DEMOGRAPHY© Volume 22, Number 4 November 1985

COMMENT ON J.-Y. PARLANGE, M. J. GUILFOYLE, AND R. E. RICKSON'S "MORTALITY LEVELS AND FAMILY FERTILITY GOALS"

S. Krishnamoorthy
Department of Sociology, Bharathiar University, Coimbatore 641 046 Tamilnadu, India
P. M. Kulkarni
Institute for Social and Economic Change, Nagarbhavi, Bangalore 560 072 Karnataka, India

Parlange, Guilfoyle and Rickson (1983), in attempting to highlight the problem of ecological fallacy in the estimation of family fertility rates from the expected probability of a child surviving to a given age of a parent, tried to examine Krishnamoorthy's (1979) model. We agree with Parlange et al. in principle that due consideration to homogeneous subgroups makes the group that is under study more representative of an individual family and hence a more accurate predictor of its behavior. But in their attempt to refine Krishnamoorthy's model to demonstrate this, they have made serious mistakes in mathematical logic. We believe that such mistakes should not go unnoticed.

Krishnamoorthy in his model assumes $G_o m(x)$ as the age-specific paternity rate, the number of sons born to fathers at age x. Therefore, $G_o m(x)\Delta x$ is the number of boys born to a father when he lives through the age interval $(x, x + \Delta x)$. The probability that all these sons will be dead by the time the father reaches his age Y $(x < Y)$ is:

$$[1 - \ell(Y - x)]^{-G_o m(x)\Delta x} \tag{1}$$

where $\ell(Y - x)$ is the probability that a boy just born will survive $(Y - x)$ years. Mathematical logic permits this. Parlange et al. attempt to improve this model by introducing order-specific birth rates. They define $m_1(x,Y)$ the normalized age-specific male birth rate of the ith son born to a father of age Y. Once we specify the order of the son born, we are dealing with a probability density. Then $m_1(x,Y)\Delta x$ has to be interpreted as the probability that the first son is born in the age interval $(x, x + \Delta x)$ of the father and not as the fraction of first sons being born in that age interval. Therefore, the probability that the first son will be born to a father between age x and $(x + \Delta x)$ and will die before the father reaches age Y is

$$[1 - \ell(Y - x)]\, m_1(x, Y)\Delta x, \tag{2}$$

and not, as Parlange et al. inappropriately following Krishnamoorthy, write,

$$[1 - \ell(Y - x)]^{m_1(x,Y)\Delta x} \tag{3}$$

Integrating (2) over the reproductive age range of (α, Y) of the father, we get the probability that a father of age Y has his first son already dead as

$$\int_{\alpha}^{Y} [1 - \ell(Y - x)]\, m_1(x, Y)\, dx. \tag{4}$$

The approach of Parlange et al. would result in

$$\exp \int_{\alpha}^{Y} m_1(x, Y) \ln[1 - \ell(Y - x)]\, dx, \tag{5}$$

which can be verified by setting n equal to unity in the expression (5) of Parlange et al. Certainly this is incorrect.

Even if we assume the wrong formulation of the model by Parlange et al. to be correct, the second major drawback in their model is that there is no provision for the natural condition that the ith son cannot be born before the $(i - 1)$th son. Their model permits the birth of ith son before $(i - 1)$th. Krishnamoorthy escapes from this awkward situation by having the order of the birth determined by the time of occurrence of the event.

The basic problem in Krishnamoorthy's (1979) model according to Parlange et al. (1983:536) is that it is formulated "in terms of the average family in the population rather than the individual couple." Krishnamoorthy (1980) attempted to rectify this by developing a model using a nonhomogeneous Poisson process. This again is not completely free from stringent assumptions.

The correct solution to the problem can be obtained only when a model incorporates the joint probability density of ages of father at births of sons of different order. Such an attempt would be purely academic in nature, since joint distributions are hardly available for any population.

REFERENCES

Krishnamoorthy, S. 1979. Mortality level, desire for surviving son and rate of increase. Population Studies 33:565–571.

———. 1980. Effects of fertility and mortality on extinction of family and number of living children. Social Biology 27:62–69.

Parlange, J. Y., M. J. Guilfoyle, and R. E. Rickson. 1983. Mortality levels and family fertility goals. Demography 20:535–540.