}\left[\frac{1}{2}(1 + \cos i)\right]^2$$

$$F_{31} = \left(\frac{-3}{a^3}\right)\left[\frac{1}{2}(1 + \cos i) - \frac{5}{8}\sin^2 i(1 + 3\cos i)\right]$$

$$F_{33} = \left(\frac{45}{a^3}\right)\left[\frac{1}{2}(1 + \cos i)\right]^3$$

$$F_{42} = \left(\frac{-15}{a^4}\right)\left[\frac{1}{4}(1 + \cos i)^2 - \frac{7}{4}\sin^2 i \cos i(1 + \cos i)\right]$$

$$F_{44} = \left(\frac{420}{a^4}\right)\left[\frac{1}{2}(1+\cos i)\right]^4 , \qquad (3a)$$

and, (for the 12-hour data, orbits of any eccentricity):

$$\ddot{\lambda} = 12\pi^2 \sum_{\ell, m \text{ relevant}} C_{\ell m} \left[m a^{-\ell} (FG)_{\ell m} \begin{vmatrix} \sin m\left(\lambda + \frac{\gamma_{\ell m}}{m}\right) \\ -\cos m\left(\lambda + \frac{\gamma_{\ell m}}{m}\right) \end{vmatrix}^{\ell - m \text{ even}}_{\ell - m \text{ odd}} \right]$$

$$+ S_{\ell m} \left[m a^{-\ell} (FG)_{\ell m} \begin{vmatrix} -\cos m\left(\lambda + \frac{\gamma_{\ell m}}{m}\right) \\ -\sin m\left(\lambda + \frac{\gamma_{\ell m}}{m}\right) \end{vmatrix}^{\ell - m \text{ even}}_{\ell - m \text{ odd}} \right] , \text{ rad./sid-day}^2 . \qquad (4)$$

The $(FG)_{\ell m}$ and the $\gamma_{\ell m}$ are the amplitudes and phase angles of composite harmonic vectors which depend on the inclination, eccentricity and argument of perigee of the resonant orbits.[11] For the eighteen 12-hour observations, these orbit-harmonic functions are listed in Table 1 for the relevant harmonics ($H_{\ell m}$) through degree five: H_{22}, H_{32}, H_{42}, H_{44}, H_{52}, and H_{54}. In equations (3) and (4), a is the semimajor axis of the satellite's orbit in earth radii, and i is its inclination. The observation data in Table 1, along with the condition equations (3) and (4), is suitable for incorporation into existing solutions for the gravity field if the data is weighted according to the given estimates of their standard deviations. These estimates are felt to be quite realistic. They take into account many sources of small non-resonant gravity model errors including sun and moon effects and atmospheric drag. In fact, I have solved (in a weighted least squares sense) for a set of ten low degree coefficients from this data using equations (3) and (4). The characteristics of this solution are discussed in the next section.

SOLUTION FOR TEN GRAVITY COEFFICIENTS

Previous calculation has shown[15] that resonance effects of degree higher than four should have negligible influence on the 24-hour satellite. Similarly, considering the higher errors for the observations on the Russian satellites, it appears that the rapid decline in the anomalous potential at 12-hour altitudes will result in small or negligible effects for 5th and higher degree harmonics. Detailed calculations show that of the two harmonics of 4th degree resonant on these satellites, H_{42} should at best be only barely discernible from the data, while H_{44} should have strong influence on the 12-hour observations. Murphy and Victor,[16] though, have measured the influence of the 5th degree harmonics H_{52} and H_{54} on the 12-hour satellites and it seems that though their effects are small, at least H_{54} should be included in future solutions from observations on these objects.

In Table 2, I list the results of the weighted least squares solution of equations (3) and (4) according to the data in Table 1. The solution is for the ten unnormalized resonant coefficients which appear to be most sensitive to this data: $(C,S)_{22}$, $(C,S)_{31}$, $(C,S)_{32}$, $(C,S)_{33}$ and $(C,S)_{44}$. The internal quality of this solution can be gauged by comparing the accelerations computed from it (as shown in Table 1 in the column labeled "Wagner 1968") with the actual observations. The residuals of the solution, shown in Table 1 also, are normalized with respect to the estimated standard deviation of the measured observations. These normalized residuals are also displayed against the data longitude in Figure 1. The normalized solution is listed in Table 3, and displayed in Figure 2 for comparison of individual coefficients with other recent combined-data geoids.

For two cases I have computed drift accelerations for these satellites as predicted (according to equations (3) and (4)) from the published geoids of Kohnlein-SAO 1967,[4] and Kaula 1966.[7] The results and residuals of these predictions are also listed in Table 1 and displayed in Figure 1. These comparison computations included the effects of H_{42} on the 24-hour but not on the 12-hour data. However, inclusion of these effects on the 12-hour satellites, as well as those of 5th and possibly higher degree, is not expected to alter the major results of this study.

DISCUSSION OF THE SOLUTION

The chief point to be made about the solution presented in the last section is that it agrees quite well overall with the recent satellite-surface gravity results for the low degree field. On the other hand, it is also clear from the excess of large residuals, that the full field geoid investigations would benefit from an incorporation of the acceleration data presented here. These comparison residuals (Figure 1) show no glaring bias with respect to longitude, except perhaps for the very well determined Early Bird observations, but do average about twice the residuals of the resonant solution. The Kohnlein-1967 solution is marginally superior to that of Kaula. This is interesting because the Kohnlein solution does not incorporate any high altitude resonant orbit data while the Kaula solution does, albeit with small weight. It also appears that the Bjerhammar-1967 solution (Table 3 and Figure 2) is of about the same quality as the Kohnlein and Kaula geoids with respect to this resonant data. The Bjerhammar field [17] was constructed by combining, accumulating and solving normal equations representing satellite data from the work of Kozai, Anderle and Gaposhkin, and uses the same surface gravity data as in Kaula's studies.

The concordance of the resonant solution with the recent comparison geoids appears best with regard to H_{22} and H_{32} and worst with regard to H_{31} and H_{33} (see Figure 2). It is interesting, and perhaps significant, that the strongest correlation coefficient in the resonant solution, 0.84, is between C_{33} and S_{31} (see Table 2). This attests to the relative uncertainty of the discrimination of these harmonics by this data. Other than this, all other correlations have absolute values less than 0.60. Absolute bounds (1σ) only on H_{22} and H_{32} are found in Table 3. They are somewhat greater than those given by the statistics of the solution (Table 2). They appear justified by the resonably low correlations in the resonant solution and by the good agreement with the new, independently derived geoids.

There is some evidence from Murphy and Victor's study [17] that the amplitude of H_{44} for the resonant solution will be reduced when the effects of H_{54} on the 12-hour data are considered. If true, this would improve the agreement of this harmonic and the 12-hour residuals with respect to the recent more complex geoids. Future gravity data reductions of these high altitude resonant orbit observations will include the effects (either as known or as solved for) of all relevant harmonics through at least the 5th degree.

CONCLUSIONS

Data reduction of over 600 orbits of 12 and 24-hour satellites has established a set of drift accelerations which appear to be strong determinants for many components of low degree in the geopotential. Recent complex geoids, independently derived, are quite compatible with most of this data. But inclusion of the best of the high altitude resonant orbit accelerations in the complex geoid solutions should result in considerable improvement in the overall precision and accuracy of those solutions.

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TABLE 1

MEASURED AND PREDICTED ACCELERATIONS ON 12 AND 24 HOUR SATELLITES

	SATELLITE	LONGITUDE, $\lambda°$	SEMIMAJOR AXIS, E. R.	INCLINATION (°)	ECCENTRICITY e	ARG. OF PER: w	MEASURED ACCELERATION, $\ddot{\lambda}$; 10^{-5} RAD/SID. DAY^2	PREDICTED ACCELERATIONS*: WAGNER-1968 (10^{-5} R/D^2)	RESIDUAL (σ)	KOHNLEIN SAO-1967 (10^{-5}R/D^2)	RESIDUAL (σ)	KAULA-1966 (10^{-5} R/D^2)	RESIDUAL (σ)
24 HOUR SATELLITES	SYNCOM 2	65.30	6.611	31.78	0.00		1.000 ± 0.053	1.006	-0.11	1.049	-0.92	1.070	-1.32
	SYNCOM 2	65.31	6.611	31.79	0.00		1.017 ± 0.053	1.005	0.23	1.048	-0.58	1.069	-0.98
	SYNCOM 2	66.12	6.611	31.89	0.00		0.968 ± 0.063	0.932	0.57	0.972	-0.06	0.993	-0.40
	SYNCOM 2	66.39	6.611	31.89	0.00		0.845 ± 0.086	0.907	-0.72	0.945	-1.16	0.967	-1.42
	SYNCOM 2	69.00	6.611	31.4	0.00		0.676 ± 0.047	0.669	0.15	0.696	-0.43	0.718	-0.89
	SYNCOM 2	73.25	6.611	31.4	0.00		0.235 ± 0.038	0.260	-0.66	0.268	-0.87	0.289	-1.42
	SYNCOM 2	77.60	6.611	31.4	0.00		-.152 ± 0.041	-0.168	0.39	-0.181	0.71	-0.161	0.22
	SYNCOM 2	104.50	6.618	32.15	0.00		-2.319 ± 0.067	-2.328	0.13	-2.424	1.57	-2.433	1.70
	SYNCOM 2	130.00	6.617	32.3	0.00		-2.550 ± 0.093	-2.452	-1.05	-2.504	-0.49	-2.541	-0.10
	SYNCOM 3	160.74	6.611	1.3	0.00		-0.090 ± 0.036	-0.111	0.58	-0.153	1.75	-0.181	2.53
	SYNCOM 2	161.00	6.617	32.40	0.00		-0.194 ± 0.067	-0.142	-0.78	-0.108	-1.28	-0.133	-0.91
	SYNCOM 3	168.29	6.611	0.53	0.00		0.699 ± 0.040	0.711	-0.30	0.668	0.78	0.654	1.12
	SYNCOM 3	169.10	6.611	0.53	0.00		0.831 ± 0.039	0.797	0.85	0.753	2.00	0.740	2.34
	SYNCOM 3	172.75	6.611	0.00	0.00		1.072 ± 0.099	1.172	-1.01	1.120	-0.49	1.114	-0.42
	SYNCOM 3	176.80	6.612	0.27	0.00		1.577 ± 0.177	1.563	0.08	1.496	0.46	1.499	0.44
	SYNCOM 3	178.71	6.611	0.11	0.00		1.683 ± 0.060	1.736	-0.88	1.660	0.38	1.668	0.25
	SYNCOM 2	-140.00	6.620	32.58	0.00		2.153 ± 0.085	2.225	-0.85	2.066	1.02	2.114	0.46
	SYNCOM 2	-140.00	6.620	32.58	0.00		2.366 ± 0.159	2.225	0.89	2.066	1.89	2.114	1.59
	SYNCOM 2	-61.00	6.612	32.83	0.00		-2.288 ± 0.122	-2.266	-0.18	-2.104	-1.51	-2.138	-1.23
	SYNCOM 2	-60.94	6.612	32.83	0.00		-2.320 ± 0.058	-2.266	-0.93	-2.105	-3.71	-2.139	-3.12
	SYNCOM 2	-55.24	6.611	33.02	0.00		-2.255 ± 0.086	-2.232	-0.27	-2.119	-1.58	-2.146	-1.27
	SYNCOM 2	-55.22	6.611	33.02	0.00		-2.238 ± 0.035	-2.232	-0.17	-2.119	-3.40	-2.146	-2.63
	EARLY BIRD	-35.98	6.609	0.74	0.00		-1.976 ± 0.023	-1.954	-0.96	-1.889	-3.78	-1.889	-3.78
	EARLY BIRD	-34.57	6.612	0.64	0.00		-1.849 ± 0.036	-1.865	0.44	-1.812	-1.03	-1.811	-1.06
	EARLY BIRD	-31.86	6.612	0.55	0.00		-1.682 ± 0.038	-1.688	0.16	-1.657	-0.66	-1.652	-0.79
	EARLY BIRD	-31.46	6.610	0.85	0.00		-1.633 ± 0.014	-1.662	2.07	-1.633	0.00	-1.628	-0.36
	EARLY BIRD	-31.29	6.610	0.90	0.00		-1.644 ± 0.012	-1.650	0.50	-1.623	-1.75	-1.617	-2.25
	EARLY BIRD	-29.83	6.610	0.85	0.00		-1.527 ± 0.015	-1.547	1.33	-1.530	0.20	-1.522	-0.33
	EARLY BIRD	-28.83	6.610	0.90	0.00		-1.494 ± 0.012	-1.474	-1.67	-1.463	-2.58	-1.455	-3.25
	EARLY BIRD	-28.70	6.610	0.20	0.00		-1.472 ± 0.014	-1.465	-0.50	-1.455	-1.21	-1.446	-1.86
	EARLY BIRD	-28.54	6.611	0.43	0.00		-1.453 ± 0.038	-1.452	-0.03	-1.444	-0.24	-1.435	-0.53
	EARLY BIRD	-28.37	6.611	0.43	0.00		-1.448 ± 0.014	-1.440	-0.57	-1.432	-1.14	-1.423	-1.79
					e	ARG. OF PER: w							
12 HOUR SATELLITES	[1]COSMOS 41	8.	4.1517	65.55	.735	321.77	7.586 ± 0.640	8.043	-0.71	8.115	-0.82	7.578	0.01
	[3]MOLNIYA 1	18.	4.1731	65.38	.729	318.81	8.270 ± 0.395	7.932	0.86	7.260	2.56	6.904	3.46
	[3]MOLNIYA 1	28.	4.1731	65.38	.729	318.81	5.783 ± 0.826	6.569	-0.95	5.542	0.29	5.396	0.47
	[2]MOLNIYA 1	38.	4.1740	65.18	.737	322.25	4.320 ± 1.750	4.601	-0.16	3.493	0.47	3.535	0.45
	[2]MOLNIYA 1	48.	4.1740	65.18	.737	322.25	1.348 ± 1.658	0.729	0.37	0.182	0.70	0.411	0.57
	[1]COSMOS 41	58.	4.1517	65.55	.735	321.77	-3.524 ± 0.349	-3.552	0.08	-3.265	-0.74	-2.975	-1.57
	[1]COSMOS 41	68.	4.1517	65.55	.735	321.77	-6.926 ± 0.582	-7.118	0.33	-6.148	-1.34	-5.932	-1.71
	[3]MOLNIYA 1	78.	4.1731	65.38	.729	318.81	-9.241 ± 0.381	-8.992	-0.65	-7.916	-3.48	-7.851	-3.65
	[3]MOLNIYA 1	88.	4.1731	65.38	.729	318.81	-7.784 ± 0.799	-9.150	1.71	-8.389	0.76	-8.416	0.78
	[1]COSMOS 41	98.	4.1517	65.55	.735	321.77	-8.296 ± 0.367	-8.210	-0.23	-8.011	-0.78	-8.060	-0.64
	[1]COSMOS 41	108.	4.1517	65.55	.735	321.77	-5.764 ± 0.577	-5.868	0.18	-6.448	1.18	-6.408	1.12
	[2]MOLNIYA 1	118.	4.1740	65.18	.737	322.25	-5.447 ± 1.608	-3.284	-1.35	-4.365	-0.67	-4.164	-0.80
	[1]COSMOS 41	128.	4.1517	65.55	.735	321.77	-0.681 ± 0.380	-0.924	0.64	-1.967	3.38	-1.646	2.54
	[3]MOLNIYA 1	138.	4.1731	65.38	.729	318.81	0.971 ± 0.396	1.060	-0.22	0.656	0.80	0.946	0.06
	[3]MOLNIYA 1	148.	4.1731	65.38	.729	318.81	2.000 ± 0.861	2.581	-0.67	2.847	-0.98	3.020	-1.18
	[2]MOLNIYA 1	158.	4.1740	65.18	.737	322.25	4.685 ± 1.505	3.695	0.66	4.589	0.06	4.592	0.06
	[2]MOLNIYA 1	168.	4.1740	65.18	.737	322.25	6.260 ± 1.594	5.206	0.66	6.330	-0.04	6.056	0.13
	[2]MOLNIYA 1	178.	4.1740	65.18	.737	322.25	7.633 ± 1.608	6.797	0.52	7.609	0.01	7.123	0.32
									RMS: .765		RMS: 1.536		RMS: 1.604

NOTES:

1 ARC 1, COSMOS 41, HARMONIC FUNCTIONS:
2 ARC 2, MOLNIYA, HARMONIC FUNCTIONS:
3 ARC 3, MOLNIYA, HARMONIC FUNCTIONS:

ARC	(FG)$_{\ell m}$ 22	32	42	44	52	54	$\gamma_{\ell m}$ (RADIANS) 22	32	42	44	52	54
1	3.482	11.553	37.16	869.5	228.7	7996.	.8141	-2.1056	-1.244	.9457	3.056	-1.766
2	3.491	11.923	39.00	890.2	238.7	8362.	.8063	-2.1388	-1.299	.9289	3.023	-1.794
3	3.409	10.642	35.20	769.5	217.2	7013.	.8711	-2.0355	-1.214	1.017	3.065	-1.684

*From Equations (3) & (4) with the gravity constants of Table 3.

Table 2

Statistics of the Solution for 10 Resonant Gravity Coefficients (Unnormalized) From 12 & 24 Hour Satellite Accelerations

	$10^6 C_{22}$	$10^6 S_{22}$	$10^6 C_{31}$	$10^6 S_{31}$	$10^6 C_{32}$	$10^6 S_{32}$	$10^6 C_{33}$	$10^6 S_{33}$	$10^6 C_{44}$	$10^6 S_{44}$
Solution	1.55	-0.92	-0.46	-1.71	0.24	-0.18	0.022	.153	0.04	1.47
(Standard Deviation)	.01	.01	.90	.66	.05	.05	.015	.010	.11	.16

σ (Experiment Error) = 2.74×10^{-5} Radians/Sid. Day^2

CORRELATION COEFFICIENTS

C_{22}	S_{22}	C_{31}	S_{31}	C_{32}	S_{32}	C_{33}	S_{33}	C_{44}	S_{44}	
1.000	-.49	.47	.11	-.08	-.40	.12	.43	-.01	.01	C_{22}
	1.000	.38	-.56	.28	.25	-.33	-.35	-.18	-.05	S_{22}
		1.000	-.52	.18	-.13	-.09	.06	-.14	-.08	C_{31}
			1.000	-.13	-.08	.84	.54	.10	-.05	S_{31}
				1.000	.13	-.06	-.05	.01	-.55	C_{32}
					1.000	-.06	-.17	.03	-.12	S_{32}
						1.000	.55	-.03	-.05	C_{33}
							1.000	.10	-.04	S_{33}
								1.000	-.07	C_{44}
									1.000	S_{44}

Table 3
Resonant Longitude Gravity Coefficients* From Recent Studies

Geoid**	C_{22} (10^{-6})	S_{22} (10^{-6})	C_{31} (10^{-6})	S_{31} (10^{-6})	C_{32} (10^{-6})	S_{32} (10^{-6})	C_{33} (10^{-6})	S_{33} (10^{-6})	C_{42} (10^{-6})	S_{42} (10^{-6})	C_{44} (10^{-6})	S_{44} (10^{-6})
Kohnlein S.A.O, 1967	2.38	-1.35	1.71	0.23	0.84	-0.51	0.66	1.43	0.35	0.48	0.04	0.30
Kaula, 1966	2.42	-1.36	1.79	0.18	0.78	0.75	0.57	1.42	0.30	0.60	-0.06	0.32
Bjerhammar, 1967	2.39	-1.32	1.84	0.11	0.69	-0.50	0.58	1.60	0.33	0.67	-0.03	0.36
Wagner, C. -1968	2.40 ± .03	-1.43 ± .03	-0.42	-1.58	0.69 ± .20	-0.53 ± .20	0.16	1.10	0.0	0.0	0.02	0.70

*Fully normalized coefficients.
**Kohnlein SAO, 1967: Table 1 in Reference 4
Kaula, 1966: Solution CA (combined satellite solutions with surface gravity data) in Reference 7
Bjerhammar, 1967: Final combined satellite and terrestrial gravity solution in Reference 17
Wagner, 1968: 12 and 24 hr. satellite resonant solution of this report.

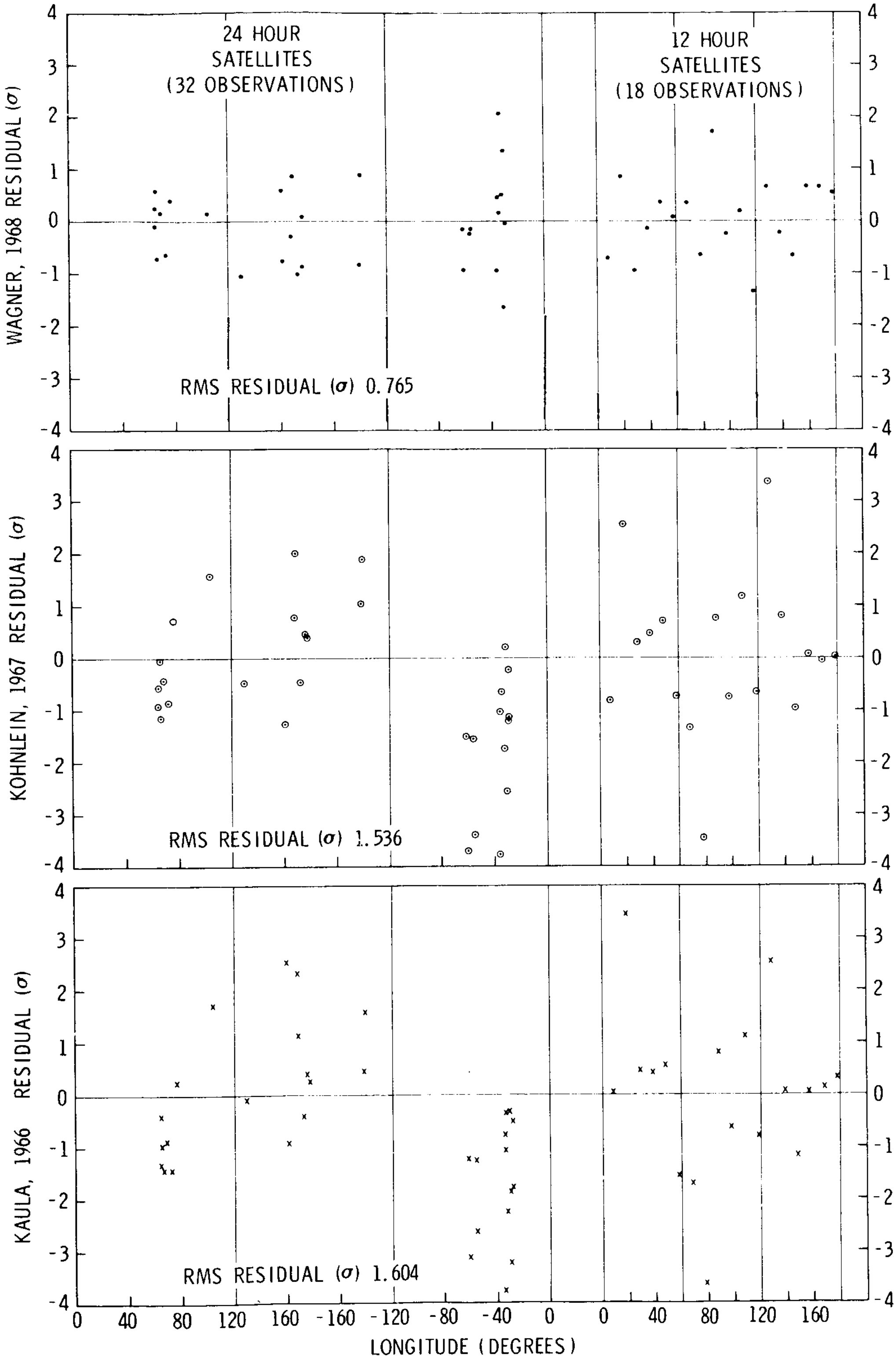

Figure 1. Observation Residuals for 12 and 24 Hour Satellite Accelerations From Recent Geoids.

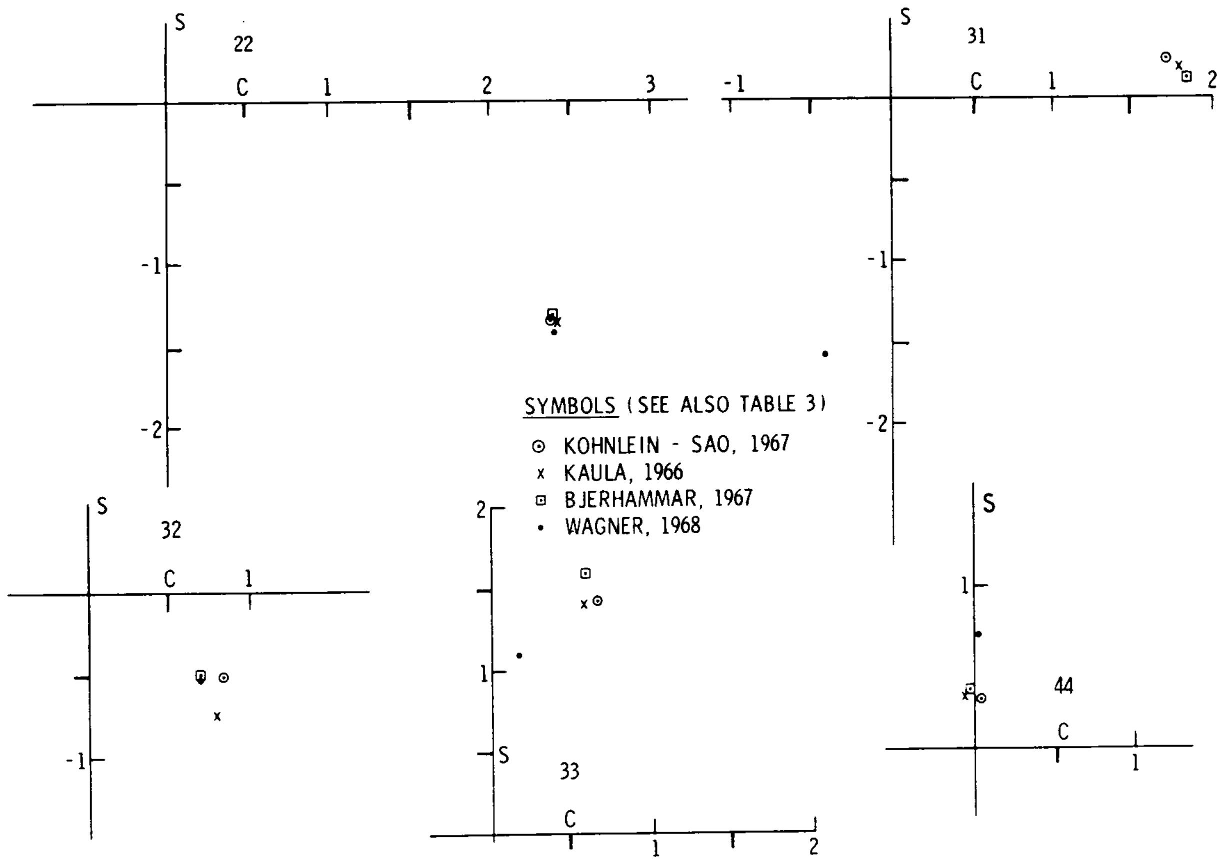

Figure 2. Normalized Longitude Gravity Coefficients (In Units of 10^{-6})