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COMMENT ON MITRA'S GENERALIZATION

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In a recent article in this journal, Mitra (1983) extended work by Espenshade, et al. (1982) who examined the long-run implications for population size and composition if below-replacement fertility rates remain fixed in the face of a constant influx of immigrants. By differentiating the fundamental renewal equation for annual births, Mitra developed a unified framework for studying the existence of long-term equilibrium solutions when constant annual immigration is combined with fertility rates that are below, at, or above replacement.

When fertility is below replacement, Mitra confirmed the conclusion reached by Espenshade et al. showing that a stationary population is the long-run outcome. He showed that annual births, and therefore total population, grow linearly if fertility is at replacement. For fertility above replacement, Mitra found that population growth rates tend asymptotically to the intrinsic rate they would exhibit in the absence of immigration.

In the case where fertility is below replacement, it would be more accurate to say that Espenshade et al. have generalized Mitra's results. Mitra assumes that immediately upon their arrival, immigrant women adopt the fertility behavior of native born women. Espenshade et al. have shown that a stationary population will materialize in the long run even if immigrant women and successive generations of their descendants have fertility above replacement. All that is needed for a stationary population to result is that, at some point in the chain of immigrant descendants, one generation and all those following it must adopt below-replacement fertility. Moreover, in the case where fertility is at replacement and a constant number of immigrants is assumed to enter the population each year, Ansley Coale (1972) preceded Mitra in showing that population growth will be linear in equilibrium.

In the last part of his paper, Mitra studies a population with two homogeneous groups, one having fertility rates above replacement and one with fertility rates below replacement. He then examines "the mechanisms by which the growing population may adopt the reproductive norms of the declining population to result in the eventual stationarity of both groups" (113). It is not entirely clear how this section of Mitra's paper relates to his previous results on immigration. To link the two, it is useful to introduce migration explicitly into the analysis.

To do so, imagine a closed population of females divided into two separate geographic regions. Suppose that women with below-replacement fertility reside in the north and those with fertility above replacement live in the south. Let $m_H(a)$ and $m_L(a)$ be the age-specific fertility schedules of the high and low fertility women, respectively. Assume that both regions are characterized by the same force of mortality schedule, $\mu_d(a)$. Assume further that women from the south can migrate to the north (but not vice versa), and that they migrate at a rate which is constant through time, but not necessarily across age. Once in the north, these migrant females take on the fertility schedule $m_L(a)$. The rate at which women move from the high fertility area to the low fertility one is formally equivalent to Mitra's concept of the rate at which high fertility women adopt the reproductive norms of the other group.

Suppose we represent the rate of outmigration from the south by $\mu_m(a)$. Then, since the demographic effect of emigrating at age a is the same as that of dying at age a, the influence of emigration on population growth and structure can be analyzed in terms of changes in mortality. If we assume that dying and emigrating are independent events, for females in the south we can define the combined "death" rate, $\mu(a)$, as

$$\mu(a) = \mu_d(a) + \mu_m(a).$$

It follows that

$$p(a) = p_d(a) \cdot p_m(a),$$

where $p_d(a)$ is the probability of surviving from birth to age a, given the death rates implied by $\mu_d(a)$, $p_m(a)$ is the probability of not having emigrated prior to age a given $\mu_m(a)$, and $p(a)$ is the probability of both being alive at age a *and* living in the south.

The population in the southern region will eventually become stationary if rates of outmigration to the north are such that

$$\int_0^\beta p_d(a)p_m(a)m_H(a)da = 1,$$

since

$$\int_0^\beta e^{-ra}p(a)m(a)da = 1$$

is the general formula for determining the intrinsic growth rate, r. The proportionate age distribution of this stationary population is given by

$$\Pi(a) = b \cdot p_d(a) \cdot p_m(a),$$

where b, the crude birth rate in the stationary population, can be computed from

$$b = \left[\int_0^\omega p_d(a) \cdot p_m(a)da\right]^{-1},$$

where ω is the oldest age attained by the population. The relationship between this analysis and Mitra's is that the emigrant function $p_m(a)$ occupies the same role as $1 - c(a)$ does for Mitra, where $c(a)$ is defined as the cumulative proportion in the high-fertility group that has adopted the reproductive norms of the low-fertility group by age a.

Once a stationary population is established in the south, there will be a constant annual number of northbound migrants, whose age distribution is also fixed because $\mu_m(a)$ is fixed. Incidentally, Mitra's equation (34) establishes the fact that the annual number of women from the high fertility group who adopt the low fertility norms will be constant, but he fails to show that their age composition is also fixed—an important omission, since his earlier results apply only if both the annual number and the age distribution of immigrants are constant. Given the below replacement fertility of women in the north, we know from Espenshade et al. and from Mitra that a stationary population will eventually be installed there as well. Formulas for the size and age composition of the stationary population in the north are summarized in equations (2) and (9) in Espenshade et al.

In short, by explicitly linking the high and low fertility populations through migration, we are better able to relate Mitra's findings in the second half of his paper to previous research on immigration and stable population models, and to demonstrate that the age distribution of outmigrants from the high fertility area is asymptotically fixed.

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