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COMMENTS ON THE 6-INDEX PHOTON IN D = 11 SUPERGRAVITY AND THE GAUGING OF FREE DIFFERENTIAL ALGEBRAS

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ABSTRACT

The structural theorem of D. Sullivan on the decomposition of a free differential algebra into a contractible and minimal one is utilized to give an intrinsic definition of the concepts of curvatures and potentials (= connections) in gauge theories including antisymmetric tensors. Applying this idea to D = 11 supergravity I clarify the role of the 6-index photon showing that all D = 11 field equations are a consequence of the principle of rheonomy inserted into the complete differential algebra encompassing both the 3- and 6-forms. Moreover the 6-index photon is dual to the 3-index one as a consequence of the algebra. This is in close analogy with the relation among the axion and the dilaton in conformal supergravity. Finally the action of the D = 11 theory has a simpler formulation in terms of the curvatures of the full differential algebra.

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This paper does not claim to present any new result in D = 11 supergravity [1] or any new D = 11 theory. My purpose is only to clarify the formal structure of the standard D = 11 theory in connection with the concept of free differential algebra and the role played by the 6-index photon which so far has eluded our efforts to fit it into the theory [2,3]. As I will show the relation among the 3-index photon $A_{\mu\nu\rho}$ and the 6-index one $B_{\lambda\mu\nu\rho\sigma\tau}$ in eleven dimensional Poincaré supergravity is formally analogue to the relation among the axial photon A_μ and the dilaton D_μ in four-dimensional conformal supergravity [4,5]. In both cases only one of the two eligible fields appears in the second-order Lagrangian and propagates ($A_{\mu\nu\rho}$ and A_μ respectively) yet the other field ($B_{\lambda\mu\nu\rho\sigma\tau}$ and D_μ respectively) sits in the algebra, gauges a symmetry of the action and its curl is dual to the curl of the physical field. Moreover in the same way as the Lagrangian of the Weyl multiplet (e^a_μ, ψ_μ, A_μ) corresponds to the gauging of the full super conformal group, the Lagrangian of the D = 11 Poincaré multiplet (V^a_μ, ψ_μ, $A_{\mu\nu\rho}$) can be thought as the gauging of the full free differential algebra [3] possessing also the 6-form B. The concept of free differential algebra is the main motivation of this letter.

It was introduced and developed in the mathematical literature by D. Sullivan [6], who showed that it is the obvious generalization of the Lie algebra concept, but it remained unknown to physicists for some time. Totally unaware of Sullivan's work, R. D'Auria and myself reinvented the same concept under the name of Cartan integrable systems [3] in our effort of geometrizing D = 11 supergravity. Later, once the identification of our Cartan integrable systems with Sullivan's free differential algebras was accomplished, few expositions, for the physicists' benefit, of the two main structural theorems for these algebras were given by P. van Nieuwenhuizen [7] and P. van Nieuwenhuizen and R. Stora [8]. Although I have been aware of Sullivan's theorems for two years, only recently have I realized their relevance for the physical concept of curvature and hence for the gauging of a rigid algebra.

This is the main point which I would like to bring to the reader's attention. The example of D = 11 supergravity I will discuss should be regarded as instrumental with respect to the new viewpoint I want to propose. Recalling Eq. (2.9) of [3], a free differential algebra (= Cartan Integrable system CIS) is described by a generalized Maurer-Cartan equation of the following type:

$$d\Theta^{A(p)} + \sum_{n=1}^{N} \frac{1}{n} C^{A(p)}_{B_1(p_1)\ldots B_n(p_n)} \Theta^{B_1(p_1)} \wedge \ldots \wedge \Theta^{B_n(p_n)} = 0 \qquad (1)$$

where $\{\Theta^{A(p)}\}$ is a collection of p-forms of various degree whose exterior derivative $d\,\Theta^{A(p)}$ can still be expressed as a polynomial in $\Theta^{A(p)}$ with constant coefficients. In [3] we took the point of view that Eq. (1) is the analogue of the Maurer–Cartan equations satisfied by the gauge null fields and therefore we assumed that it describes the vacuum state of the physical theory, i.e. flat space. In order to introduce physics we considered therefore soft-forms $\Pi^{A(p)}$ which are in one-to-one correspondence to the rigid $\Theta^{A(p)}$ but no longer fulfill the Maurer–Cartan equation (1). The deviation from zero of the r.h.s. of (1) was then named the curvature of the set $\{\Pi^{A(p)}\}$:

$$R^{A(p+1)} = d\Pi^{A(p)} + \sum_{n=1}^{N} \frac{1}{n} C^{A(p)}_{B_1(p_1)\ldots B_n(p_n)} \Pi^{B_1(p_1)} \wedge \ldots \wedge \Pi^{B_n(p_n)} \qquad (2)$$

The integrability of Eq. (1) implies that the curvature forms $\{R^{A(p+1)}\}$ satisfy the following Bianchi identity:

$$dR^{A(p+1)} + \sum_{m=1}^{N} C^{A(p)}_{B_1(p_1)\ldots B_m(p_m)} R^{B_1(p+1)} \wedge \Pi^{B_2(p_2)} \wedge \ldots \wedge \Pi^{B_m(p_m)} = 0 \quad (3)$$

The physical theory was then constructed by regarding the soft forms $\Pi^{A(p)}$ as dynamical variables, requiring that the flat space $\{R^{A(p+1)} = 0\}$ be a solution and imposing the principle of rheonomy. This procedure is evidently the analytic continuation of what one does in gauging Lie algebras but it is too naive for the following reason: if we introduce a new set of forms $\{\tilde{\Theta}^{A(p)}\}$, defined in the following way:

$$\tilde{\Theta}^{A(p)} = \Pi^{A(p)} \quad (4a)$$

$$\tilde{\Theta}^{A(p+1)} = R^{A(p+1)} \quad (4b)$$

we see that we can reinterpret the definition of the curvatures (2) plus the Bianchi identity (3) as a single Maurer–Cartan equation for a larger set of rigid forms $\{\tilde{\Theta}^{A(p)}\}$. It follows that the concept of free differential algebra is already large enough to accommodate both the rigid fields and the curvatures and that the definition of these latter as deviation from zero of the Maurer–Cartan equations is ambiguous and ill-posed. A distinction between potentials and curvatures is on the other hand vital for any application to physics and we desperately need a consistent one. Fortunately it is provided in an intrinsic fashion by Sullivan's structural theorem.

According to this theorem every free differential algebra A can be decomposed into a contractible algebra C and a minimal algebra M. By definition a contractible algebra consists of pairs of p and p + 1 forms (both bosonic or both fermionic) satisfying

$$d\omega^{A(p)} = \tau^{A(p+1)}; \quad d\tau^{A(p+1)} = 0 \quad (5)$$

In a minimal algebra, on the other hand, $d\omega^{A(p)}$ is equal to a sum of products of forms (or equal to zero) but never equal to a single p + 1-form generator. Denoting the algebras generated by all p-forms with $p \leq k$ by C^k and M^k respectively, we have

$$dC^k \subset C^{k+1}$$

for the contractible algebra and

$$dM^k \subset M^k \wedge M^k$$

the minimal one. The decomposition of A into $C \otimes M$ can be obtained via an iterative redefinition of the generators which is explicitly discussed in Ref. [8]. We shall not deal with the details: for us the relevant point is that to every A there is associated a unique minimal algebra M. It is this latter which plays the same role as the rigid Lie algebra in the gauging of groups and describes the symmetry of the vacuum. Indeed the second structural theorem by Sullivan (see [7], [9] for example) shows that M always contains a normal Lie subalgebra (or superalgebra) $L \subset M$ and all the other generators of M are essentially determined by the cohomology of L. The generators of A which do not sit in M and which therefore we shall call the contractible generators are to be identified with the curvatures. Hence we propose the following identification between mathematical and physical concepts.

Mathematics		**Physics**
Contractible algebra C	$\longleftrightarrow$	Bianchi identities
Contractible generators	$\longleftrightarrow$	Curvatures
Minimal algebra M	$\longleftrightarrow$	Symmetry of vacuum (rigid algebra)
Minimal generators	$\longleftrightarrow$	Yang-Mills potential gauging vacuum symmetry

The practical outcome of this discussion is that we always have to start from a minimal algebra but, at the moment of gauging, we do not have to call curvature the deviation from zero of the minimal Maurer–Cartan equations rather we have to be more subtle and allow the appearance of contractible generators wherever they can be introduced. From now on we exemplify these ideas with D = 11 supergravity. In [3] it was found that in D = 11 we can write the following minimal algebra, containing besides the left-invariant 1-forms of the super Poincaré group (ω^{ab}, V^a, ψ) a 3-form A and a 6-form B:

$$d\omega^{ab} - \omega^{ac} \wedge \omega_c{}^{\cdot b} = 0 \tag{6a}$$

$$dV^a - \omega^{ab} \wedge V_b - \frac{i}{2} \bar{\psi} \wedge \Gamma^a \psi = 0 \tag{6b}$$

$$d\psi - \frac{1}{4} \omega^{ab} \wedge \Gamma_{ab} \psi = 0 \tag{6c}$$

$$dA - \frac{1}{2} \bar{\psi} \wedge \Gamma^{ab} \psi \wedge V_a \wedge V_b = 0 \tag{6d}$$

$$\begin{aligned} dB &- \frac{i}{2} \bar{\psi} \wedge \Gamma_{a_1 \ldots a_5} \psi \wedge V^{a_1} \wedge \ldots \wedge V^{a_5} \\ &- \frac{15}{2} \bar{\psi} \wedge \Gamma_{a_1 a_2} \psi \wedge V^{a_1} \wedge V^{a_2} \wedge A = 0 \end{aligned} \tag{6e}$$

In [3] the curvatures $\{R^{ab}, R^a, \rho, R^{\square}, R^{\otimes}\}$ were introduced, according to the old point of view, as the deviation from zero of Eqs. (6). Following our new ideas the curvatures, namely the contractible generators, can also appear multiplied by minimal generators if this is allowed by the symmetries of the minimal algebra. In our example there are two symmetries which we need to respect, one is SO(1,10) Lorentz invariance, the other is the global scale invariance of Eqs. (6) under the following replacements:

$$\begin{aligned} \omega^{ab} &\longmapsto \omega^{ab} \quad ; \quad V^a \mapsto e V^a \; ; \quad \psi \longmapsto \sqrt{e}\, \psi \\ A &\longmapsto e^3 A \quad ; \quad B \longmapsto e^6 B \end{aligned} \tag{7}$$

already discussed in Eq. (4.3) of [3].

This implies that the contractible generators $\{R^{ab}, R^{a}, \rho, R^{\square}, R^{\otimes}\}$ will have the same scaling weights as their corresponding minimal generators and can be placed in the algebra only where their scaling weight allows them to stay. Taking this into account we find that the most general decontraction of the minimal algebra (6) is given by the following equations:

$$d\omega^{ab} - \omega^{ac} \wedge \omega^{cb} = R^{ab} \tag{8a}$$

$$\mathcal{D}V^{a} - \frac{i}{2}\bar{\psi}\wedge\Gamma^{a}\psi = R^{a} \tag{8b}$$

$$\mathcal{D}\psi = \rho \tag{8c}$$

$$dA - \frac{1}{2}\bar{\psi}\wedge\Gamma^{ab}\psi\wedge V_a \wedge V_b = R^{\square} \tag{8d}$$

$$\begin{aligned} dB &- \frac{i}{2}\bar{\psi}\wedge\Gamma_{a_1\ldots a_5}\psi\wedge V^{a_1}\wedge\ldots\wedge V^{a_5} \\ &- \frac{15}{2}\bar{\psi}\wedge\Gamma_{a_1a_2}\psi\wedge V^{a_1}\wedge V^{a_2}\wedge A - \alpha R^{\square}\wedge A = R^{\otimes} \end{aligned} \tag{8e}$$

$$\begin{aligned} &dR^{ab} + \ldots = 0 \;;\; d\rho + \ldots = 0 \;;\; dR^{\otimes} + \ldots = 0 \\ &dR^{a} + \ldots = 0 \;;\; dR^{\square} + \ldots = 0 \end{aligned} \tag{8f}$$

where α is a free parameter and where the r.h.s. of Eq. (8f) has not been written in order to emphasize that it follows from Eqs. (8a)–(8d). Altogether Eqs. (8) describe a free differential algebra but they are nicely separated in two sets, corresponding to the splitting in a minimal algebra $\oplus$ a contractible algebra (Bianchi identities). The whole difference among the new and old approach is the freedom in the choice of α. In [3] we automatically fixed $\alpha = 0$; now we want a better criterion to fix this number and therefore to choose among the infinity of free differential algebras which correspond to the same minimal model. This criterion is provided by the following observation: the minimal algebra (6) is invariant under the following gauge transformation:

$$A \longmapsto A + d\varphi^{\square} \tag{9a}$$

$$B \longmapsto B + d\varphi^{\otimes} + \frac{15}{2}\varphi^{\square}\wedge\bar{\psi}\wedge\Gamma^{ab}\psi\wedge V_a\wedge V_b \tag{9b}$$

where $\phi^{\square}$ is any 2-form and $\phi^{\otimes}$ is any 5-form. We would like to promote this gauge invariance to an invariance of the decontracted algebra (8). Hence we modify Eqs. (9) into the only possible way which is consistent with the scaling weights and which reduces to (9) when all contractible generators are set to zero:

$$A \longmapsto A + d\varphi^{\square} \tag{10a}$$

$$B \rightarrow B + d\varphi^{\otimes} + \frac{15}{2}\varphi^{\square}\wedge\bar{\psi}\wedge\Gamma^{ab}\psi\wedge V_a\wedge V_b + \beta\,\varphi^{\square}\wedge R^{\square} \tag{10b}$$

Then we require that the contractible generators (i.e. the curvatures R^a, R^{ab}, $R^{\square}$, $R^{\otimes}$, ρ) be invariant under the transformation (10). For R^a, R^{ab}, $R^{\square}$, ρ this is trivially true, while for $R^{\otimes}$ we obtain

$$\delta R^{\otimes} = (-15+\beta)\varphi^{\square}\wedge\bar{\psi}\wedge\Gamma_{ab}\rho\wedge V^a\wedge V^b + (15-\beta)\varphi^{\square}\wedge \wedge\bar{\psi}\wedge\Gamma_{ab}\psi\wedge R^a\wedge V^b + (\beta-\alpha)\,d\varphi^{\square}\wedge R^{\square} \tag{11}$$

Therefore $R^{\otimes}$ is gauge invariant if and only if

$$\alpha = \beta = 15 \tag{12}$$

We conclude that there is a unique decontraction of the minimal algebra (6) which preserves all of its symmetries and it is the free differential algebra given in Table I. There the first five equations can be taken as the definitions of the curvatures while the second five may be reinterpreted as the Bianchi identities.

Fixed the free differential algebra it might seem that we still have the problem of constructing the physical theory namely the action, the supersymmetry transformation rules and the field equations. This is not so. As it has already been pointed out in the most updated reviews on the so called "group-manifold approach" [10] the action is only an after thought since the correct field equations are already uniquely determined by the free differential algebra plus the principle of rheonomy. In the mathematical language of Sullivan this has to do with what he calls a realization of the abstract free algebra by a set of differential forms living on a manifold. In our case the manifold is $D = 11$ superspace and the principle of rheonomy is the requirement of "analyticity" for the differential forms: we require that the integral of any p-form on any bosonic p-surface should not depend on the choice of the surface in superspace. This implies the "Cauchy–Riemann conditions" (i.e. rheonomic conditions), namely the fact that the components of the curvatures (= contractible generators) in the direction of one or more ψ, should be linear combination of the components of the curvatures in V-directions. With standard procedures [10] we find that the unique rheonomic solution of the Bianchi identities of Table 1 is given by the curvature parametrization of Table 2. We note that the components of $R^{\otimes}$ are the dual of the components of $R^{\square}$. This is not an arbitrary choice but it is forced on us by the Bianchi identities (iv)′ and (v)′. Indeed Eq. (iv)′ of Table 1 implies Eq. (iii) of Table 2 and this, inserted into (v)′ of Table 1 implies Eq. (v) of Table 2. Once this is established the characteristic non linear field equation $DF = \varepsilon FF$ is seen to be nothing else but the V ... V-projection of the Bianchi identity (v)′ of Table 2. Hence all the dynamical field equations of $D = 11$ supergravity are nothing else but the yield of rheonomy in the correct free differential algebra. This ceases to be true, however, if we omit the 6-index photon B and its associated Bianchi identity. We could, at this point stop, since we have just given a complete algebraic derivation of the theory. However, in order to see the analogy with conformal supergravity in $D = 4$, let us reconsider problem of the action. This has to be determined in such a way that $R^{ab} = R^a = R^{\square} = R^{\otimes} = \rho = 0$ be a solution and that it admits, as most general non trivial solution, the parametrization of Table 2. To this purpose we can just start from the result given in Eqs. (4.19a) and (4.19b) of Ref. [3]. Indeed the Lagrangian $L_0(\gamma_1, \gamma_2, k)$ obtained there is the most general one whose equations of motion admit the solution $R^{ab} = R^{a} = R^{\square} = R^{\otimes} = \rho = 0$, namely are consistent with the minimal algebra which has not been changed. For our purposes it is just sufficient to make the following replacement

$$R^{\otimes}_{old} = R^{\otimes}_{new} + 15\, R^{\square} \wedge A \tag{13}$$

in Eq. (4.19b). This yields the form of $L_0^{(new)}(\gamma'_1, \gamma_2, k)$:

$$\begin{aligned}
\mathcal{L}_0^{new}(\gamma'_1, \gamma_2, k) = & -\frac{1}{9} R^{a_1 a_2} \wedge V^{a_3} \wedge \ldots \wedge V^{a_{11}} \varepsilon_{a_1 \ldots a_{11}} \\
& + \frac{7}{30} i\, R^{a} \wedge V_a \wedge \bar{\psi} \wedge \Gamma_{b_1 \ldots b_5} \psi \wedge V_{b_6} \wedge \ldots \wedge V_{b_{11}} \varepsilon^{b_1 \ldots b_{11}} \\
& + k\, R^{\otimes}_{new} \wedge \bar{\psi} \wedge \Gamma_{ab} \psi \wedge V^a \wedge V^b + i(k-84) R^{\square} \wedge \bar{\psi} \wedge \Gamma_{a_1 \ldots a_5} \psi \wedge V^{a_1} \wedge \ldots \wedge V^{a_5} \\
& + (30k - 840) R^{\square} \wedge A \wedge \bar{\psi} \wedge \Gamma_{ab} \psi \wedge V^a \wedge V^b + \\
& + 2 \bar{\varrho} \wedge \Gamma_{c_1 \ldots c_8} \psi \wedge V^{c_1} \wedge \ldots \wedge V^{c_8} + \frac{1}{4}\left(1 + \frac{k}{28}\right) \bar{\psi} \Gamma_{a_1 a_2} \wedge \psi \wedge \bar{\psi} \Gamma_{a_3 a_4} \wedge \psi \wedge \\
& \wedge V_{a_5} \wedge \ldots \wedge V_{a_{11}} \varepsilon^{a_1 \ldots a_{11}} + \gamma'_1 R^{\square} \wedge R^{\square} \wedge A \\
& + \gamma_2 R^{\otimes}_{new} \wedge R^{\square} + \frac{15}{2}(k - 28) \bar{\psi} \wedge \Gamma_{a_1 a_2} \psi \wedge \bar{\psi} \wedge \Gamma_{a_3 a_4} \psi \wedge V^{a_1} \wedge \ldots \wedge V^{a_4} \wedge A
\end{aligned} \tag{14}$$

The relation among the new parameter γ'_1 and the old parameter γ_1 is the following one:

$$\gamma_1' = \gamma_1 + 15\gamma_2 \tag{15}$$

At this point in Ref. [3] we required gauge invariance of L_0 under the gauge transformation

$$A \mapsto A + d\varphi^{\square} \quad ; \quad B \to B \tag{16}$$

This was obviously wrong since the correct gauge transformation is given by Eqs. (10) with $\beta = 15$. Imposing this gauge invariance on $L_0^{new}(\gamma'_1, \gamma_2, k)$ we find the condition:

$$\gamma_1' = -840 + 30k \quad ; \quad \gamma_2 = \text{arbitrary} \tag{17}$$

to be compared with the conditions obtained in Eq. (4.24) of [3] which fixed γ_1 and the ratio between γ_2 and k.

If we vary the action L_0^{new} with respect to all the fields we find that its equations of motion are always incompatible with the parametrization of Table 2. Hence we resort once more to the standard trick and we add a detrivializing Lagrangian based on a 0-form, which we want to identify with the $F_{a_1 \dots a_4}$ tensor appearing in Table 2:

$$\mathcal{L}'(m,n,p) = m F_{a_1..a_4} R^{\square} \wedge V_{a_5} \wedge .. \wedge V_{a_{11}} \varepsilon^{a_1 \dots a_{11}}$$
$$+ p F_{a_1..a_4} R^{\otimes} \wedge V^{a_1} \wedge \dots \wedge V^{a_4} + n F_{a_1..a_4} F^{a_1..a_4} V^{b_1} \wedge .. \wedge V^{b_{11}} \varepsilon_{b_1..b_{11}} \tag{18}$$

Now varying the action

$$\mathcal{A} = \int \mathcal{L}_0^{new}(\gamma_1' = -840 + 30k, \gamma_2, k) + \mathcal{L}'(m,n,p) \tag{19}$$

and demanding that Table 2 be a solution we obtain the following conditions on the parameters.

$$\delta F_{a_1..a_4} \Longrightarrow n = -\frac{1}{660}\left(m + \frac{p}{84}\right) \tag{20a}$$

$$\delta \varphi \Longrightarrow \begin{cases} p = 2k - \gamma_2 \\ m = -2 + \frac{p}{84} \end{cases} \tag{20b}$$

$$\delta B \Longrightarrow \text{no new condition} \tag{20c}$$

$$\delta A \Longrightarrow \gamma_2 = 2k \tag{20d}$$

$$\left.\begin{matrix} \delta \omega^{ab} \\ \delta V^a \end{matrix}\right\} \Longrightarrow \text{no new condition} \tag{20e}$$

At the end of the day we might be disappointed since the Lagrangian we have found is essentially the same as that of Ref. [3] since the freedom in the parameter k we are left with is that of a total derivative. Thinking for a moment, however, we realize that it could not happen anything else; if two Lagrangians yield the same field equations they must differ by a total derivative. The relevant point is another one: what happens is that the dynamical equations of a certain smaller multiplet of gauge fields are, actually contained in the Bianchi identities of a larger gauge algebra. This implies that:

i) The curl of some gauge field not appearing in the Lagrangian is dual to the curl of the physical field appearing in the Lagrangian.

ii) The Lagrangian can be written in the most elegant and economical way using the curvatures of the full algebra, rather than the subalgebra corresponding to the physical fields.

iii) The action is invariant under the full gauge algebra although some gauge fields appear only through total derivatives.

All this, as already pointed out, happens in conformal supergravity [4, 5] with the axion and the dilaton and it is repeated here with $A_{\mu\nu\rho}$ and $B_{\mu_1 \ldots \mu_6}$.

To make point (ii) evident it is sufficient to choose $k = 28 \rightarrow \gamma_2 = 56$. This choice simplifies the Lagrangian dramatically since it eliminates all bare A-terms, the three-linear term $R^{\square}_{\wedge} R^{\square}_{\wedge} A$ and the so-called generalized cosmological terms which have no curvature factor. The result is shown in Table 3.

It is may be useful to stress once more that the $D = 11$ case was in my intention instrumental as an illustration of the general concept: "Free differential algebras already contain both potentials and curvatures and supplemented with the principle of rheonomy (= analiticity) yield the full information of an action, namely field equations and supersymmetry transformation rules." The search of new theories is therefore reduced to the study of free differential algebra structures.

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Table 1
Free differential algebra of D = 11 supergravity

i) $$R^{ab} = d\omega^{ab} - \omega^{ac} \wedge \omega_c^{\cdot\, b}$$

ii) $$R^a = dV^a - \omega^{ab} \wedge V_b - \frac{i}{2}\bar{\psi} \wedge \Gamma^a \psi = \mathcal{D}V^a - \frac{i}{2}\bar{\psi} \wedge \Gamma^a \psi$$

iii) $$\rho = d\psi - \frac{1}{4}\omega^{ab} \wedge \Gamma_{ab}\psi = \mathcal{D}\psi$$

iv) $$R^{\square} = dA - \frac{1}{2}\bar{\psi} \wedge \Gamma^{ab}\psi \wedge V_a \wedge V_b$$

v) $$R^{\otimes} = dB - \frac{i}{2}\bar{\psi} \wedge \Gamma_{a_1 \dots a_5}\psi \wedge V^{a_1} \wedge \dots \wedge V^{a_5} - \frac{15}{2}\bar{\psi} \wedge \Gamma_{ab}\psi \wedge V^a \wedge V^b \wedge A - 15\, R^{\square} \wedge A$$

i)′ $$\mathcal{D}R^{ab} = 0$$

ii)′ $$\mathcal{D}R^a + R^{ab} \wedge V_b - i\bar{\psi} \wedge \Gamma^a \rho = 0$$

iii)′ $$\mathcal{D}\rho + \frac{1}{4}\Gamma_{ab}\psi \wedge R^{ab} = 0$$

iv)′ $$dR^{\square} - \bar{\psi} \wedge \Gamma_{a_1 a_2}\rho \wedge V^{a_1} \wedge V^{a_2} + \bar{\psi} \wedge \Gamma_{a_1 a_2}\psi \wedge R^{a_1} \wedge V^{a_2} = 0$$

v)′ $$dR^{\otimes} - i\bar{\psi} \wedge \Gamma_{a_1 \dots a_5}\rho \wedge V^{a_1} \wedge \dots \wedge V^{a_5} - \frac{5}{2} i\bar{\psi} \wedge \Gamma_{a_1 \dots a_5}\psi \wedge R^{a_1} \wedge V^{a_2} \wedge \dots \wedge V^{a_5} - 15\,\bar{\psi} \wedge \Gamma^{a_1 a_2}\psi \wedge R^{\square} \wedge V_{a_1} \wedge V_{a_2} - 15\, R^{\square} \wedge R^{\square} = 0$$

Table 2

Rheonomic solution of the Bianchi identities

ii) $R^{a} = 0$

iv) $R^{\square} = F_{a_1 \dots a_4} V^{a_1} \wedge \dots \wedge V^{a_4}$

v) $R^{\otimes} = \frac{1}{84} \varepsilon^{a_1 \dots a_4 b_1 \dots b_7} F_{a_1 \dots a_4} V_{b_1} \wedge \dots \wedge V_{b_7}$

iii) $\rho = \rho_{ab} V^{a} \wedge V^{b} - \frac{i}{3} \left(\Gamma^{a_1 a_2 a_3} \psi \wedge V^{a_4} + \frac{1}{8} \Gamma^{a_1 \dots a_4 m} \psi \wedge V_m \right) F_{a_1 \dots a_4}$

i) $$R^{ab} = R^{ab}{}_{\cdot mn} V^{m} \wedge V^{n} + i \bar{\rho}_{mn} \left(\frac{1}{2} \Gamma^{abmn} - \frac{2}{9} \Gamma^{mn[a} \delta^{b]c} + \right.$$
$$\left. + 2 \Gamma^{ab[m} \delta^{n]c} \right) \psi \wedge V_c + \bar{\psi} \wedge \Gamma_{mn} \psi F^{mnab}$$
$$+ \frac{1}{24} \bar{\psi} \wedge \Gamma^{abc_1 \dots c_4} \psi F_{c_1 \dots c_4}$$

where the inner components satisfy:

iv) $\mathcal{D}_m F^{mc_1 \dots c_3} + \frac{1}{96} \varepsilon^{c_1 \dots c_3 a_1 \dots a_8} F_{a_1 \dots a_4} F_{a_5 \dots a_8} = 0$

iii) $\Gamma^{abc} \rho_{bc} = 0$

i) $R^{am}{}_{bm} = 6 F^{ac_1 \dots c_3} F_{bc_1 \dots c_3} - \frac{1}{2} \delta^{a}_{b} F^{c_1 \dots c_4} F_{c_1 \dots c_4} = 0$

Table 3
Action of D = 11 supergravity with the choice k = 28

$$\mathcal{A} = \int_{M_{11}} \mathcal{L}$$

$$\mathcal{L} = -\frac{1}{9} R^{a_1 a_2} \wedge V^{a_3} \wedge \ldots \wedge V^{a_{11}} \varepsilon_{a_1 \ldots a_{11}}$$

$$+ 2 \bar{\rho} \wedge \Gamma_{c_1 \ldots c_8} \psi \wedge V^{c_1} \wedge \ldots \wedge V^{c_8}$$

$$+ \frac{7}{30} i \, R^m \wedge V_m \wedge \bar{\psi} \wedge \Gamma_{a_1 \ldots a_5} \psi \wedge V_{a_6} \wedge \ldots \wedge V_{a_{11}} \varepsilon^{a_1 \ldots a_{11}}$$

$$+ 28 R^{\otimes} \wedge \bar{\psi} \wedge \Gamma_{ab} \psi \wedge V^a \wedge V^b - 56 i R^{\square} \wedge \bar{\psi} \Gamma_{a_1 \ldots a_5} \wedge \psi \wedge V^{a_1} \wedge \ldots \wedge V^{a_5}$$

$$+ 56 R^{\otimes} \wedge R^{\square}$$

$$- 2 F_{a_1 \ldots a_4} R^{\square} \wedge V_{a_5} \wedge \ldots \wedge V_{a_{11}} \varepsilon^{a_1 \ldots a_{11}}$$

$$+ \frac{1}{330} F_{a_1 \ldots a_4} F^{a_1 \ldots a_4} \varepsilon_{b_1 \ldots b_{11}} V^{b_1} \wedge \ldots \wedge V^{b_{11}}$$