

Commutation Code Index DCSK Modulation Technique for High-Data-Rate Communication Systems

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Abstract—In this work, a novel non-coherent modulation system named Commutation Code Index Differential Chaos Shift Keying (CCI-DCSK) is presented. In the proposed scheme, the benefits of index modulation (IM) are exploited so that the system energy and spectral efficiencies are improved. We are putting forward a CCI-DCSK scheme in which the reference sequence along with its corresponding data bearing orthogonal version are sent within the same time slot to enhance spectral efficiency. Specifically, at the transmitter p additional bits are mapped into the index of the repetition commutation which is performed on the reference signal for the sake of forming its orthogonal version. Furthermore, the modulated bit is spread by the corresponding commutated replica of the reference signal. The reference and orthogonal data bearing signals are then summed within the same time slot prior to transmission. At the receiver, the received signal is first correlated with all possible combinations of the commutated replicas in order to find the index of the correlator output with the maximum magnitude. Subsequently, the maximizing index is then used to estimate the mapped bits, while the output of the corresponding correlator is used to despread the modulated bit via a zero-threshold comparator. To complete the study, analytical expressions for the bit error rate (BER) are derived for both additive white Gaussian noise (AWGN) and multipath Rayleigh fading channels. Finally, performance results of the proposed CCI-DCSK scheme is compared with the state-of-the-art IM-based non-coherent chaotic modulations and is found to be superior and competitive.

Index Terms—Commutation code index modulation, bit error rate (BER), non-coherent detection.

I. INTRODUCTION

EXCELLENT properties of chaotic signals have made them a perfect candidate for spread spectrum (SS) communications. The resistance to multipath propagation, low cross and auto-correlation values, easy generation and noise-like behaviour distinguish them from other codes used in SS systems. In particular, differential chaos shift keying (DCSK) [1] is the most popular non-coherent chaotic modulation. In DCSK, each data frame is divided into two time slots, where a reference signal is sent in the first slot followed by a time-delayed data modulated reference signal in the second slot.

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The main drawback of such approach is the halving of the data rate due to the time multiplexing of the reference and data bearing signals, and the reduction of energy efficiency caused by introducing a reference signal in association with the implementation of wideband RF delay lines. To tackle these drawbacks, many papers proposed elegant solutions aiming to improve the data rate and the energy efficiency of transmit reference system [2]–[7].

Recently, a promising modulation technique called index modulation (IM) is proposed to increase data rate, energy and spectral efficiencies, while keeping a relatively simple hardware complexity. The proposed IM uses indices of different system parameters to convey additional data. The IM concept was used for the first time in spatial modulation (SM) [8], which is an energy efficient version of multiple-input multiple-output (MIMO) systems. In SM, transmission takes place via multiple antennas where a single antenna transmits at a time, and the index of the active antenna is exploited to convey an extra information. Furthermore, in orthogonal frequency division multiplexing index modulation (OFDM-IM) [9] additional information is mapped in the indices of active sub-carriers. While the former method increases the hardware complexity of the transmitter, the latter uses the allocated spectrum with poor efficiency as many subcarriers are nulled at each transmission. On the other hand, in [10], [11] the authors propose for the first time an IM technique for SS schemes. This system, called code index modulation (CIM), aims to increase the data rate by mapping additional data in the indices of different codes, which are used for spreading modulated bits. In contrast to OFDM-IM, CIM uses the whole allocated spectrum efficiency. Recently, IM was adopted for usage in non-coherent chaotic modulations. In particular, in carrier index DCSK (CI-DCSK) [12] and orthogonal multi-level DCSK (OM-DCSK) [13], the indexing of active sub-carriers and different Walsh codes combined with Hilbert transform is used for data conveying. The combination of CIM and CS-DCSK scheme, called CIM-CS-DCSK, is proposed in [14].

In this brief, an efficient modulation called commutation code index DCSK (CCI-DCSK) is proposed. CCI-DCSK scheme exploits the benefits of IM by mapping p extra bits in distinct commutated replicas of the reference chaotic sequence, which is then used for spreading a modulated bit. At the transmitter, a block of p mapped bits selects an index of the corresponding commutation operator which will be used

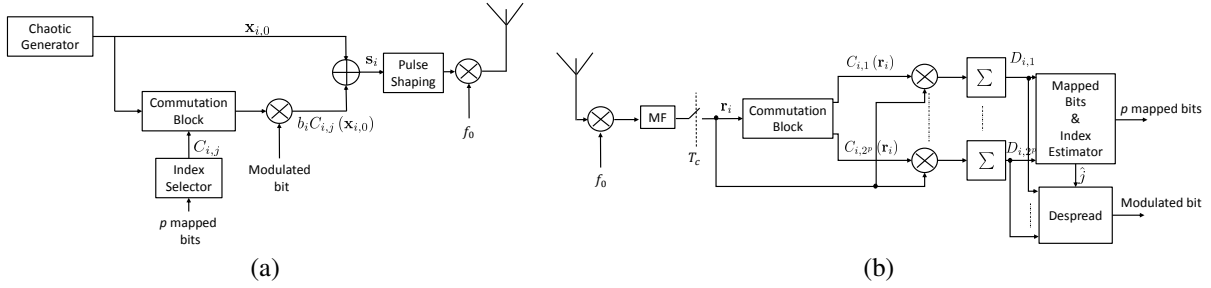


Fig. 1: A block diagram of the general structure of CCI-DCSK communication system a) transmitter, and b) receiver.

to transform the reference signal into its orthogonal version. Subsequently, the formed commutated orthogonal version of the reference signal is used to spread one modulated symbol. Then prior to transmission, the reference and data bearing signals are summed and sent within the very same time slot. At the receiver side, all possible commutations are performed on the received signal and the outcome of each commutation is independently correlated with the non-commutated received signal. The largest correlator output magnitude is used to estimate the index of the commutation which has been performed at the transmitter side. This estimates the mapped bits while the sign of the corresponding correlator output is used to despread the modulated bit. Conclusively, in the proposed CCI-DCSK scheme, the spectral and energy efficiencies are enhanced by mapping p extra bits per transmission in the indices of distinct independent commutations performed on the reference sequence. In the proposed architecture, having the reference signal added up to the data-bearing signal and sent on half the time duration boosts spectral efficiency. Furthermore, nulling out RF delay lines improves the overall system design. This article also presents analytical bit error rate (BER) expressions over Rayleigh multipath fading and additive white Gaussian noise (AWGN) channels, which are derived and validated with simulation results to show the accuracy of the developed analytical model. Finally, the performance of CCI-DCSK is analyzed and compared with state-of-the-art modulation techniques to show its superiority.

II. CCI-DCSK SYSTEM MODEL

In the proposed method, the orthogonal signal set is constructed by commutating different parts of the reference signal. As shown in Fig 2, the reference signal is divided into 2^n parts, where n is a deterministic integer. Then, the different parts of the divided reference sequence are commutated to form different orthogonal sequences. Additionally, in order to maintain a strong orthogonality between the commutated replicas of the reference sequence, all of the 2^n parts of the reference signal should be placed in distinct positions within the different commutated replicas in order to avoid overlapping chaotic parts when performing commutations at the receiver. Moreover, it should be mentioned that the orthogonality between the reference and the commutated signals is credible for high β values. As illustrated in Fig. 1 (a), a block of p mapped bits selects one of the $2^p \leq 2^n$ possible commutations performed on the reference signal to form its orthogonal version. Then, one extra bit is spread with obtained

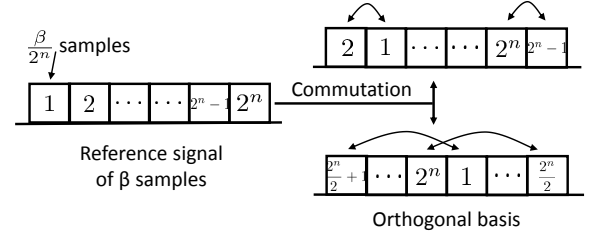


Fig. 2: Design of orthogonal basis signals.

commutated sequence. Subsequently, the reference and data bearing signal are summed within the same time slot prior the transmission. Therefore, the baseband discrete CCI-DCSK i^{th} symbol can be written as

$$\mathbf{s}_i = [\mathbf{x}_{i,0} + b_i C_{i,j}(\mathbf{x}_{i,0})], \quad (1)$$

where b_i is the i^{th} symbol modulated bit, $C_{i,j}(\cdot)$ is the j^{th} commutation operator of the i^{th} data frame, where $j \in \{1, 2, \dots, 2^p\}$. More, $\mathbf{x}_{i,\tau_{i,l}}$ is the i^{th} vector of β chaotic samples with a time delay $\tau_{i,l}$, defined as

$$\mathbf{x}_{i,\tau_{i,l}} = [x_{i,1-\tau_{i,l}}, x_{i,2-\tau_{i,l}}, \dots, x_{i,\beta-\tau_{i,l}}]. \quad (2)$$

In this brief, a commonly used channel model with Rayleigh distributed channel coefficients is used [7]. Furthermore, the channel coefficients are considered to be flat over each symbol duration and that the maximum channel path delay is much lower than $\beta/2^n$, i.e. $\beta/2^n \gg \tau_{i,\max}$. Considering a Rayleigh probability density function (PDF) of the channel coefficients α is given in [7], the received signal would be expressed as

$$\mathbf{r}_i = \sum_{l=1}^L \alpha_{i,l} [\mathbf{x}_{i,\tau_{i,l}} + b_i C_{i,j}(\mathbf{x}_{i,\tau_{i,l}})] + \mathbf{n}_i, \quad (3)$$

where L is the number of paths, while $\alpha_{i,l}$ and $\tau_{i,l}$ represent the l^{th} channel coefficient and path delay, respectively. The vector \mathbf{n}_i is the AWGN component of β samples that contaminates the i^{th} data frame, where each sample has zero mean and variance $N_0/2$. At the receiver, the received signal is correlated with all possible commutations, forming 2^p number of decision variables described as

$$\begin{aligned} D_{i,m} &= \mathbf{r}_i C_{i,m}(\mathbf{r}_i)^T \\ &= \left(\sum_{l=1}^L \alpha_{i,l} [\mathbf{x}_{i,\tau_{i,l}} + b_i C_{i,j}(\mathbf{x}_{i,\tau_{i,l}})] + \mathbf{n}_i \right) \\ &\quad \times C_{i,m} \left(\sum_{l=1}^L \alpha_{i,l} [\mathbf{x}_{i,\tau_{i,l}} + b_i C_{i,j}(\mathbf{x}_{i,\tau_{i,l}})] + \mathbf{n}_i \right)^T, \end{aligned} \quad (4)$$

$$1 \leq m \leq 2^p,$$

where $(\cdot)^T$ denotes transposition. Subsequently, the commutation index is determined by selecting the correlator output with the maximum magnitude out of 2^p possible values

$$\hat{j} = \arg \max_m \{|D_{i,m}|\}, m \in \{1, 2, \dots, 2^p\}. \quad (5)$$

The estimated commutation index \hat{j} is used for determining the inherently transmitted p number of mapped bits. Furthermore, the modulated bit is recovered by computing the sign of the selected correlator output, i.e., $\hat{b}_i = \text{sgn}(D_{i,\hat{j}})$.

Generally, for higher number of mapped bits p , the reference sequence should be divided into 2^n parts satisfying $2^p \leq 2^n$. Subsequently, increasing p will cause the reference sequence to be split into shorter $\beta/2^n$ parts, which increases the destructive influence of ISI on the BER performance as $(\beta/2^n) \leq \tau_{i,max}$. In addition, a higher number of mapped bits demands more correlators at the receiver. Accordingly, p has to be chosen to satisfy the following two constraints: low receiver complexity and the length of a given commutated reference part being larger than the maximum channel delay spread, i.e. $(\beta/2^n) \gg \tau_{i,max}$.

On the other hand, for a chosen p satisfying the above constraints, CCI-DCSK scheme can be realized with lower hardware complexity compared CI-DCSK or OM-DCSK. In general, CCI-DCSK and CI-DCSK need $2^{p_{tot}-1}$ correlators at the receiver to demodulate the received signal, while OM-DCSK needs $2^{p_{tot}}$. However, CCI-DCSK scheme needs only one channel i.e, I or Q for data transmission, while both CI-DCSK and OM-DCSK need $2^p + 1$ and 2 channels, respectively to transmit the same number of bits. Furthermore, the generation of Walsh codes is required at the OM-DCSK receiver which additionally increases its hardware complexity.

III. PERFORMANCE ANALYSIS

In this work, Chebyshev polynomial function (CPF) is employed for generating chaotic samples due to its excellent correlation properties and its ease of implementation in radio hardware platforms [15], [16]. The CPF map is defined as $x_{i,k} = 1 - 2x_{i,k-1}^2$. For simplicity the chip time T_c is set to unity while the chaotic samples are normalized to have a zero mean value $E(x_{i,k}) = 0$ and a unit variance $E(x_{i,k}^2) = 1$, where $E(\cdot)$ is the expectation operator.

A. BER Analysis of CCI-DCSK

By using the signalling scheme described in Section II, it can be easily shown that

$$C_{i,j} (C_{i,j}(\mathbf{x}_{i,\tau_{i,l}})) = \mathbf{x}_{i,\tau_{i,l}}. \quad (6)$$

Furthermore, for large spreading factors β , the following approximation can be made [4]

$$\mathbf{x}_{i,\tau_{i,l'}} (\mathbf{x}_{i,\tau_{i,l}})^T \approx 0, \quad \text{for } l \neq l'. \quad (7)$$

Therefore, by using expression (6) and (7) the mean and variance of the decision variable at the output of the j^{th} correlator, for i^{th} symbol, can be approximated as

$$E(D_{i,j}) \approx \sum_{l=1}^L \alpha_{i,l}^2 b_i E_s, \quad (8)$$

$$\text{var}(D_{i,j}) \approx \left(\sum_{l=1}^L \alpha_{i,l}^2 \right)^2 \frac{E_s^2}{2\beta} + 2 \sum_{l=1}^L \alpha_{i,l}^2 N_0 E_s + \frac{N_0^2 \beta}{2}, \quad (9)$$

where $E_s = 2 \sum_{n=1}^{\beta} E(x_{i,n}^2)$ represents the transmitted symbol energy. The first term in (9) arises from the fact that the expectation of the squared product of two differently commutated sequences results in

$$\begin{aligned} E \left([\mathbf{x}_{i,\tau_{i,l}} C_{i,j}(\mathbf{x}_{i,\tau_{i,l}})]^2 \right) &= E \left([C_{i,z}(\mathbf{x}_{i,\tau_{i,l}}) C_{i,j}(\mathbf{x}_{i,\tau_{i,l}})]^2 \right) \\ &= \frac{E_s^2}{4\beta} \quad \text{for } z \neq j. \end{aligned} \quad (10)$$

Subsequently, the mean and variance at the output of the m^{th} correlator for the i^{th} symbol and $m \neq j$ is approximated as

$$E(D_{i,m}) \approx 0, \quad (11)$$

$$\text{var}(D_{i,m}) \approx \left(\sum_{l=1}^L \alpha_{i,l}^2 \right)^2 \frac{E_s^2}{\beta} + 2 \sum_{l=1}^L \alpha_{i,l}^2 N_0 E_s + \frac{N_0^2 \beta}{2}. \quad (12)$$

As stated earlier, in every CCI-DCSK transmission p mapped bits and a single modulated bit are sent. Accordingly, the overall CCI-DCSK BER P_T is composed of the BER of the mapped bits P_{map} and the BER of modulated bit P_{mod} . Hence, the P_T can be written as

$$P_T = \frac{p}{p_{\text{tot}}} P_{\text{map}} + \frac{1}{p_{\text{tot}}} P_{\text{mod}}, \quad (13)$$

where $p_{\text{tot}} \triangleq p + 1$ is total number of bits per symbol.

B. Probability of Error for Mapped Bits

An error occurs in detecting the mapped bits if the largest absolute value of the m^{th} correlator output is larger than the absolute value of j^{th} (desired) correlator output for any $m \neq j$. Therefore, the probability that an incorrect correlator output will be selected P_{id} is equal to

$$P_{\text{id}} = P(|D_{i,j}| < \max_{m \neq j} (|D_{i,m}|)), \quad \text{for } 1 \leq m \leq 2^p. \quad (14)$$

Since all correlator output variables are statistically independent, (14) can be reexpressed as [11]

$$P_{\text{id}} = 1 - \int_0^{\infty} (F_{|D_{i,m}|}(y))^{2^p-1} f_{|D_{i,j}|}(y) dy, \quad (15)$$

where $F_{|D_{i,m}|}(y)$ is cumulative density function (CDF) of $|D_{i,m}|$ and $f_{|D_{i,j}|}(y)$ is PDF of $|D_{i,j}|$. Since the variables $|D_{i,m}|$ and $|D_{i,j}|$ follow folded normal distributions, $F_{|D_{i,m}|}(y)$ and $f_{|D_{i,j}|}(y)$ can be written as

$$F_{|D_{i,m}|}(y) = \text{erf} \left(\frac{y}{\sqrt{2\text{var}(D_{i,m})}} \right), \quad (16)$$

$$\begin{aligned} f_{|D_{i,j}|}(y) &= \frac{1}{\sqrt{2\pi\text{var}(D_{i,j})}} \left[\exp \left(-\frac{(y-E(D_{i,j}))^2}{2\text{var}(D_{i,j})} \right) \right. \\ &\quad \left. + \exp \left(-\frac{(y+E(D_{i,j}))^2}{2\text{var}(D_{i,j})} \right) \right]. \end{aligned} \quad (17)$$

By using (8), (9), (12), (15), (16) and (17) the probability P_{id} may be stated as

$$P_{id} = 1 - \frac{1}{\sqrt{\pi(p_{tot}^2 \gamma_b^2 / \beta + 4p_{tot} \gamma_b + \beta)}} \times \int_0^\infty \left(\operatorname{erf} \left(\frac{y}{\sqrt{2p_{tot}^2 \gamma_b^2 / \beta + 4p_{tot} \gamma_b + \beta}} \right) \right)^{2^p - 1} \times \left[\exp \left(-\frac{(y - p_{tot} \gamma_b)^2}{p_{tot}^2 \gamma_b^2 / \beta + 4p_{tot} \gamma_b + \beta} \right) + \exp \left(-\frac{(y + p_{tot} \gamma_b)^2}{p_{tot}^2 \gamma_b^2 / \beta + 4p_{tot} \gamma_b + \beta} \right) \right] dy, \quad (18)$$

$\gamma_b = E_b/N_0 \sum_{l=1}^L \alpha_{i,l}^2$ and $E_b = E_s/p_{tot}$ is the bit energy of the transmitted signal. The bit energy E_b can be assumed to be constant for higher spreading factors β . Subsequently, the probability of an erroneous commutation detection can be converted to the corresponding mapped bits BER by using [17]

$$P_{map} = \frac{2^{p-1}}{2^p - 1} P_{id}. \quad (19)$$

C. Probability of Error for the Modulated Bit

An error in the estimation of the modulated bit occurs in two cases. The first case is when the commutation index is correctly detected but an error happens in the despreading process of the modulated bit, e.g. while comparing the corresponding correlator output $D_{i,j}$ with a zero threshold. Subsequently, the probability of the despreading process error can be given as

$$P_{despread} = \frac{1}{2} \operatorname{erfc} \left(\frac{E(D_{i,j})}{\sqrt{2\operatorname{var}(D_{i,j})}} \right), \quad (20)$$

where $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$ is the complementary error function. Substituting (8) and (9) in (20) states the probability $P_{despread}$ as

$$P_{despread} = \frac{1}{2} \operatorname{erfc} \left(\frac{\gamma_b}{\sqrt{p_{tot}^2 \gamma_b^2 / \beta + 4p_{tot} \gamma_b + \beta}} \right). \quad (21)$$

The second case is when the commutation index is erroneously detected and the despreading process is performed on a wrong correlator output. The eventual probability of correct detection in this case will be equal to 0.5, (i.e. the same as the priori probability of binary signalling) [18]. Due to the fact that P_{mod} strongly depends on P_{id} , the overall BER of the modulated bit may be expressed as

$$P_{mod} = P_{despread}(1 - P_{id}) + 0.5P_{id}. \quad (22)$$

Finally, by inserting (19) and (22) in (13), the instantaneous BER of CCI-DCSK over Rayleigh fading channels is obtained. Subsequently, the averaged BER of the CCI-DCSK system over multipath Rayleigh fading channels is given as

$$\bar{P}_T = \int_0^\infty P_T f(\gamma_b) d\gamma_b, \quad (23)$$

where the integral in (23) is evaluated numerically, while $f(\gamma_b)$ is the PDF of γ_b , which for dissimilar path gains can be found in [17].

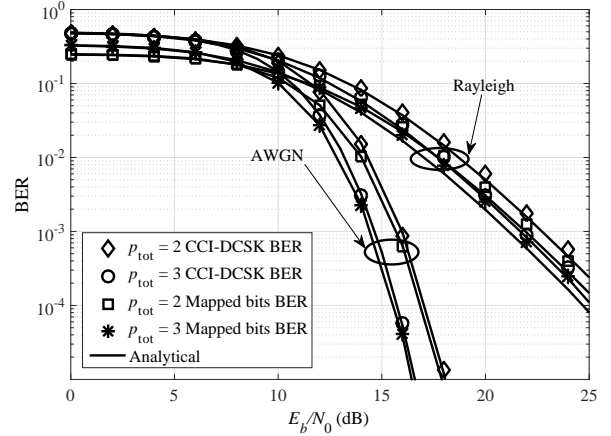


Fig. 3: The BER performance of CCI-DCSK for $\beta = 256$ and $n = 3$.

IV. RESULTS AND DISCUSSIONS

In this section, the performance of the proposed CCI-DCSK modulation is evaluated over AWGN and multipath Rayleigh fading channels. For the multipath Rayleigh fading channel, a three-ray channel model is used with average power gains of $E(|\alpha_1|^2) = 4/7$, $E(|\alpha_2|^2) = 2/7$ and $E(|\alpha_3|^2) = 1/7$, and with time delays that are uniformly distributed in the range 0 and $4T_c$. In all simulation, CCI-DCSK reference signal is split in eight parts i.e. $n = 3$ and commutated replicas of the reference sequence are obtained as follows: [2 1 4 3 6 5 8 7], [3 4 1 2 7 8 5 6], [5 6 7 8 1 2 3 4] and [8 7 6 5 4 3 2 1], where each number within the vector represents a part of the reference sequence of length $\beta/2^n$.

In order to demonstrate the accuracy of the obtained analytical BER expressions for the proposed CCI-DCSK approach, a verification vs Monte Carlo simulation has been made and the result is depicted in Fig. 3 for the case of $p_{tot} = 2, 3$, $n = 3$ and $\beta = 256$ over AWGN and multipath Rayleigh fading channels. As clearly seen in Fig. 3, analytical and simulation results are in perfect match over both channel scenarios. Additionally, the BER of mapped bits, which corresponds to the first part of eq. (13), is plotted to show its contribution to the overall CCI-DCSK BER.

In Fig. 4, a comparison of CCI-DCSK with CI-DCSK, OM-DCSK and DCSK is shown for $p_{tot} = 3$ and $\beta = 256$. As observed, CCI-DCSK outperforms OM-DCSK and DCSK in both AWGN and multipath Rayleigh fading channel scenarios. For instance, at a BER of 10^{-3} , CCI-DCSK outperforms OM-DCSK for approximately 1.2 dB while it outperforms DCSK for almost 3 dB over multipath Rayleigh fading channel. Furthermore, CCI-DCSK outperforms CI-DCSK over multipath Rayleigh channels for all E_b/N_0 , while it has a better BER performance for $E_b/N_0 < 16$ dB in AWGN channels. Despite the cross-signal interference which occurs as a consequence of summing the reference and data bearing signals, the BER improvement of CCI-DCSK arises from the fact that the energy of the desired signal, noted in (8), is twice larger in comparison with the desired energy of CI-DCSK, OM-DCSK and DCSK. Further, CCI-DCSK transmits twice faster than DCSK as the symbol duration is only half of the others.

In Fig. 5, a BER performance comparison between CCI-DCSK, CI-DCSK, OM-DCSK and DCSK is demonstrated for $p_{\text{tot}} = 3$ and $\beta = 320$. As observed, the CCI-DCSK outperforms CI-DCSK, OM-DCSK and DCSK for all E_b/N_0 ratios over both multipath Rayleigh fading and AWGN channels. This performance improvement is a consequence of the decrement in the cross-signal interference caused by the proportionally increased orthogonality between the reference signal and its commutated replica. Furthermore, in order to show the influence of the chaotic maps on the CCI-DCSK performance, the BER of this latter is plotted using a piecewise linear (PWL) map [15] having a flat invariant probability density distribution, as spreading code. As clearly witnessed, the BER performance of the CCI-DCSK scheme with either CPF or PWL is similar. This is due to the fact that for longer spreading codes, both maps will tend to have similar cross-correlation profiles and the variance of the bit energy spread by the chaotic map becomes negligible. Note that the impact of the chaotic map on the BER performance behavior is well studied in [15].

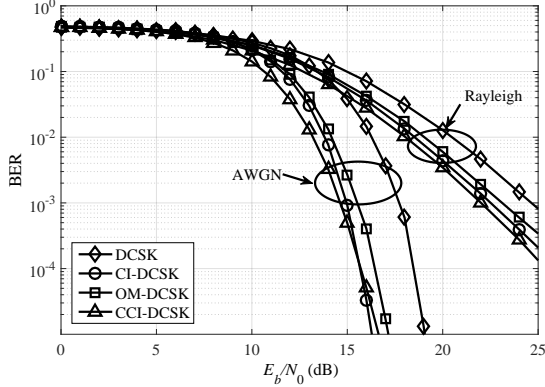


Fig. 4: BER comparison of CCI-DCSK to DCSK, CI-DCSK and OM-DCSK for $p_{\text{tot}} = 3$ and $\beta = 256$.

V. CONCLUSIONS

In this brief, a new multi-level, non-coherent CCI-DCSK scheme where IM principle is used to increase energy and spectral efficiencies. In particular, p extra bits per transmission are mapped in the indices of different commutations of the reference signal. Then, the commutated replica of the reference signal is used to spread a single modulated bit. The commutation operation is performed to construct orthogonality between the reference and data bearing signals. Due to the achieved orthogonality the reference and data bearing signals could be summed within same time slot, which additionally augments the spectral efficiency. Moreover, analytical expressions for CCI-DCSK BER over AWGN and Rayleigh multipath channels are derived, the outcome of which is confirmed with Monte-Carlo simulation. Finally, the CCI-DCSK BER performances are compared with DCSK, CI-DCSK and OM-DCSK. The obtained results show a promising BER performance of this scheme in comparison to its rivals, specially for high spreading factors.

REFERENCES

[1] G. Kolumbán, B. Vizvari, W. Schwarz, and A. Abel, "Differential chaos shift keying: a robust coding for chaos communication," in *Proc. NDES*, Seville, Spain, 1996, pp. 92–97.

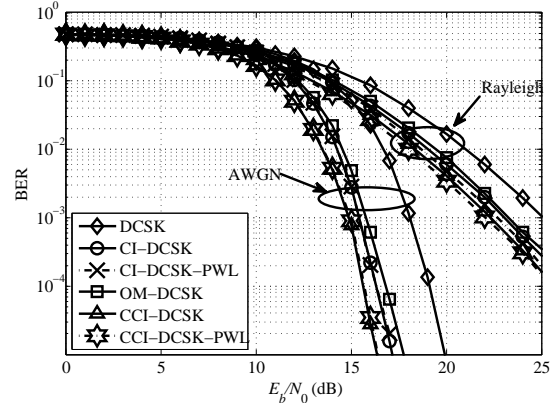


Fig. 5: BER comparison of CCI-DCSK to DCSK, CI-DCSK and OM-DCSK for $p_{\text{tot}} = 3$ and $\beta = 320$.

[2] H. Yang and G.-P. Jiang, "High-efficiency differential-chaos-shift-keying scheme for chaos-based noncoherent communication," *IEEE Trans. Circuits Syst. II: Exp. Briefs*, vol. 59, no. 5, pp. 312–316, May 2012.

[3] T. Huang, L. Wang, W. Xu, and F. C. M. Lau, "Multilevel code-shifted differential-chaos-shift-keying system," *IET Commun.*, vol. 10, no. 10, pp. 1189–1195, 2016.

[4] G. Kaddoum and F. Gagnon, "Design of a high-data-rate differential chaos-shift keying system," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 59, no. 7, pp. 448–452, 2012.

[5] M. Herceg, K. Miličević, and T. Matic, "Frequency-translated differential chaos shift keying for chaos-based communications," *J. Franklin Inst.*, vol. 353, no. 13, pp. 2966–2979, 2016.

[6] G. Kaddoum, E. Soujeri, C. Arcila, and K. Eshteivi, "I-DCSK: An improved non-coherent communication system architecture," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 62, no. 9, pp. 901–905, 2015.

[7] G. Kaddoum, F. Richardson, and F. Gagnon, "Design and analysis of a multi-carrier differential chaos shift keying communication system," *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 3281–3291, Aug. 2013.

[8] R. Y. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228–2241, July 2008.

[9] E. Başar, U. Aygözü, E. Panayrc, and H. V. Poor, "Orthogonal frequency division multiplexing with index modulation," *IEEE Trans. Signal Process.*, vol. 61, no. 22, pp. 5536–5549, Nov 2013.

[10] G. Kaddoum, M. F. A. Ahmed, and Y. Nijssure, "Code index modulation: A high data rate and energy efficient communication system," *IEEE Commun. Lett.*, vol. 19, no. 2, pp. 175–178, Feb 2015.

[11] G. Kaddoum, Y. Nijssure, and H. Tran, "Generalized code index modulation technique for high-data-rate communication systems," *IEEE Trans. Veh. Technol.*, vol. 65, no. 9, pp. 7000–7009, Sept 2016.

[12] G. Cheng, L. Wang, W. Xu, and G. Chen, "Carrier index differential chaos shift keying modulation," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. PP, no. 99, pp. 1–1, 2016.

[13] H. Yang, W. K. Tang, G. Chen, and G.-P. Jiang, "System design and performance analysis of orthogonal multi-level differential chaos shift keying modulation scheme," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 63, no. 1, pp. 146–156, 2016.

[14] W. Xu, T. Huang, and L. Wang, "Code-shifted differential chaos shift keying with code index modulation for high data rate transmission," *IEEE Trans. Commun.*, vol. PP, no. 99, pp. 1–1, 2017.

[15] G. Kaddoum, P. Chargé, D. Roviras, and D. Fournier-Prunaret, "A methodology for bit error rate prediction in chaos-based communication systems," *Birkhäuser, Circuits, Syst. Signal Process.*, vol. 28, pp. 925–944, Aug. 2009.

[16] G. Kaddoum, J. Olivain, G. Beaufort Samson, P. Giard, and F. Gagnon, "Implementation of a differential chaos shift keying communication system in gnu radio," in *International Symposium on Wireless Communication Systems (ISWCS)*, 2012, 2012, pp. 934–938.

[17] J. G. Proakis, *Digital communications systems*. McGraw-Hill, New York, 2001.

[18] M. Herceg, G. Kaddoum, D. Vranjes, and E. Soujeri, "Permutation index dcsk modulation technique for secure multi-user high-data-rate communication systems," *IEEE Trans. Veh. Technol.*, 2017.