*Animal Learning & Behavior*
*1979, 7 (2), 267-268*

# Notes and Comment

## Comment on Heyman and Luce: "Operant matching is not a logical consequence of maximizing reinforcement rate"

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In the first part of their paper, Heyman and Luce (1979) deal with the standard concurrent variable-interval schedules in which the alternatives are *independent*. Consider the following facts about such schedules: (1) As the authors demonstrate, maximization is incompatible with matching to *programmed* reinforcement rates on independent VI schedules. (2) Programmed reinforcement rates differ from obtained reinforcement rates. (3) The empirical matching relationship applies to *obtained*, not programmed, reinforcement rates.

Since a precise maximizing theory would predict only one result, it cannot predict matching to *both* programmed and obtained rates as Heyman and Luce imply it should. They state that the "reinforcement contingencies in independent and interdependent *conc* VI VI schedules are different. Accordingly . . . the maximizing solutions for each are different. Yet is is well established that subjects match in both procedures." But subjects in both procedures match *obtained* rates of reinforcement, whereas their demonstration in Figure 3 is constructed wholly on the basis of *programmed* rates.

That maximizing is incompatible with matching to programmed reinforcement rates (Heyman & Luce, 1979, Figure 3) will be good news to those who, like myself, believe that maximization of some kind is the best theoretical approach to the analysis of operant performance. The usual finding with concurrent independent VI schedules is that relative rate of response *overmatches* relative rate of *programmed* reinforcement, exactly what is predicted by Heyman and Luce's maximizing model. At extreme values of their parameter I, certainly, matching and maximizing say the same thing. When $I = 0$, the changeover rate is infinite and the subject collects all reinforcements. At this point, obtained rate of reinforcement would equal programmed rate; it is exactly at this point that Heyman and Luce's model predicts matching. At the other extreme, when $I = \infty$, the subject does not change over. It stays only on one alternative or another. Given this pattern, it makes sense from the point of view of maximizing to stay at the alternative which is delivering reinforcements at a faster rate. Matching to obtained rates of reinforcement is a trivial consequence of this behavior. Thus, the two extreme points of Figure 3 are exactly predicted by a theory that identifies maximizing with matching to *obtained* rates of reinforcement. It would not be surprising if relative obtained rates of reinforcement for various intermediate values of I fell on or close to the triangles of Figure 3. Thus, to the extent that it makes any predictions about concurrent independent variable-interval schedules, the theory of Luce and Heyman shows that maximizing reinforcement rate *does* imply matching.

Almost all of the evidence for matching comes from such independent concurrent variable-interval schedules. Occasionally, *interdependent* schedules have been used in studies of concurrent behavior but, because there have been relatively fewer reports with such schedules, universality of matching is necessarily less certain in those cases. With interdependent schedules, Heyman and Luce are better able to discuss *obtained* rates of reinforcement. Their Figure 4 shows deviation from matching in the direction of undermatching. (One must assume, because they do not say, that the positions of the triangles on the lines were determined analytically.) And published results do show some undermatching. Two of Stubbs and Pliskoff's (1969) three pigeons and three of Baum's (1975) four people undermatched, according to de Villiers (1977), as Figure 4 predicts they should.

But let us assume that Figure 4 indeed shows a deviation from matching more extreme than would be obtained experimentally. The problem here is with an underlying assumption.

As with the independent schedules, the Heyman-Luce model relies critically on the assumption that probability of changeover is constant. Looking again at their Figures 1 and 2, based on *interdependent* schedules, while it is clear that over the long run there is no upward trend, the first few points in each panel do seem to be heading upward. These points are *not* included in the statistical tests for stationarity, and therefore one may assume that they are as nonhorizontal as they seem.

These first few points may be very important. Interreinforcement times are assumed (correctly) to be exponentially distributed. An exponential distribution has most of its weight on the first few bins. If changeover responses were also exponentially distributed as assumed (probably incorrectly), the interaction in the first few seconds might account for much of the reinforcement and have a strong influence on the shape of Figure 4. It is evident that only very slight alteration of Figure 4 would be required to shift the maxima widely in one direction

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or another. The authors claim (consideration 2 under the rubric, "Generality of the Exponential Model and its Implications") that other distributions had little effect on the implications of Equation 11, which is the basis for Figure 3. But, as I argued above, Figure 3 shows no inconsistency between matching and maximizing of obtained rates. Their claim would be more convincing if it were made with respect to Equation 13, which is the basis for Figure 4. But even if such a claim could be made, it would still not establish their argument. It is not sufficient to show that "there is a wide range of other switching patterns for which matching and maximizing are different." Certainly this is true. But are there switching patterns for which matching and maximizing are the same? What is the relation of these patterns to those actually observed (rather than assumed) in Figures 1 and 2? Until such questions are answered, one cannot take seriously the authors' version of maximizing.

## REFERENCES

Baum, W. M. Time allocation and human vigilance. *Journal of the Experimental Analysis of Behavior*, 1975, **23**, 45-53.

de Villiers, P. A. Choice in concurrent schedules and quantitative formulations of the law of effect. In W. K. Honig & J. Staddon (Eds.), *Handbook of operant behavior*. Englewood Cliffs, N.J: Prentice-Hall, 1977.

Heyman, G. M., & Luce, R. D. Operant matching is not a logical consequence of maximizing reinforcement rate. *Animal Learning & Behavior*, 1979, **7**, 133-140.

Stubbs, D. A., & Pliskoff, S. S. Concurrent responding with fixed relative rate of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1969, **12**, 887-895.

(Received for publication September 5, 1978;
accepted December 14, 1978.)