

COMMUTATIVITY WITH DERIVATIONS OF SEMIPRIME RINGS

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Abstract

Let R be a 2-torsion free semiprime ring with the centre $Z(R)$, U be a non-zero ideal and $d: R \rightarrow R$ be a derivation mapping. Suppose that R admits

- (1) a derivation d satisfying one of the following conditions:
 - (i) $[d(x), d(y)] - [x, y] \in Z(R)$ for all $x, y \in U$,
 - (ii) $[d^2(x), d^2(y)] - [x, y] \in Z(R)$ for all $x, y \in U$,
 - (iii) $[d(x^2), d(y^2)] - [x, y] \in Z(R)$ for all $x, y \in U$,
 - (iv) $[d(x^2), d(y^2)] - [x, y] \in Z(R)$ for all $x, y \in U$,
 - (v) $[d(x), d(y)] - [x^2, y^2] \in Z(R)$ for all $x, y \in U$.
- (2) a non-zero derivation d satisfying one of the following conditions:
 - (i) $d([d(x), d(y)]) - [x, y] \in Z(R)$ for all $x, y \in U$,
 - (ii) $d([d(x), d(y)]) + [x, y] \in Z(R)$ for all $x, y \in U$.

Then R contains a non-zero central ideal.

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