Comment on "Temperature dependence of the Casimir effect"

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Recently, Brevik *et al.* [Phys. Rev. E, **71**, 056101 (2005)] adduced arguments against the traditional approach to the thermal Casimir force between real metals and in favor of one of the alternative approaches. The latter assume zero contribution from the transverse electric mode at zero frequency in qualitative disagreement with unity as given by the thermal quantum field theory for ideal metals. Those authors claim that their approach is consistent with experiments as well as with thermodynamics. We demonstrate that these conclusions are incorrect. We show specifically that their results are contradicted by four recent experiments and also violate the third law of thermodynamics (the Nernst heat theorem).

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The paper [1] is devoted to an important problem which has been actively discussed in the literature starting in 2000, and which has created much controversy among various research groups. The authors of Ref. [1] try to justify an alternative approach to the theoretical description of the thermal Casimir force, which was first suggested in Ref. [2] and later supported and further developed in their own Refs. [3–5]. (Henceforth, this approach will be called BSBAHM after the principal authors Boström, Sernelius, Brevik, Aarseth, Høye, and Milton.) However, in Ref. [1] no attention is paid to the serious shortcomings of this approach, and the traditional approaches in their discussion are misrepresented.

The authors of Ref. [1] claim that their calculations of the thermal Casimir force between a Cu plate and a Au sphere are (a) consistent with current experiments (their wording is "not inconsistent"), and (b) consistent with the third law of thermodynamics. Below we demonstrate that both of these conclusions are incorrect.

(a) The first main claim contained in Ref. [1] is that the approach by BSBAHM is consistent with current experiments (Sec. III). In Ref. [6] the opposite conclusion was drawn, namely that this approach is experimentally excluded. Note that Ref. [6] contains the results of two experiments performed using a micromechanical torsional oscillator: a static measurement of the Casimir force between a sphere and a plate, and a dynamic measurement of the effective Casimir pressure between two plane parallel plates. The static measurement was less precise than the dynamic measurement over a wide separation range. The conclusion that the BSBAHM approach is inconsistent with the experimental data in the separation region from $z=260$ nm to $z=700$ nm was made in Ref. [6] on the basis of the dynamic measurement. This conclusion is illustrated in Fig. 12 of Ref. [6] where the differences between the theoretical $P_C^{th,1}$ (calculated in the framework of BSBAHM approach) and experimental P_C^{exp} values of the Casimir pressure are plotted as a function of separation for one set of measurements containing 235 experimental points. For comparison, in Fig. 11 of Ref. [6] the same differences are plotted for the theoretical Casimir pressures calculated in the framework of the impedance approach of Ref. [7]. (Recall that this approach, as well as the approach using the dielectric function of the plasma model, are called "traditional" since they yield results in qualitative agreement with the case of ideal metals; the thermal corrections predicted by the alternative approach of Refs. [1–5] are many times greater at short separations.) From these two figures it becomes apparent that the BSBAHM approach is excluded by experiment, whereas the impedance approach is consistent with experimental data.

Surprisingly, in order to demonstrate the consistency of their approach with current experiments, the authors of Ref. [1] discuss at length only the experiments of Refs. [8,9] and the static measurement of Refs. [6,10]. These are at present not the most accurate, and were not used in the literature to exclude their approach. As to the dynamic measurement of Ref. [6] (which contradicts the BSBAHM approach), only a brief mention of this experiment is made at the end of Sec. III, even though this is the most precise current measurement. Reference [1] claims without proof that there are systematic theoretical and experimental uncertainties in Ref. [6] connected, in particular, with a systematic shift of position as discussed earlier in Refs. [5,11]. It is easy to verify, however, that the reasoning of Refs. [5,11] is incapable of avoiding the conclusion of Ref. [6] that the BSBAHM approach is excluded experimentally. According to Refs. [5,11], even a small experimental error Δz in separation distances between two parallel plates ($\Delta z=1$ nm in Ref. [6]) leads to an error in the theoretical Casimir pressure given by $\Delta P_C^{th,1} \approx -P_C^{th,1}(4\Delta z/z)$. At short separations this error may be rather large. At separations under consideration in Ref. [6] it

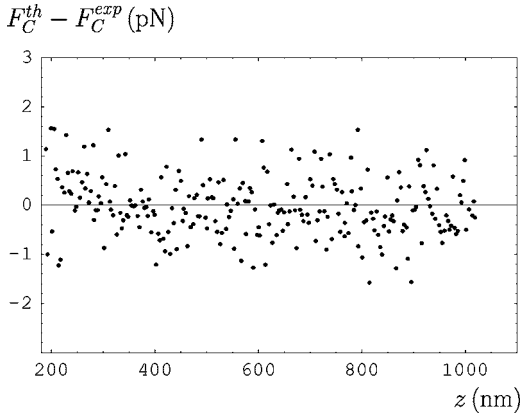

FIG. 1. Differences of theoretical and experimental Casimir forces between the sphere and plate vs separation in the static experiment calculated and measured in Ref. [6].

is, however, much less than the discrepancies between the BSBAHM theory and experiment. Thus, at the shortest separation $z=260$ nm, $\Delta P_C^{th,1}=3.7$ mPa, which compares with the 5.5 mPa mean deviation of the BSBAHM prediction from experiment as shown in Fig. 12 of Ref. [6]. At separations of $z=300$, 400, 500, and 600 nm the above error is equal to 1.5, 0.36, 0.12, and 0.05 mPa, respectively, which should be compared with much larger mean deviations between the BSBAHM approach and experiment at these separations (5, 2, 0.8, and 0.4 mPa, respectively). This demonstrates that the uncertainties in separations discussed in Refs. [5,11] do not affect the conclusion of Ref. [6] that the BSBAHM approach is excluded experimentally.

Note that even the comparison of the static experiment in Sec. III of Ref. [1] with the BSBAHM approach is incorrect. Only one point at a separation $z=200$ nm in Fig. 3 of Ref. [10] is considered. Instead of using the original measured force values, Ref. [1] uses a maximum (not a mean) difference of -1 pN between the theoretical (as calculated in Ref. [10] at zero temperature) and experimental values of the force. The authors of Ref. [1] explain this difference by the existence of a thermal correction equal to 1 pN. They compare this 1 pN with their predicted thermal correction of 2.56 pN at $z=200$ nm and conclude that the result "is encouraging." In fact, however, they have a deviation of 1.56 pN between their theory and the static measurement instead of a -1 pN deviation between the traditional theory of Ref. [10] and the same measurement. Reference [1], however, does not inform the reader that in a later publication [6] the preliminary theoretical result of Ref. [10] was recalculated using a more precise roughness correction (this is clearly explained in the left column on p. 2 of Ref. [6]). It was demonstrated that the static measurement of the Casimir force F_C^{exp} is in fact in agreement with the theoretical values F_C^{th} given by traditional theory with a more precise roughness correction (see Fig. 1). Thus, if one removes this misunderstanding, Ref. [1] must reconcile the zero mean deviation between the traditional theory and static experiment (see Fig. 1) with an extra 2.56 pN thermal correction predicted by the BSBAHM alternative approach. This leads to the evident failure of their approach.

Moreover, the BSBAHM theoretical approach disagrees significantly [12,13] with the first modern measurement of the Casimir force between Au surfaces of a plate and a spherical lens by means of a torsion pendulum [14]. In Ref. [14] the experimental data were found to be consistent with the theoretical Casimir force between ideal metals. A net deviation between the Casimir forces at a temperature $T=300$ K and at a separation $z=1$ μm, computed for ideal metals and using the BSBAHM approach, is about 25% of the Casimir force between ideal metals (recall that for ideal metals at $z=1$ μm, $T=300$ K the thermal correction is equal to only 1.2% of the zero-temperature force). Of this deviation, 19% is due to the large thermal correction predicted by BSBAHM. In spite of the fact that the experimental uncertainty in Ref. [14] at 1 μm is less than 10%, the effect predicted by the BSBAHM approach was not observed. No mention of this important experiment is made in Ref. [1].

Quite recently the dynamic experiment of Ref. [6] was repeated (see Ref. [15]) with many important improvements, including a significant suppression of the surface roughness on the interacting surfaces, and a decrease by a factor of 1.7 (down to $\Delta z=0.6$ nm) of the experimental error in the measurement of the absolute separations between the zero roughness levels. An improvement in detection sensitivity, together with a reduction of the coupling between the micromachined oscillator and the environment, yielded measurements at smaller separations between the test bodies (160 nm instead of 260 nm).

From the new results in Ref. [15] the conclusion was drawn that the BSBAHM approach to the thermal Casimir force is excluded experimentally in the separation region from 170 nm to 700 nm at 95% confidence. In the separation region from 300 nm to 500 nm the BSBAHM approach is excluded experimentally at even higher confidence of 99% [15]. Here we illustrate these conclusions in Fig. 2(a) where the differences of the theoretical (calculated in the BSBAHM approach) and experimental Casimir pressures are plotted versus separation for 14 sets of measurements containing 4066 experimental points. By contrast, in Fig. 2(b) the same differences are plotted in the case that the theoretical Casimir pressures are calculated using the impedance approach. In both figures the solid lines represent the 95% confidence interval for the differences between theoretical and experimental Casimir pressures as a function of separation. It should be particularly emphasized that this confidence interval takes into account all experimental and theoretical errors, including in full measure the previously discussed error in the Casimir pressures due to experimental errors in separation distances as suggested in Refs. [5,11]. The comparison of Figs. 2(a) and 2(b) clearly demonstrates that the BSBAHM approach is excluded by the improved dynamic experiment measuring the Casimir pressure, whereas the impedance approach is in excellent agreement with experiment. The traditional approach using the dielectric function of the plasma model is also consistent with the data (see Ref. [15] for more details).

Thus, the authors' [1] approach is in contradiction not only with a dynamic experiment by means of a micromechanical oscillator [6] and the torsion pendulum experiment [14] (which they leave out of their discussion), but also with

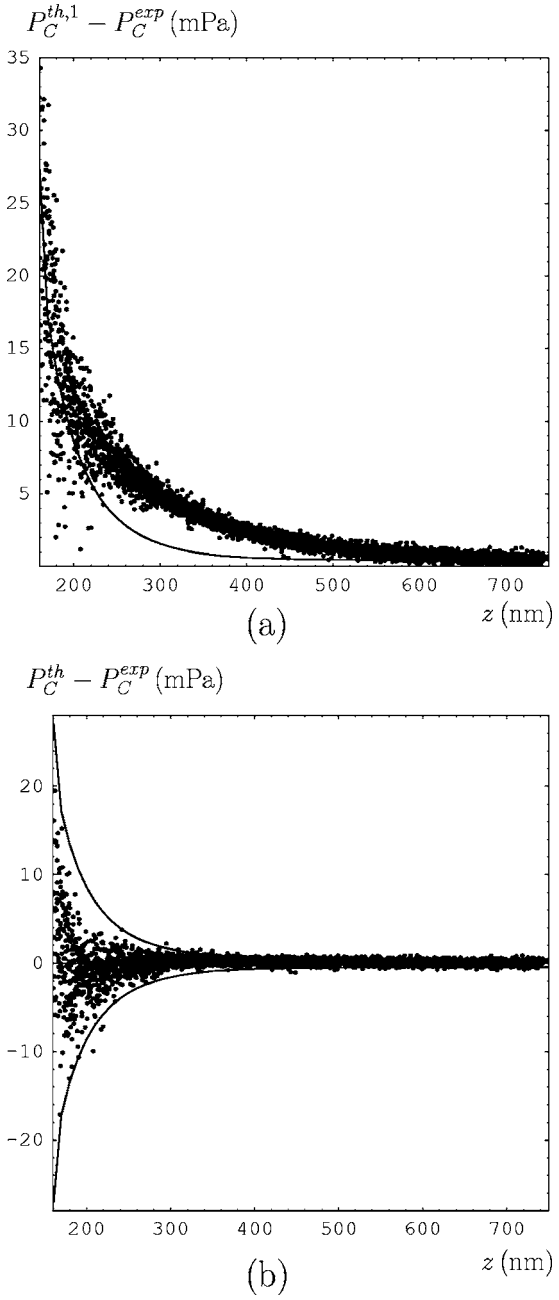

FIG. 2. Differences of theoretical and experimental parallel plate Casimir pressures vs separation obtained from the improved dynamic measurement of Ref. [15]. The theoretical values for $P_C^{th,1}$ are calculated as in Refs. [1–5] (a), and for P_C^{th} as in Refs. [7,15] (b). Solid lines represent the 95% confidence interval.

a static experiment [6] and an improved dynamic experiment [15] which measure the Casimir force and pressure, respectively. It should be emphasized that this conclusion cannot be refuted by introducing an unaccounted systematic error in the measurement of the surface separation which might be present in the experiments of Refs. [6,15] in addition to the ones discussed above. The reason is that the influence of such an error (if it existed) decreases with an increase in separation whereas the contribution of the thermal correction, as predicted in the BSBAHM approach, increases with separation at moderate separations. Bearing in mind that the

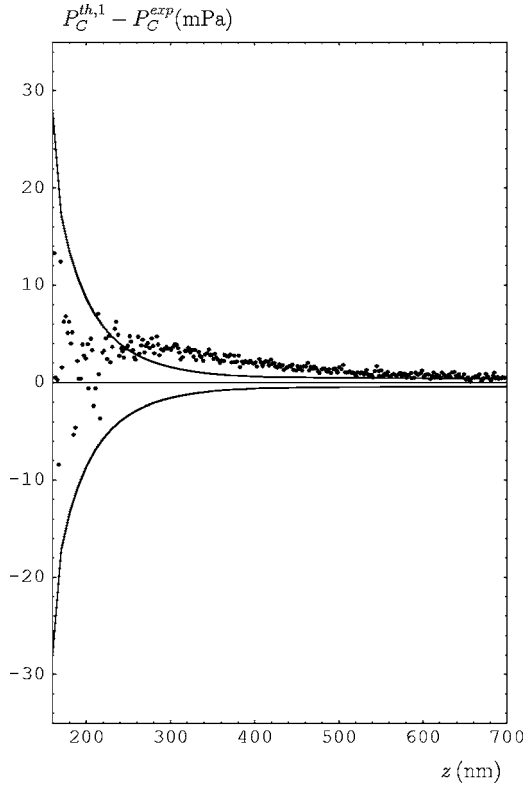

FIG. 3. Differences of theoretical and experimental parallel plate Casimir pressures vs separation obtained from the improved dynamic measurement of Ref. [15] after all separation distances have been decreased by 1 nm to take into account the hypothetical systematic error suggested by Ref. [16]. The theoretical values for $P_C^{th,1}$ are calculated as in Refs. [1–5]. Solid lines represent the 95% confidence interval.

BSBAHM approach significantly disagrees with experiment in a wide separation range for several different experiments, it is easy to check that no unaccounted fixed systematic error is capable of bringing this approach into agreement with data within the whole range of measurements. As an example, in Fig. 3 we present the pressure differences $(P_C^{th,1} - P_C^{exp})$ versus separation for one typical set of measurements where all separations are decreased by 1 nm as suggested recently in Ref. [16]. From Fig. 3 it is clearly seen that such hypothetical systematic error is incapable of bringing the data into agreement with the BSBAHM approach within a wide separation range from 240 nm to 700 nm. Note also that the first version of preprint [16] obtains slightly larger magnitudes of the Casimir pressure than those computed in Ref. [15] in the framework of the BSBAHM approach. (The greatest difference is at $z=160$ nm, $T=300$ K where, according to Ref. [16], $P_C^{th,1}=-1132$ mPa against -1125.5 mPa in Ref. [15].) The reason for this difference is that Refs. [1,16] used data for $\varepsilon(i\xi)$ from the work of Ref. [17]. The computations in Ref. [17] contain an error in a conversion coefficient from eV to rad/s (on p. 313, left column the value of 1.537×10^{15} is used instead of the correct value 1.51927×10^{15} [15]). In addition, the computations of Ref. [17] utilize [18] slightly different values of the Au plasma frequency and relaxation parameter $[\omega_p=9.03$ eV, $\nu(T=300$ K)

$=0.0345$ eV] than were cited in Refs. [1,16], and actually used in Ref. [15] [$\omega_p=9$ eV, $\nu(T=300\ \mathrm{K})=0.035$ eV]. When both these changes are taken into account in the computations of Refs. [1,16], the result $P_C^{th,1}=-1125.5$ mPa of Ref. [15] is recovered. (We note that the abovementioned error does not invalidate the computations of the reduction factors in Ref. [17], which were performed up to only two significant figures.)

(b) The second main claim contained in Ref. [1] is that the approach by BSBAHM is consistent with thermodynamics. The calculations of Ref. [1] are based on the Lifshitz formula for the free energy of a fluctuating field [19]. The dielectric permittivites of Au and Cu at nonzero imaginary Matsubara frequencies are taken from Ref. [17]. The contribution of the zero-frequency term is obtained by the substitution of the Drude dielectric function along the imaginary frequency axis,

$$\varepsilon(i\xi)=1+\frac{\omega_p^2}{\xi[\xi+\nu(T)]}, \tag{1}$$

into the Lifshitz formula. This substitution leads to an absence of the zero-frequency contribution of the transverse electric mode, and, in application to metals, results in a serious inconsistency with thermodymanics. The point is that for perfect lattices with no defects or impurities but with a nonzero relaxation $\nu(T)$ and finite conductivity for nonzero T, the Bloch-Grüneisen law leads to $\nu(0)=0$. (This important property for perfect lattices remains even when the effects of electron-electron collisions are included.) As proven analytically in Ref. [20], for such perfect lattices the approach by BSBAHM leads to a nonzero entropy of a fluctuating field at zero temperature given by

$$S(z,0)=\frac{k_B}{16\pi z^2}\int_0^\infty y\,dy\,\ln\left[1-\left(\frac{y-\sqrt{\frac{4\omega_p^2z^2}{c^2}+y^2}}{y+\sqrt{\frac{4\omega_p^2z^2}{c^2}+y^2}}\right)^2 e^{-y}\right]<0, \tag{2}$$

which depends on the parameter of the system under consideration, i.e., on the separation distance z (k_B is the Boltzmann constant).

In an attempt to avoid this serious problem, the authors of Ref. [1] use a nonzero value of the Drude relaxation parameter at zero temperature arising from the presence of impurities. They can then obtain in Secs. II A and IV a zero value of entropy at zero temperature (a result obtained first by Boström and Sernelius [21,22]). This, however, does not solve the inconsistency of the BSBAHM approach with thermodynamics, as is claimed in Ref. [1], because it is still violated for a perfect lattice having finite conductivity and nonzero relaxation at any $T>0$. The unstated assumption of the authors is that perfect crystals with no defects or impurities do not exist and, therefore, that the laws of thermodynamics need not apply to them. This assumption is unphysical. Nernst and Planck formulated their famous theorem for the case of a perfect lattice which is truly an equilibrium system. Later this theorem was proven in the framework of quantum statistical physics for any system with a nondegenerate dynamical state of lowest energy (see, for instance, Refs. [23,24]). Consequently, it is valid for both perfect lattices and lattices with impurities. The violation of the Nernst heat theorem for a perfect lattice would lead to the failure of the theory of electron-phonon interactions, and to the eventual abandonment of much of condensed matter physics, statistical physics, and thermodynamics. For this reason the approach advocated by the authors is, in fact, in violation of thermodynamics.

In addition to the above serious problems, Ref. [1] contains several misleading statements, including the following:

(1) The authors discuss the so-called modified ideal metal model (MIM), but do not mention that it violates thermodynamics. Namely, for the MIM model the free energy in the case of two parallel plates [Eq. (3.2) of Ref. [1]] can be identically rearranged to the form

$$\mathcal{F}^{\mathrm{MIM}}(z,T)=\mathcal{F}^{\mathrm{IM}}(z,T)+\zeta(3)k_BT/(16\pi z^2), \tag{3}$$

where $\mathcal{F}^{IM}$ is the Casimir free energy for the usual ideal metal [25], and $\zeta(z)$ is the Riemann zeta function. By differentiating both sides of Eq. (3) with respect to T one obtains

$$S^{\mathrm{MIM}}(z,T)=S^{\mathrm{IM}}(z,T)-\zeta(3)k_B/(16\pi z^2), \tag{4}$$

where S^{MIM} and S^{IM} are the entropies for the MIM and the ideal metal (IM) models, respectively. Taking into account that for the usual ideal metal $S^{\mathrm{IM}}\to 0$ when $T\to 0$ [26], we come to the conclusion

$$S^{\mathrm{MIM}}(a,0)=-\zeta(3)k_B/(16\pi z^2)<0, \tag{5}$$

i.e., the MIM model violates the Nernst heat theorem. Remarkably the result for real metals obtained in Ref. [1] coincides with that for MIM (which violates thermodynamics) at large separations and does not coincide with the classical limit based on Kirchhoff's law [27].

(2) In Sec. IV of Ref. [1] it is claimed that "a transverse electric zero mode… should not be present according to Maxwell's equations of electromagnetism." This is not correct. Maxwell's equations alone do not lead to a contribution of the transverse electric mode at zero frequency, unless they are supplemented by an adequate characterization of the material boundaries. More importantly, the characterization by means of the Drude dielectric function, used in Ref. [1], is inadequate to describe virtual photons.

(3) In Sec. VI the authors repeat their argument of Ref. [4] that the exact impedances, which depend on transverse momentum, lead to a zero contribution of the transverse electric mode to the Casimir force, as does the Drude dielectric function. Ref. [1] claims that Refs. [7,20], where the nonzero contribution to this mode was obtained, completely disregard the transverse-momentum dependence. This, however, is incorrect. In Ref. [20] the dependence of the impedance on a transverse momentum is considered in detail. As was demonstrated in Ref. [20], this dependence disappears in the limit of zero frequency if the dispersion equation for the determination of photon proper frequencies is taken into account in the boundary conditions, and, as a consequence, the reflection properties of virtual photons on a classical boundary coincide with those of real photons. The transverse-

momentum dependence in the impedance is, however, preserved at nonzero frequencies and in the reflection coefficients at all frequencies. Reference [1] fails to account for this important property, thus resulting in a violation of the Nernst heat theorem. Reference [16] recalls that the Lifshitz formula contains not real frequencies but the imaginary Matsubara frequencies for which there is no mass-shell equation. However, Ref. [16] disregards the fact that the impedance boundary conditions are prior to the Lifshitz formula. They are imposed on any real or virtual electromagnetic wave of real frequency. The next step is the derivation of the dispersion equation and only after the application of the argument principle do we arrive at the Lifshitz formula with the imaginary Matsubara frequencies [7]. If we admit that the reflection properties of virtual photons on a real boundary coincide with those of real photons, we must impose the dispersion equation on the impedance function in the boundary condition formulated in terms of real frequencies. Therefore, the speculations in Ref. [16] that for imaginary frequencies the angle of incidence becomes meaningless are misleading.

(4) In the end of Sec. VI of Ref. [1], the claim of Ref. [5] is repeated against the extrapolation [6,7,20] of the impedance function from the infrared region to zero frequency. No mention is made of Ref. [28] containing the justification for this extrapolation, and demonstrating that the treatment of the zero-frequency mode as in Ref. [5] results once again in a violation of the Nernst heat theorem. Preprint [16] claims that according to Ref. [15] real data cannot be used in the description. Reference [15], however, uses exactly the same real data and in the same frequency range as Ref. [1] does. In fact, the disagreement between Refs. [1,15] is only in the method of extrapolating real data to zero frequency.

To conclude, the theoretical approach of Ref. [1] is excluded by four already performed experiments, namely, by the measurements of the Casimir force using a torsion pendulum [12–14], a micromechanical oscillator [6], and by using two determinations of the Casimir pressure in a micromechanical system [6,15]. The main results of Ref. [1] are also in contradiction with fundamental physical principles such as the laws of thermodynamics.

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