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COMMON FIXED POINT THEOREMS FOR OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS VIA IMPLICIT RELATIONS

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Abstract

We prove common fixed point theorems for four mappings satisfying implicit relations in symmetric spaces using the concept of occasionally weakly compatible mappings. Our Theorems generalize results of [1], [3], [4] and [7].

1 Introduction and Preliminaries

Let A and S be self-mappings of a metric space (X, d) and $C(A, S)$ the set of coincidence points of A and S.

Definition 1.1 [5]. A and S are said to be weakly compatible if $SAu = ASu$ for all $u \in C(A, S)$.

Definition 1.2 [2]. A and S are said to be occasionally weakly compatible if $SAu = ASu$ for some $u \in C(A, S)$.

Remark 1.3 [2] If A and S are weakly compatible, then they are occasionally weakly compatible, but the following Example shows that the converse is not true in general.

Example 1.4. Let $X = [1, \infty)$ with the usual metric. Define A, $S : X \to X$ by: $Ax = 3x - 2$ and $Sx = x^2$. We have $Ax = Sx$ iff $x = 1$ or $x = 2$ and $AS(1) = SA(1) = 1$, but $AS(2) \neq SA(2)$. Therefore, A and S are occasionally weakly compatible, but they are not weakly compatible.

Lemma 1.5 [6]. If A and S have a unique coincidence point $w = Ax = Sx$, then w is the unique common fixed point of A and S.

Definition 1.6. Let X be a set. A symmetric on X is a mapping $d : X \times X \to [0, \infty)$ such that

$d(x, y) = 0$ iff $x = y$ and $d(x, y) = d(y, x)$ for all $x, y \in X$.

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Let K_6 the family of all continuous mappings $F(t_1, t_2, t_3, t_4, t_5, t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ with $t_3 + t_4 \neq 0$ satisfying the following conditions:

(K_1) : K is decreasing in variables t_5 and t_6

(K_2) : there exists $0 \leq h < 1$ such that for all $u, v, w \geq 0$ with

(K_a) : $F(u, v, v, u, u + v, 0) \leq 0$ or

(K_b) : $F(u, v, u, v, 0, u + v) \leq 0$

we have $u \leq hv$.

Let F_6 the family of all continuous mappings $F(t_1, t_2, t_3, t_4, t_5, t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ with $t_3 + t_4 \neq 0$ satisfying the following condition:

(F_1) : there exists $0 \leq h < 1$ such that for all $u, v, w \geq 0$ with

(F_a) : $F(u, v, v, u, w, 0) \leq 0$ or

(F_b) : $F(u, v, u, v, 0, w) \leq 0$

we have $u \leq hv$.

Let H_6 the family of all continuous mappings $H(t_1, t_2, t_3, t_4, t_5, t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ with $t_5 + t_6 \neq 0$ satisfying the following conditions.

(H_1) : there exists $0 \leq h < 1$ such that for all $u, v, w \geq 0$ with

(H_a) : $H(u, v, v, u, w, 0) \leq 0$ or

(H_b) : $H(u, v, u, v, 0, w) \leq 0$

we have $u \leq hv$.

(H_2) : $H(u, u, 0, 0, u, u) > 0$ for all $u > 0$.

Let G_6 the family of all continuous mappings $G(t_1, t_2, t_3, t_4, t_5, t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ with $t_2 + t_4 \neq 0$ satisfying the following conditions:

(G_1) : there exists $0 \leq h < 1$ such that for all $u, v \geq 0$ with

(G_a) : $G(u, v, v, u, w, 0) \leq 0$ or

(G_b) : $G(u, v, u, v, 0, w) \leq 0$

we have $u \leq hv$.

(G_2) : $G(u, u, 0, 0, u, u) > 0$ for all $u > 0$.

The following Theorems were proved by [1].

Theorem 1.7. *Let f, g, S and T be self-mappings of a metric space (X, d) satisfying the following conditions:*

$$S(X) \subset g(X) \text{ and } T(X) \subset f(X) \tag{1.1}$$

$$F(d(Sx, Ty), d(fx, gy), d(fx, Sx), d(gy, Ty), d(fx, Ty), d(Sx, gy)) \leq 0$$

for all $x, y \in X$ if $d(fx, Sx) + d(gy, Ty) \neq 0$, where $F \in F_6$ satisfies (F_1), or

$$d(Sx, Ty) = 0 \text{ if } d(fx, Sx) + d(gy, Ty) = 0.$$

Suppose that one of $S(X), T(X), f(X)$ and $g(X)$ is a complete subspace of X and the pairs (S, f) and (T, g) are weakly compatible. Then, f, g, S and T have a unique common fixed point in X.

Theorem 1.8. *Let f, g, S and T be self-mappings of a metric space (X, d) satisfying* (1.1) *and*

$$H(d(Sx, Ty), d(fx, gy), d(fx, Sx), d(gy, Ty), d(fx, Ty), d(Sx, gy)) \leq 0$$

for all $x, y \in X$ *if* $d(fx, Ty) + d(Sx, gy) \neq 0$, where $H \in H_6$ *satisfies* (H_1) *and* (H_2), *or*

$$d(Sx, Ty) = 0 \text{ if } d(fx, Ty) + d(Sx, gy) = 0.$$

Suppose that one of $S(X), T(X), f(X)$ *and* $g(X)$ *is a complete subspace of* X *and the pairs* (S, f) *and* (T, g) *are weakly compatible. Then,* f, g, S *and* T *have a unique common fixed point in* X.

Theorem 1.9. *Let* f, g, S *and* T *be self-mappings of a metric space* (X, d) *satisfying* (1.1) *and*

$$G(d(Sx, Ty), d(fx, gy), d(fx, Sx), d(gy, Ty), d(fx, Ty), d(Sx, gy)) \leq 0$$

for all $x, y \in X$ *if* $d(fx, gy) + d(gy, Ty) \neq 0$, *where* $G \in C_6$ *satisfies* (G_1) *and* (G_2), *or*

$$d(Sx, Ty) = 0 \text{ if } d(fx, gy) + d(gy, Ty) = 0.$$

Suppose that one of $S(X), T(X), f(X)$ *and* $g(X)$ *is a complete subspace of* X *and the pairs* (S, f) *and* (T, g) *are weakly compatible. Then,* f, g, S *and* T *have a unique common fixed point in* X.

The following Theorems was proved by [7].

Theorem 1.10. *Let* f, g, S *and* T *be self-mappings of a metric space* (X, d) *satisfying* (1.1) and

$$K(d(Sx, Ty), d(fx, gy), d(fx, Sx), d(gy, Ty), d(fx, Ty), d(Sx, gy)) \leq 0$$

for all $x, y \in X$ *if* $d(fx, Sx) + d(gy, Ty) \neq 0$, *where* $K \in K_6$ *satisfies* (K_1) and (K_2), *or*

$$d(Sx, Ty) = 0 \text{ if } d(fx, Sx) + d(gy, Ty) = 0.$$

Suppose that one of $S(X), T(X), f(X)$ *and* $g(X)$ *is a complete subspace of* X *and the pairs* (S, f) *and* (T, g) *are weakly compatible. Then,* f, g, S *and* T *have a unique common fixed point in* X.

It is our purpose in this paper to prove common fixed point theorems for occasionally weakly compatible mappings satisfying implicit relations in symmetric spaces. Our Theorems generalize results of [1], [3], [4] and [7].

2 Implicit relations

Let F_6 the family of all functions $F(t_1, t_2, t_3, t_4, t_5, t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ with $t_3 + t_4 \neq 0$

Example 2.1. $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - a\dfrac{t_2 t_3}{t_3 + t_4} - b\dfrac{t_4 t_5}{t_5 + t_6 + 1}$, $a, b > 0$.

Example 2.2. $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - a\dfrac{t_2 t_4}{t_3 + t_4} - b\dfrac{t_3 t_6}{t_5 + t_6 + 1}$, $a, b > 0$.

Example 2.3. $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - a\dfrac{t_3 t_5 + t_4 t_6}{t_3 + t_4} - bt_2$, $a, b > 0$.

Example 2.4. $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \dfrac{a t_3 t_4 + b t_5 t_6}{t_3 + t_4} - ct_2$, $a, b, c > 0$.

Let H_6 the family of all functions $H(t_1,t_2,t_3,t_4,t_5,t_6):\mathbb{R}_+^6\to\mathbb{R}$ with $t_5+t_6\neq 0$ satisfying the following condition:

(H_1) : $H(u,u,0,0,u,u)>0$ for all $u>0$.

Example 2.5. $H(t_1,t_2,t_3,t_4,t_5,t_6)=t_1-a\dfrac{t_3t_6+t_4t_5}{t_5+t_6}-bt_2$, $a,b>0$ and $b<1$.

Example 2.6. $H(t_1,t_2,t_3,t_4,t_5,t_6)=t_1-\dfrac{at_3t_5+bt_4t_6}{t_5+t_6}-ct_2$, $a,b,c>0$ and $c<1$.

Let G_6 the family of all functions $G(t_1,t_2,t_3,t_4,t_5,t_6):\mathbb{R}_+^6\to\mathbb{R}$ with $t_2+t_4\neq 0$ satisfying the following condition:

(G_1) : $G(u,u,0,0,u,u)>0$ for all $u>0$.

Example 2.7. $G(t_1,t_2,t_3,t_4,t_5,t_6)=t_1-a_1\dfrac{t_2t_5}{t_2+t_4}-a_2(t_3+t_4)-a_3(t_5+t_6)-a_4t_2$, $a_1,a_2,a_3,a_4>0$ and $a_1+2a_3+a_4<1$.

Example 2.8. $G(t_1,t_2,t_3,t_4,t_5,t_6)=t_1-a\dfrac{t_2t_4}{t_2+t_4}-b\dfrac{t_3t_5}{t_5+t_6+1}$, $a,b>0$.

Example 2.9. $G(t_1,t_2,t_3,t_4,t_5,t_6)=t_1-a\dfrac{t_2t_4}{t_2+t_4}-b\dfrac{t_3t_6}{t_5+t_6+1}$, $a,b>0$.

3 Main Results

Theorem 3.1. *Let f,g,S and T be self-mappings of a symmetric space (X,d) satisfying the following condition:*

$$F(d(Sx,Ty),d(fx,gy),d(fx,Sx),d(gy,Ty),d(fx,Ty),d(Sx,gy))\leq 0 \tag{3.1}$$

for all $x,y\in X$ if $d(fx,Sx)+d(gy,Ty)\neq 0$, where $F\in F_6$, or

$$d(Sx,Ty)=0 \text{ if } d(fx,Sx)+d(gy,Ty)=0. \tag{3.2}$$

Suppose that the pairs (S,f) and (T,g) are occasionally weakly compatible. Then, f,g,S and T have a unique common fixed point in X.

Proof. Since the pairs (S,f) and (T,g) are occasionally weakly compatible, there exist $u,v\in X$ such that $fu=Su$ and $gv=Tv$. As $d(fu,Su)+d(gv,Tv)=0$, it follows from (3.2) that $Su=Tv$ and so $fu=Su=gv=Tv$. Moreover, if there is another point u' such that $fu'=Su'$, using (3.2) it follows that $fu'=Su'=gv=Tv$. Therefore, $z=fu=Su$ is the unique point of coincidence of f and S. By Lemma 1.5, z is the unique common fixed point of f and S. Similarly, z' is the unique common fixed point of g and T. On the other hand, $d(fz,Sz)+d(gz',Tz')=0$ implies that $d(Sz,Tz')=0$, hence $z=fz=Sz=gz'=Tz'=z'$. Therefore, z is the unique common fixed point of f,g,S and T.

Corollary 3.2. (Theorem 1.7).

Proof. By Theorem 1.7, there exists u,v in X such that $z=Tv=gv=Su=fu$. Since weakly compatible mappings are occasionally weakly compatible, then the conclusion follows from Theorem 3.l.

Corollary 3.3. (Theorem 1.10).

Theorem 3.4. *Let f, g, S and T be self-mappings of a symmetric space (X, d) satisfying: the following condition:*

$$H(d(Sx,Ty), d(fx,gy), d(fx,Sx), d(gy,Ty), d(fx,Ty), d(Sx,gy)) \leq 0 \tag{3.3}$$

for all $x, y \in X$ if $d(fx,Ty) + d(Sx,gy) \neq 0$, where *$H \in H_6$ satisfies (H_1), or*

$$d(Sx,Ty) = 0 \text{ if } d(fx,Ty) + d(Sx,gy) = 0. \tag{3.4}$$

Suppose that the pairs (S, f) and (T, g) are occasionally weakly compatible. Then, f, g, S and T have a unique common fixed point in X.

Proof. Since the pairs (S, f) and (T, g) are occasionally weakly compatible, there exist $u, v \in X$ such that $fu = Su$ and $gv = Tv$. Assume that $Su \neq Tv$. As $d(fu,Tv) + d(Su,gv) \neq 0$, using (3.3) we have

$$H(d(Su,Tv), d(Su,Tv), 0, 0, d(Su,Tv), d(Su,Tv)) \leq 0$$

which is a contradiction of (H_1) and so $fu = Su = gv = Tv$. The rest of the proof follows as in Theorem 3.1.

Corollary 3.5 (Theorem 1.8).

Theorem 3.6. *Let f, g, S and T be self-mappings of a symmetric space (X, d) satisfying the following condition:*

$$G(d(Sx,Ty), d(fx,gy), d(fx,Sx), d(gy,Ty), d(fx,Ty), d(Sx,gy)) \leq 0 \tag{3.5}$$

for all $x, y \in X$ if $d(fx,gy) + d(gy,Ty) \neq 0$, where $G \in G_6$ satisfies (G_1), or

$$d(Sx,Ty) = 0 \text{ if } d(fx,gy) + d(gy,Ty) = 0. \tag{3.6}$$

Suppose that the pairs (S, f) and (T, g) are occasionally weakly compatible. Then, f, g, S and T have a unique common fixed point in X.

Proof. It follows as in Theorem 3.4.

Corollary 3.7 (Theorem 1.9).

Corollary 3.8 (Theorem of [4]).

If $T = S$ and $g = f$ in Theorems 3.1, 3.3 and 3.6, we obtain the following Corollaries which generalize Corollaries of Theorems 1.7, 1.8 and 1.9.

Corollary 3.9. *Let f and S be self-mappings of a symmetric space (X, d) satisfying the following condition:*

$$F(d(Sx,Sy), d(fx,fy), d(fx,Sx), d(fy,Sy), d(fx,Sy), d(Sx,fy)) \leq 0$$

for all $x, y \in X$ if $d(fx,Sx) + d(fy,Sy) \neq 0$, where $F \in F_6$, or

$$d(Sx,Sy) = 0 \text{ if } d(fx,Sx) + d(fy,Sy) = 0.$$

Suppose that the pair (S, f) is occasionally weakly compatible. Then, f and S have a unique common fixed point in X.

Corollary 3.10. *Let f and S be self-mappings of a symmetric space (X, d) satisfying the following condition:*

$$H(d(Sx, Sy), d(fx, fy), d(fx, Sx), d(fy, Sy), d(fx, Sy), d(Sx, fy)) \leq 0$$

for all $x, y \in X$ if $d(fx, Sy) + d(Sx, fy) \neq 0$, where $H \in H_6$ *satisfies (H_1), or*

$$d(Sx, Sy) = 0 \text{ if } d(fx, Sy) + d(Sx, fy) = 0.$$

Suppose that the pair (S, f) is occasionally weakly compatible. Then, f and S have a unique common fixed point in X.

Corollary 3.11. *Let f and S be self-mappings of a symmetric space (X, d) satisfying the following condition:*

$$G(d(Sx, Sy), d(fx, fy), d(fx, Sx), d(fy, Sy), d(fx, Sy), d(Sx, fy)) \leq 0$$

for all $x, y \in X$ if $d(fx, fy) + d(fy, Sy) \neq 0$, where $G \in G_6$ satisfies (G_1), or

$$d(Sx, Sy) = 0 \text{ if } d(fx, fy) + d(fy, Sy) = 0.$$

Suppose that the pair (S, f) is occasionally weakly compatible. Then, f and S have a unique common fixed point in X.

Now, we give Examples to support our Theorems.

Example 3.12. Let $X = [1, \infty)$, $d(x, y) = (x - y)^2$, f, g, S and T are self mappings of X defined by:

$Sx = 3x - 2$, $fx = x^2$, $Tx = 3x^2 - 2$, $gx = x^4$ and $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - a\dfrac{t_3t_5 + t_4t_6}{t_3 + t_4} - bt_2$, $a, b > 0$. It is easy to see that the pairs (f, S) and (g, T) are occasionally weakly compatible, but they are not weakly compatible. We have for all $x, y \in X$

$$\begin{aligned} d(fx, gy) &= (x^2 - y^4)^2 \\ &= (x - y^2)^2(x + y^2)^2 \\ &\geq 4(x - y^2)^2 \\ &= \frac{4}{9}d(Sx, Ty). \end{aligned}$$

Therefore

$$\begin{aligned} & d(Sx, Ty) \leq \frac{9}{4}d(fx, gy) \\ \leq\ & a\frac{d(fx, Sx)d(fx, Ty) + d(gy, Ty)d(Sx, gy)}{d(fx, Sx) + d(gy, Ty)} \\ & + \frac{9}{4}d(fx, gy),\ a > 0, \\ \text{if } d(fx, Sx) + d(gy, Ty) &\neq 0. \\ d(Sx, Ty) &= 0 \text{ if } d(fx, Ty) + d(Sx, gy) = 0 \end{aligned}$$

and so for all $x, y \in X$

$$\begin{array}{rcl} F(d(Sx,Ty), d(fx,gy), d(fx,Sx), d(gy,Ty), d(fx,Ty), d(Sx,gy)) & \leq & 0 \\ \text{if } d(fx,Sx) + d(gy,Ty) & \neq & 0. \end{array}$$

and

$$d(Sx,Ty) = 0 \text{ if } d(fx,Sx) + d(gy,Ty) = 0.$$

Then, all conditions of Theorem 3.1 hold and 1 is the unique common fixed point of f, g, S and T.

Example 3.13. Let $X = \{1\} \cup [\sqrt[3]{3}, \infty)$, $d(x,y) = (x-y)^2$, f, g, S and T are self-mappings of X defined by:

$$Sx = \begin{cases} x^3+1 & \text{if} \quad x \in \{1\} \cup [\sqrt[3]{3}, \infty) \text{ and } x \neq 4 \\ 4 & \text{if} \quad x = 4 \end{cases},$$

$$fx = \begin{cases} 2x^6 & \text{if} \quad x \in \{1\} \cup [\sqrt[3]{3}, \infty) \text{ and } x \neq 4 \\ 4 & \text{if} \quad x = 4 \end{cases},$$

$$Tx = \begin{cases} x^2+1 & \text{if} \quad x \in \{1\} \cup [\sqrt[3]{3}, \infty) \text{ and } x \neq 4 \\ 4 & \text{if} \quad x = 4 \end{cases},$$

$$gx = \begin{cases} 2x^4 & \text{if} \quad x \in \{1\} \cup [\sqrt[3]{3}, \infty) \text{ and } x \neq 4 \\ 4 & \text{if} \quad x = 4 \end{cases}$$

and $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - a\dfrac{t_3t_6 + t_4t_5}{t_5 + t_6} - bt_2$, $a, b > 0$ and $b < 1$.

It is easy to see that the pairs (f, S) and (g, T) are occasionally weakly compatible, but they are not weakly compatible

If $x = y = 4$ or $x = y = 1$, we have

$$d(Sx,Ty) = 0 \text{ since } d(fx,Ty) + d(Sx,gy) = 0.$$

If $x \in [\sqrt[3]{3}, \infty)$, $x \neq 4$ and $y \in \{1\} \cup [\sqrt[3]{3}, \infty)$, $y \neq 4$, we get

$$\begin{array}{rcl} d(fx,gy) & = & 4(x^6 - y^4)^2 \\ & = & 4(x^3 - y^2)^2(x^3 + y^2)^2 \\ & \geq & 64d(Sx,Ty). \end{array}$$

Therefore

$$d(Sx,Ty) \leq \frac{1}{64}d(fx,gy).$$

If $x \in [\sqrt[3]{3}, \infty)$, $x \neq 4$ and $y = 4$, we get

$$d(fx,gy) = 4(x^6 - 2)^2$$

and

$$d(Sx,Ty) = (x^3 - 3)^2.$$

It follows that

$$\begin{aligned}\frac{d(fx,gy)}{d(Sx,Ty)} &= 4(x^3+3+\frac{7}{x^3-3})^2\\ &> 64 \text{ if } x\neq \sqrt[3]{3}.\end{aligned}$$

Hence

$$d(Sx,Ty) < \frac{1}{64}d(fx,gy).$$

Similarly, if $x=4$ and $y\in[\sqrt[3]{3},\infty)$, $y\neq 4$ we get

$$d(Sx,Ty) < \frac{1}{64}d(fx,gy).$$

Then, for all $x,y\in X$

$$\begin{aligned} d(Sx,Ty) &\leq \frac{1}{64}d(fx,gy)+\\ a\frac{d(fx,Sx)d(fx,Ty)+d(gy,Ty)d(Sx,gy)}{d(fx,Sx)+d(gy,Ty)},\ a &> 0 \text{ if } d(fx,Ty)+d(Sx,gy)\neq 0.\\ d(Sx,Ty) &= 0 \text{ if } d(fx,Ty)+d(Sx,gy)=0.\end{aligned}$$

and so for all $x,y\in X$

$$\begin{aligned} H(d(Sx,Ty),d(fx,gy),d(fx,Sx),d(gy,Ty),d(fx,Ty),d(Sx,gy)) &\leq 0\\ \text{if } d(fx,Ty)+d(Sx,gy) &\neq 0.\end{aligned}$$

and

$$d(Sx,Ty)=0 \text{ if } d(fx,Ty)+d(Sx,gy)=0.$$

Then, all conditions of Theorem 3.4 hold and 4 is the unique common fixed point of f,g,S and T.

Taking Example 3.13, It can be verified that for all $x,y\in X$

$$\begin{aligned} G(d(Sx,Ty),d(fx,gy),d(fx,Sx),d(gy,Ty),d(fx,Ty),d(Sx,gy)) &\leq 0\\ \text{if } d(fx,gy)+d(gy,Ty) &\neq 0.\end{aligned}$$

and

$$d(Sx,Ty)=0 \text{ if } \ d(fx,gy)+d(gy,Ty)=0,$$

where $G(t_1,t_2,t_3,t_4,t_5,t_6) = t_1 - a_1\frac{t_2t_5}{t_2+t_4} - a_2(t_3+t_4) - a_3(t_5+t_6) - a_4t_2$, $a_1,a_2,a_3,a_4>0$ and $a_1+2a_3+a_4<1$.

Then, all conditions of Theorem 3.6 hold and 4 is the unique common fixed point of f,g,S and T.

Remark 3.14. Theorems of [1], [3], [4] and [7] can not be applicable since the pairs (f,S) and (g,T) are not weakly compatible and the function d defined in Examples 3.12 and 3.13 is not a metric.

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