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Letter to the Editor

Comment on "Rough Multisets and Information Multisystems"

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We show that some results introduced in Girish and John (2011) are incorrect. Moreover, a counterexample is given to confirm our claim. Furthermore, the correction form of the incorrect results in Girish and John (2011) is presented.

1. Introduction

In addition to Girish and John (2011) [1], many authors were recently interested in studying the extensions of results and properties of rough set to rough multiset [1–3]. There exist many of the applications on rough multisets in several fields such as the medicine field in [3]. Additionally, the concept of rough multisets and the basic definitions of relations in multiset context are introduced by Girish and John [4, 5]. Therefore, the notion of multisets (briefly, msets) was introduced by Yager [6], and Blizard [7, 8] and Jena et al. [9] have mentioned them as well.

2. Preliminaries

The aim of this section is to present the basic concepts and properties of msets. At the end of this section, rough msets and the definitions and notions of relations in msets are introduced.

Definition 1 (see [9]). An mset M drawn from the set X is represented by a count function C_M defined as $C_M\colon X \to N$, where N represents the set of nonnegative integers.

Here $C_M(x)$ is the number of occurrences of the element x in the mset M. The mset M is drawn from the set $X = \{x_1, x_2, x_3, \ldots, x_n\}$ and it is written as $M = \{m_1/x_1, m_2/x_2, m_3/x_3, \ldots, m_n/x_n\}$, where m_i is the number of occurrences of the element x_i, $i = 1, 2, 3, \ldots, n$, in the mset M.

Definition 2 (see [9]). A domain X is defined as a set of elements from which msets are constructed. The mset space $[X]^w$ is the set of all msets whose elements are in X such that no element in the mset occurs more than w times.

The mset space $[X]^\infty$ is the set of all msets over a domain X such that there is no limit on the number of occurrences of an element in an mset. If $X = \{x_1, x_2, \ldots, x_k\}$, then $[X]^w = \{\{m_1/x_1, m_2/x_2, \ldots, m_k/x_k\} : m_i \in \{0, 1, 2, \ldots, w\}, i = 1, 2, \ldots, k\}$.

Definition 3 (see [9]). Let M and N be two msets drawn from a set X. Then,

(1) $M = N$ if $C_M(x) = C_N(x)$ for all $x \in X$,

(2) $M \subseteq N$ if $C_M(x) \leq C_N(x)$ for all $x \in X$,

(3) $P = M \cup N$ if $C_P(x) = \max\{C_M(x), C_N(x)\}$ for all $x \in X$,

(4) $P = M \cap N$ if $C_P(x) = \min\{C_M(x), C_N(x)\}$ for all $x \in X$,

(5) $P = M \oplus N$ if $C_P(x) = \min\{C_M(x) + C_N(x), w\}$ for all $x \in X$,

(6) $P = M \ominus N$ if $C_P(x) = \max\{C_M(x) - C_N(x), 0\}$ for all $x \in X$, where $\oplus$ and $\ominus$ represent mset addition and mset subtraction, respectively.

Let M be an mset drawn from a set X. The support set of M denoted by M^* is a subset of X and $M^* = \{x \in X : C_M(x) > 0\}$; that is, M^* is an ordinary set.

Definition 4 (see [9]). Let M be an mset drawn from the set X. If $C_M(x) = 0$ for all $x \in X$, then M is called an empty mset and denoted by ϕ; that is, $\phi(x) = 0$ for all $x \in X$.

Definition 5 (see [9]). Let M be an mset drawn from the set X and $[X]^w$ be the mset space defined over X. Then, for any mset $M \in [X]^w$, the complement M^c of M in $[X]^w$ is an element of $[X]^w$ such that $C_{M^c}(x) = w - C_M(x)$ for every $x \in X$.

Definition 6 (see [4]). Let M_1 and M_2 be two msets drawn from a set X. Then, the Cartesian product of M_1 and M_2 is defined as $M_1 \times M_2 = \{(m/x, n/y)/mn : x \in^m M_1,\ y \in^n M_2\}$.

The Cartesian product of three or more nonempty msets can be defined by generalizing the definition of the Cartesian product of two msets. Thus, the Cartesian product $M_1 \times M_2 \times \cdots \times M_n$ of the nonempty msets $M_1, M_2, \ldots, M_n$ is the mset of all ordered n-tuples $(m_1, m_2, \ldots, m_n)$, where $m_i \in^{r_i} M_i$, $i = 1, 2, \ldots, n$, and $(m_1, m_2, \ldots, m_n) \in^p M_1 \times M_2 \times \cdots \times M_n$ with $p = \prod r_i$, where $r_i = C_{M_i}(m_i)$ and $i = 1, 2, \ldots, n$.

Definition 7 (see [4]). A submset R of $M \times M$ is said to be an mset relation on M if every member $(m/x, n/y)$ of R has a count (mn). Then, m/x related to n/y is denoted by $m/xRn/y$.

Definition 8 (see [4]). The domain and range of the mset relation R on M are defined as follows, respectively.

$\operatorname{Dom} R = \{x \in^r M : \exists y \in^s M$ such that $(r/x)R(s/y)\}$, where $C_{\operatorname{Dom} R}(x) = \sup\{C_1(x, y) : x \in^r M\}$.

$\operatorname{Ran} R = \{y \in^s M : \exists x \in^r M$ such that $(r/x)R(s/y)\}$, where $C_{\operatorname{Ran} R}(y) = \sup\{C_2(x, y) : y \in^s M\}$.

Definition 9 (see [1]). An m-equivalence class in R containing an element $x \in^m M$ is denoted by $[m/x]$. The pair (M, R) is called an mset approximation space. For any $N \subseteq M$, the lower mset approximation and upper mset approximation of N are defined, respectively, by

$$R_L(N) = \left\{x \in^m M : \left[\frac{m}{x}\right] \subseteq N\right\}, \tag{1}$$

$$R_U(N) = \left\{x \in^m M : \left[\frac{m}{x}\right] \cap N \neq \phi\right\}. \tag{2}$$

The pair $(R_L(N), R_U(N))$ is referred to as the rough mset of N.

3. Counterexample

In this section, we point out where the errors occur in [1] and then give counterexamples to confirm our claim. Finally, the correction form of these errors is presented.

In [[1], Theorem 4.5, p. 12], the authors introduced the fact that, for any submsets M_1 and M_2 of M,

(1) $R_L[(M_1 \ominus M_2)^c] = R_L({M_1}^c) \oplus R_L(M_2)$,
(2) $R_L[(M_1 \oplus M_2)^c] = R_L({M_1}^c) \ominus R_L(M_2)$.

The following example shows that

(1) $R_L[(M_1 \ominus M_2)^c] \nsubseteq R_L({M_1}^c) \oplus R_L(M_2)$,
(2) $R_L[(M_1 \oplus M_2)^c] \nsupseteq R_L({M_1}^c) \ominus R_L(M_2)$.

Example 1. Let $M = \{3/x, 2/y, 4/z, 8/r\}$ and $R = \{(3/x, 3/x)/9, (2/y, 2/y)/4, (4/z, 4/z)/16, (8/r, 8/r)/64, (3/x, 2/y)/6, (2/y, 3/x)/6, (3/x, 4/z)/12, (4/z, 3/x)/12, (2/y, 4/z)/8, (4/z, 2/y)/8\}$. Then, $[3/x] = [2/y] = [4/z] = \{3/x, 2/y, 4/z\}$ and $[8/r] = \{8/r\}$. If $M_1, M_2 \subseteq M$ such that

(1) $M_1 = \{3/x, 4/z, 8/r\}$ and $M_2 = \{3/x, 4/z\}$, then $R_L[(M_1 \ominus M_2)^c] = \{3/x, 2/y, 4/z\}$, $R_L(M_1^c) = \phi$, and $R_L(M_2) = \phi$. Thus, $R_L({M_1}^c) \oplus R_L(M_2) = \phi$. Hence, $R_L[(M_1 \ominus M_2)^c] \nsubseteq R_L({M_1}^c) \oplus R_L(M_2)$,
(2) $M_1 = \phi$ and $M_2 = \{2/x, 2/y, 8/r\}$, then $R_L[(M_1 \oplus M_2)^c] = \phi$, $R_L(M_1^c) = M$, and $R_L(M_2) = \{8/r\}$. Thus, $R_L({M_1}^c) \ominus R_L(M_2) = \{3/x, 2/y, 4/z\}$. Hence, $R_L[(M_1 \oplus M_2)^c] \nsupseteq R_L({M_1}^c) \ominus R_L(M_2)$.

The following theorem is the correction form of [Theorem 4.5, p. 12] in [1].

Theorem 2. *For any submsets M_1 and M_2 of M,*

(1) $R_L[(M_1 \ominus M_2)^c] \supseteq R_L({M_1}^c) \oplus R_L(M_2)$,
(2) $R_L[(M_1 \oplus M_2)^c] \subseteq R_L({M_1}^c) \ominus R_L(M_2)$.

Proof.

(1)

$$\begin{aligned} R_L\left[(M_1 \ominus M_2)^c\right] &= \left\{x \in^m M : \left[\frac{m}{x}\right] \subseteq (M_1 \ominus M_2)^c\right\} \\ &= \left\{x \in^m M : \left[\frac{m}{x}\right] \subseteq M_1^c \oplus M_2\right\} \\ &\supseteq \left\{x \in^m M : \left[\frac{m}{x}\right] \subseteq M_1^c\right\} \\ &\quad \oplus \left\{x \in^m M : \left[\frac{m}{x}\right] \subseteq M_2\right\} \\ &= R_L\left({M_1}^c\right) \oplus R_L\left(M_2\right). \end{aligned} \tag{3}$$

(2)

$$\begin{aligned} R_L\left[(M_1 \oplus M_2)^c\right] &= \left\{x \in^m M : \left[\frac{m}{x}\right] \subseteq (M_1 \oplus M_2)^c\right\} \\ &= \left\{x \in^m M : \left[\frac{m}{x}\right] \subseteq M_1^c \ominus M_2\right\} \\ &\subseteq \left\{x \in^m M : \left[\frac{m}{x}\right] \subseteq M_1^c\right\} \\ &\quad \ominus \left\{x \in^m M : \left[\frac{m}{x}\right] \subseteq M_2\right\} \\ &= R_L\left({M_1}^c\right) \ominus R_L\left(M_2\right). \end{aligned} \tag{4}$$

□

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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