Sādhanā Vol. 27, Part 3, June 2002, pp. 405–407. © Printed in India

Comments on "Surface waves in fibre-reinforced anisotropic elastic media" by Sengupta and Nath [*Sādhanā* 26: 363–370 (2001)]

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MS received 1 March 2002

Abstract. In the paper under discussion, the problem of surface waves in fibre-reinforced anisotropic elastic media has been studied. The authors express the plane strain displacement components in terms of two scalar potentials to decouple the plane motion into P and SV waves. In the present note, we show that, for wave propagation in fibre-reinforced anisotropic media, this decoupling cannot be achieved by the introduction of the displacement potentials. In fact, the expressions for the displacement potentials used by the authors do not satisfy one of the equations of motion. Consequently, most of the equations and results of the subject paper are either irrelevant or incorrect.

Keywords. Decoupling; displacement potentials; fibre-reinforced media; surface waves; transversely isotropic media.

1. Introduction

In a recent paper, Sengupta & Nath (2001), henceforth referred to simply as SN, studied the problem of surface waves in fibre-reinforced elastic media. The reinforcement introduces anisotropy in the medium which becomes transversely isotropic. For wave propagation in an isotropic homogeneous medium, the introduction of displacement potentials leads to the decoupling of P, SV and SH motions. This decoupling cannot be achieved for wave propagation in transversely isotropic media (see, e.g., Rahman & Ahmad 1996). However, SN attempt decoupling by expressing the displacements in terms of two scalar functions. On close examination it is found that their expressions for the potentials do not satisfy one of the equations of motion. Consequently, most of the results and conclusions of SN are unacceptable.

2. Discussion

Throughout this note, the notation used by SN is preserved. As can be verified from (1) of SN, the expressions for τ_{12} and τ_{13} given in (2) of SN are incorrect. The correct expressions are (Mal & Singh 1991, p. 150)

$$\tau_{12} = 2\mu_L e_{12}, \tau_{13} = 2\mu_L e_{13}. \tag{1}$$

The expression for u_2 given in (9) of SN is also incorrect. The correct expression is (Mal & Singh 1991, p. 293)

$$u_1 = \frac{\partial \phi}{\partial X_1} - \frac{\partial \psi}{\partial X_2}, u_2 = \frac{\partial \phi}{\partial X_2} + \frac{\partial \psi}{\partial X_1}. \tag{2}$$

Using the representation of u_1 and u_2 given in (2) above, (6) and (7) of SN become

$$\begin{aligned} &\frac{\partial}{\partial X_1}\left[(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\frac{\partial^2 \phi}{\partial X_1^2} + (\alpha + \lambda + 2\mu_L)\frac{\partial^2 \phi}{\partial X_2^2} - \rho\frac{\partial^2 \phi}{\partial t^2}\right] \\ &\quad - \frac{\partial}{\partial X_2}\left[(\alpha + 3\mu_L - 2\mu_T + \beta)\frac{\partial^2 \psi}{\partial X_1^2} + \mu_L\frac{\partial^2 \psi}{\partial X_2^2} - \rho\frac{\partial^2 \psi}{\partial t^2}\right] = 0, \end{aligned} \tag{3}$$

$$\begin{aligned} &\frac{\partial}{\partial X_2}\left[(\alpha + \lambda + 2\mu_L)\frac{\partial^2 \phi}{\partial X_1^2} + (\lambda + 2\mu_T)\frac{\partial^2 \phi}{\partial X_2^2} - \rho\frac{\partial^2 \phi}{\partial t^2}\right] \\ &\quad + \frac{\partial}{\partial X_1}\left[\mu_L\frac{\partial^2 \psi}{\partial X_1^2} + (2\mu_T - \mu_L - \alpha)\frac{\partial^2 \psi}{\partial X_2^2} - \rho\frac{\partial^2 \psi}{\partial t^2}\right] = 0. \end{aligned} \tag{4}$$

For an isotropic medium without reinforcement,

$$\alpha = \beta = 0, \quad \mu_L = \mu_T = \mu, \tag{5}$$

and, therefore, (3) and (4) reduce to

$$\frac{\partial}{\partial X_1}\left[(\lambda + 2\mu)\nabla^2\phi - \rho\frac{\partial^2 \phi}{\partial t^2}\right] - \frac{\partial}{\partial X_2}\left[\mu\nabla^2\psi - \rho\frac{\partial^2 \psi}{\partial t^2}\right] = 0, \tag{6}$$

$$\frac{\partial}{\partial X_2}\left[(\lambda + 2\mu)\nabla^2\phi - \rho\frac{\partial^2 \phi}{\partial t^2}\right] + \frac{\partial}{\partial X_1}\left[\mu\nabla^2\psi - \rho\frac{\partial^2 \psi}{\partial t^2}\right] = 0. \tag{7}$$

Equations (6) and (7) are identically and simultaneously satisfied if

$$(\lambda + 2\mu)\nabla^2\phi = \rho\frac{\partial^2 \phi}{\partial t^2}, \tag{8}$$

$$\mu\nabla^2\psi = \rho\frac{\partial^2 \psi}{\partial t^2}. \tag{9}$$

However, no such conclusion can be drawn about (3) and (4) relevant to fibre-reinforced anisotropic media. While (3) leads to (10) and (11) of SN, (4) is satisfied if

$$(\alpha + \lambda + 2\mu_L)\frac{\partial^2 \phi}{\partial X_1^2} + (\lambda + 2\mu_T)\frac{\partial^2 \phi}{\partial X_2^2} = \rho\frac{\partial^2 \phi}{\partial t^2}, \tag{10}$$

$$\mu_L\frac{\partial^2 \psi}{\partial X_1^2} + (2\mu_T - \mu_L - \alpha)\frac{\partial^2 \psi}{\partial X_2^2} = \rho\frac{\partial^2 \psi}{\partial t^2}. \tag{11}$$

The method of potentials fails in the present context because (10) is not consistent with (10) of SN and (11) is not consistent with (11) of SN. The assertion of SN that the potentials ϕ and ψ must satisfy (10) and (11) of SN is invalid. In fact, the expresstions for ϕ and ψ given in (15) of SN do not satisfy the equation of motion (7) of SN. Therefore, the subsequent treatment of SN regarding Rayleigh and Stoneley waves is meaningless.

3. Conclusion

The method of potentials is not suitable for studying wave propagation in fibre-reinforced anisotropic elastic media. SN apply this method incorrectly, and, therefore, their results regarding Rayleigh and Stoneley waves in fibre-reinforced elastic media are incorrect.

The author thanks the Council of Scientific and Industrial Research for financial support through its Emeritus Scientist Scheme.

References

Mal A, Singh S J 1991 *Deformation of elastic solids* (Englewood Cliffs, NJ: Prentice-Hall)
Rahman A, Ahmad F 1998 Representation of the displacement in terms of scalar functions for use in transversely isotropic materials. *J. Acoust. Soc. Am.* 104: 3675–3676
Sengupta P R, Nath S 2001 Surface waves in fibre-reinforced anisotropic elastic media. *Sādhanā* 26: 363–370