*American Journal of Educational Research, 2015, Vol. 3, No. 12, 1519-1523*
Available online at http://pubs.sciepub.com/education/3/12/6

DOI:10.12691/education-3-12-6

# Common and Flexible Use of Mathematical Non Routine Problem Solving Strategies

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**Abstract** This study aims to investigate whether high-achieving sixth, seventh and eighth graders can exhibit strategy flexibility while they are solving non-routine problems. In this context, four students from each grade level participated in the study. Four non routine problems were represented to the students one by one in separate papers. Students worked in pairs and all interviews were videotaped. These records, pupils' scripts, and notes taken by the researchers were used in data analysis. Four criteria (selection and use of the most appropriate strategy, changing strategies when it does not work for the solution of a problem, using multiple strategies for the solution of a problem and changing strategies between problems) were established to determine students' flexibility levels. Each answer given by pairs was evaluated based on these criteria and scored as 0, 1 or 2. Results showed that students usually can select the most appropriate strategy, and use multiple strategies in one problem. Students were comfortable in using "look for a pattern" and "make a drawing" strategies. On the other hand, the most unfavorable strategy for them was "simplify the problem". Additionally, there were enterprises to use "write an equation" strategy. Besides, it was observed that students did not need to make a significant change in their thinking ways when their first attempts were wrong and they rarely change their strategies between problems. A longitudinal study including more students at different achievement levels and different kind of non-routine problems will give in-depth information about this subject.

***Keywords:*** *problem solving, strategic flexibility, non-routine problems, problem solving strategies, mathematics education*

**Cite This Article:** Cigdem Arslan, and Yeliz Yazgan, "Common and Flexible Use of Mathematical Non Routine Problem Solving Strategies." *American Journal of Educational Research*, vol. 3, no. 12 (2015): 1519-1523. doi: 10.12691/education-3-12-6.

## 1. Introduction

One of the important issues in the psychology of (mathematics) education is how students can be taught curricular subjects so that they develop "adaptive expertise", namely "the ability to apply meaningfully learned procedures flexibly and creatively" [14]. Moreover, one basic indicator of proficiency in problem-solving is the development of flexible knowledge [6,11]. Mathematics educators emphasize the educational importance of recognizing and stimulating flexibility in children's self-constructed strategies. Predominant focus in prior works about this flexibility is on the description of it. Therefore, there are different definitions about flexibility in different papers. According to Verschaffel, Luwel, Torbeyns, and Van Dooren [13] a flexible strategy choice includes "the conscious or unconscious selection and use of the most appropriate solution strategy on a given mathematical item or problem, for a given individual, in a given context." Besides, Star and Rittle-Johnson [9] attribute a broader meaning to flexibility: They define flexibility as knowledge of multiple strategies and the relative efficiency of these strategies. Based on this definition, the first important characteristic of flexibility is to have information about multiple strategies. Flexible problem solvers know more than one way to complete tasks. Secondly, flexibility is required knowledge of strategy efficiency. This means that flexible problem solvers can recognize which strategies are more efficient than others under particular circumstances. Another researcher, Selter [7] defines the flexibility as an ability to switch between different strategies.

In addition to these definitions, Krems [5] states that a flexible problem solver must have following abilities:

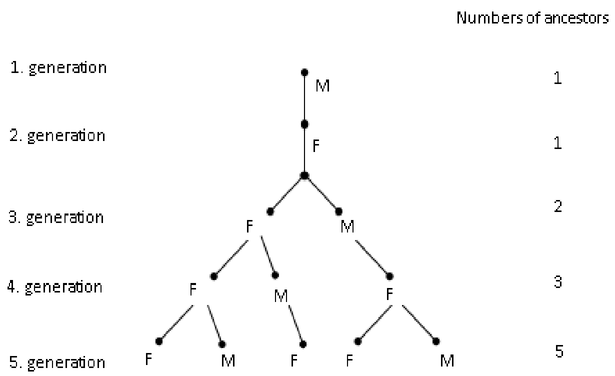
The first is multiple interpretations of data. A flexible problem solver is able to consider several alternative interpretations of a given situation. When the situation warrants a change, the problem solver is able to switch from one interpretation to another.

Second is modifying representations. A flexible problem solver chooses an appropriate representation for the task and current situation, for example, between a concrete or abstract representation, a functional or structural representation, or a principle oriented or surface-oriented representation.

Third is modification of strategies. A flexible problem solver can change strategies to reflect changes in resources and task demands. These strategy changes might reflect resource usage, or the basic problem solving approach (e.g., from a more goal oriented to a more data-oriented approach, from a top down to a bottom up, from an exploratory to a confirmatory strategy (p.209).

Studies on flexibility are mostly about specific subject matters such as arithmetic and algebra [4]. For example, in Heinze, Marschick and Lipowsky [3]'s study, the adaptive use of strategies of different groups of third graders is investigated in relation to the instructional approach of their textbooks. Their study is limited with addition and subtraction of three digit numbers. In their study, Star and Newton [8] search the nature and development of experts' use of strategies in equation solving. Star, Rittle-Johnson, Lynch and Perova [10], reports about two intervention studies on individual factors influencing the learning of the flexible use of estimation strategies.

Although studies mentioned in previous paragraph, there are a few studies which examine this issue in the terms of non-routine problem solving. The most conspicuous theoretical support in this domain was provided by Elia, van den Heuvel-Panhuizen, and Kolovou [1]'s study. In this study, the authors explore the strategies used by high-achieving students for non-routine problems, and they distinguish two types of strategy flexibility: inter-task flexibility (changing strategies across problems) and intra task flexibility (changing strategies within problems). They found that these two types of flexibility are not displayed to a large extent by the students in their sample. Moreover, according to their findings, students who show inter-task strategy flexibility are more successful than students who persevere with the same strategy, while intra-task strategy flexibility does not help the students to arrive at the correct answer. Similarly, Zhang [15] carried out a study in which four non-routine problem were used. The major goal of the researcher was to determine whether the individual's performances were consistent across different subject areas and problem types that could be solved different heuristics, and to identify possible factors that influenced children's choices and strategy use in different contexts. Results showed that intra-task strategy flexibility does not imply success at reaching correct answers to tasks, yet further proposed that the level of intra-task strategy flexibility might depend largely on the individual's confidence and preference for the use of certain strategies. Additionally, inconsistency in the same individual's mathematics problem solving behaviors across different subject areas and/or heuristics usage was revealed.




**Picture 1.** Solution of the sample for non-routine problem

In connection with all aforementioned studies, "strategy use" and "strategy flexibility" terms can be exemplified with the following non-routine problem: "A male bee is born from an unfertilized egg, a female bee from a fertilized one. So, in other words, a male bee only has a mother, while a female bee has a mother and a father. How many total ancestors does a male bee have going ten generations back?" This problem can be solved with the help of the diagram like in Picture 1. In this diagram F means female and M means male.

Without completing this diagram until the tenth generation, we can find out the solution by using the pattern among numbers of bees in every generation: Add the first and second number (1 + 1 = 2) to get the third number, add the second and third number to get the fourth number (1 + 2 = 3) and so on. According to this pattern, solution of the problem is 55.

If a person solves this problem in this way, he/she uses three strategies here: When a drawing is made use of as a visual support in solving process, "draw a diagram" strategy emerges. In the next step, "simplify the problem" strategy is used when simple versions of problem including smaller numbers are firstly solved. At the point that the regularity among outcomes obtained from each simple version is sought, this is an application of "look for a pattern" strategy. In this solution, there is an attempt to develop and choose most reasonable strategy or strategies. Common use of these three strategies is a sign of intra task strategy flexibility.

**Aim and research questions**

This study generally focuses on strategy use and strategy flexibility in non-routine problem solving. Based on remarks of Verschaffel et al. [13], and Star & Riddle-Johnson [9] about flexible strategy use, in the present study, the term of strategy flexibility means selection and use of the most appropriate solution strategy, and ability of using multiple strategies and changing them (when needed) during problem solving. Besides, we will utilize the terms of two types of strategy flexibility, namely inter-task flexibility and intra-task flexibility which are distinguished by Elia et al [1] in their study.

On the basis of theoretical framework given in Introduction, this study aims to investigate whether high achievers at sixth, seventh and eighth grade level (11 to 14 years old) can exhibit strategy flexibility in solving a non-routine problem. In this context, specific research questions were determined as follows:

- Can high-achieving sixth, seventh and eighth graders choose and apply proper strategy (or strategies) in non-routine problem solving?
- At which level do high-achieving sixth, seventh and eighth graders exhibit inter and intra task strategy flexibility?

## 2. Method

### 2.1. Participants

Since this study is about sixth, seventh and eighth graders' strategy use and strategic flexibility, four students (two pairs) from each grade level participated in the present study. Namely, there were 12 participants in total. These students were selected by their mathematics teachers on the basis of their mathematical achievement level. All students were above average in terms of their mathematics scores in school. Besides, teachers took consideration their views and observations about mathematical attitudes and dispositions of student during selection process. All students were voluntary to participate in the study.

## 2.2. Procedure

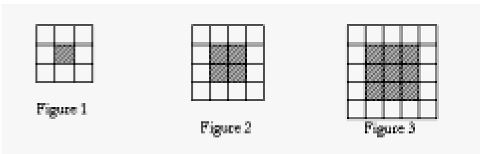
A problem by problem performance model was used in the study. Each non-routine problem was represented to students in a separate paper. Students worked in pairs so that they could create and discuss their own solution procedures. It was supposed that this kind of interaction between students reliably will lead to increased procedural flexibility [11]. Researchers did not intervene in students' solving processes unless they had a problem about understanding question or they spent so much time since they persevered on the wrong strategy. Each interview consisted of approximately 45-55 minutes. All interviews were carried out in a separate place and video-taped by the researchers. During interviews, students were especially asked to think aloud. These records, pupils' scripts, and notes taken by the researchers were used in data analysis.

## 2.3. Questions Used in the Study

There is a need to explain why non routine problems were chosen in the present study to assess strategy flexibility in problem solving. Students who develop flexibility in problem solving are more likely to use existing strategies when faced with unfamiliar transfer problems [9,12]. Therefore, we thought that strategy flexibility of students could be more observable while they are solving non routine problems, since they do not know a direct way of reaching to the solutions of these problems.

Four non routine problems represented in the following were asked to students:

P1.



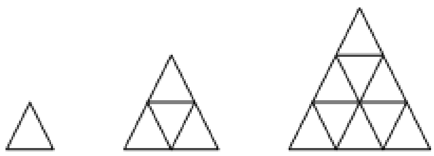

a) Draw the fourth figure.

b) How many small shaded squares are there in the fifth figure? Explain how you found your answer.

c) How many small unshaded squares are there in the fifth figure? Explain how you found your answer.

d) Figure 1 has 8 unshaded squares. Figure 3 has 16 unshaded squares. If a figure has 44 unshaded squares, which figure is it? Explain how you found your answer [2]

P2. Each of following shapes consists of small triangles like the first one.



How many small triangles do you need to make fifteenth shape?

P3. If there are 10 people in a group, and each people shakes everyone else's hands only once, how many handshakes take place?

P4. How many squares are there in a chessboard?

## 2.4. Assesment

By taking into consideration definitions and explanations about flexibility, four criteria were established to determine levels of students:

C1. Selection and use of the most appropriate strategy

C2. Changing strategies when it does not work for the solution of a problem (intra task strategy flexibility)

C3. Using multiple strategies for the solution of a problem (intra task strategy flexibility)

C4. Changing strategies between problems (inter task strategy flexibility).

Each answer given by student pairs to each problem was evaluated based on these criteria, and scored as 0 (if students don't show this behaviour), 1 (if students show this behaviour partly or after intervention) and 2 (if students show directly this behaviour). However, there were some exceptions about coding: when students did not need to change their strategy since they were able to complete the solution with it, we did not evaluate second criterion. To be able to evaluate Criterion 4 (C4), it was required to examine the strategy change between problems. So, all problems have taken into consideration, and this criterion was scored on the basis of each student pair, instead of each problem.

With regard to third criteria, a deeper analysis was made. Since questions used in the current study are feasible to use "look for a pattern", "simplify the problem" and "make a drawing" strategies commonly in one problem, we established a similar coding system to determine use of each strategies for each problem: 0 (if student don't use the strategy), 1 (if students use the strategy partly or after intervention), 2 (if students use the strategy directly).

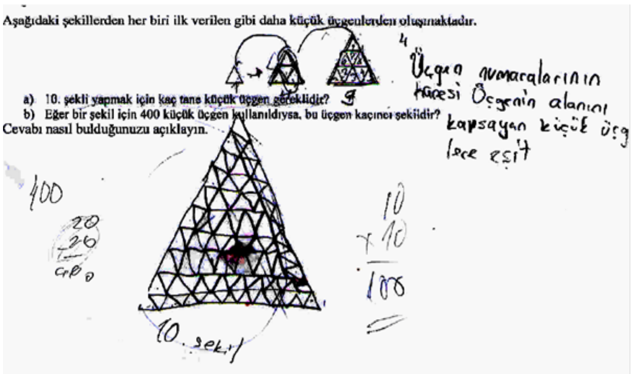
# 3. Results

As stated with research questions, it was aimed to investigate strategy use and flexibility of sixth, seventh and eighth grade students in this study. In Table 1, results related to raw scores of pairs for each criterion of flexibility level are represented.

**Table 1. Raw scores of pairs for each criterion of flexibility**

| Grade | Pair Number | Problem 1 | | | Problem 2 | | | Problem 3 | | | Problem 4 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | C1 | C2 | C3 | C1 | C2 | C3 | C1 | C2 | C3 | C1 | C2 | C3 | C4 |
| 6 | I | 2 | 0 | 1 | 1 | 1 | 1 | 2 | - | 2 | 2 | - | 1 | 0 |
| | II | 2 | - | 2 | 2 | - | 2 | 2 | - | 0 | 1 | - | 1 | 0 |
| 7 | III | 2 | - | 2 | 2 | 2 | 2 | 2 | - | 0 | 1 | - | 1 | 1 |
| | IV | 2 | - | 2 | 2 | - | 2 | 2 | - | 0 | 1 | - | 0 | 0 |
| 8 | V | 2 | - | 2 | 2 | - | 2 | 2 | - | 2 | 1 | - | 1 | 1 |
| | VI | 2 | - | 2 | 2 | - | 2 | 2 | - | 2 | 2 | - | 2 | 0 |

As seen from the Table 1, scores of pairs for Criterion 1 (C1) show that students' first strategies were mostly appropriate and led them to the right answer. So, almost all students in pairs tended to stay with their initial strategies until they solved the problem. Since they could reach to the right answer with their first strategies, their scores about Criterion 2 (C2) mostly did not appear in Table 1. If students' first strategies were wrong and they insisted on using it, then they were encouraged to change their point of view. An example of this situation is given in Picture 2. In this answer given by Pair I to Problem 2, students tried to draw tenth shape, and they were stuck to this strategy although it did not work. After researchers' advice to think on it again, they could use the "simplify the problem" strategy.




**Picture 2.** Answer given by Pair I to Problem 2

Besides, students were not directly asked to think on other alternative strategies, but opportunities were given to them with questions like "Is there anything else that you want to discuss about problem or your solution?" Notwithstanding, they did not use these opportunities, and mostly considered the problem was finished whenever they reached an answer.

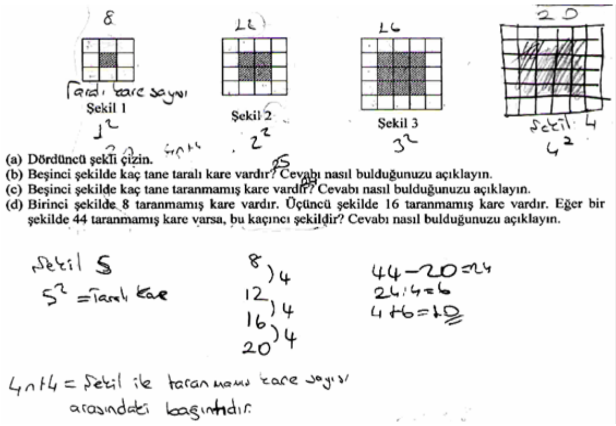
As for third criterion (C3) about using multiple strategies in one problem, it seems that students had no problem about this criterion. Scores about different strategies used for one problem can be seen in Table 2.

**Table 2. Scores about different strategies used for one problem**

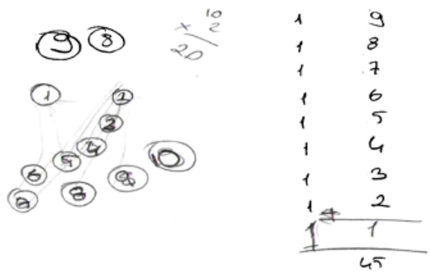
| Grade | Pair Number | Problem 1 MD[a] | Problem 1 LP[b] | Problem 1 SP[c] | Problem 2 MD | Problem 2 LP | Problem 2 SP | Problem 3 MD | Problem 3 LP | Problem 3 SP | Problem 4 MD | Problem 4 LP | Problem 4 SP |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | I | 2 | 1 | 1 | 2 | 1 | 0 | 2 | 2 | 0 | 2 | 1 | 1 |
| | II | 0 | 2 | 2 | 0 | 2 | 2 | 0 | 2 | 0 | 2 | 1 | 1 |
| 7 | III | 0 | 2 | 2 | 0 | 2 | 2 | 0 | 2 | 0 | 2 | 1 | 1 |
| | IV | 2 | 2 | 2 | 0 | 2 | 2 | 0 | 2 | 0 | 2 | 0 | 0 |
| 8 | V | 0 | 2 | 2 | 0 | 2 | 2 | 0 | 2 | 0 | 2 | 2 | 2 |
| | VI | 0 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 0 | 1 | 1 | 1 |

MD [a]: Make a drawing, LP[b]: Look for a pattern, SP [c]: Simplifying the problem

With regard to strategies used by students in their problem solving processes, it can be said that they could effectively benefit from "look for a pattern" strategy. They had no difficulty in seeing patterns in Problem 1, Problem 2 and Problem 3. But, in Problem 4, they needed help to count each size of square in chessboard and to recognize the relation between size of square and number of size. Besides, it was observed that usage level of "simplify the problem" strategy was really low in solving processes of all problems. If first three simple versions of problem with smaller numbers were directly given to the students (like Problem 1 and Problem 3) in the problem, they took advantage of this strategy. Otherwise, it did not occurred to them that simplifying the problem could facilitate to reach to answer. In Picture 3 and Picture 4, examples of multiple strategy use are presented.




**Picture 3.** Answer given by Pair VI to Problem 1



**Picture 4.** Answer given by Pair II to Problem 3

Lastly, scores about C4 in Table 1 show that only two pairs preferred to change strategies across problems. Still, they needed an encouragement to show this behavior, that is why they got 1 point from this criterion.

Generally speaking, students did not have much difficulty in selecting and using appropriate strategies. That is, they mostly exhibited C1. On the contrary, they had trouble with changing strategies for different problems. So, they were weak at C4.

# 4. Discussion

The present study questioned strategy use and strategy flexibility of high achieving students in sixth, seventh and eighth grades. Unlike many other research studies in which students' problem solving performance was examined either on routine tasks or word problems, non-routine problems were used in this research. Thus, this study broadens the domain in which flexibility is explored from arithmetic and algebra to non-routine problem solving like Elia et al [1] did: Both studies are related to strategy flexibility in non-routine problem solving, and participants were high achievers. Nevertheless, students who joined our study were older, and there were fewer participants in our study since it has both qualitative and quantitative structure. Moreover, students did not solve the problems individually, and their solving processes were videotaped in addition to their written answers in the current study.

With regard to the first research question, the most outstanding finding was that students could easily develop appropriate strategies for non-routine problems. This was an expected result, because, non-routine problems and strategies to solve them are partly included in Turkish math curricula and mathematics textbooks, although they were not specially taught to students or elaborated on as a separate subject. However, this finding is not in line with the findings of Elia et al [1] and Verschaffel et al [12] suggesting that strategies are hardly used by students when confronted with non-routine problems. This situation can be explained by two reasons: Firstly, in these two studies, grade levels of participants were four and five, while our participants were at sixth, seventh and eighth grade levels. Moreover, strategies used in Elia et al [1]'s study were different from strategies which we focused in our study. It seems that strategic flexibility can be affected by characteristics and contexts of problems.

In concern to intra task flexibility, the majority of students who made a solution attempt did not use mainly one strategy as stated by Elia et al [1]. They were able to use multiple (or complementary) strategies in one problem. But, there was a weakness about this kind of flexibility: In accordance with the findings of Zhang [15] and Elia et al. [1], we observed that students' self-confidence and preference has a negative impact on intra-task flexibility. Most of the students participated in our study had a high confidence in the strategy that they currently used. Therefore, they did not need to make a significant change in level of understanding the problem or in their way of thinking about solution without researchers' intervention.

About different strategies used by students, we can say that students were really comfortable in using "look for a pattern" and "make a drawing" strategies. Actually, they did not need to draw so many diagrams since they can easily see the patterns without any visual support due to questions' structure. On the other hand, as it is stated in Findings, the most unfavorable strategy for them was "simplify the problem". Additionally, there were enterprises to use "write an equation" strategy as seen in Picture 3. Any other strategy except abovementioned ones was not used, meaning that these students has not very rich repertoire to use all strategies effectively, so they had no much chance to change their strategies between problems. For example, for Problem 3, they could apply "make a systematic list" strategy, if they identified each person with a symbol like a number or a letter and represented each handshake with these symbols in an organized way. In other words, students were not often found to show traces of inter-task flexibility.

Despite the fact that this study provided us some new insights concerning the strategy flexibility, number and types of non-routine problems used in the current study are limited. Moreover, number and characteristics of students involved is not enough to get a robust idea about students' level of flexibility. It is supposed that a longitudinal study including more students at different achievement levels and different kind of non-routine problems will give in-depth information about this subject.

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