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Comment on "Generalized Hall-effect measurement geometries and limitations of van der Pauw-type Hall-effect measurements"

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The general validity of the van der Pauw-Hall measurement technique for arbitrarily shaped samples has recently been disputed, and the claim has been made that the samples must have mirror symmetry. Here, it is shown that the analysis leading to the mirror symmetry requirement is more restrictive than necessary and that arbitrary shapes can be used. It is also shown that even with nonsymmetric shapes, magnetoresistance effects are eliminated by reversing the magnetic field and averaging the data.

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The general validity of the van der Pauw–Hall measurement technique for arbitrarily shaped samples[1] has recently been disputed by Boerger, Kramer, and Partain (BKP).[2] In their paper they claim that the van der Pauw theory (1) requires alternating voltage and current contacts, although these were not specified by van der Pauw, (2) does not consider and eliminate the effects of magnetoresistance on the measured Hall voltage, and (3) is not valid except in cases where the sample has mirror symmetry about a line connecting the current contacts.

Because of the importance and wide use of the van der Pauw technique, it is necessary to elaborate on the first two contentions and refute the third. First, it is true that alternating voltage and current contacts are required for the Hall voltage measurement. However, this was implied by van der Pauw's notation in his paper[1] when he refers to measuring $\Delta R_{BD,AC}$ on a sample with *successive* contacts A, B, C, and D. Nonalternating contacts ($R_{AB,CD}$ and $R_{BC,DA}$) were specified for measuring resistivity.

Second, as BKP show, the van der Pauw treatment[1] does not explicitly take into account magnetoresistance effects. As shown below, however, for crystals that exhibit isotropic conduction in the plane of the sample, the standard technique of averaging the measured values for the two magnetic field directions eliminates this effect even with arbitrary sample shapes.

To see that mirror symmetry is not required, one only needs to use a more general version of the analysis presented by BKP. That analysis is based on finding the voltage that appears across the voltage contacts by integrating the electric field along some path $P(c,d)$ between the voltage contacts. Using $\mathbf{J} = \boldsymbol{\sigma}\mathbf{E}$ with

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \tag{1}$$

yields

$$E_y = \left(\frac{1}{\sigma_{xx}^2 + \sigma_{xy}^2}\right)(\sigma_{xy} J_x + \sigma_{xx} J_y), \tag{2a}$$

$$E_x = \left(\frac{1}{\sigma_{xx}^2 + \sigma_{xy}^2}\right)(\sigma_{xx} J_x - \sigma_{xy} J_y), \tag{2b}$$

where J is the current density, E is the electric field, and σ is the conductivity tensor. The symmetry implications of this tensor are discussed at the end of this correspondence. Note the negative sign in Eq. (2b), which differs from Eq. (7) in the paper by BKP. The voltage difference between the contacts is then

$$V_m = -\int_c^d \mathbf{E}\cdot d\mathbf{l} = -\int_c^d E_x(x,y)\,dx - \int_c^d E_y(x,y)\,dy, \tag{3}$$

where dl lies along the path, and the other notation follows BKP. Using the symmetry relations for reversal of the magnetic field strength B

$$\sigma_{xx}(-B) = \sigma_{xx}(B), \tag{4a}$$

$$\sigma_{xy}(-B) = -\sigma_{xy}(B), \tag{4b}$$

it can be seen that the Hall voltage V_H is

$$V_H = \tfrac{1}{2}[V_m(B) - V_m(-B)] = -\left(\frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}\right)\left(\int_c^d J_x(x,y)\,dy - \int_c^d J_y(x,y)\,dx\right). \tag{5}$$

BKP now require

$$\int_c^d J_y(x,y)\,dx = 0 \tag{6}$$

and identify the total current I

$$I = w\int_c^d J_x(x,y)\,dy, \tag{7}$$

where w is the sample thickness, so that the Hall coefficient is

$$R_H = -\frac{wV_H}{BI} = \frac{1}{B}\frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}. \tag{8}$$

However, there is no need to use Eq. (6) to arrive at this result because, for thin slabs,

$$I = w\int_c^d \mathbf{J}\cdot\hat{n}\,dl, \tag{9}$$

where $\hat{n}$ is a unit vector in the plane of the sample and perpendicular to the path. Thus, by using

$$\hat{n} = \hat{l} \times \hat{z} = l_y\hat{x} - l_x\hat{y}, \tag{10}$$

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where ^ indicates unit vectors, one gets

$$I = w\left(\int_c^d J_x(x,y)\,dy - \int_c^d J_y(x,y)\,dx\right), \qquad (11)$$

and substitution into Eq. (5) yields Eq. (8). Elimination of Eq. (6) removes the requirement of mirror symmetry and confirms the van der Pauw formulation. Note also that the field averaging used in Eq. (5) eliminated the effect of ohmic and magnetoresistances, even for arbitrary sample shapes.

These results are expected, of course, because of the arguments given by van der Pauw in the section of his paper[1] that discusses the Hall voltage across arbitrary, thin, flat shapes. The arguments given in this comment are an alternative proof for cases where the sample conductivity can be described by the tensor in Eq. (1). Such a description requires only that conductivity be isotropic within the plane of the thin film or wafer. Since the nonlinear current paths which occur in the van der Pauw configuration cause an averaging effect over a range of directions, the van der Pauw method cannot be used to determine the direction dependent mobilities of a sample with nonisotropic conduction.

The largest errors that BKP observed when using the van der Pauw configuration occurred when nonalternating contacts were used. As mentioned, these are not consistent with the van der Pauw method for measuring mobility, but rather are used for measuring resistivity. In fact, since the net current which flows across any path connecting adjacent voltage contacts is zero, Eqs. (5) and (11) show that the field averaged V_H equals zero for nonalternating contacts. The lesser errors can be ascribed to contact size effects and to their sample shape, rather than to lack of mirror symmetry. The square cross-section bars that they used do not satisfy the van der Pauw requirement that the thickness of the sample (in the direction of the magnetic field) be much less than the transverse dimensions. As a result, the current density would not be uniform along the magnetic field direction, and Eq. (7) or Eq. (9) cannot be used.

It has been shown that the analysis of Boerger, Kramer, and Partain was more restrictive than necessary and that, for thin planar samples with isotropic conduction in the plane, the use of arbitrarily shaped van der Pauw samples is valid. Furthermore, under the same conditions, the effects of magnetoresistance are formally eliminated by averaging data taken for normal and reversed magnetic fields.

[1] L. J. van der Pauw, Philips Res. Rep. **13**, 1 (1958).
[2] D. M. Boerger, J. J. Kramer, and L. D. Partain, J. Appl. Phys. **52**, 269 (1981).