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## COMMON FIXED POINT THEOREM FOR FOUR MAPPINGS IN NON-ARCHIMEDEAN PM-SPACES

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ABSTRACT. We define the concept of weakly  $f$ -compatible pair  $(f, S)$  in non-Archimedean Menger probabilistic metric spaces and obtain a common fixed point theorem for four maps which improves a theorem of Y.J.Cho.et.al.

### Introduction

Recently Y.J.Cho et.al [4] introduced the concepts of compatible mappings and compatible mappings of type (A) in non-Archimedean Menger probabilistic metric spaces and obtained some common fixed point theorems in the space. In this paper we prove a common fixed point theorem which generalizes a theorem of Y.J.Cho et.al [4] by introducing the notion of weakly compatible pair of mappings in non -Archimedean PM-Space. For terminologies, notations and properties of probabilistic metric spaces, refer to [1], [2], [3] and [4].

DEFINITION 1: A distribution function is a mapping  $F: IR^+ \rightarrow IR^+$  which is non decreasing and left continuous with  $\inf F = 0$  and  $\sup F = 1$ . We will denote  $D$  by the set of all distribution functions.

DEFINITION 2: Let  $X$  be any non empty set. An ordered pair  $(X, \mathbb{F})$  is called a non-Archimedean probabilistic metric space (briefly a N.A. PM-space) if  $\mathbb{F}$  is a mapping from  $X \times X$  into  $D$  satisfying the following conditions (We shall denote the distribution function  $\mathbb{F}(x, y)$  by  $F(x, y)$  for all  $x, y \in X$ ):

(2.1)  $F(x, y, t) = 1$  for all  $t > 0$  if and only if  $x = y$ ,

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$$(2.2) F(x, y) = F(y, x),$$

$$(2.3) F(x, y, 0) = 0,$$

$$(2.4) \text{ If } F(x, y, t_1) = 1 \text{ and } F(y, z, t_2) = 1 \text{ then } F(x, y, \max\{t_1, t_2\}) = 1.$$

DEFINITION 3: A t-norm is a function  $\Delta: [0,1] \times [0,1] \rightarrow [0,1]$  satisfying the following conditions:

$$(3.1) \Delta(a, b) \geq \Delta(c, d) \text{ for } a \geq c, b \geq d,$$

$$(3.2) \Delta(a, b) = \Delta(b, a)$$

$$(3.3) \Delta(a, 1) = a,$$

$$(3.4) \Delta(\Delta(a, b), c) = \Delta(a, \Delta(b, c))$$

DEFINITION 4: A non-Archimedean Menger PM-space is an ordered triplet  $(X, \mathbb{F}, \Delta)$  where  $\Delta$  is t-norm and  $(X, \mathbb{F})$  is a non-Archimedean PM-space satisfying the following condition:

$$(4.1) F(x, z, \max\{t_1, t_2\}) \geq \Delta(F(x, y, t_1), F(y, z, t_2)) \text{ for all } x, y, z \in X \text{ and } t_1, t_2 \geq 0.$$

DEFINITION 5: A PM-space  $(X, \mathbb{F})$  is said to be type  $(C)_g$  if there exists a  $g \in \Omega$  such that

$$(5.1) g(F(x, y, t)) \leq g(F(x, z, t)) + g(F(z, y, t)) \text{ for all } x, y, z \in X \text{ and } t \geq 0$$

where  $\Omega = \{g/g : [0, 1] \rightarrow [0, \infty) \text{ is continuous, strictly decreasing, } g(1)=0\}$ .

DEFINITION 6: A non-Archimedean Menger PM-space  $(X, \mathbb{F}, \Delta)$  is said to be type  $(D)_g$  if there exists a  $g \in \Omega$  such that

$$(6.1) g(\Delta(s, t)) \leq g(s) + g(t) \text{ for all } s, t \in [0, 1].$$

Note : If a N.A. PM-space  $(X, \mathbb{F}, \Delta)$  is of type  $(D)_g$  then it is of type  $(C)_g$ . Throughout this paper, let  $(X, \mathbb{F}, \Delta)$  be a N.A. PM-space of type  $(D)_g$  with a continuous strictly increasing t-norm  $\Delta$ . Here afterwards we denote  $g(F(x, y, t))$  by  $\theta(x, y, t)$ .

DEFINITION 7: Let  $f, S : X \rightarrow X$  be mappings. The pair  $(f, S)$  is said to be partially commuting (or coincidentally commuting or weak-compatible) at  $z$  if  $fz = Sz$  provided there exists  $w \in X$  such that  $fw = Sw = z$ .

DEFINITION 8 ([4]) : Let  $f, S : X \rightarrow X$  be mappings.  $f$  and  $S$  are said to be compatible if  $\lim_{n \rightarrow \infty} \theta(fSx_n, Sfx_n, t) = 0$  for all  $t > 0$ , whenever  $\{x_n\}$  is a sequence in  $X$  such that  $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} Sx_n$  for some  $z \in X$ .

DEFINITION 9 ([4]) : Let  $f, S : X \rightarrow X$  be mappings.  $f$  and  $S$  are said to be compatible of type(A) if  $\lim_{n \rightarrow \infty} \theta(fSx_n, SS_n, t) = 0$  and

$$\lim_{n \rightarrow \infty} \theta(Sfx_n, ffx_n, t) = 0 \text{ for all } t > 0, \text{ whenever } \{x_n\} \text{ is a sequence in } X \text{ such that } \lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} Sx_n \text{ for some } z \in X.$$

Now we give the following definition.

DEFINITION 10: Let  $f, S : X \rightarrow X$  be mappings. The ordered pair  $(f, S)$  is said to be weakly  $f$ -compatible at  $z$  if either  $\lim_{n \rightarrow \infty} \theta(Sfx_n, fz, t) = 0$  or  $\lim_{n \rightarrow \infty} \theta(SSx_n, fz, t) = 0$  whenever  $\{x_n\}$  is a sequence in  $X$  such that  $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} Sx_n = z$  and  $\lim_{n \rightarrow \infty} fSx_n = \lim_{n \rightarrow \infty} ffx_n = fz$  for some  $z \in X$ .

REMARK 11: (i) If  $(f, S)$  is weakly  $f$ -compatible at  $z$  then it is partially commuting at  $z$ .

(ii) If  $f$  and  $S$  are compatible or compatible of type (A) then the ordered pair  $(f, S)$  is weakly  $f$ -compatible. The converse need not be true in view of the following example in metric space.

EXAMPLE 12: Let  $X = [0,1]$  with usual metric  $d$ . Define  $f, S : X \rightarrow X$  by  $fx = 1 - x$  and

$$Sx = \begin{cases} x & \text{if } 0 \leq x \leq 1/2, \\ 1 & \text{if } 1/2 < x \leq 1. \end{cases}$$

Let  $\{x_n\}$  be a sequence in  $X$  such that  $x_n < 1/2 \forall n$  and  $x_n \rightarrow 1/2$ .

Then  $fx_n = 1 - x_n \rightarrow 1/2$  and  $Sx_n = x_n \rightarrow 1/2$ .

Also  $fSx_n = 1 - x_n \rightarrow 1/2 = f(1/2)$ ,  $ffx_n = x_n \rightarrow 1/2 = f(1/2)$ ,

$Sfx_n = 1$ ,  $SSx_n = x_n \rightarrow 1/2$ .

Clearly  $(f, S)$  is weakly  $f$ -compatible at  $1/2$ .

Since  $d(fSx_n, Sfx_n) = x_n \rightarrow 1/2$ , it follows that  $f$  and  $S$  are not compatible.

Since  $d(Sfx_n, ffx_n) = 1 - x_n \rightarrow 1/2$ , it follows that  $f$  and  $S$  are not compatible of type (A).

We need the following Lemma.

LEMMA 13(Lemma 1.2.of Cho.et.al.[4]): Let  $\{y_n\}$  be a sequence in  $X$  such that  $F(y_n, y_{n+1}, t) = 1$  for all  $t > 0$ . If the sequence  $\{y_n\}$  is not a Cauchy sequence in  $X$ , then there exist  $\varepsilon_0 > 0$ ,  $t_0 > 0$ , two sequences  $\{m_k\}$ ,  $\{n_k\}$  of positive integers such that

(13.1)  $m_k > n_k + 1$  and  $n_k \rightarrow \infty$  as  $k \rightarrow \infty$ ,

(13.2)  $F(y_{m_k}, y_{n_k}, t_0) < 1 - \varepsilon_0$  and  $F(y_{m_k-1}, y_{n_k}, t_0) \geq 1 - \varepsilon_0$ ,  $k = 1, 2, \dots$

Main Theorem:

THEOREM 14: Let  $A, B, S$  and  $T$  be self maps on  $X$  satisfying

(14.1)  $\theta(Ax, By, t) \leq \Psi(\theta(Sx, Ty, t))$  for all  $t > 0$  and for all  $x, y \in X$  with  $Ax = Ty$  or  $By = Sx$  and

(14.2)  $\theta(Ax, By, t) \leq \Psi(\max\{\theta(Sx, Ty, t) + \theta(Ax, Sx, t) + \theta(Bx, Ty, t), \theta(Sx, By, t), \theta(Bx, Ty, t) + \theta(Ax, Ty, t)\})$

for all  $t > 0$  and for all  $x, y \in X$ , where  $\Psi : IR^+ \rightarrow IR^+$  is monotonically increasing and  $\Psi(t) < t$  for all  $t > 0$ .

Suppose that for some  $x_0 \in X$ , there exists a sequence  $\{x_n\}$  in  $X$  such that  $Ax_{2n} = Tx_{2n+1}(= y_{2n}, \text{ say})$  and  $Bx_{2n+1} = Sx_{2n+2}(= y_{2n+1}, \text{ say})$  for  $n = 0, 1, \dots$ . Then  $\{y_n\}$  is a Cauchy sequence in  $X$ .

Further assume that  $\{y_n\}$  converges to some  $z \in X$ . Then  $z$  is the unique common fixed point of  $A, B, S$  and  $T$  if one of the following statements is true.

(i)  $(A, S)$  is  $A$ -continuous at  $z$  and  $(A, S)$  is weakly  $A$ -compatible at  $z$ ,  $(B, T)$  is partially commuting at  $z$ ,  $Az \in T(X)$  and  $Bz \in S(X)$ .

(ii)  $(B, T)$  is  $B$ -continuous at  $z$  and  $(B, T)$  is weakly  $B$ -compatible at  $z$ ,  $(A, S)$

is partially commuting at  $z$ ,  $Az \in T(X)$  and  $Bz \in S(X)$ .

(iii)  $(A, S)$  is  $S$ -continuous at  $z$  and  $(A, S)$  is weakly  $S$ -compatible at  $z$ ,  $(B, T)$

is partially commuting at  $z$  and  $Az \in T(X)$ .

(iv)  $(B, T)$  is  $T$ -continuous at  $z$  and  $(B, T)$  is weakly  $T$ -compatible at  $z$ ,  $(A, S)$

is partially commuting at  $z$  and  $Bz \in S(X)$ .

PROOF: Since  $Ax_{2n} = Tx_{2n+1}$  from (14.1) we have

$$\theta(y_{2n}, y_{2n+1}, t) = \theta(Ax_{2n}, Bx_{2n+1}, t) \leq \Psi(\theta(y_{2n-1}, y_{2n}, t)).$$

Since  $Sx_{2n} = Bx_{2n-1}$  from (14.1) we have

$$\theta(y_{2n}, y_{2n-1}, t) = \theta(Ax_{2n}, Bx_{2n-1}, t) \leq \Psi(\theta(y_{2n-1}, y_{2n-2}, t)).$$

Thus  $\theta(y_n, y_{n+1}, t) \leq \Psi(\theta(y_{n-1}, y_n, t))$  for  $n = 1, 2, \dots$

Hence  $\theta(y_n, y_{n+1}, t) \leq \Psi^n(\theta(y_0, y_1, t))$  for  $n = 1, 2, \dots$

Since  $\Psi$  is monotonically increasing and  $\Psi(t) < t$  for all  $t > 0$  it follows that  $\Psi^n(t) \rightarrow 0$  as  $n \rightarrow \infty$  for any  $t > 0$ . Hence

(I)  $\theta(y_n, y_{n+1}, t) \rightarrow 0$  as  $n \rightarrow \infty$ .

Suppose  $\{y_n\}$  is not a Cauchy sequence. Since  $g$  is strictly decreasing, by Lemma (13), there exist  $\varepsilon_0 > 0$ ,  $t_0 > 0$  and two sequences  $\{m_k\}, \{n_k\}$  of positive integers such that

(a)  $m_k > n_k + 1$  and  $n_k \rightarrow \infty$  as  $k \rightarrow \infty$ ,

(b)  $\theta(y_{m_k}, y_{n_k}, t_0) > g(1 - \varepsilon_0)$  and  $\theta(y_{m_k-1}, y_{n_k}, t_0) \leq g(1 - \varepsilon_0)$  for  $k = 1, 2, \dots$

Now

$$\begin{aligned} g(1 - \varepsilon_0) &< \theta(y_{m_k}, y_{n_k}, t_0) \leq \\ &\leq \theta(y_{m_k}, y_{m_k-1}, t_0) + \theta(y_{m_k-1}, y_{n_k}, t_0) \leq \theta(y_{m_k}, y_{m_k-1}, t_0) + g(1 - \varepsilon_0). \end{aligned}$$

Letting  $k \rightarrow \infty$  we get

$$(II) \lim_{n \rightarrow \infty} \theta(y_{m_k}, y_{n_k}, t_0) = g(1 - \varepsilon_0)$$

On the otherhand, we have

$$(III) g(1 - \varepsilon_0) < \theta(y_{m_k}, y_{n_k}, t_0) \leq \theta(y_{m_k}, y_{n_k+1}, t_0) + \theta(y_{n_k+1}, y_{n_k}, t_0)$$

Without loss of generality assume that both  $m_k$  and  $n_k$  are even.

$$\begin{aligned} \theta(y_{m_k}, y_{n_k+1}, t_0) &= \theta(Ax_{m_k}, Bx_{n_k+1}, t_0) \\ &\leq \Psi(\max\{\theta(y_{m_k-1}, y_{n_k}, t_0) + \theta(y_{m_k}, y_{m_k-1}, t_0) + \theta(y_{n_k+1}, y_{n_k}, t_0), \\ &\quad \theta(y_{m_k}, y_{m_k-1}, t_0) + \theta(y_{m_k-1}, y_{n_k+1}, t_0), \\ &\quad \theta(y_{n_k+1}, y_{n_k}, t_0) + \theta(y_{m_k}, y_{n_k}, t_0)\}) \\ &\leq \Psi(\max\{g(1 - \varepsilon_0) + \theta(y_{m_k}, y_{m_k-1}, t_0) + \theta(y_{n_k+1}, y_{n_k}, t_0), \\ &\quad \theta(y_{m_k}, y_{m_k-1}, t_0) + g(1 - \varepsilon_0) + \theta(y_{n_k}, y_{n_k+1}, t_0), \\ &\quad \theta(y_{n_k+1}, y_{n_k}, t_0) + \theta(y_{m_k}, y_{n_k}, t_0)\}) \end{aligned}$$

Substituting this in (III), letting  $k \rightarrow \infty$  and using (I), (II)

we get  $g(1 - \varepsilon_0) \leq \Psi(g(1 - \varepsilon_0)) < g(1 - \varepsilon_0)$  which is a contradiction.

Hence  $\{y_n\}$  is a Cauchy sequence in  $X$ .

Further assume that  $\{y_n\}$  converges to some  $z \in X$ .

(i) Suppose that the statement (i) is true.

Since  $\{Ax_{2n}\}$  and  $\{Sx_{2n}\}$  converge to  $z$  and  $(A, S)$  is  $A$ -continuous at  $z$  we have  $\{AAx_{2n}\}$  and  $\{ASx_{2n}\}$  converge to  $Az$ .

Since  $(A, S)$  is weakly  $A$ -compatible at  $z$  we have either  $\{SAx_{2n}\}$  or  $\{SSx_{2n}\}$  converge to  $Az$ .

Case :- Suppose  $\{SAx_{2n}\}$  converges to  $Az$ .

$$\begin{aligned} \theta(AAx_{2n}, Bx_{2n+1}, t) &\leq \Psi(\max\{\theta(SAx_{2n}, Tx_{2n+1}, t) + \theta(AAx_{2n}, SAx_{2n}, t) + \\ &\theta(Bx_{2n+1}, Tx_{2n+1}, t), \theta(AAx_{2n}, SAx_{2n}, t) + \theta(SAx_{2n}, Bx_{2n+1}, t), \\ &\theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(AAx_{2n}, Tx_{2n+1}, t)\}). \end{aligned}$$

Letting  $n \rightarrow \infty$  we get

$$\begin{aligned} \theta(Az, z, t) &\leq \Psi(\max\{\theta(Az, z, t) + \theta(Az, Az, t) + \theta(z, z, t), \theta(Az, Az, t) + \\ &+ \theta(Az, z, t), \theta(z, z, t) + \theta(Az, z, t)\}) \end{aligned}$$

Case:- Suppose  $\{SSx_{2n}\}$  converges to  $Az$ .

$$\begin{aligned} \theta(ASx_{2n}, Bx_{2n+1}, t) &\leq \Psi(\max\{\theta(SSx_{2n}, Tx_{2n+1}, t) + \theta(ASx_{2n}, SSx_{2n}, t) + \\ &\theta(Bx_{2n+1}, Tx_{2n+1}, t), \theta(ASx_{2n}, SSx_{2n}, t) + \theta(SSx_{2n}, Bx_{2n+1}, t), \\ &\theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(ASx_{2n}, Tx_{2n+1}, t)\}). \end{aligned}$$

Letting  $n \rightarrow \infty$  we get

$$\begin{aligned} \theta(Az, z, t) &\leq \Psi(\max\{\theta(Az, z, t) + \theta(Az, Az, t) + \theta(z, z, t), \\ &\theta(Az, Az, t) + \theta(Az, z, t), \theta(z, z, t) + \theta(Az, z, t)\}) \\ &= \Psi(\theta(Az, z, t)) \text{ which implies that } Az = z. \end{aligned}$$

Since  $z = Az \in T(X)$ , there exists  $w \in X$  such that  $z = Tw$ .

$$\begin{aligned} \theta(Ax_{2n}, Bw, t) &\leq \Psi(\max\{\theta(Sx_{2n}, Tw, t) + \theta(Ax_{2n}, Sx_{2n}, t) + \theta(Bw, Tw, t), \\ &\theta(Ax_{2n}, Sx_{2n}, t) + \theta(Sx_{2n}, Bw, t), \\ &\theta(Bw, Tw, t) + \theta(Ax_{2n}, Tw, t)\}) \end{aligned}$$

Letting  $n \rightarrow \infty$  we get

$$\begin{aligned} \theta(z, Bw, t) &\leq \Psi(\max\{\theta(z, z, t) + \theta(z, z, t) + \theta(Bw, z, t), \theta(z, z, t) + \theta(z, Bw, t), \\ &\theta(Bw, z, t) + \theta(z, z, t)\}) \\ &= \Psi(\theta(z, Bw, t)) \text{ which implies that } Bw = z. \end{aligned}$$

Since  $(B, T)$  is partially commuting at  $z$  and  $Bw = Tw = z$ . We have  $Bz = Tz$ .

$$\begin{aligned} \theta(Ax_{2n}, Bz, t) &\leq \Psi(\max\{\theta(Sx_{2n}, Tz, t) + \theta(Ax_{2n}, Sx_{2n}, t) + \theta(Bz, Tz, t), \\ &\theta(Ax_{2n}, Sx_{2n}, t) + \theta(Sx_{2n}, Bz, t), \\ &\theta(Bz, Tz, t) + \theta(Ax_{2n}, Tz, t)\}). \end{aligned}$$

Letting  $n \rightarrow \infty$  we get

$$\begin{aligned} \theta(z, Bz, t) &\leq \Psi(\max\{\theta(z, Bz, t) + \theta(z, z, t) + \theta(Bz, Bz, t), \theta(z, z, t) + \theta(z, Bz, t), \\ &\theta(Bz, Bz, t) + \theta(z, Bz, t)\}) \\ &= \Psi(\theta(z, Bz, t)) \text{ which implies that } Bz = z. \end{aligned}$$

Thus  $Bz = z = Tz$ .

Now  $z = Bz \in S(X)$ , there exists  $v \in X$  such that  $Sv = z$ .

$$\begin{aligned} \theta(Av, Bx_{2n+1}, t) &\leq \Psi(\max\{\theta(Sv, Tx_{2n+1}, t) + \\ &+ \theta(Av, Sv, t) + \theta(Bx_{2n+1}, Tx_{2n+1}, t), \theta(Av, Sv, t) + \\ &+ \theta(Sv, Bx_{2n+1}, t), \theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(Av, Tx_{2n+1}, t)\}). \end{aligned}$$

Letting  $n \rightarrow \infty$  we get

$$\begin{aligned} \theta(Av, z, t) &\leq \Psi(\max\{\theta(z, z, t) + \theta(Av, z, t) + \theta(z, z, t), \theta(Av, z, t) + \theta(z, z, t), \\ &\theta(z, z, t) + \theta(Av, z, t)\}) \\ &= \Psi(\theta(Av, z, t)) \text{ which implies that } Av = z. \end{aligned}$$

Thus  $Av = Sv = z$ .

Since  $(A, S)$  is weakly  $A$ -compatible at  $z$  it is partially commuting at  $z$ .

Hence  $Az = Sz$  so that  $z = Az = Sz$ .

Thus  $z$  is a common fixed point of  $A, B, S$  and  $T$ .

Uniqueness of common fixed point follows easily from (14.2).

(ii) Proof follows as in (i).

(iii) Suppose the statement (iii) is true.

Since  $\{Ax_{2n}\}$  and  $\{Sx_{2n}\}$  converge to  $z$  and  $(A, S)$  is  $S$ -continuous at  $z$  we have  $\{SAx_{2n}\}$  and  $\{SSx_{2n}\}$  converge to  $Sz$ .

Since  $(A, S)$  is weakly  $S$ -compatible at  $z$  it follows that  $\{ASx_{2n}\}$  or  $\{AAx_{2n}\}$  converges to  $Sz$ .

Case:- Suppose  $\{ASx_{2n}\}$  converges to  $Sz$ .

$$\theta(ASx_{2n}, Bx_{2n+1}, t) \leq \Psi(\max\{\theta(SSx_{2n}, Tx_{2n+1}, t) + \theta(ASx_{2n}, SSx_{2n}, t) + \theta(Bx_{2n+1}, Tx_{2n+1}, t),$$

$$\theta(ASx_{2n}, SSx_{2n}, t) + \theta(SSx_{2n}, Bx_{2n+1}, t), \\ \theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(ASx_{2n}, Tx_{2n+1}, t)\})$$

Letting  $n \rightarrow \infty$  we get

$$\theta(Sz, z, t) \leq \Psi(\max\{\theta(Sz, z, t) + \theta(Sz, Sz, t) + \theta(z, z, t), \theta(Sz, Sz, t) + \theta(Sz, z, t), \\ \theta(z, z, t) + \theta(Sz, z, t)\}) \\ = \Psi(\theta(Sz, z, t)) \text{ which implies that } Sz = z.$$

Case:- Suppose  $\{AAx_{2n}\}$  converges to  $Sz$ .

$$\theta(AAx_{2n}, Bx_{2n+1}, t) \leq \Psi(\max\{\theta(SAx_{2n}, Tx_{2n+1}, t) + \theta(AAx_{2n}, SAx_{2n}, t) + \theta(Bx_{2n+1}, Tx_{2n+1}, t),$$

$$\theta(AAx_{2n}, SAx_{2n}, t) + \theta(SAx_{2n}, Bx_{2n+1}, t), \\ \theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(AAx_{2n}, Tx_{2n+1}, t)\}).$$

Letting  $n \rightarrow \infty$  we get

$$\theta(Sz, z, t) \leq \Psi(\max\{\theta(Sz, z, t) + \theta(Sz, Sz, t) + \theta(z, z, t), \theta(Sz, Sz, t) + \theta(Sz, z, t), \\ \theta(z, z, t) + \theta(Sz, z, t)\}) \\ = \Psi(\theta(Sz, z, t)) \text{ which implies that } Sz = z.$$

Now

$$\theta(Az, Bx_{2n+1}, t) \leq \Psi(\max\{\theta(Sz, Tx_{2n+1}, t) + \theta(Az, Sz, t) + \\ + \theta(Bx_{2n+1}, Tx_{2n+1}, t), \theta(Az, Sz, t) + \theta(Sz, Bx_{2n+1}, t), \\ \theta(Bx_{2n+1}, Tx_{2n+1}, t) + \theta(Az, Tx_{2n+1}, t)\}).$$

Letting  $n \rightarrow \infty$  we get

$$\theta(Az, z, t) \leq \Psi(\max\{\theta(z, z, t) + \theta(Az, z, t) + \theta(z, z, t), \theta(Az, z, t) + \theta(z, z, t), \\ \theta(z, z, t) + \theta(Az, z, t)\}) \\ = \Psi(\theta(Az, z, t)) \text{ which implies that } Az = z.$$

Since  $z = Az \in T(X)$  and  $(B, T)$  is partially commuting at  $z$  it follows as in (i) that  $Bz = Tz = z$ .

Thus  $z$  is a common fixed point of  $A, B, S$  and  $T$ .

(iv) Proof follows as in (iii).

Theorem 14 is an improvement of the following theorem.

**THEOREM 15** (Theorem 3.2 of [4]) : Let  $A, B, S, T : X \rightarrow X$  be mappings satisfying

$$(15.1) \quad A(X) \subset T(X) \text{ and } B(X) \subset S(X),$$

$$(15.2) \quad \theta(Ax, By, z) \leq \Psi(\max\{\theta(Sx, Ty, t), \theta(Ax, Sx, t), \theta(By, Ty, t), \\ 1/2[\theta(Sx, By, t) + \theta(Ty, Ax, t)]\})$$

for all  $t > 0$  and for all  $x, y \in X$  where  $\Psi : IR^+ \rightarrow IR^+$  is upper semi continuous from the right and  $\Psi(t) < t$  for all  $t > 0$ .

(15.3)  $S$  or  $T$  is continuous,

(15.4) the pairs  $(A, S)$  and  $(B, T)$  are compatible of type  $(A)$ .

Then  $A, B, S$  and  $T$  have a unique common fixed point in  $X$ .

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