FAKULTÄT FÜR INFORMATIK
Faculty of Informatics

Compact Argumentation Frameworks

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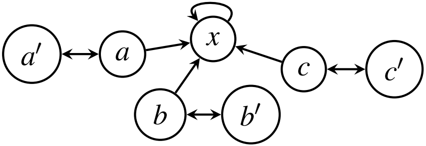
Der Wissenschaftsfonds.

Introduction

- Abstract Argumentation Framework [Dung, 1995]:



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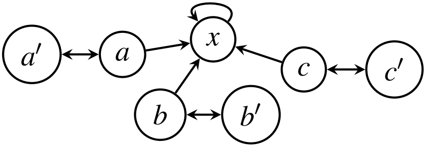

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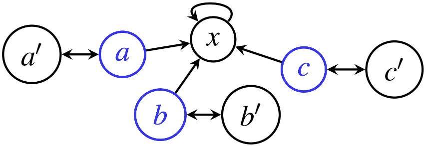

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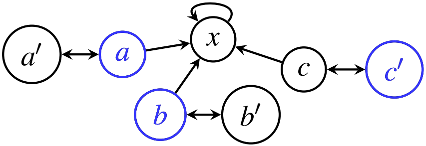

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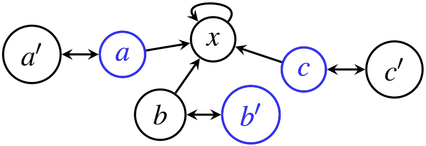

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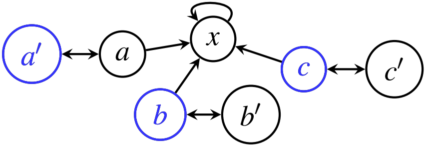

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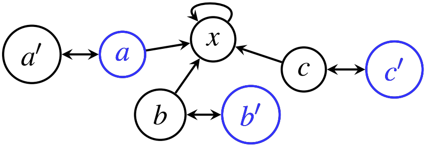

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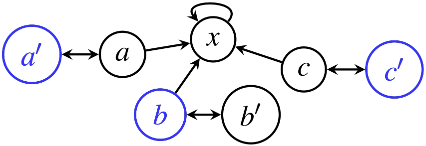

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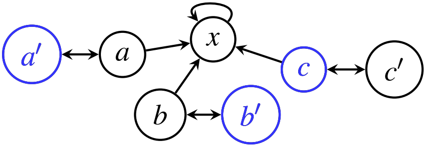

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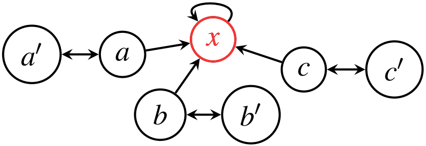

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Problem

Can we find an equivalent AF F' without argument x?

Motivation

- Realizability [Dunne et al., 2014]
  - Structural analysis of the expressiveness of argumentation semantics.
  - Unlimited use of auxiliary arguments.

$\Rightarrow$ Compact Realizability

- Compact Argumentation Frameworks
  - Each argument occurs in at least one extension.
  - “Semantic” subclass.
  - Attractive for normal-forms.

Background

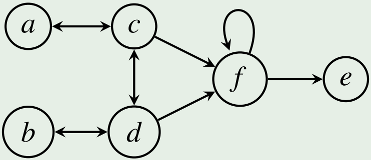
Countably infinite set of arguments $\mathfrak{A}$.

Definition

An argumentation framework (AF) is a pair (A, R) where

- $A \subseteq \mathfrak{A}$ is a finite set of arguments and
- $R \subseteq A \times A$ is the attack relation representing conflicts.

Example



$$F = \big(\{a, b, c, d, e, f\},$$
$$\{(a, c), (c, a), (c, d), (d, c), (d, b), (b, d), (c, f), (d, f), (f, f), (f, e)\}\big)$$

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Definition

Given an AF $F = (A, R)$, a set $S \subseteq A$ is

- conflict-free if for each $a, b \in S$, $(a, b) \notin R$,
- naive extension if $S \in cf(F)$ and $\nexists T \in cf(F) : T \supset S$,
- stable extension if $S \in cf(F)$ and $\forall b \in A \setminus S\ \exists a \in S : (a, b) \in R$.

Signature of semantics σ: $\Sigma_\sigma = \{\sigma(F) \mid F \text{ is an AF}\}$

Compact Realizability

Definition

Given a semantics σ, an extension-set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$ is called compactly σ-realizable if there exists an AF $F = (\mathit{Args}_{\mathbb{S}}, R)$ such that $\sigma(F) = \mathbb{S}$.

C-Signature: $\Sigma_\sigma^c = \{\sigma(F) \mid F = (A, R) \text{ is an AF}, \mathit{Args}_{\sigma(F)} = A\}$.

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Given an extension-set $\mathbb{S}$,

- $\mathit{Args}_{\mathbb{S}} = \bigcup_{S \in \mathbb{S}} S$, and
- $\mathit{Pairs}_{\mathbb{S}} = \{(a, b) \mid \exists S \in \mathbb{S} : \{a, b\} \subseteq S\}$.

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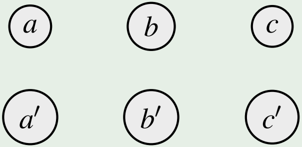
Canonical Argumentation Framework

$$F_{\mathbb{S}} = (\mathit{Args}_{\mathbb{S}}, (\mathit{Args}_{\mathbb{S}} \times \mathit{Args}_{\mathbb{S}}) \setminus \mathit{Pairs}_{\mathbb{S}})$$

Compact Realizability: Naive Semantics

Example

$\mathbb{T} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \{a, b, c\}\}$.



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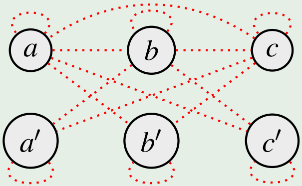

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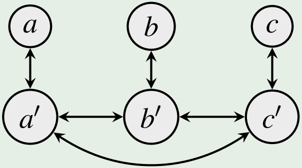

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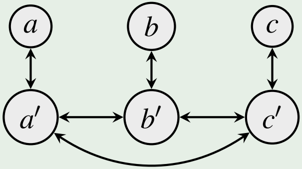
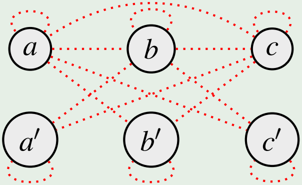


naive$(F_{\mathbb{T}}) = \mathbb{T}$.

Compact Realizability: Naive Semantics

Example

$\mathbb{U} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \sout{\{a, b, c\}}\}.$



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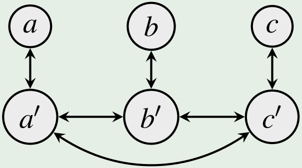

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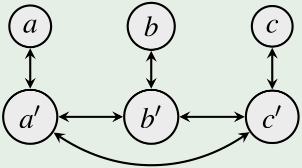

$naive(F_{\mathbb{U}}) = \mathbb{T} \neq \mathbb{U}$.

- $\mathbb{S}^+ = naive(F_{\mathbb{S}}) = stb(F_{\mathbb{S}})$
- $\mathbb{S}^- = \mathbb{S}^+ \setminus \mathbb{S}$

Theorem

$\Sigma^c_{naive} = \{\mathbb{S} \subseteq 2^{\mathfrak{A}} \mid \mathbb{S} \neq \emptyset, \mathbb{S} = \mathbb{S}^+\} = \Sigma_{naive}$.

Compact Realizability: Stable Semantics

Proposition

For every extension-set $\mathbb{S} \in \Sigma_{stb}$ it holds that if $|\mathbb{S}| \leq 3$ then $\mathbb{S} \in \Sigma^c_{stb}$.

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For every extension-set $\mathbb{S} \in \Sigma_{stb}$ such that for each $S \in \mathbb{S}$ there is an $a \in S$ with $\forall T \in (\mathbb{S} \setminus \{S\}) : a \notin T$ then $\mathbb{S} \in \Sigma^c_{stb}$.

Compact Realizability: Stable Semantics

Proposition

$\Sigma^c_{naive} = \Sigma_{naive} \subset \Sigma^c_{stb} \subset \Sigma_{stb}$

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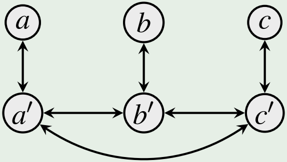
$\mathbb{U} = \{\{a,b,c'\},\{a,b',c\},\{a',b,c\},\cancel{\{a,b,c\}}\} \subset \mathbb{U}^+$



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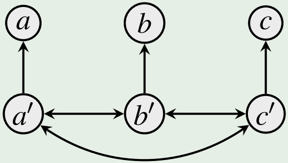

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Compact Realizability: Stable Semantics

Definition

Given an extension-set $\mathbb{S}$, an exclusion-mapping is the set

$$\mathfrak{R}_{\mathbb{S}} = \bigcup_{S \in \mathbb{S}^{-}} \{(s, \mathfrak{f}_{\mathbb{S}}(S)) \mid s \in S \text{ s.t. } (s, \mathfrak{f}_{\mathbb{S}}(S)) \notin \mathit{Pairs}_{\mathbb{S}}\}$$

where $\mathfrak{f}_{\mathbb{S}} : \mathbb{S}^{-} \to \mathit{Args}_{\mathbb{S}}$ is a function with $\mathfrak{f}_{\mathbb{S}}(S) \in (\mathit{Args}_{\mathbb{S}} \setminus S)$.

An extension-set $\mathbb{S}$ is called independent if there exists an exclusion-mapping $\mathfrak{R}_{\mathbb{S}}$ such that

- $\mathfrak{R}_{\mathbb{S}}$ is antisymmetric, and
- $\forall S \in \mathbb{S} \forall a \in (\mathit{Args}_{\mathbb{S}} \setminus S) : \exists s \in S : (s, a) \notin (\mathfrak{R}_{\mathbb{S}} \cup \mathit{Pairs}_{\mathbb{S}})$.

Theorem

For every independent extension-set $\mathbb{S} \in \Sigma_{stb}$ it holds that $\mathbb{S} \in \Sigma^{c}_{stb}$.

Compact Realizability: Stable Semantics

Definition

We call an AF $F = (A, R)$ conflict-explicit under semantics σ iff for each $a, b \in A$ such that $(a, b) \notin Pairs_{\sigma(F)}$, we find $(a, b) \in R$ or $(b, a) \in R$ (or both).

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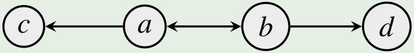

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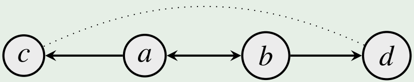

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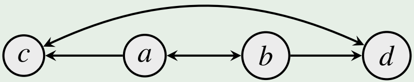

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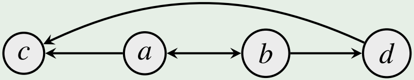

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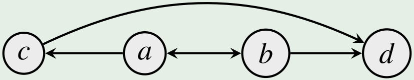

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Compact Realizability: Stable Semantics

Explicit-Conflict-Conjecture

For each AF $F = (A, R)$ there exists an AF $F' = (A, R')$ which is conflict-explicit under the stable semantics such that $stb(F) = stb(F')$.

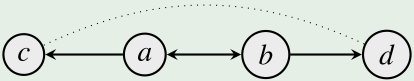
Theorem

Under the assumption that the EC-conjecture holds,

$$\Sigma^c_{stb} = \{\mathbb{S} \in \Sigma_{stb} \mid \mathbb{S} \text{ is independent}\}.$$

Explicit-Conflict-Conjecture

Example



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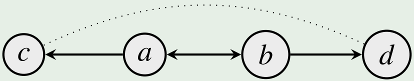

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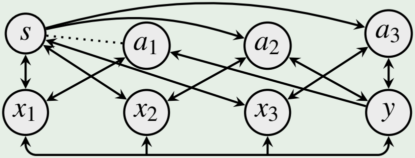

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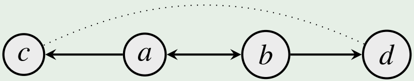

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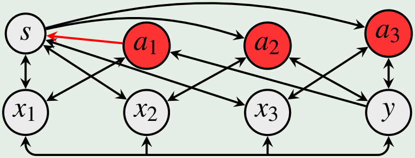

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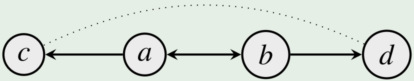

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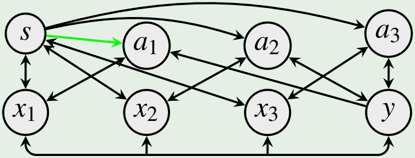

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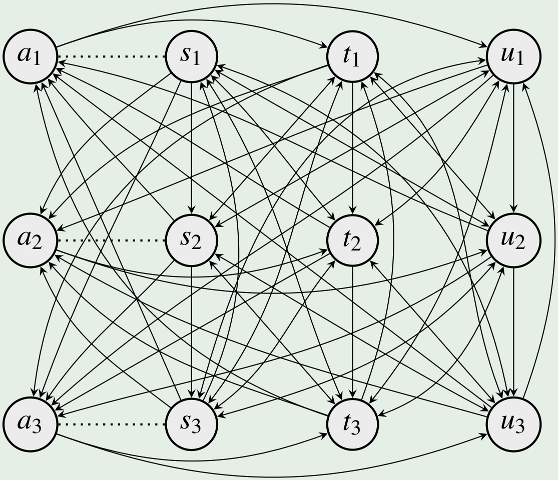

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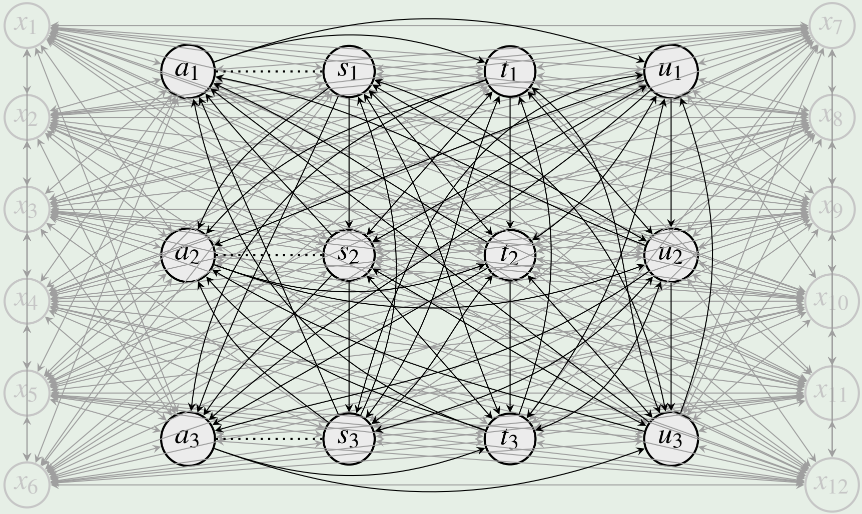

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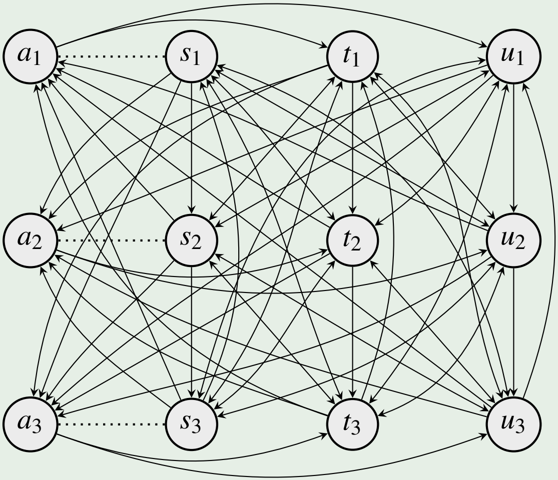


Impossibility Results

- Decision procedure for compact realizability supposed to be hard.
- Shortcuts can be achieved by impossible numbers.
- Maximal numbers for non-compact frameworks: [Baumann and Strass, 2014].
- Based on results for maximal independent sets [Griggs et al., 1988].

- Subsequent results hold for $\sigma \in \{\mathit{stb}, \mathit{sem}, \mathit{pref}, \mathit{stage}, \mathit{naive}\}$.

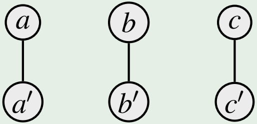
Impossibility Results

Proposition

Given an extension-set $\mathbb{S}$, the component-structure $\mathcal{K}(\mathbb{S})$ of any AF F compactly realizing $\mathbb{S}$ under σ is given by the equivalence classes of the transitive closure of $\overline{Pairs_{\mathbb{S}}}$, i.e. $\left(\overline{Pairs_{\mathbb{S}}}\right)^*$.

Example

$\mathbb{S}=\{\{a,b,c\},\{a,b',c'\},\{a',b,c'\},\{a',b',c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\}\}$.



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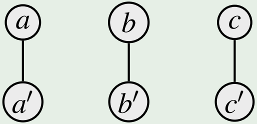


Proposition

Given an extension-set $\mathbb{S}$ where $|\mathbb{S}|$ is odd, it holds that if $\exists K \in \mathcal{K}(\mathbb{S}) : |K| = 2$ then $\mathbb{S}$ is not compactly realizable under semantics σ.

Impossibility Results

$\sigma_{\text{max}}^{\text{con}}(n) = \max\{|\sigma(F)| \mid F \in \mathsf{AF}_n, F \text{ connected}\}$

Theorem

$$\sigma_{\text{max}}^{\text{con}}(n) = \begin{cases} n, & \text{if } n \leq 5, \\ 2 \cdot 3^{s-1} + 2^{s-1}, & \text{if } n \geq 6 \text{ and } n = 3s, \\ 3^s + 2^{s-1}, & \text{if } n \geq 6 \text{ and } n = 3s+1, \\ 4 \cdot 3^{s-1} + 3 \cdot 2^{s-2}, & \text{if } n \geq 6 \text{ and } n = 3s+2. \end{cases}$$

Definition

We denote the set of possible numbers of σ-extensions of a compact and connected AF with n arguments as $\mathcal{P}^c(n)$.

- $\forall p \in \mathcal{P}^c(n) : p \leq \sigma_{\text{max}}^{\text{con}}(n)$.
- Exact contents of $\mathcal{P}^c(n)$ unknown.

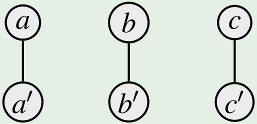
Impossibility Results

Proposition

Let $\mathbb{S}$ be an extension-set that is compactly realizable under semantics σ where $\mathcal{K}_{\geq 2}(\mathbb{S}) = \{K_1, \ldots, K_n\}$. Then for each $1 \leq i \leq n$ there is a $p_i \in \mathcal{P}^c(|K_i|)$ such that $|\mathbb{S}| = \prod_{i=1}^{n} p_i$.

Example

$\mathbb{V} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \{a', b', c'\}\}$.



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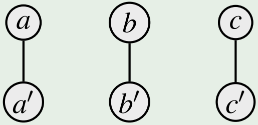

Corollary

Let extension-set $\mathbb{S}$ with $|Args_{\mathbb{S}}| = n$ be compactly realizable under σ. If $|\mathbb{S}|$ is a prime number, then $|\mathbb{S}| \leq \sigma^{\text{con}}_{\text{max}}(n)$.

Theorem

1. $\mathit{CAF}_{sem} \subset \mathit{CAF}_{pref}$
2. $\mathit{CAF}_{stb} \subset \mathit{CAF}_{\sigma} \subset \mathit{CAF}_{naive}$ for $\sigma \in \{pref, sem, stage\}$
3. $\mathit{CAF}_{\theta} \not\subseteq \mathit{CAF}_{stage}$ and $\mathit{CAF}_{stage} \not\subseteq \mathit{CAF}_{\theta}$ for $\theta \in \{pref, sem\}$

Theorem

For $\sigma \in \{pref, sem, stage\}$, AF $F = (A, R) \in \mathit{CAF}_{\sigma}$ and $E \subseteq A$, it is coNP-complete to decide whether $E \in \sigma(F)$.

Conclusion

Summary

- Compact realizability
  - Exact characterizations hard to find
  - Missing step for stable semantics: EC-Conjecture
- Shortcuts via impossible numbers of extensions
- Full picture of relations between compact AFs under the considered semantics

Future Work

- Exact characterizations of compact signatures.
- Closing the gap between general and compact realizability with fragments of ADFs.
- Explicit-Conflict-Conjecture.

References I

- Baumann, R. and Strass, H. (2014).
  On the Maximal and Average Numbers of Stable Extensions.
  In TAFA 2013, volume 8306 of LNAI, pages 111–126.
- Dung, P. M. (1995).
  On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games.
  AIJ, 77(2):321–357.
- Dunne, P. E., Dvořák, W., Linsbichler, T., and Woltran, S. (2014).
  Characteristics of multiple viewpoints in abstract argumentation.
  In KR, pages 72–81.
- Griggs, J. R., Grinstead, C. M., and Guichard, D. R. (1988).
  The number of maximal independent sets in a connected graph.
  Discrete Mathematics, 68(2–3):211–220.

$\Sigma_{stage} \not\subseteq \Sigma_{sem}$