

COMMON FIXED POINT THEOREMS OF COMPATIBLE MAPPING OF TYPE (P) IN INTUITIONISTIC FUZZY METRIC SPACES

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The aim of this paper is to point out results of M. Koireng and Yumnam Rahen [10] on compatible mappings of type (P) in fuzzy metric spaces into intuitionistic fuzzy metric spaces with same terminology and notations.

1. Introduction

Fuzzy set theory was introduced by Zadeh in 1965 [23]. Many authors have introduced and discussed several notions of fuzzy metric space in different ways [11], [5], [6] and also proved fixed point theorems with interesting consequent results in the fuzzy metric spaces [7]. Recently the concept of intuitionistic fuzzy metric space was given by Park [13] and the subsequent fixed point results in the intuitionistic fuzzy metric spaces are investigated by Alaca et al. [1] and Mohamad [12] (see, also [2], [3], [18] and [22]).

2. Preliminaries

The study of fixed points of various classes of mappings have been the focus of vigorous research for many Mathematicians. Among them one of the impor-

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tant result in theory of fixed points of compatible mappings was obtained by G. Jungck [8] in 1986. Since then there have been a flood of research papers involving various types of compatibility such as Compatible mappings of type (A) [9], Semi-compatibility [4], compatible mappings of type (P) [14], compatible mappings of type (B) [16] and compatible mappings of type (C) [17] etc.

The following definitions, lemma and examples are useful for our presentations.

Definition 2.1 (See [19]). A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t -norm if the binary operation $*$ satisfying the following conditions:

- (i) $*$ is commutative and associative,
- (ii) $*$ is continuous,
- (iii) $a * 1 = a$ for all $a \in [0, 1]$,
- (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.2 (See [19]). A binary operation \diamond : $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t -conorm if the binary operation \diamond satisfying the following conditions:

- (i) \diamond is commutative and associative,
- (ii) \diamond is continuous,
- (iii) $a \diamond 0 = a$ for all $a \in [0, 1]$,
- (iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.3 (See [1]). A 5- tuple $(X, M, N, *, \diamond)$ is called a intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t -norm, \diamond is a continuous t -conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$

- (IFM - 1) $M(x, y, t) + N(x, y, t) \leq 1$,
- (IFM - 2) $M(x, y, 0) = 0$,
- (IFM - 3) $M(x, y, t) = 1$ if and only if $x = y$,
- (IFM - 4) $M(x, y, t) = M(y, x, t)$,
- (IFM - 5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
- (IFM - 6) $M(x, y, \cdot): (0, \infty) \rightarrow (0, 1]$ is left continuous,
- (IFM - 7) $\lim_{t \rightarrow \infty} M(x, y, t) = 1$,
- (IFM - 8) $N(x, y, 0) = 1$,
- (IFM - 9) $N(x, y, t) = 0$ if and only if $x = y$,
- (IFM - 10) $N(x, y, t) = N(y, x, t)$,
- (IFM - 11) $N(x, y, t) \diamond N(y, z, s) \leq N(x, z, t + s)$,
- (IFM - 12) $N(x, y, \cdot): (0, \infty) \rightarrow (0, 1]$ is right continuous,
- (IFM - 13) $\lim_{t \rightarrow \infty} N(x, y, t) = 0$.

Then (M, N) is called an intuitionistic fuzzy metric on X . The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t , respectively.

Remark 2.4. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \diamond)$ such that t -norm $*$ and t -conorm \diamond are associated, that is, $x \diamond y = 1 - ((1 - x) * (1 - y))$ for all $x, y \in X$.

Example 2.5. (Induced intuitionistic fuzzy metric space) Let (X, d) be a metric space. Define $a * b = ab$ and $a \diamond b = \min\{1, a + b\}$ for all $a, b \in [0, 1]$ and let M_d and N_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \quad N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}.$$

Then $(X, M_d, N_d, *, \diamond)$ is an intuitionistic fuzzy metric induced by metric d the standard intuitionistic fuzzy metric space.

Definition 2.6 (See [1]). Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then

(a) A sequence $\{x_n\}$ in X is said to be convergent to a point x in X if and only if $\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$ and $\lim_{n \rightarrow \infty} N(x_n, x, t) = 0$ for each $t > 0$.

(b) A sequence $\{x_n\}$ in X is called Cauchy sequence if $\lim_{n \rightarrow \infty} M(x_{n+p}, x, t) = 1$ and $\lim_{n \rightarrow \infty} N(x_{n+p}, x, t) = 0$ for each $p > 0$ and $t > 0$.

(c) An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent in X .

Lemma 2.7 (See [20]). Let $\{x_n\}$ be a sequence in an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ with $t * t \geq t$ and $(1 - t) \diamond (1 - t) \leq (1 - t)$ for all $t \in [0, 1]$. If there exists a number $q \in (0, 1)$ such that $M(x_{n+2}, x_{n+1}, qt) \geq M(x_{n+1}, x_n, t)$ and $N(x_{n+2}, x_{n+1}, qt) \leq N(x_{n+1}, x_n, t)$ for all $t > 0$ and $n \in \mathbb{N}$, then $\{x_n\}$ is a Cauchy sequence in X .

Proof. For $t > 0$ and $q \in (0, 1)$ we have,

$$M(x_2, x_3, qt) \geq M(x_1, x_2, t) \geq M(x_0, x_1, \frac{t}{q}),$$

or

$$M(x_2, x_3, t) \geq M(x_0, x_1, \frac{t}{q^2}).$$

By simple induction, we have for all $t > 0$ and $n \in \mathbb{N}$,

$$M(x_{n+1}, x_{n+2}, t) \geq M(x_1, x_2, \frac{t}{q^n}).$$

Thus for any positive number p and real number $t > 0$, we have

$$\begin{aligned} M(x_n, x_{n+p}, t) &\geq M(x_n, x_{n+1}, \frac{t}{p}) * \dots * M(x_{n+p-1}, x_{n+p}, \frac{t}{p}), \text{ by (IFM-5),} \\ &\geq M(x_1, x_2, \frac{t}{pq^{n-1}}) * \dots * M(x_1, x_2, \frac{t}{pq^{n+p-2}}). \end{aligned}$$

Therefore by (IFM – 7), we have

$$M(x_n, x_{n+p}, t) \geq 1 * \dots * 1 \geq 1.$$

Similarly, for $t > 0$ and $q \in (0, 1)$, we have

$$N(x_2, x_3, qt) \leq N(x_1, x_2, t) \leq N(x_0, x_1, \frac{t}{q}),$$

or

$$N(x_2, x_3, t) \leq N(x_0, x_1, \frac{t}{q^2}).$$

By simple induction, we have for all $t > 0$ and $n \in \mathbb{N}$,

$$N(x_{n+1}, x_{n+2}, t) \leq N(x_1, x_2, \frac{t}{q^n}).$$

Thus for any positive number p and real number $t > 0$, we have

$$\begin{aligned} N(x_n, x_{n+p}, t) &\leq N(x_n, x_{n+1}, \frac{t}{p}) \diamond \dots \diamond N(x_{n+p-1}, x_{n+p}, \frac{t}{p}), \text{ by (IFM – 11),} \\ &\leq N(x_1, x_2, \frac{t}{pq^{n-1}}) \diamond \dots \diamond N(x_1, x_2, \frac{t}{pq^{n+p-2}}). \end{aligned}$$

Therefore by (IFM – 13), we have

$$N(x_n, x_{n+p}, t) \leq 0 \diamond \dots \diamond 0 \leq 0.$$

This implies that $\{x_n\}$ is a Cauchy sequence in X . This completes the proof. \square

Lemma 2.8 (See [20]). *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. If $\forall x, y \in X$ and $t > 0$ with positive number $q \in (0, 1)$ and $M(x, y, qt) \geq M(x, y, t)$ and $N(x, y, qt) \leq N(x, y, t)$, then $x = y$.*

Proof. If for all $t > 0$ and some constant $q \in (0, 1)$, then we have

$$M(x, y, t) \geq M(x, y, \frac{t}{q}) \geq M(x, y, \frac{t}{q^2}) \geq \dots \geq M(x, y, \frac{t}{q^n}) \geq \dots,$$

and

$$N(x, y, t) \leq N(x, y, \frac{t}{q}) \leq N(x, y, \frac{t}{q^2}) \leq \dots \leq N(x, y, \frac{t}{q^n}) \leq \dots,$$

$n \in \mathbb{N}$ and for all $t > 0$ and $x, y \in X$. When $n \rightarrow \infty$, we have $M(x, y, t) = 1$ and $N(x, y, t) = 0$ and thus $x = y$. This completes the proof. \square

Definition 2.9 (See [21]). Two self mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ are called compatible if

$$\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(ASx_n, SAx_n, t) = 0$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x$ for some $x \in X$.

Definition 2.10. Two self mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ are called compatible of type (P) if

$$\lim_{n \rightarrow \infty} M(AAx_n, SSx_n, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(AAx_n, SSx_n, t) = 0$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = x$ for some $x \in X$.

Theorem 2.11. Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be self mappings of X satisfying the following conditions:

- (i) $A(X) \subseteq T(X), B(X) \subseteq S(X)$,
 - (ii) S and T are continuous,
 - (iii) the pairs $\{A, S\}$ and $\{B, T\}$ are compatible mappings of type (P) on X ,
 - (iv) there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Bx, Ty, t) * M(Ax, Ty, t).$$
- Then A, B, S and T have a unique common fixed point in X .

The aim of this paper is to extends Theorem 2.11 in the framework of intuitionistic fuzzy metric space.

3. Main Results

Theorem 3.1. Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying the following conditions:

- (i) $A(X) \subseteq T(X), B(X) \subseteq S(X)$,
- (ii) S and T are continuous,
- (iii) the pairs $\{A, S\}$ and $\{B, T\}$ are compatible mappings of type (P) on X ,
- (iv) there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Bx, Ty, t) * M(Ax, Ty, t)$$

and

$$N(Ax, By, qt) \leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(Bx, Ty, t) \diamond N(Ax, Ty, t).$$

Then A, B, S and T have a unique common fixed point in X .

Proof. Since $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$. We define a sequence $\{y_n\}$ such that

$$y_{2n-1} = Tx_{2n-1} = Ax_{2n-2} \quad \text{and} \quad y_{2n} = Sx_{2n} = Bx_{2n-1}, \quad \forall n \in \mathbb{N}.$$

We shall prove that $\{y_n\}$ is a Cauchy sequence. From (iv), we have

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, qt) &= M(Ax_{2n}, Bx_{2n+1}, qt) \\ &\geq M(Sx_{2n}, Tx_{2n+1}, t) * M(Ax_{2n}, Sx_{2n}, t) \\ &* M(Bx_{2n+1}, Tx_{2n+1}, t) * M(Ax_{2n}, Tx_{2n+1}, t) \\ &= M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n}, t) \\ &* M(y_{2n+2}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+1}, t) \\ &\geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t), \end{aligned}$$

which implies

$$M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t).$$

Similarly, we have

$$M(y_{2n+2}, y_{2n+3}, qt) \geq M(y_{2n+1}, y_{2n+2}, t).$$

Hence, we have

$$M(y_{n+1}, y_{n+2}, qt) \geq M(y_n, y_{n+1}, t). \quad (1)$$

Now

$$\begin{aligned} N(y_{2n+1}, y_{2n+2}, qt) &= N(Ax_{2n}, Bx_{2n+1}, qt) \\ &\leq N(Sx_{2n}, Tx_{2n+1}, t) \diamond N(Ax_{2n}, Sx_{2n}, t) \\ &\diamond N(Bx_{2n+1}, Tx_{2n+1}, t) \diamond N(Ax_{2n}, Tx_{2n+1}, t) \\ &= N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n}, t) \\ &\diamond N(y_{2n+2}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n+1}, t) \\ &\leq N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n+2}, t), \end{aligned}$$

which implies

$$N(y_{2n+1}, y_{2n+2}, qt) \leq N(y_{2n}, y_{2n+1}, t).$$

Similarly, we have

$$N(y_{2n+2}, y_{2n+3}, qt) \leq N(y_{2n+1}, y_{2n+2}, t).$$

Hence, we have

$$N(y_{n+1}, y_{n+2}, qt) \leq N(y_n, y_{n+1}, t). \quad (2)$$

Equations (1) and (2) show that $\{y_n\}$ is a Cauchy sequence.

Since X is complete, $\{y_n\}$ converges to some point $z \in X$ and so sequences $\{Ax_{2n-2}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ also converge to z .

Then, we have

$$AAx_{2n-2} \rightarrow Sz \quad \text{and} \quad SSx_{2n} \rightarrow Az, \quad (3)$$

and

$$BBx_{2n-1} \rightarrow Tz \quad \text{and} \quad TTx_{2n-1} \rightarrow Bz. \quad (4)$$

From (iv), we get

$$\begin{aligned} M(AAx_{2n-2}, BBx_{2n-1}, qt) &\geq M(SAx_{2n-2}, TBx_{2n-1}, t) * M(AAx_{2n-2}, SAx_{2n-1}, t) \\ &\quad * M(BBx_{2n-1}, TBx_{2n-1}, t) * M(AAx_{2n-2}, TBx_{2n-1}, t). \end{aligned}$$

Using (3) and (4) and taking the limit as $n \rightarrow \infty$, we have

$$\begin{aligned} M(Sz, Tz, qt) &\geq M(Sz, Tz, t) * M(Sz, Sz, t) \\ &\quad * M(Tz, Tz, t) * M(Sz, Tz, t) \\ &\geq M(Sz, Tz, t) * 1 * 1 * M(Sz, Tz, t) \\ &\geq M(Sz, Tz, t) \end{aligned}$$

implies

$$M(Sz, Tz, qt) \geq M(Sz, Tz, t).$$

Similarly,

$$\begin{aligned} N(AAx_{2n-2}, BBx_{2n-1}, qt) &\leq N(SAx_{2n-2}, TBx_{2n-1}, t) \diamond N(AAx_{2n-2}, SAx_{2n-1}, t) \\ &\quad \diamond N(BBx_{2n-1}, TBx_{2n-1}, t) \diamond N(AAx_{2n-2}, TBx_{2n-1}, t). \end{aligned}$$

Using (3) and (4) and taking the limit as $n \rightarrow \infty$, we have

$$\begin{aligned} N(Sz, Tz, qt) &\leq N(Sz, Tz, t) \diamond N(Sz, Sz, t) \\ &\quad \diamond N(Tz, Tz, t) \diamond N(Sz, Tz, t) \\ &\leq N(Sz, Tz, t) \diamond 0 \diamond 0 \diamond N(Sz, Tz, t) \\ &\leq N(Sz, Tz, t) \end{aligned}$$

implies

$$N(Sz, Tz, qt) \leq N(Sz, Tz, t).$$

It follows that

$$Sz = Tz. \quad (5)$$

Now, again from (iv), we have

$$\begin{aligned} M(Az, BTx_{2n-1}, qt) &\geq M(Sz, TTx_{2n-1}, t) * M(Az, Sz, t) \\ &\quad * M(BTx_{2n-1}, TTx_{2n-1}, t) * M(Az, TTx_{2n-1}, t). \end{aligned}$$

Using (3) and (4) and taking the limit as $n \rightarrow \infty$, we have

$$\begin{aligned} M(Az, Tz, qt) &\geq M(Sz, Sz, t) * M(Az, Tz, t) \\ &\quad * M(Tz, Tz, t) * M(Az, Tz, t) \\ &\geq 1 * M(Az, Tz, t) * 1 * M(Az, Tz, t) \\ &\geq M(Az, Tz, t) \end{aligned}$$

implies

$$M(Az, Tz, qt) \geq M(Az, Tz, t).$$

Similarly,

$$\begin{aligned} N(Az, BTx_{2n-1}, qt) &\leq N(Sz, TTx_{2n-1}, t) \diamond N(AAx_{2n-2}, SAx_{2n-1}, t) \\ &\quad \diamond N(BTx_{2n-1}, TTx_{2n-1}, t) \diamond N(Az, TTx_{2n-1}, t). \end{aligned}$$

Using (3) and (4) and taking the limit as $n \rightarrow \infty$, we have

$$\begin{aligned} N(Az, Tz, qt) &\leq N(Sz, Sz, t) \diamond N(Az, Tz, t) \\ &\quad \diamond N(Tz, Tz, t) \diamond N(Az, Tz, t) \\ &\leq 0 \diamond N(Az, Tz, t) \diamond 0 \diamond N(Az, Tz, t) \\ &\leq N(Az, Tz, t) \end{aligned}$$

implies

$$N(Az, Tz, qt) \leq N(Az, Tz, t).$$

It follows that

$$Az = Tz. \tag{6}$$

Now from (iv) and using (5) and (6), we have

$$\begin{aligned} M(Az, Bz, qt) &\geq M(Sz, Tz, t) * M(Az, Sz, t) \\ &\quad * M(Bz, Tz, t) * M(Az, Tz, t) \\ &= M(Az, Az, t) * M(Az, Az, t) \\ &\quad * M(Bz, Az, t) * M(Az, Az, t) \\ &\geq 1 * 1 * M(Bz, Az, t) * 1 \\ &\geq M(Az, Bz, t) \end{aligned}$$

implies

$$M(Az, Bz, qt) \geq M(Az, Bz, t).$$

Similarly,

$$\begin{aligned} N(Az, Bz, qt) &\leq N(Sz, Tz, t) \diamond N(Az, Sz, t) \\ &\quad \diamond N(Bz, Tz, t) \diamond N(Az, Tz, t) \\ &= N(Az, Az, t) \diamond N(Az, Az, t) \\ &\quad \diamond N(Bz, Az, t) \diamond N(Az, Az, t) \\ &\leq 0 \diamond 0 \diamond N(Bz, Az, t) \diamond 0 \\ &\leq N(Az, Bz, t) \end{aligned}$$

implies

$$N(Az, Bz, qt) \leq N(Az, Bz, t).$$

It follows that

$$Az = Bz. \tag{7}$$

From (5), (6) and (7), we have

$$Az = Bz = Tz = Sz.$$

Now, we shall show that $Bz = z$.

Again from (iv), we have

$$\begin{aligned} M(Ax_{2n}, BTz, qt) &\geq M(Sx_{2n}, Tz, t) * M(Ax_{2n}, Sx_{2n}, t) \\ &\quad * M(BTz, Tz, t) * M(z, Tz, t). \end{aligned}$$

Using (5) and (6) and taking the limit as $n \rightarrow \infty$, we have

$$\begin{aligned} M(z, Bz, qt) &\geq M(z, Tz, t) * M(z, z, t) \\ &\quad * M(Bz, Tz, t) * M(z, Tz, t) \\ &= M(z, Bz, t) * 1 * M(Az, Az, t) * M(z, Bz, t) \\ &\geq M(z, Bz, t) \end{aligned}$$

implies

$$M(z, Bz, qt) \geq M(z, Bz, t).$$

Similarly,

$$N(Ax_{2n}, Bz, qt) \leq N(Sx_{2n}, Tz, t) \diamond N(Ax_{2n}, Sx_{2n}, t) \diamond N(Bz, Tz, t) \diamond N(z, Tz, t).$$

Using (5) and (6) and taking the limit as $n \rightarrow \infty$, we have

$$\begin{aligned} N(z, Bz, qt) &\leq N(z, Tz, t) \diamond N(z, z, t) \diamond N(Bz, Tz, t) \diamond N(z, Tz, t) \\ &= N(z, Bz, t) \diamond 0 \diamond N(Az, Az, t) \diamond N(z, Bz, t) \\ &\leq N(z, Bz, t) \end{aligned}$$

implies

$$N(z, Bz, qt) \leq N(z, Bz, t).$$

It follows that

$$Bz = z. \quad (8)$$

Thus from (8), $z = Az = Bz = Tz = Sz$ and hence z is a common fixed point of the mappings A, B, S and T .

Uniqueness

Let w be another common fixed point of A, B, S and T . Then

$$\begin{aligned} M(z, w, qt) &= M(Az, Bw, qt) \\ &\geq M(Sz, Tw, t) * M(Az, Sz, t) \\ &\quad * M(Bw, Tw, t) * M(Az, Tw, t) \\ &\geq M(z, w, t) \end{aligned}$$

implies

$$M(z, w, qt) \geq M(z, w, t).$$

And

$$\begin{aligned} N(z, w, qt) &= N(Az, Bw, qt) \\ &\leq N(Sz, Tw, t) \diamond N(Az, Sz, t) \\ &\quad \diamond N(Bw, Tw, t) \diamond N(Az, Tw, t) \\ &\leq N(z, w, t) \end{aligned}$$

implies

$$N(z, w, qt) \leq N(z, w, t).$$

Hence $z = w$. This completes the proof. \square

Corollary 3.2. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying the conditions*

(i)-(iii) of Theorem 3.1 and there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$\begin{aligned} &M(Ax, By, qt) \\ &\geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t) * M(Ax, Ty, t) \end{aligned}$$

and

$$\begin{aligned} &N(Ax, By, qt) \\ &\leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(By, Sx, 2t) \diamond N(Ax, Ty, t). \end{aligned}$$

Then A, B, S and T have a unique common fixed point in X .

Corollary 3.3. Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying the conditions (i)-(iii) of Theorem 3.1 and there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Ax, By, qt) \geq M(Sx, Ty, t)$$

and

$$N(Ax, By, qt) \leq N(Sx, Ty, t).$$

Then A, B, S and T have a unique common fixed point in X .

Corollary 3.4. Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying the conditions (i)-(iii) of Theorem 3.1 and there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t)$$

and

$$N(Ax, By, qt) \leq N(Sx, Ty, t) \diamond N(Sx, Ax, t) \diamond N(Ax, Ty, t).$$

Then A, B, S and T have a unique common fixed point in X .

Theorem 3.5. Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space. If S and T are continuous self mappings of X , then mappings S and T have a common fixed point in X if and only if there exists a self mapping A of X satisfying the following conditions:

- (i) $A(X) \subseteq T(X) \cap S(X)$,
- (ii) the pairs $\{A, S\}$ and $\{A, T\}$ are compatible mappings of type (P) on X ,
- (iii) there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t)$$

and

$$N(Ax, Ay, qt) \leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(Ay, Ty, t) \diamond N(Ax, Ty, t).$$

Then A , S and T have a unique common fixed point in X .

Proof. Necessary part. Let S and T have a common fixed point in X , say z , then $Sz = z = Tz$. Let $Ax = z$ for all $x \in X$, then $A(X) \subseteq T(X) \cap S(X)$ and we know that $\{A, S\}$ and $\{A, T\}$ are compatible mappings of type (P) , in fact $A \circ S = S \circ A$ and $A \circ T = T \circ A$ and hence the conditions (i) and (ii) are satisfied. For some $q \in (0, 1)$, we have

$$M(Ax, Ay, qt) = 1 \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t)$$

and

$$N(Ax, Ay, qt) = 0 \leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(Ay, Ty, t) \diamond N(Ax, Ty, t)$$

for all $x, y \in X$ and $t > 0$. Hence the condition (iii) is satisfied.

Sufficient part. Let $A = B$ in Theorem 3.1. Then A , S and T have a unique common fixed point in X . This completes the proof. \square

Corollary 3.6. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space. If S and T are continuous self mappings of X , then mappings S and T have a common fixed point in X if and only if there exists a self mapping A of X satisfying the conditions (i)-(ii) of Theorem 3.5 and there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,*

$$\begin{aligned} M(Ax, Ay, qt) \\ \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Sx, 2t) * M(Ax, Ty, t) \end{aligned}$$

and

$$\begin{aligned} N(Ax, Ay, qt) \\ \leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(Ay, Ty, t) \diamond N(Ax, Sx, 2t) \diamond N(Ax, Ty, t). \end{aligned}$$

Then A , S and T have a unique common fixed point in X .

Corollary 3.7. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space. If S and T are continuous self mappings of X , then mappings S and T have a common fixed point in X if and only if there exists a self mapping A of X satisfying the conditions (i)-(ii) of Theorem 3.5 and there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,*

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t)$$

and

$$N(Ax, Ay, qt) \leq N(Sx, Ty, t).$$

Then A, S and T have a unique common fixed point in X .

Corollary 3.8. Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space. If S and T are continuous self mappings of X , then mappings S and T have a common fixed point in X if and only if there exists a self mapping A of X satisfying the conditions (i)-(ii) of Theorem 3.5 and there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t)$$

and

$$N(Ax, Ay, qt) \leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(Ax, Ty, t).$$

Then A, S and T have a unique common fixed point in X .

Example 3.9. Let $X = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$ with $*$ continuous t -norm and \diamond continuous t -conorm defined by $a * b = ab$ and $a \diamond b = \min\{1, a + b\}$ respectively, for $a, b \in [0, 1]$. For each $t \in [0, \infty)$ and $x, y \in X$, define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t + |x - y|}, & \text{if } t > 0, \\ 0, & \text{if } t = 0, \end{cases}$$

and

$$N(x, y, t) = \begin{cases} \frac{|x - y|}{t + |x - y|}, & \text{if } t > 0, \\ 1, & \text{if } t = 0. \end{cases}$$

Clearly $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space.

Define $A(x) = B(x) = \frac{x}{6}$ and $S(x) = T(x) = \frac{x}{2}$ on X . It is clear that $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$.

Now

$$\begin{aligned} M(Ax, By, \frac{t}{3}) &= \frac{\frac{t}{3}}{\frac{t}{3} + \frac{|x - y|}{6}} = \frac{2t}{2t + |x - y|} \\ &\geq \frac{t}{t + \frac{|x - y|}{2}} = \frac{2t}{2t + |x - y|} \\ &= M(Tx, Sy, t), \end{aligned}$$

and

$$\begin{aligned} N(Ax, By, \frac{t}{3}) &= \frac{\frac{|x-y|}{6}}{\frac{t}{3} + \frac{|x-y|}{6}} = \frac{|x-y|}{2t + |x-y|} \\ &\leq \frac{\frac{|x-y|}{2}}{t + \frac{|x-y|}{2}} = \frac{|x-y|}{2t + |x-y|} \\ &= N(Tx, Sy, t). \end{aligned}$$

Thus all the conditions of Theorem 3.1 are satisfied and so A , B , S and T have a unique common fixed point 0 .

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